Status of heavy-quarkonium photoproduction at NLO in High Energy Factorization

Maxim Nefedov[1]

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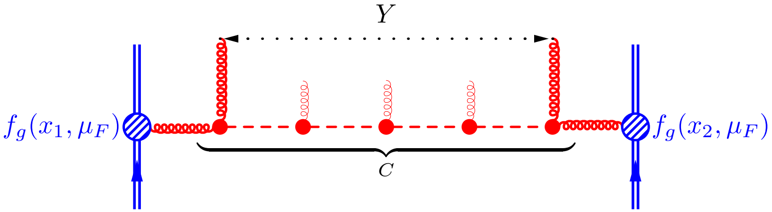
[1]Samara National Research University, Samara, Russia

High-Energy factorization in a nutshell

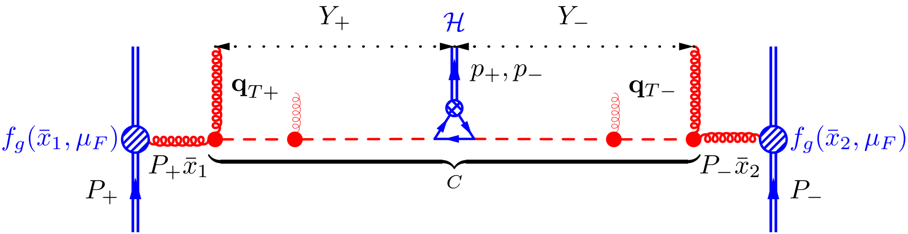
<u>Reminder:</u> Müller-Navelet dijet production (p_T of both jets is fixed) and BFKL (see also talk by M.Fucilla):



Hard-scattering coefficient C contains higher-order corrections $\propto (\alpha_s Y)^n$ (LLA) or $\alpha_s(\alpha_s Y)^n$ (NLLA), which can be resummed at leading power w.r.t. e^{-Y} using BFKL-formalism.

High-Energy factorization in a nutshell

High-Energy Factorization [Collins, Ellis, 91'; Catani, Hautmann, 94']:



Using the same formalism one can resum corrections to C enhanced by

$$Y_\pm = \ln\left(\frac{\mu_Y}{|\mathbf{q}_{T\pm}|}\frac{1-z_\pm}{z_\pm}\right) \simeq \ln\frac{\mu_Y}{|\mathbf{q}_{T\pm}|} + \ln\frac{1}{z_\pm}, \text{ in LP w.r.t. } \frac{|\mathbf{q}_{T\pm}|}{\mu_Y}\frac{z_\pm}{1-z_\pm}$$

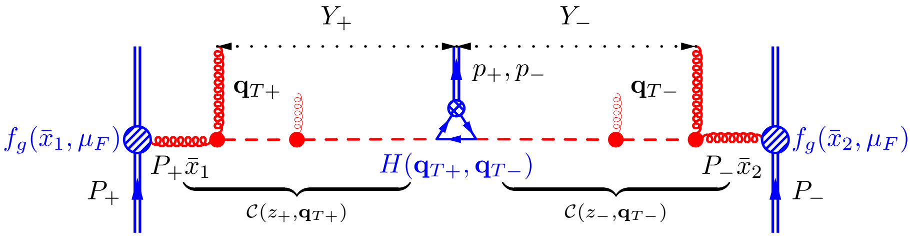
in inclusive observables (e.g. inclusive quarkonium production). Here

$$z_+ = \frac{p_+}{P_+\bar{x}_1}, \; z_- = \frac{p_-}{P_-\bar{x}_2} \text{ and } \mu_Y = p_+ e^{-y_{\mathcal{H}}} = p_- e^{y_{\mathcal{H}}},$$

e.g. $\mu_Y^2 = m_{\mathcal{H}}^2 + \mathbf{p}_T^2$.

