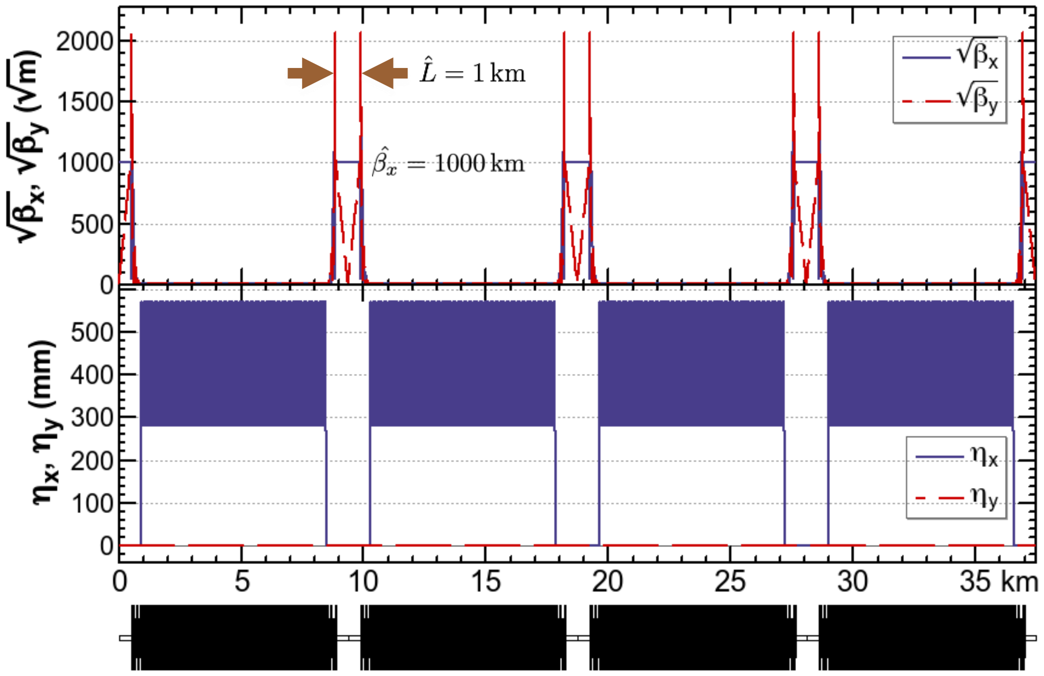
GW detection by resonant betatron oscillations (K. Oide)
Response of a storage-ring beam to a gravitational wave

ring optics for a GR antenna



In the case of such an "antenna", the response to GR in the arc is small compared to the $\hat{\beta}_x$ sections, to we ignore below. Thus Eq. 7 is reduced to

$$\Delta\hat{x} = -\frac{k^2 R\hat{L}}{2} h \sum_{m=0}^{n} \hat{\beta}_x \sin\left(2(n-m)\pi\Delta\nu_x\right) \cos\left(\frac{m\pi}{2}kR\right) \cos m\pi\,, \qquad (11)$$

where $\Delta\mu_x$ is the phase advance between two $\hat{L}$ sections, and we have used $\theta = m\pi/2$ at the m-th $\hat{L}$ section.

Equation 11 has resonances at

$$2\Delta\nu_x = \pm kR + N\,, \qquad (12)$$

where N is an integer. At the resonance, the horizontal amplitude accumulates by

$$\Delta\hat{x} \approx -\frac{k^2 R\hat{L}}{2}\hat{\beta}_x h \qquad (13)$$

per turn.

If we set $\omega_{\mathrm{GR}} = 2\pi \times 1\,\mathrm{kHz}$ and with parameters above, Eq. 13 gives $\Delta\hat{x} \approx 1000h\,\mathrm{m}$, which seems still too small to detect. As the amplitude is proportional to ω_{GR}^2, there may exist a possibility to detect a GR in $\omega_{\mathrm{GR}} = 2\pi \times 1\,\mathrm{MHz}$ range, giving $\Delta\hat{x} \approx 10^9 h$ m (0.1 pm for $h = 10^{-22}$), if there are sources.

$$\Delta x \approx \frac{\omega^2 R\hat{L}\hat{\beta}}{2c^2} h$$

signal over 1 turn (K. Oide)

