Predicting the Response of p-type Silicon Sensors to Hadron Radiation: a Hamburg Model Simulation

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Introduction

- Silicon detection is a mature technology for registering the passage of charged particles.
  - It continues to evolve toward increasing radiation tolerance as well as precision and adaptability.
  - Damage is known to be caused by non-ionizing energy loss.
- The high energy physics community has gradually shifted to the use of p-type (n-in-p) silicon sensors in place of n-type (p-in-n).
  - p-type sensors are potentially more radiation hard and have reduced fabrication costs
- The Hamburg Model simulation code developed for the prediction of n-type silicon sensors in the experiments at the LHC is being adapted for p-type silicon sensors.
  - The alterations of the model and code base* will be discussed.

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* ATLAS Collaboration, Hamburg Model Simulation Code, (2019), https://gitlab.cern.ch/atlas/athena/tree/master/InnerDetector/InDetCalibAlgs/PixelCalibAlgs/RadDamage/HamburgModel

Hamburg Model: Leakage Current Simulations

- The Hamburg Model[*] is based on this relationship:

$$\Delta I_{\text{leak}} = \alpha\, \Phi_{\text{eq}}\, V$$

- Here, ΔI_{leak} is the difference in leakage current at fluence Φ_{eq} relative to the value before irradiation of the sensor depleted volume V, and α is the current-related damage coefficient
- And by replacing α the equation becomes[**]:

$$\Delta I = (\Phi_{\text{eq}}/L_{\text{int}}) \times V \cdot \sum_{i=1}^{n} L_{\text{int},i} \cdot \left[\alpha_{\text{I}} \exp\left(-\sum_{j=i}^{n} \frac{t_j}{\tau(T_j)}\right) + \alpha_0^* - \beta \log\left(\sum_{j=i}^{n} \frac{\Theta(T_j) \cdot t_j}{t_0}\right)\right]$$

- Where the variables are:

t_i is the time, and $t_0 = 1$ min

$\alpha_I = (1.23 \pm 0.06) \times 10^{-17}$ A/cm

$\tau^{-1} = (1.2^{+5.3}_{-1.0}) \times 10^{13}\ \text{s}^{-1} \times e^{(-1.11 \pm 0.05)\ \text{eV}/k_{\text{B}}T}$

$\alpha_0^* = 7.07 \cdot 10^{-17}$ A/cm

$\beta = (3.29 \pm 0.18) \times 10^{-18}$ A/cm

and $\Theta(T) = \exp\left[-\frac{E_I^*}{k_{\text{B}}}\left(\frac{1}{T} - \frac{1}{T_{\text{ref}}}\right)\right]$

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[*] M. Moll et al., Leakage Current of Hadron Irradiated Silicon Detectors - Material Dependence. Nucl. Instrum. Meth. A , 426(87), 1999.

[**] ATLAS Collaboration, "Modelling radiation damage to pixel sensors in the ATLAS detector," JINST 14, P06012 (2019).

Hamburg Model: Depletion Voltage Simulations

- The Hamburg Model[*] proposes that the impact on depletion voltage is dependent on irradiation and temperature and can be determined through a parameterized set of equations that can be used to compute the effective doping concentration:

$$V_{\text{dep}} = \frac{qd^2}{2\epsilon\epsilon_0} \times |N_{\text{eff}}|$$

- The effective doping concentration can change through stable damage and initial dopant removal:

$$N_{\text{C}} = N_{\text{C},0}(1 - c\Phi_{\text{eq}}) + g_{\text{C}}\Phi_{\text{eq}}$$

- It may also undergo short term beneficial annealing and long term annealing effects as (respectively):

$$N_{\text{A}} = g_{\text{A}}\Phi_{\text{eq}}e^{-t/\tau_{\text{a}}}$$

$$N_{\text{Y}} = g_{\text{Y}}\Phi_{\text{eq}}(1 - e^{t/\tau_{\text{Y}}})$$

- These effects are used to describe the change in effective doping concentration with respect to the initial doping concentration of the sensor

$$\Delta N_{\text{eff}} = N_{\text{C}} + N_{\text{A}} + N_{\text{Y}}$$

here τ_{a} and τ_{Y} ($1/\,k_{\text{a}}$ and $1/\,k_{\text{Y}}$, respectively) are defined by Arrhenius equations and have a temperature dependence. The constants g_{C}, g_{A}, g_{Y} are the "introduction rates" and c is the removal constant.

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[*] M. Moll, "Radiation damage in silicon particle detectors: Microscopic defects and macroscopic properties," Doctoral dissertation (University of Hamburg, Hamburg, Germany, 1999)

Hamburg Model as Differential Equations

- The Hamburg Model can be written as a set of differential equations that describe the change in the donor and acceptor concentrations
- This set of equations describe (initially) **n-type silicon sensors***

$$
\begin{aligned}
\Delta N_{\text{eff}} &= N_{\text{donor}} - N_{acceptor} \\
&= N_{\text{donor}}^{\text{non-removable}} + N_{\text{donor}}^{\text{removable}} - N_{\text{acceptor}}^{\text{stable}} - N_{\text{acceptor}}^{\text{beneficial}} - N_{\text{acceptor}}^{\text{reverse}}
\end{aligned}
$$

Possible for some initial donors to be "non-removable"