High-Energy factorization in a nutshell

High-Energy Factorization [Collins, Ellis, 91'; Catani, Hautmann, 94']:



Hard-scattering coefficient is re-factorized, *unintegrated PDF* is introduced:

$$\Phi_g(x, \mathbf{q}_T, \mu_Y) = f_g\left(\frac{x}{z}, \mu_F\right) \otimes \mathcal{C}(z, \mathbf{q}_T, \mu_F, \mu_Y).$$

- *Collinear divergences* from additional emissions are subtracted inside UPDF.
- New coefficient function H depends on $x_{1,2}$ as well as $\mathbf{q}_{T\pm}$ (k_T*-factorization*).
- Factorization with single type of factors $\mathcal{C}$ and H is proven at LL and NLL approximation [Fadin *et.al.*, early 2000s], and known to be violated at N^{2}LL. Factorization with several types of $\mathcal{C}$ and H should be introduced then.

High-Energy factorization in a nutshell

Structure of the LL BFKL series for $\mathcal{C}$ *after subtraction of collinear divergences:*

$$\mathcal{C}(z, \mathbf{q}_T, \mu_Y, \mu_F) = \sum_n \alpha_s^n Y^n \ln^n \frac{\mu_F}{|\mathbf{q}_T|} = \sum_n \left[\alpha_s \left(\ln \frac{\mu_Y}{|\mathbf{q}_T|} + \ln \frac{1}{z} \right) \ln \frac{\mu_F}{|\mathbf{q}_T|} \right]^n$$

$$= \underbrace{\sum_n \left[\alpha_s \ln \frac{\mu_Y}{|\mathbf{q}_T|} \ln \frac{\mu_F}{|\mathbf{q}_T|} \right]^n}_{\text{LL TMD PDF (CSS)} \simeq \text{KMRW UPDF}} + \underbrace{\sum_{n,k<n} \alpha_s^n \ln^k \frac{1}{z} \ln^{n-k} \frac{\mu_Y}{|\mathbf{q}_T|} \ln^n \frac{\mu_F}{|\mathbf{q}_T|}}_{\text{overlap with } N^k LL \text{ TMD}}$$

$$+ \underbrace{\sum_n \left[\alpha_s \ln \frac{1}{z} \ln \frac{\mu_F}{|\mathbf{q}_T|} \right]^n}_{\text{LL UPDF [Catani, Hautmann, Blümlein]}}$$

MRK LL evolution equation

Is the LO BFKL-equation with real emissions ordered in physical rapidity $y_j = \ln(k_j^+/k_j^-)/2$:

$$\mathcal{C}(x, \mathbf{q}_T, \mu_Y) = \delta(x-1)\delta(\mathbf{q}_T)\mathcal{D}(\mu_Y, \epsilon) + \leftarrow \text{ coll. initial condition}$$

$$\int\limits_x^1 \frac{dz}{z(1-z)} \left\{ \frac{\alpha_s C_A}{\pi} \int \frac{d^{D-2}\mathbf{k}_T}{\pi(2\pi)^{-2\epsilon}} \frac{1}{\mathbf{k}_T^2} \leftarrow \text{ real emission} \right.$$

$$\times \mathcal{C}\left(\frac{x}{z}, \mathbf{q}_T + \mathbf{k}_T, \frac{|\mathbf{k}_T|}{1-z} \right) \theta\left(\Delta(|\mathbf{k}_T|, \mu_Y) - z\right) \leftarrow \ y - \text{ordering}$$

$$\left. + 2\omega_g(\mathbf{q}_T^2)\mathcal{C}\left(\frac{x}{z}, \mathbf{q}_T, \mu_Y \frac{x(1-z)}{z(z-x)} \right) \theta\left(\Delta(|\mathbf{q}_T|, \mu_Y) - z\right) \right\} \leftarrow \text{ virt.part}$$

where $D = 4 - 2\epsilon$, $\Delta(q_T, \mu) = \mu/(\mu + q_T)$, $\mathcal{D}$ – collinear loop factor, $\omega_g(\mathbf{q}_T^2) = \frac{\alpha_s C_A}{2\pi} \frac{\mathbf{q}_T^{-2\epsilon}}{\epsilon} \frac{(4\pi)^\epsilon \Gamma(1+\epsilon)\Gamma^2(1-\epsilon)}{\Gamma(1-2\epsilon)}$ – one-loop gluon Regge trajectory.