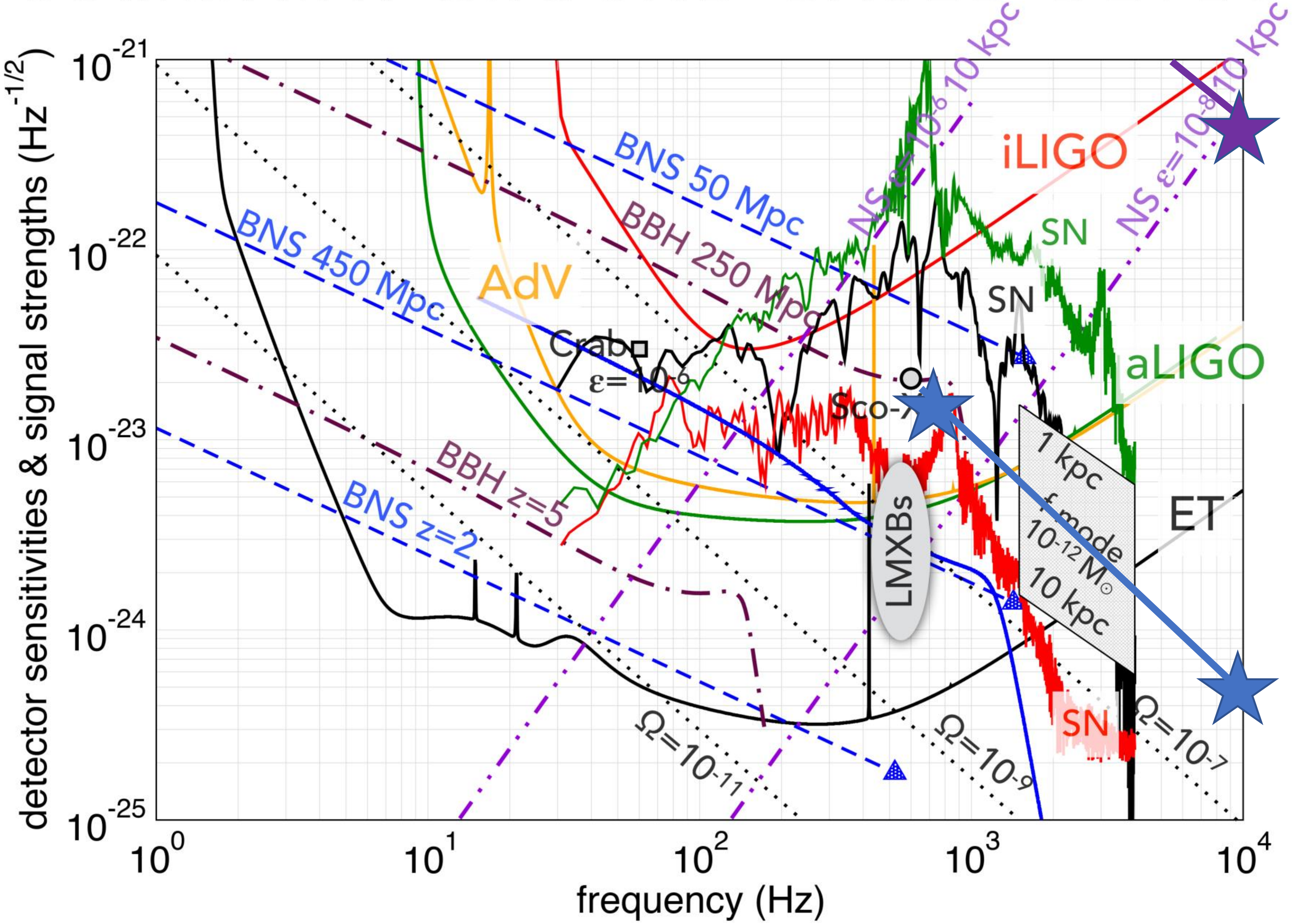
signal could be accumulated over $1/\Delta Q \sim 10^4$ turns

$$h_{sens} \approx \frac{2c^2 \Delta Q}{\omega^2 R\hat{L}\hat{\beta}} \Delta x \quad \longrightarrow \quad \frac{h_{sens}}{\sqrt{\Delta f}} \approx \frac{c^2 \Delta Q}{2\pi^2 f^2 \sqrt{\Delta f} R\hat{L}\hat{\beta}} \Delta x$$

$$\Delta Q \approx \frac{\Delta f\ C}{2c} \quad \longrightarrow \quad \frac{h_{sens}}{\sqrt{\Delta f}} \approx \frac{c\sqrt{\Delta f/f}}{2\pi f^{3/2} \hat{L}\hat{\beta}} \Delta x$$

SOURCE SUMMARY: GROUND-BASED DETECTORS

noise sources to be assessed



1 pm BPM resolution

1 fm BPM resolution

β=1000 km

L = 1 km

$\sqrt{\Delta f / f} \approx 0.01$

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