Faster induced-voltage calculation?

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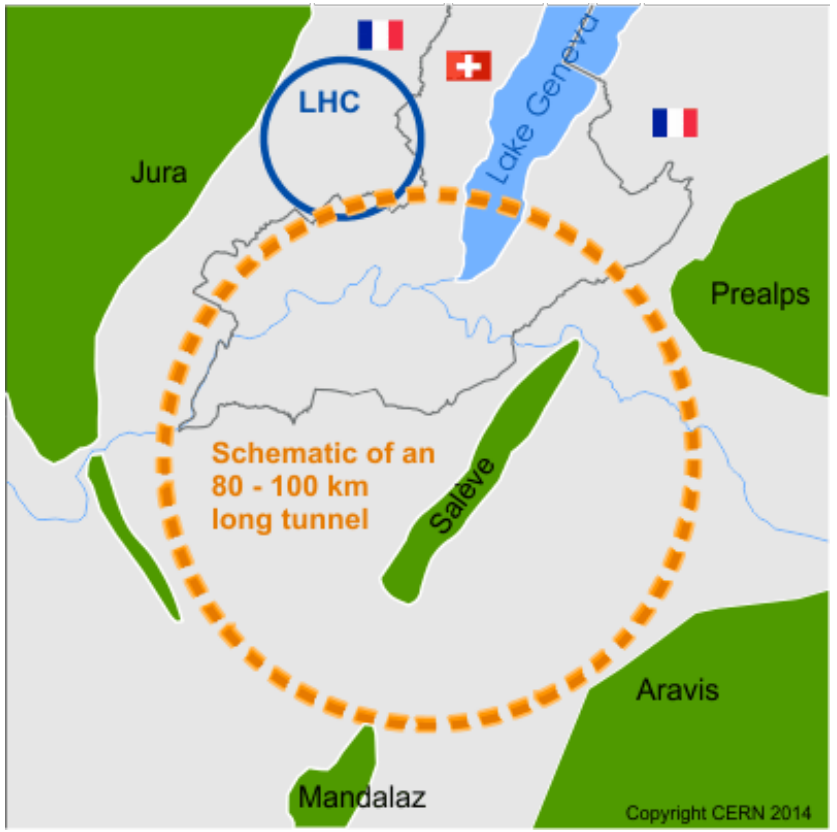
Motivation

We would like to simulate longitudinal instabilities in machines with the following parameters:

Parameter	FCC-hh	FCC-ee Z
Harmonic number	130680	
Revolution time, ms	0.33	
Number of bunches	16640	10400

The FCC transverse feedback studies were done in 2018 using PyHEADTAIL *(J. Komppula, FCC week 2018)*

Future circular colliders



Basics of induced voltage calculation

Induced voltage $$V_{\mathrm{ind}}(t) = -qN_p\,\lambda * W(t)$$

Line density — Wake function

BLonD implementations

Object	Impedance type	Method	Sampling	Complexity
InducedVoltageFreq	Any	Circular convolution	Uniform	$N \log N$
InducedVoltageTime	Any	Linear convolution	Uniform	$N \log N$
InducedVoltageResonator	Resonator	Matrix multiplication	Non-uniform	At least N^2?

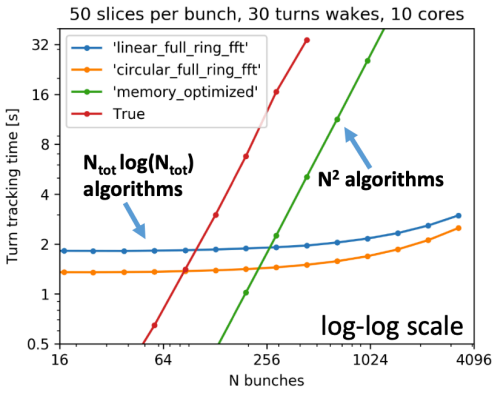
Ideally, we want to have non-uniform sampling + N log N complexity