Donors

$$\frac{d}{dt} N_{\text{donor}}^{\text{removable}}(t) = -c_{\text{D}} \Phi_{\text{eq}}(t) N_{\text{donor}}^{\text{removable}}$$

Acceptors

$$
\begin{aligned}
\frac{d}{dt} N_{\text{acceptor}}^{\text{stable}}(t) &= g_{\text{C}} \Phi_{\text{eq}}(t) \\
\frac{d}{dt} N_{\text{acceptor}}^{\text{beneficial}}(t) &= g_{\text{A}} \Phi_{\text{eq}}(t) - k_{\text{A}} N_{\text{acceptor}}^{\text{beneficial}}(t) \\
\frac{d}{dt} N_{\text{neutral}}^{\text{reverse}}(t) &= g_{\text{Y}} \Phi_{\text{eq}}(t) - k_{\text{Y}} N_{\text{neutral}}^{\text{reverse}}(t) \\
\frac{d}{dt} N_{\text{acceptor}}^{\text{reverse}}(t) &= k_{\text{Y}} N_{\text{neutral}}^{\text{reverse}}(t).
\end{aligned}
$$

Stable damage due to applied fluence

Beneficial Annealing – short term ~few days

Reverse Annealing – includes an intermediate step of neutral defect sites

Temperature Dependent

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[*] ATLAS Collaboration, "Modelling radiation damage to pixel sensors in the ATLAS detector," JINST 14, P06012 (2019).

Hamburg Model as Implemented in Code

- The differential equations describing the time (i.e., temperature and fluence) dependence can be solved (using variation of parameters) as*:

$$\begin{aligned}\Delta N_{\text{eff}} &= N_{\text{donor}} - N_{acceptor} \\ &= N^{\text{non-removable}}_{\text{donor}} + N^{\text{removable}}_{\text{donor}} - N^{\text{stable}}_{\text{acceptor}} - N^{\text{beneficial}}_{\text{acceptor}} - N^{\text{reverse}}_{\text{acceptor}}\end{aligned}$$

$$N^{\text{removable}}_{\text{donor}} = N^{\text{removable}}_{\text{donor},0}(1 - e^{-c\Phi^{\text{rate}}_{\text{eq}} t})$$

$$N^{\text{stable}}_{\text{acceptor}} = g_{\text{C}}\Phi^{\text{rate}}_{\text{eq}} t$$

$$N^{\text{beneficial}}_{\text{acceptor}} = \frac{g_{\text{A}}}{k_{\text{A}}}\Phi^{\text{rate}}_{\text{eq}}(1 - e^{-k_{\text{A}}t}) + N^{\text{beneficial}}_{\text{acceptor},0}e^{-k_{\text{A}}t}$$

$$N^{\text{reverse}}_{\text{neutral}} = \frac{g_{\text{Y}}}{k_{\text{Y}}}\Phi^{\text{rate}}_{\text{eq}}(1 - e^{-k_{\text{Y}}t}) + N^{\text{reverse}}_{\text{neutral},0}e^{-k_{\text{Y}}t}$$

$$N^{\text{reverse}}_{\text{acceptor}} = \frac{g_{\text{Y}}}{k_{\text{Y}}}\Phi^{\text{rate}}_{\text{eq}}(k_{\text{Y}}t + e^{-k_{\text{Y}}t} - 1) + N^{\text{reverse}}_{\text{neutral},0}(1 - e^{-k_{\text{Y}}t})$$

- The model is run over discrete timesteps
- The input at each timestep:
  1. Duration of the timestep (t)
  2. Fluence rate in the timestep (Φ_{eq})
  3. Temperature during the timestep (T)
- The subscript “0” refers to the concentration (N) at the beginning of the timestep

Arrhenius equation

(introduction of temperature dependence)

$$k_i = k_{i,0}e^{-E_i/k_{\text{B}}T}$$

$$E_{\text{A}} = 1.09 \pm 0.03\ \text{eV},\ E_{\text{Y}} = 1.33 \pm 0.03\ \text{eV}$$

$$k_{\text{A},0} = 2.4^{+1.2}_{-0.8} \times 10^{13}/\text{s and } k_{\text{Y},0} = 1.5^{+3.4}_{-1.1} \times 10^{15}/\text{s}$$

Donor Removal Constant†

$$c = 10.9\,\text{cm}^{-2}/N_{\text{eff},0}$$

Introduction Rates

Fit to ATLAS B-Layer Data **

$$g_C = 0.6 \times 10^{-2}/\text{cm}$$

$$g_A = 6.0 \times 10^{-2}/\text{cm}$$

$$g_Y = 0.43 \times 10^{-2}/\text{cm}.$$

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* J.C. Beyer, Doctoral dissertation, MPI Munich (Mar. 2019).