After Mellin transform w.r.t. x ($\frac{1}{z}\ln^k z \to \frac{1}{N^{k+1}}$):

$$\mathcal{C}(N, \mathbf{q}_T, \mu_Y) = \delta(\mathbf{q}_T)\mathcal{D}(\mu_Y, \epsilon) + \int\limits_0^{\mu_Y} d\bar{\mu}\ \bar{\mu}^{N-1} \Big[2\omega_g(\mathbf{q}_T^2)(\bar{\mu} + |\mathbf{q}_T|)^{-N} \mathcal{C}(N, \mathbf{q}_T, \bar{\mu} + |\mathbf{q}_T|) +$$

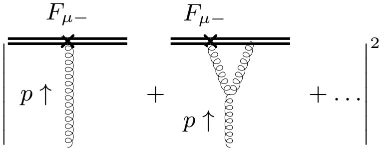
$$\frac{\alpha_s C_A}{\pi} \int \frac{d^{D-2}\mathbf{k}_T}{\pi(2\pi)^{-2\epsilon}} \frac{1}{\mathbf{k}_T^2} (\bar{\mu} + |\mathbf{k}_T|)^{-N} \mathcal{C}(N, \mathbf{q}_T + \mathbf{k}_T, \bar{\mu} + |\mathbf{k}_T|) \Big]$$

Collinear loop factor

BFKL evolution does not properly take into account interactions in the purely collinear sector ($p^- > 0$, $p^+ = |\mathbf{p}_T| = 0$, infinite rapidity).

To include them, collinear loop factor $\mathcal{D}$ should be added:

$\mathcal{D}(\epsilon, \mu_Y) =$



$$= 1 + \frac{\alpha_s(\mu) C_A}{\pi} \left[-\frac{1}{2\epsilon^2} - \frac{1}{2\epsilon} \ln \frac{\mu^2}{\mu_Y^2} - \frac{1}{4} \ln^2 \frac{\mu^2}{\mu_Y^2} \right] + O(\alpha_s^2).$$

Wilson line is tilted from light-cone:

$$n_-^\mu \to n_-^\mu + n_+^\mu e^{-2y},$$

where $e^{-2y} = \mu_Y^2 / p_-^2$ provides scale to $\mathcal{D}(\mu_Y, \epsilon)$.

Doubly-logarithmic (=Blümlein) UPDF

One has to solve the **dimensionally-regularized** evolution equation to subtract **collinear divergences.** To demonstrate how it works let's skip all $O(z)$-corrections in MRK-equation and go to $(N, \mathbf{x}_T)$-space (see [M.N. 2020, Appendix A]):

$$\mathcal{C}(N, \mathbf{x}_T) = 1 + \frac{\hat{\alpha}_s}{N} \frac{\Gamma(1-\epsilon)(\mu^2)^\epsilon}{(-\epsilon)\pi^{-\epsilon}} \int d^{2-2\epsilon}\mathbf{y}_T\, \mathcal{C}(N, \mathbf{y}_T) \times \left[(\mathbf{x}_T^2)^\epsilon \delta(\mathbf{x}_T - \mathbf{y}_T) - \frac{\epsilon\Gamma(1-\epsilon)}{\pi^{1-\epsilon}((\mathbf{x}_T - \mathbf{y}_T)^2)^{1-2\epsilon}} \right],$$

where $\hat{\alpha}_s = \alpha_s C_A (\mu^2)^{-\epsilon}/\pi$, then we solve it iteratively and collinear divergences at each order organize into:

$$Z_{\text{coll.}} = \exp\left[-\frac{1}{\epsilon} \int_0^{\hat{\alpha}_s S_\epsilon} \frac{d\alpha}{\alpha} \gamma_N(\alpha) \right],\ \gamma_N(\alpha) = \gamma_1(N)\alpha + \gamma_2(N)\alpha^2 + \ldots,$$