Compressed convolution

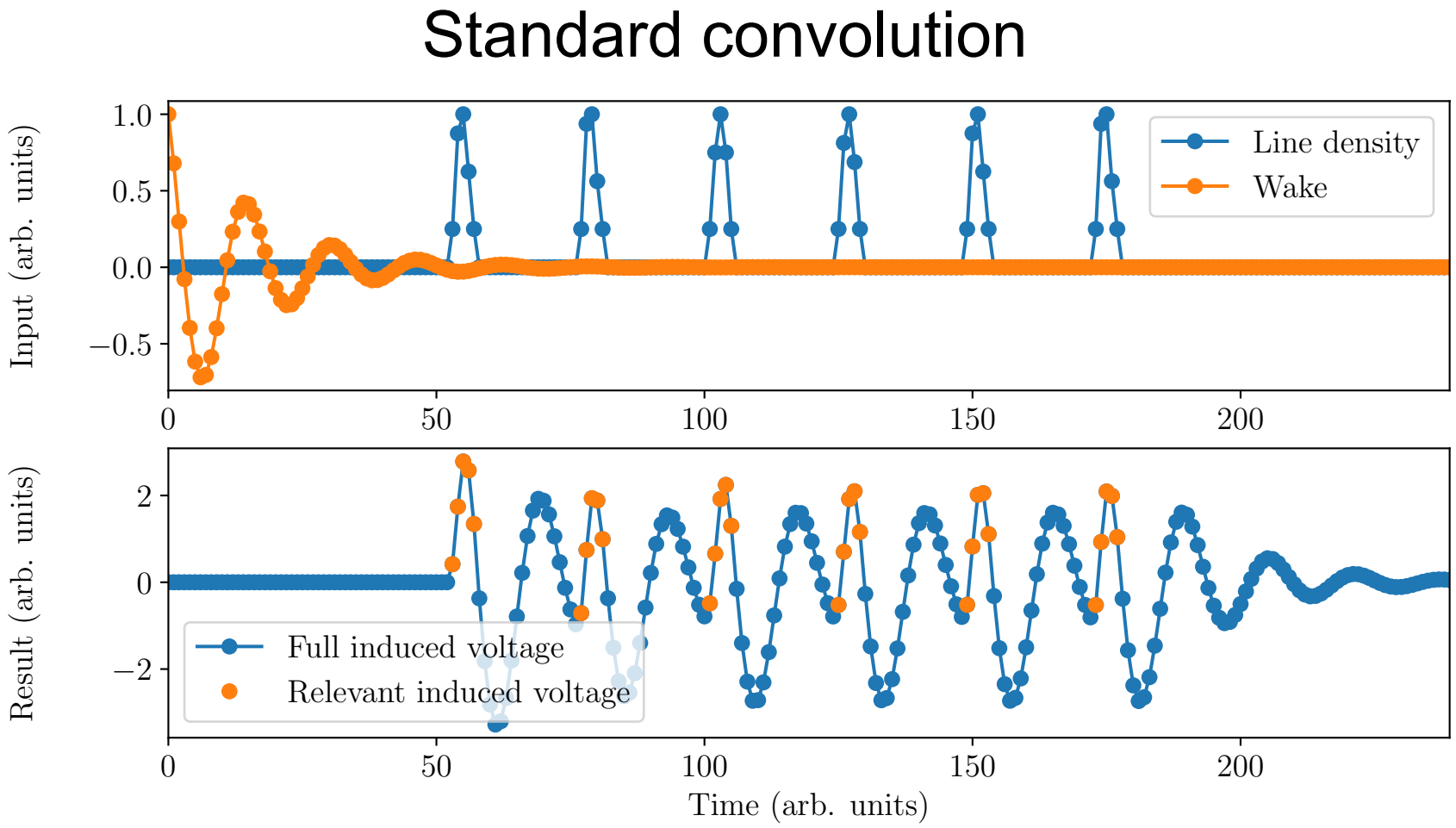
Has been implemented and used in PyHEADTAIL
(J. Komppula, K. Li, & N. Mounet, PyHEADTAIL Meeting #19, 2018)

How efficient is it?