† M. Moll, Doctoral dissertation (University of Hamburg, Hamburg, Germany, 1999)

** https://atlas.web.cern.ch/Atlas/GROUPS/PHYSICS/PLOTS/PIX-2018-005/

Hamburg Model for p-type Sensors

> "The acceptor removal process […] has become the field of high interest due to the recent shift from n-type to p-type silicon devices in the HEP community"*

- Complete characterization of p-type is still ongoing in our community.
- Some changes to the n-type model are necessary:
- Introduce an initial acceptor removal term and remove the initial donor term:

$$N_{\text{donor}}^{\text{removable}} = N_{\text{donor},0}^{\text{removable}}\left(1 - e^{-c\Phi_{\text{eq}}^{\text{rate}} t}\right) \longrightarrow N_{\text{acceptor}}^{\text{removable}} = N_{\text{acceptor},0}^{\text{removable}}\left(1 - e^{-c^{\text{acceptor}}\Phi_{\text{eq}}^{\text{rate}} t}\right)$$

- Annealing terms have generally been neglected in previous studies

$$\cancel{\boxed{\begin{aligned} N_{\text{acceptor}}^{\text{beneficial}} &= \frac{g_{\text{A}}}{k_{\text{A}}}\Phi_{\text{eq}}^{\text{rate}}\left(1 - e^{-k_{\text{A}}t}\right) + N_{\text{acceptor},0}^{\text{beneficial}} e^{-k_{\text{A}}t} \\ N_{\text{neutral}}^{\text{reverse}} &= \frac{g_{\text{Y}}}{k_{\text{Y}}}\Phi_{\text{eq}}^{\text{rate}}\left(1 - e^{-k_{\text{Y}}t}\right) + N_{\text{neutral},0}^{\text{reverse}} e^{-k_{\text{Y}}t} \\ N_{\text{acceptor}}^{\text{reverse}} &= \frac{g_{\text{Y}}}{k_{\text{Y}}}\Phi_{\text{eq}}^{\text{rate}}\left(k_{\text{Y}}t + e^{-k_{\text{Y}}t} - 1\right) + N_{\text{neutral},0}^{\text{reverse}}\left(1 - e^{-k_{\text{Y}}t}\right) \end{aligned}}}$$

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* M. Moll, IEEE Trans. Nucl. Sci. 65, 1561–1582 (2018).

** G. Kramberger et al., Initial acceptor removal in p-type silicon, TRENTO Workshop, 2015.

*** R. Wunstorf et. al, Nucl. Instr. and Meth. A 377, 228– 233 (1996).

Terms for the p-type Model

- The effective doping concentration as a function of fluence for p-type sensors can be expressed as[†,‡]:

$$N_{\text{eff}} = N_{\text{eff},0} - N_{\text{C}}(1 - \exp(-c_A\Phi_{\text{eq}})) + g_{\text{C}}\Phi_{\text{eq}}.$$

[†] B. Hiti et al., Nucl. Instruments Methods Phys. Res. Sect. A 924, 214–218 (2019).

[‡] G. Kramberger et al., Initial acceptor removal in p-type silicon, TRENTO Workshop, 2015.

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Initial doping concentration typical values are:

$$N_{\text{eff},0} = 2 \times 10^{12}\,\text{cm}^{-3}$$

for 3D p-type sensor

Stable initial acceptor removal term, expressed in the simulation code as:

$$N^{\text{removable}}_{\text{acceptor}} = N^{\text{removable}}_{\text{acceptor},0}(1 - e^{-c_{\text{A}} \Phi^{\text{rate}}_{\text{eq}} t})$$

Stable effective acceptor introduction term, expressed in the simulation code as:

$$N^{\text{stable}}_{\text{acceptor}} = g_{\text{C}} \Phi^{\text{rate}}_{\text{eq}} t$$

- Note that the annealing terms are excluded in this expression

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[*] M. Moll, IEEE Transactions on Nuclear Science, 65 8 (2018).

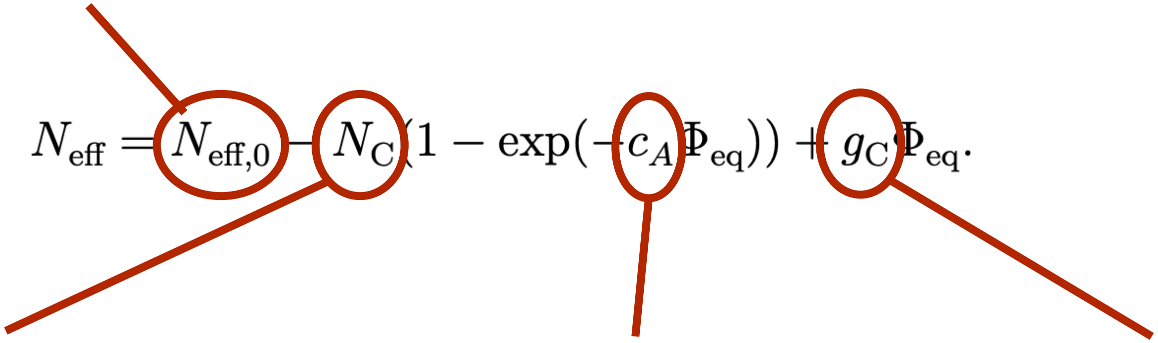
[†] B. Hiti et al., Nucl. Instruments Methods Phys. Res. Sect. A 924, 214–218 (2019).