where $S_\epsilon = \exp[\epsilon(-\gamma_E + \ln 4\pi)]$ for $\overline{MS}$-scheme and [Jaroszewicz 82', Catani, Hautmann, 94']:

$$\gamma_1 = \frac{1}{N}, \gamma_2 = \gamma_3 = 0, \gamma_4 = \frac{2\zeta_3}{N^4}, \gamma_5 = \frac{2\zeta_5}{N^5}, \ldots$$

and poles in N correspond to $\ln^k(1/z)/z$ in the DGLAP $P_{gg}(z)$.

Doubly-logarithmic (=Blümlein) UPDF

In *doubly-logarithmic apprixmation* (corrections to which start at $O(\alpha_s^3)$!), the finite part of $\mathcal{C}$ can be expressed as:

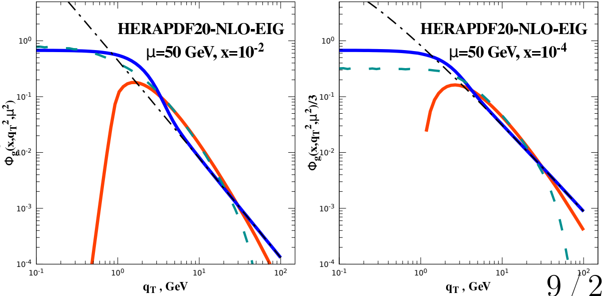
$$\mathcal{C}^{(\mathrm{ren.})}(N, \mathbf{x}_T) \underset{\mathrm{DLA}}{\simeq} \exp\left[-\gamma_0 \ln(\mu_F^2 \bar{\mathbf{x}}_T^2)\right] \Leftrightarrow \mathcal{C}_{\mathrm{DLA}}^{(\mathrm{ren.})}(N, \mathbf{q}_T) = \frac{\Gamma(1-\gamma_0)}{\mathbf{q}_T^2 \Gamma(\gamma_0)} \left(\frac{\mathbf{q}_T^2 e^{-2\gamma_E}}{\mu_F^2}\right)^{\gamma_0},$$

where $\bar{\mathbf{x}}_T = \mathbf{x}_T e^{\gamma_E}/2$, and $\gamma_0 = \frac{\alpha_s C_A}{\pi} \frac{1}{N}$. Note that:

$$\int\limits_0^{\mu_F^2} d\mathbf{q}_T^2 \ \mathcal{C}_{\mathrm{DLA}}^{(\mathrm{ren.})}(N, \mathbf{q}_T) = 1 + O(\alpha_s^3), \text{ i.e. } \int\limits_0^{\mu_F^2} d\mathbf{q}_T^2 \ \Phi_g^{\mathrm{DLA}}(x, \mathbf{q}_T) \simeq f_g(x, \mu_F^2).$$

Some numerical results:

- dash-dotted – Blümlein,
- KMRW,
- DL UPDF$\otimes \exp\left[-\mathbf{q}_T^2/(1 \text{ GeV})^2\right]$,
- Parton-Branching UPDF [Jung, Hautmann, *et.al.*]

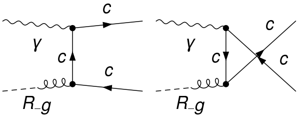


Quarkonium photoproduction in HEF

The LO coefficient functions for quarkonium photo and electro-production had been calculated in [Kniehl,Vasin,Saleev, 2006] both for **direct** and **resolved-photon** channels. The $2 \to 2$ subprocess:

$$R_-(q_1) + \gamma(q) \to c\bar{c}\left[{}^3S_1^{(1)}\right] + g,$$

where R_- – *Reggeized gluon* with momentum $q_1^\mu = x_1 P_1^\mu + q_{T1}^\mu$, $q_1^2 = -\mathbf{q}_{T1}^2$, and we take $q^2 = 0$.