- Benefits
  - FFT convolution scales as $N_{tot}\log(N_{tot})$
  - Wake field calculations can be efficiently parallelized to one turn length blocks
  - Faster than N^2 algorithms when filling factor >5-15%
- Drawbacks
  - Padding needed -> doubles the number of slices
  - Includes all the buckets, even if they are empty
  - Can be efficiently parallelized only to the total number of wake turns in the simulation (all wake kick objects in all planes)
  - More data transfer between nodes, but reasonable

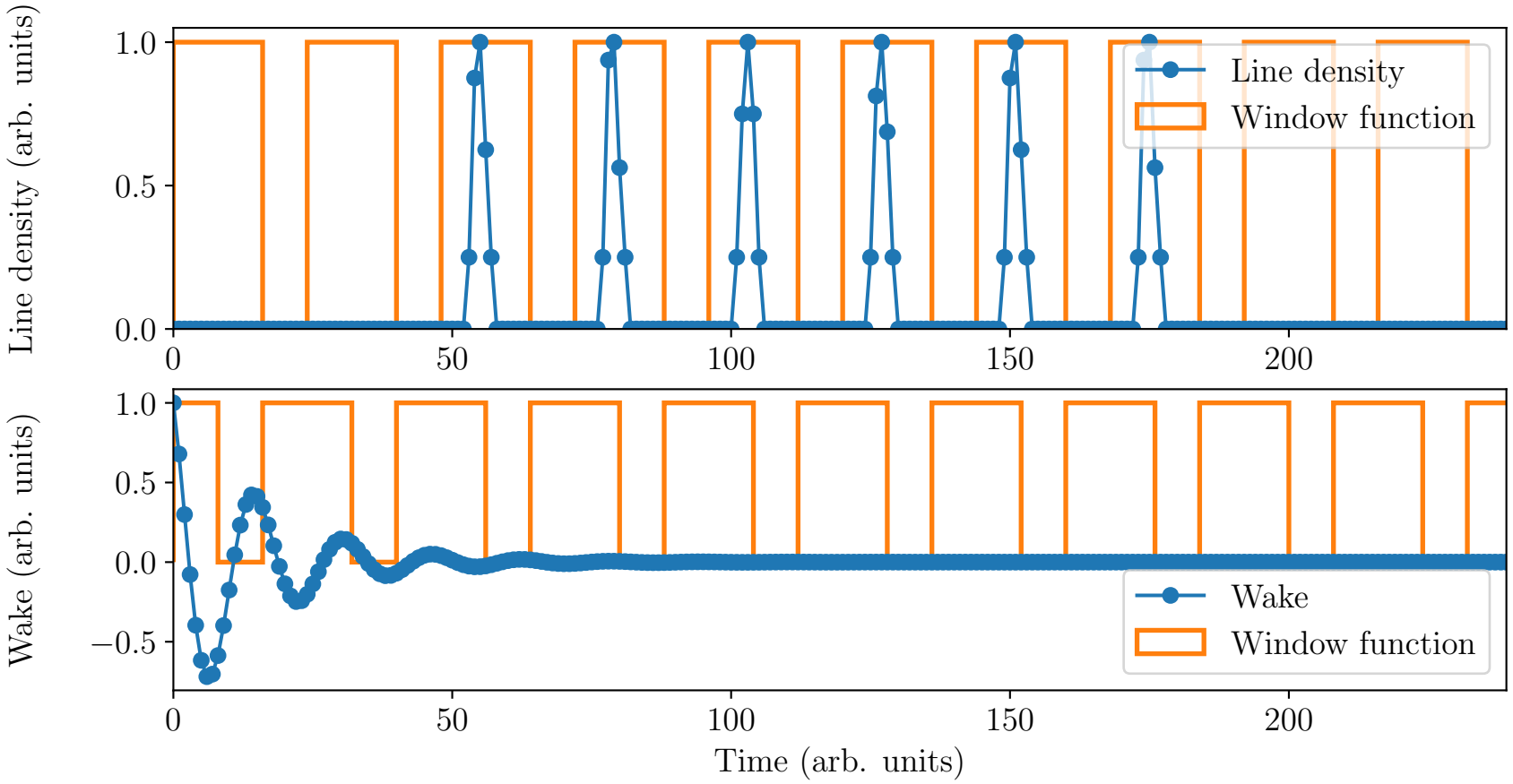


How does it work?