[‡] G. Kramberger et al., Initial acceptor removal in p-type silicon, TRENTO Workshop, 2015.

p-type Model Parameters

- There are three parameters to set in p-type model

$$N_{\mathrm{eff,0}} = 2 \times 10^{12}\,\mathrm{cm}^{-3}$$



$$N_{\mathrm{eff}} = N_{\mathrm{eff,0}} - N_{\mathrm{C}}(1 - \exp(-c_A \Phi_{\mathrm{eq}})) + g_{\mathrm{C}} \Phi_{\mathrm{eq}}.$$

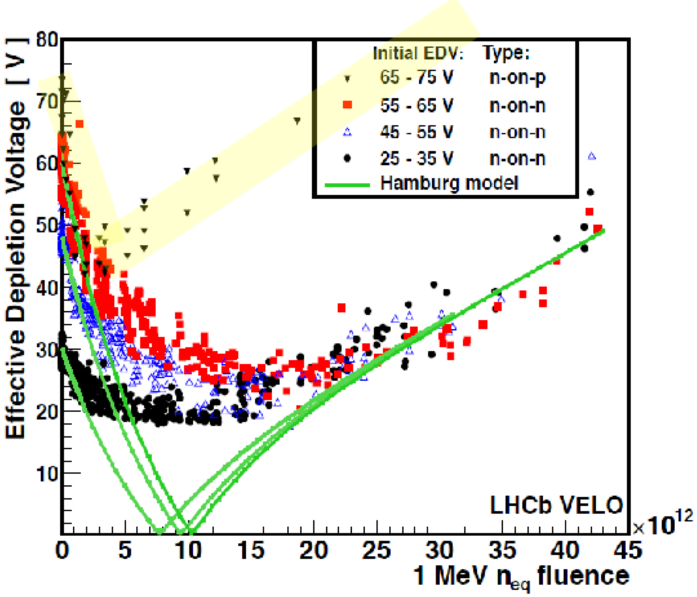
1. Number of initial acceptors that can be removed
2. Acceptor removal constant
3. Acceptor introduction rate

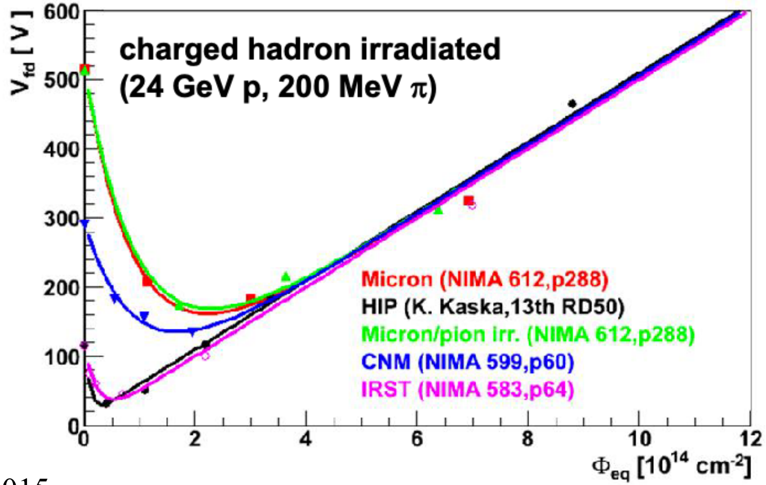
Parameter Values (1)

1. N_C - acceptors that can be removed

- Naively, we expect complete initial acceptor removal: $N_{C0}/N_{\text{eff},0} = 1$
- Incomplete acceptor removal has been observed in some devices, with values as low as: $N_{C0} = 0.1 \times N_{\text{eff},0}$
  - Studies have suggested that the value of N_C may be dependent on radiation type
- The figures[‡] on the right show (upper) the LHCb and (lower) individual experiments' depletion voltage measurements
- In the p-type sensors shown in the figures one can see the initial decrease in acceptors followed by an increase
  - At some point the acceptor removal term of the model becomes negligible

- LHCb Velo uses n-p sensor and has observed the effect of removal in V_{fd}





[‡] G. Kramberger et al., Initial acceptor removal in p-type silicon, TRENTO Workshop, 2015.

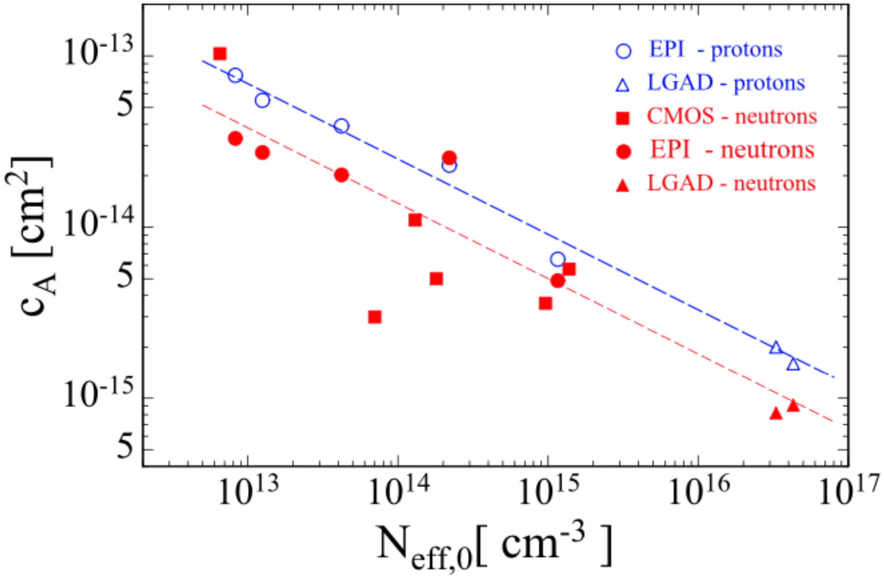
Parameter Values (2)

2. c_A –acceptor removal constant

- The value of c_A has been determined to be*:

$$c_A = 1.98 \times 10^{13}\ \mathrm{cm}^2$$

- This is consistent with the results presented in the figure† and an initial doping concentration of $N_{\mathrm{eff},0} = 2 \times 10^{12}\ \mathrm{cm}^{-3}$



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*R. Wunstorf, W. M. Bugg, J. Walter, F. W. Garber, and D. Larson, “Investigations of donor and acceptor removal and long term annealing in silicon with different boron/phosphorus ratios,” Nucl. Instr. and Meth. A 377, 228– 233 (1996)

† M. Moll, IEEE Transactions on Nuclear Science, 65 8 (2018).

Parameter Values (3)

3. g_C – acceptor introduction rate[*,†]: $0.02\,\mathrm{cm}^{-1}$

- Other values as low as 0.01 cm^{-1} are presented in [*]
- The value fit with ATLAS data[‡] for n-type sensors (on the B-Layer) is 0.0043 cm^{-1}

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[*] V. Cindro et al., “Radiation damage in p-type silicon irradiated with neutrons and protons,” NIMA 599, 60–65 (2009).

[†] G. Lindström et al., “Radiation hard silicon detectors - developments by the RD48 (ROSE) Collaboration,” Nucl. Instr. and Meth. A 466, 308 (2001).

[‡] https://atlas.web.cern.ch/Atlas/GROUPS/PHYSICS/PLOTS/PIX-2018-005/

Introducing Annealing

- Ignore short term beneficial annealing*:
  - “the resulting annealing rate was less than 1% of the acceptor removal rate, in most cases zero, which indicates that there is no significant short term room temperature annealing”
- Introduce long term reverse annealing analogous to the n-type simulation, with these two terms:

$$N^{\mathrm{reverse}}_{\mathrm{neutral}} = \frac{g_{\mathrm{Y}}}{k_{\mathrm{Y}}}\Phi^{\mathrm{rate}}_{\mathrm{eq}}(1 - e^{-k_{\mathrm{Y}}t}) + N^{\mathrm{reverse}}_{\mathrm{neutral},0}e^{-k_{\mathrm{Y}}t}$$

$$N^{\mathrm{reverse}}_{\mathrm{acceptor}} = \frac{g_{\mathrm{Y}}}{k_{\mathrm{Y}}}\Phi^{\mathrm{rate}}_{\mathrm{eq}}(k_{\mathrm{Y}}t + e^{-k_{\mathrm{Y}}t} - 1) + N^{\mathrm{reverse}}_{\mathrm{neutral},0}(1 - e^{-k_{\mathrm{Y}}t})$$

- g_{Y} is taken from the ATLAS n-type simulation: $g_Y = 6.0 \times 10^{-2}/\mathrm{cm}$

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*R. Wunstorf, W. M. Bugg, J. Walter, F. W. Garber, and D. Larson, “Investigations of donor and acceptor removal and long term annealing in silicon with different boron/phosphorus ratios,” Nucl. Instr. and Meth. A 377, 228– 233 (1996)

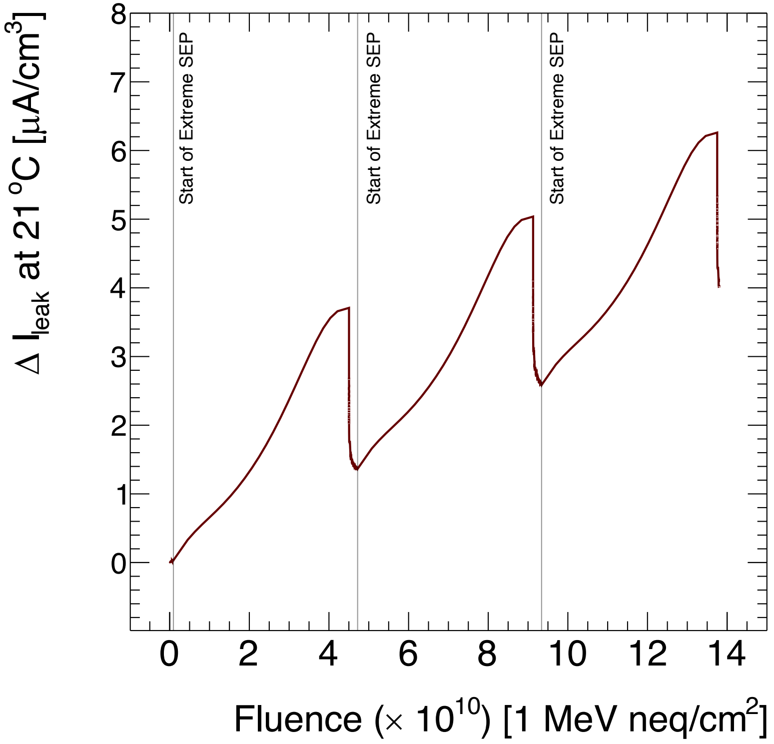
† M. Moll, IEEE Transactions on Nuclear Science, 65 8 (2018).