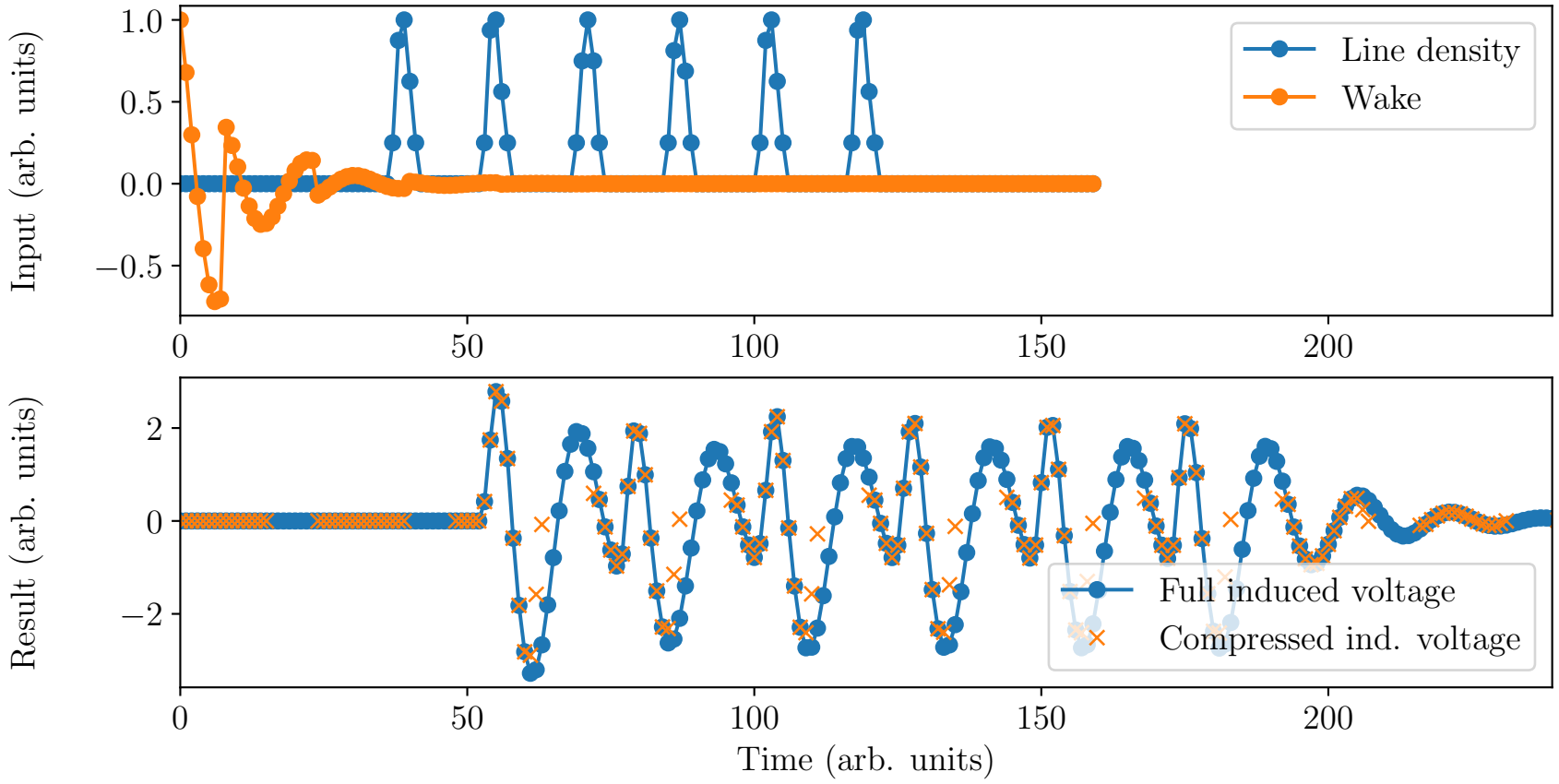
How does it work?