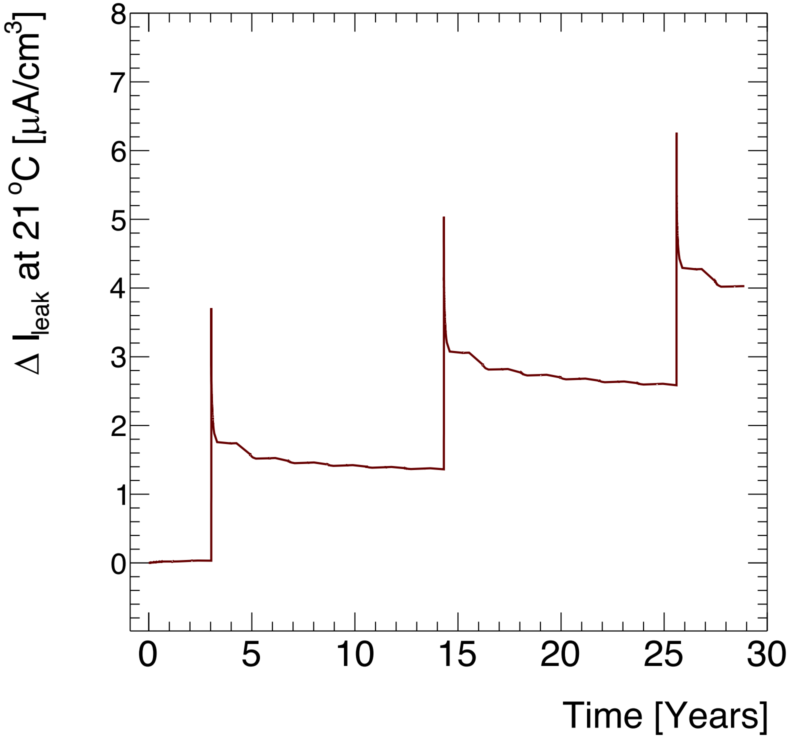
Conditions for the Simulation

- The model is run over discrete timesteps and the input at each timestep is:
  1. Duration of the timestep (t)
  2. Fluence rate in the timestep (Φ_{eq})
  3. Temperature during the timestep (T)
- The investigations presented here are for:
  - Each time step set to 1 min, for a total integrated time of tens of years
  - Continuous, low fluence rate (5.2 neq/cm^2/sec) and three higher fluence rates (4.98 × 10^5 neq/cm^2/sec) applied at equally separated time intervals
    - Total integrated fluence applied is about 1.4 × 10^{10} neq/cm^2
  - A yearly and daily temperature cycle with a mean below 0 ^{o}C and a maximum of 25 ^{o}C
- Compare simulation results for n-type and p-type sensors where the electrode spacings are set as:
  - 250 microns for n-type sensors
  - 40 microns for p-type sensors (representing an approximation to 3D sensors)

Leakage Current Results

- The leakage current results of the simulation are shown here.
  - The higher fluence rate events dominate the picture
- Results are the same for both n-type and p-type sensors

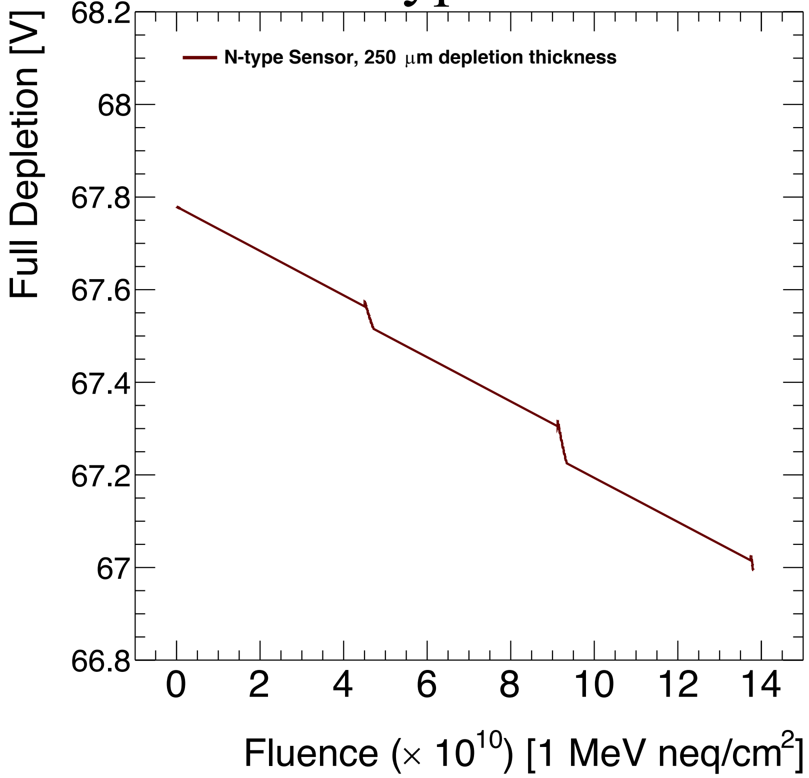




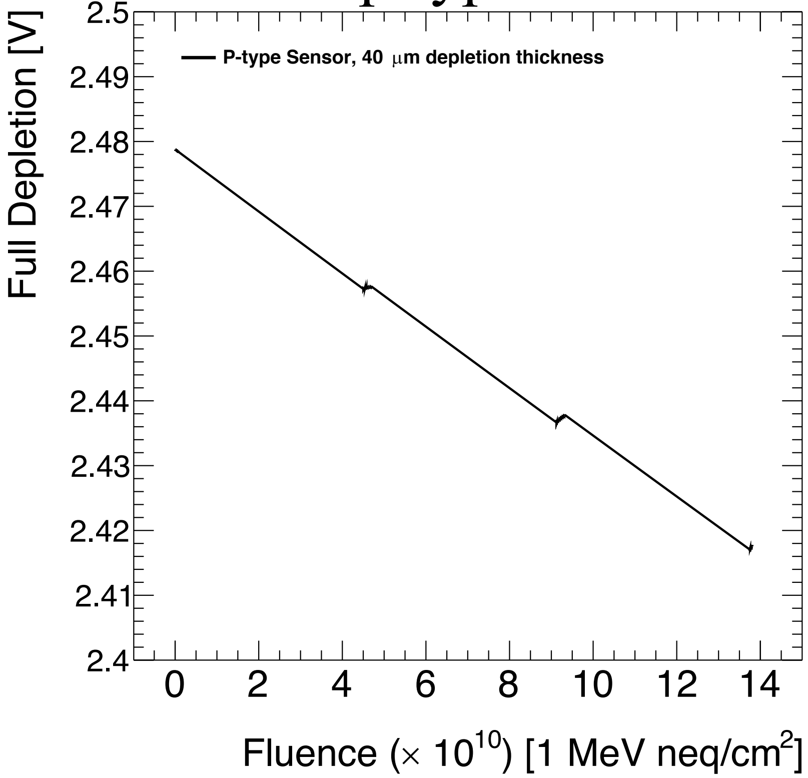
n-type and p-type Depletion Voltage Simulations

- The simulation results for (left) the n-type sensor and (right) the p-type sensor are shown here
  - The discontinuities reflect the effects of reverse annealing after cessation of the higher fluence events

n-type:

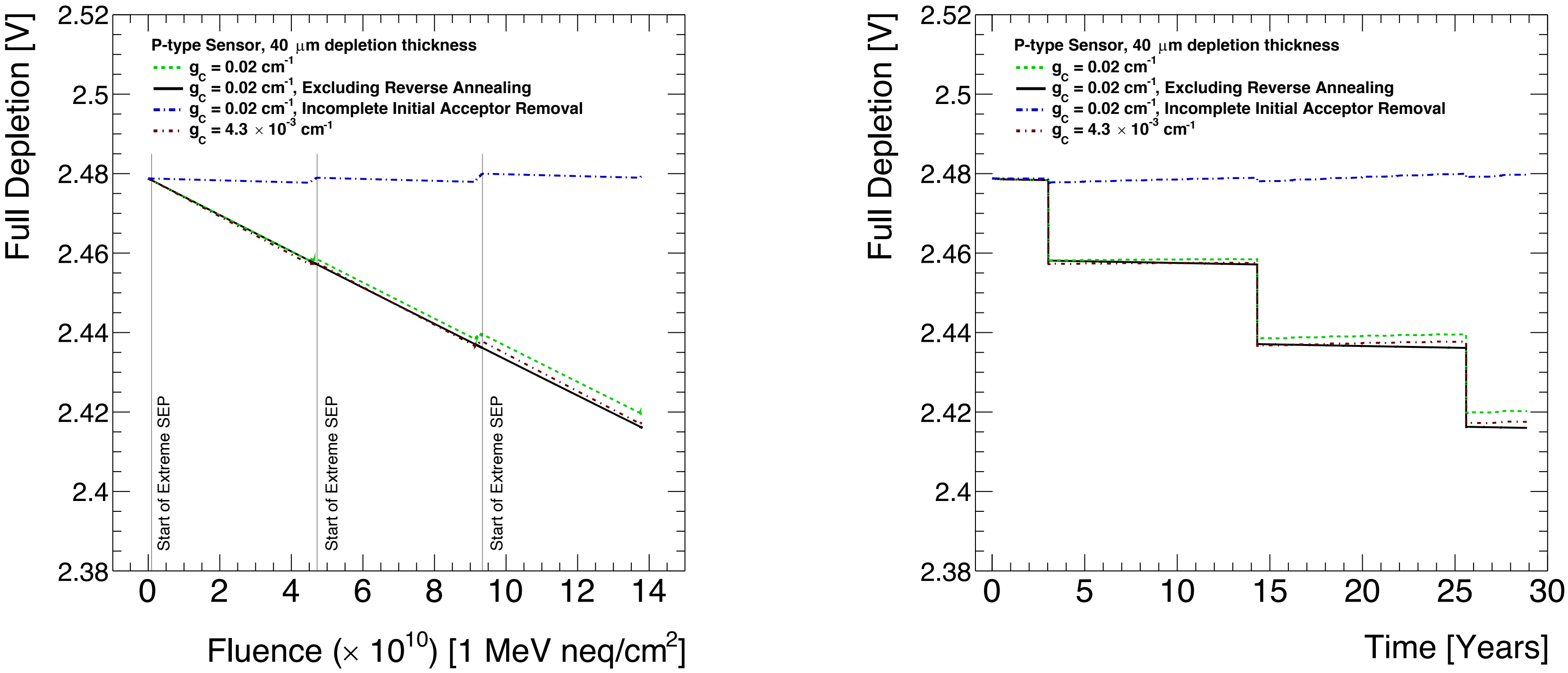


p-type:



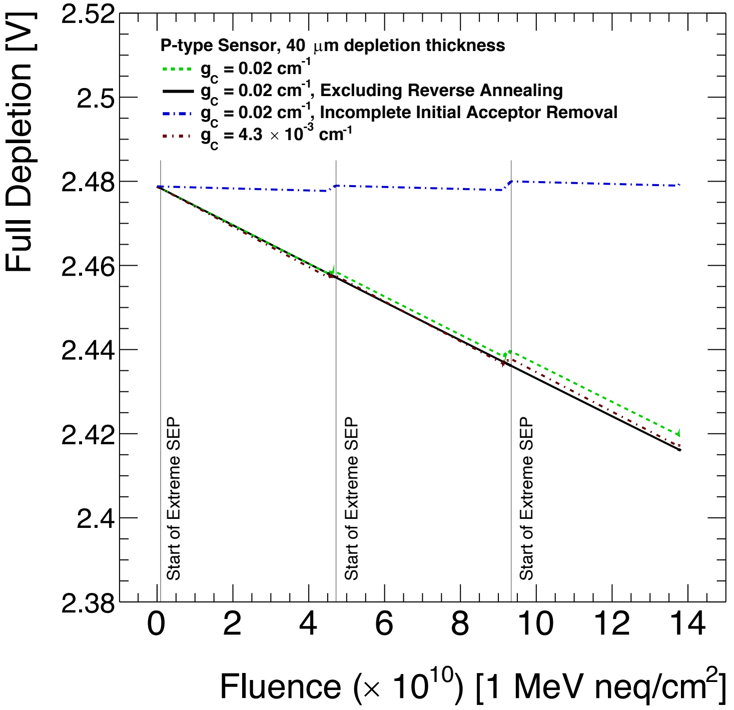
p-type Depletion Voltage Simulations

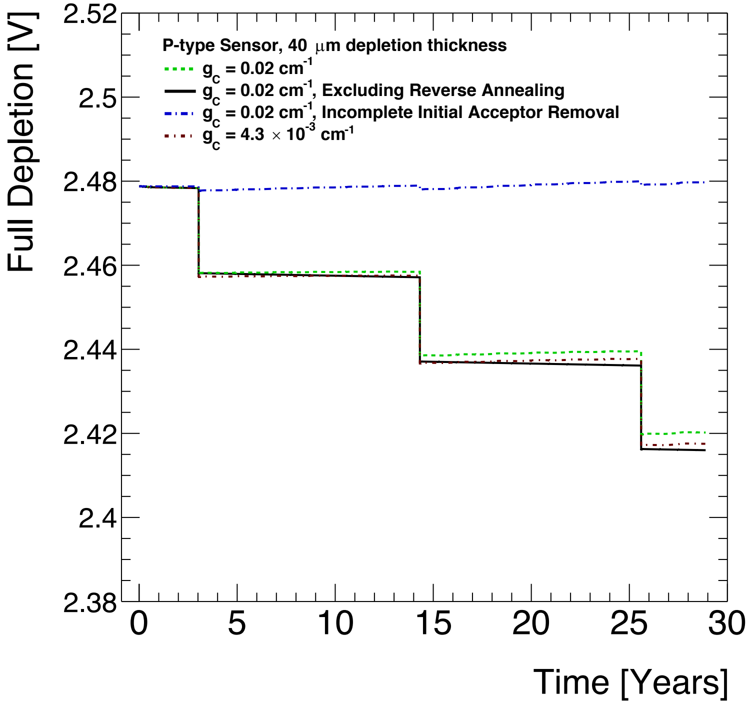
- Want to probe the simulation by using different parameter values: the p-type simulations for four different parameter settings are shown here
    - Nominally, $g_C = 0.02\ \text{cm}^{-1}$ and potential for complete acceptor removal is assumed
- (Left plot) simulation of depletion voltage for p-type sensors versus fluence
- (Right plot) simulation of depletion voltage for p-type sensors versus time



p-type Depletion Voltage Simulations - Results

- Reverse annealing plays a role for these time scales – this can be seen by comparing the black line (which excludes the reverse annealing terms) to the green dotted line
- The impact of changing g_C for the bounds of the range of values used in previous experiments is investigated – this can be seen by comparing the red dotted line ($g_C = 4.3\times10^{-3}$ cm^{-1}) to the green dotted line ($g_C = 0.02$ cm^{-1})
- Once the initial acceptor removal is exhausted, the effective acceptor introduction term will dominate subsequent behavior, and the slope will become positive – this is observed in the incomplete acceptor removal scenario in the blue dotted line





Conclusions and Outlook

- An overview of the response of p-type silicon sensors to non-ionizing energy loss and a new simulation of these effects has been discussed.
- Initial results for p-type simulations have been presented including reverse annealing in the simulation, accounting for incomplete initial acceptor removal, and varying the acceptor introduction rate for the bounds of the range of values used in previous experiments.
- Further studies of the impact due to different radiation and temperature environments will be investigated and compared to test beam data.
  - Other parameters in the model can be constrained when compared to physical data.
- Will be able to use this simulation code to describe characteristic data of irradiated silicon sensors and project the effects of further irradiation in future experiments