1. Pick relevant data and remove the rest



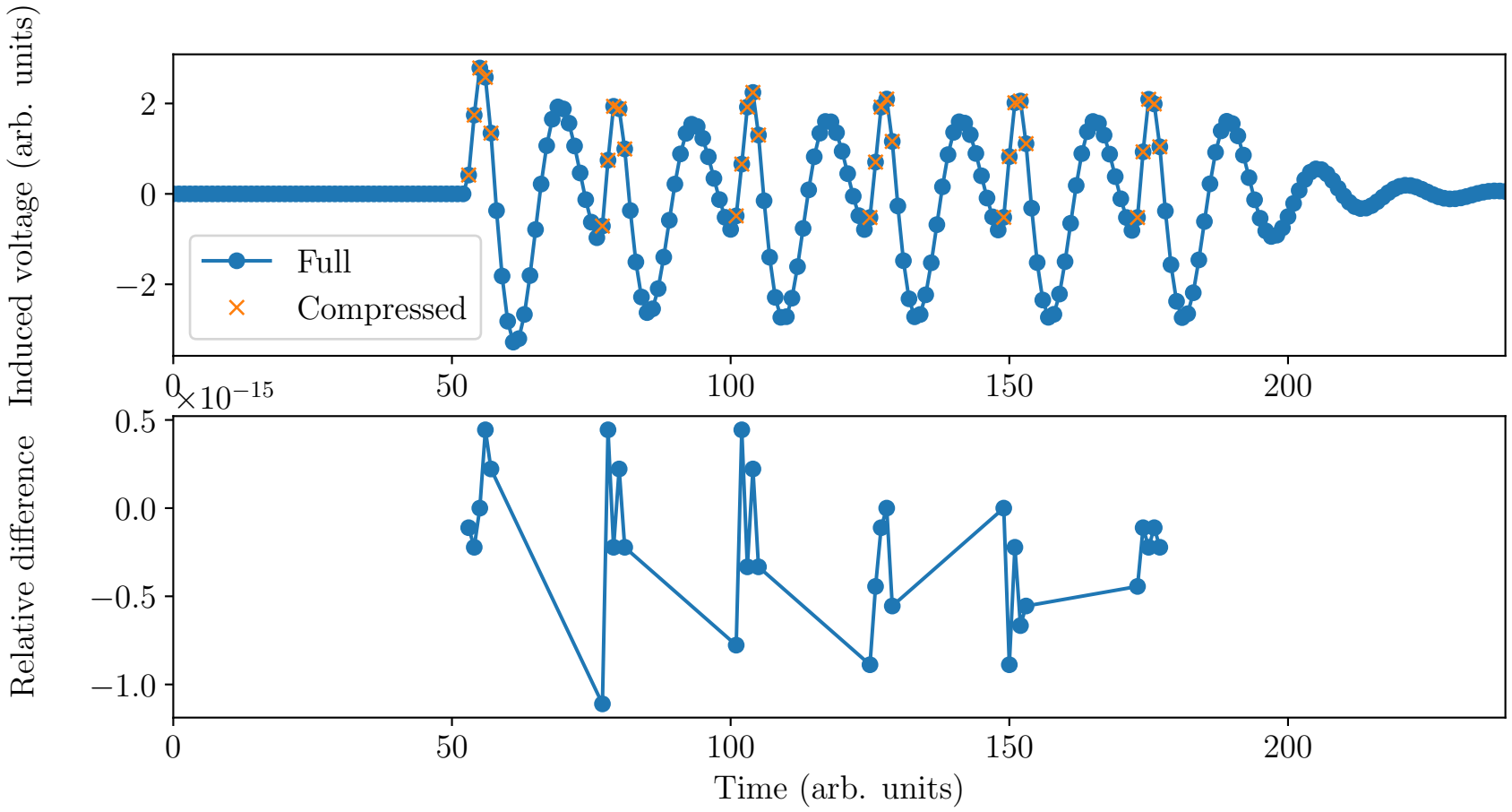
How does it work?

2. Compute convolution on compressed data



How does it work?

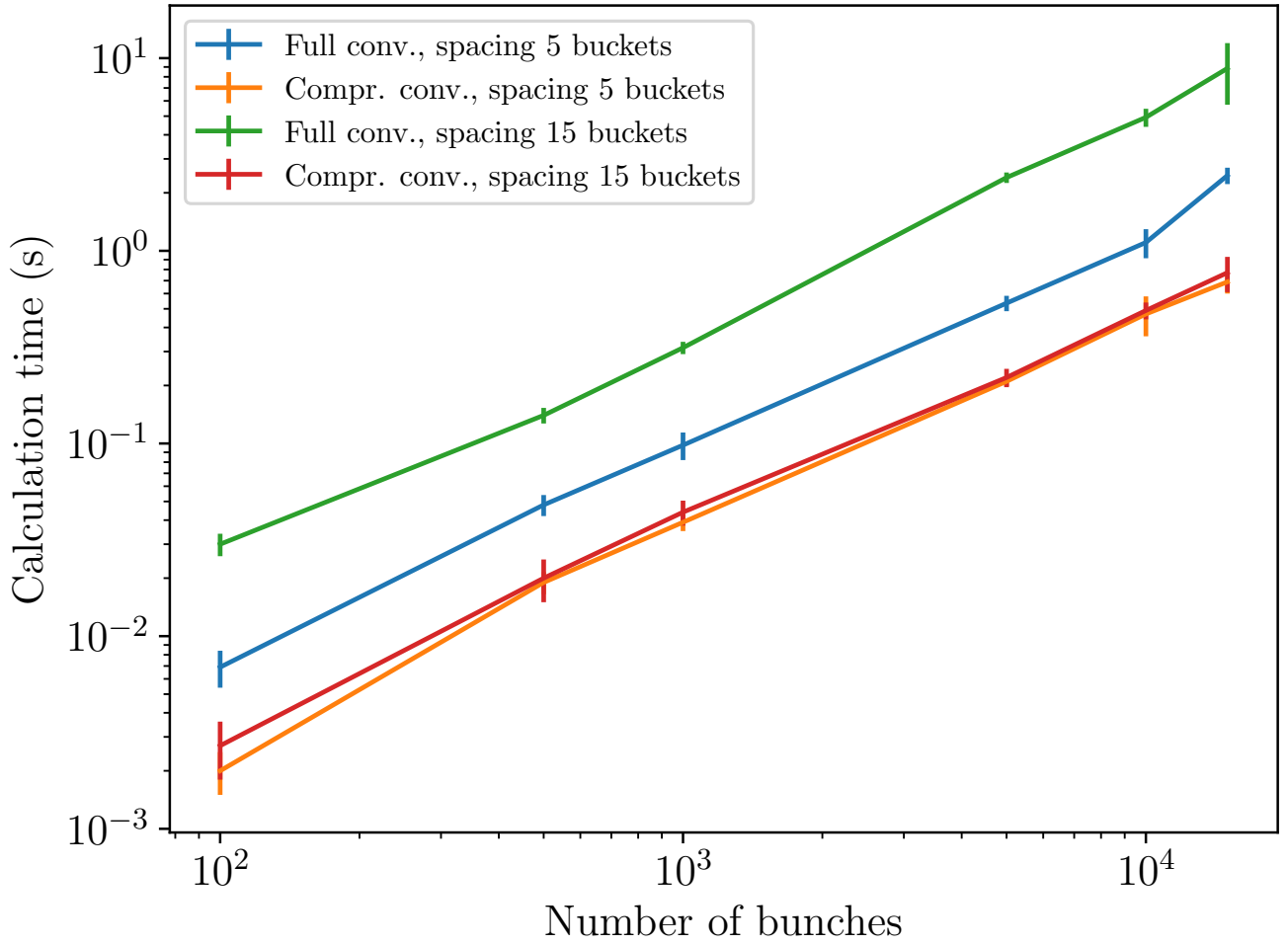
Extremely small residual error for relevant points!



Simple performance test

- Induced voltage for multi-bunch beams
- Bunches with binomial distribution for $\mu = 1.5$
- 128 slices per bucket
- Average of 50 executions

10 times faster calculations for filling!

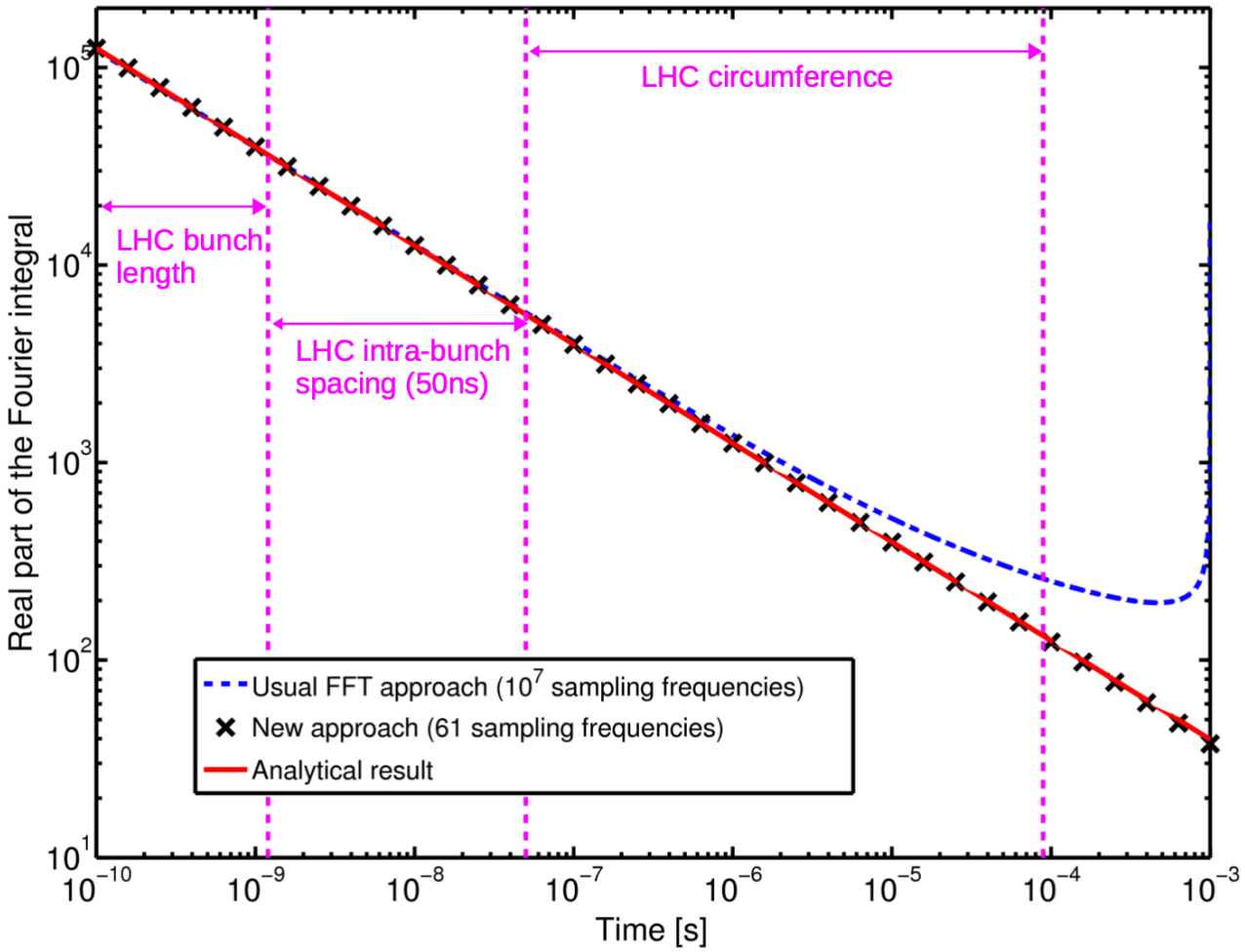


Remark on calculations of wake

Wake is given by an integral

$$W(t) \propto Re\left[\int_{\omega_{\min}}^{\infty} d\,\omega\, e^{i\omega t} Z(\omega)\right]$$

If $Z(\omega)$ is an analytic function, it could be approximated by using a cubic Hermite interpolation.
Then integral can also be analytically calculated (N. Mounet, PhD thesis, 2012).
This method is implemented in ImpedanceWake2D.



Outlook

- Compressed FFT convolution is a promising algorithm for induced voltage calculations.
- To obtain the total wake, the simplest way would be to use a resonator fit where possible. Otherwise, other analytic functions need to be properly integrated.
- Non-uniform profile is already implemented in BLonD (SparseSlices object) and can be reused.
- Another Induced voltage object can be created to use compressed line density and wake for optimized calculations.

Thank you!