The $2 \to 1$ subprocesses:

$$R_-(q_1) + \gamma(q) \to c\bar{c}\left[{}^1S_0^{(8)}, {}^3S_1^{(8)}, {}^3P_J^{(8)}\right],$$

${}^3S_1^{(8)}$ amplitude $=0$ even for $\mathbf{q}_{T1} \neq 0$.

The Reggeon-gluon "mixing" coupling ("non-sense" polarization):

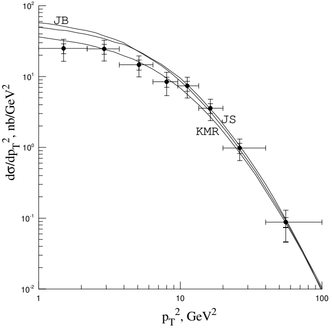
$$\frac{q_1^-}{2|\mathbf{q}_{T1}|} n_+^\mu \leftrightarrow \frac{q_{T1}^\mu}{|\mathbf{q}_{T1}|},$$

due to Slavnov-Taylor identity for amplitude $q_1^\mu \mathcal{M}_\mu = 0$ ("Gribov's trick").

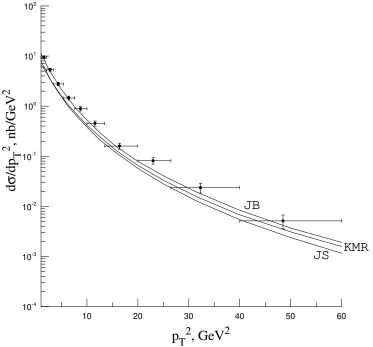
Quarkonium photoproduction in HEF

Some numerical results from [Kniehl,Vasin,Saleev, 2006]:

p_T-spectrum in DIS (H1 data, $2 < Q^2 < 110$ GeV^2)



p_T-spectrum in photo-production (ZEUS data, $Q^2 < 1$ GeV)



- ${}^3S_1^{(1)}$-state dominates, as in LO of CPM (see talk by Y.Yedelkina)
- Moderate spread between KMRW and Blümlein (JB) UPDFs.
- Large scale-unceartainty (∼factor of 2).
- photo-production cross-section is under-estimated

⇒ NLO corrections to H-function may be significant.

High-Energy EFT: Reggeon fields

Let's introduce **gauge-invariant** Reggeon fields $R_\pm(x) = T^a R^a_\pm(x)$ subject to kinematic constraints ($\Leftrightarrow$ (Q)MRK, $\partial_\pm = n^\mu_\pm \partial_\mu = 2\frac{\partial}{\partial x^\mp}$):

$$\partial_- R_+ = \partial_+ R_- = 0 \Rightarrow$$

$$R_+ \text{ carries } (k_+, \mathbf{k}_T) \text{ and } R_- \text{ carries } (k_-, \mathbf{k}_T).$$

Effective action [Lipatov, 1995]:

$$S = \int d^4x(-2R^a_+ \partial^2_\perp R^a_-) + \sum_{\text{rap. ints.}} \int d^2\mathbf{x}_T \left\{ \int \frac{dx_+ dx_-}{2} L_{\text{QCD}}(x, A_\mu, \psi) \right.$$

$$\left. + \int \frac{dx_+}{2} \,\mathrm{tr}\left[R^a_-(x_+, \mathbf{x}_T) \mathcal{T}_+[x, A_\mu] \right] + \int \frac{dx_-}{2} \,\mathrm{tr}\left[R^a_+(x_-, \mathbf{x}_T) \mathcal{T}_-[x, A_\mu] \right] \right\},$$

what are the interaction operators $\mathcal{T}_\pm$?

High-Energy EFT: light-like Wilson lines

Constraints we have:

- At leading power in energy, partons highly separated in rapidity perceive each-other as infinite light-like Wilson lines [Mueller, Nikolaev, Zakharov, 1990s; Balitsky, 2000s;...; Caron-Huot, 2013],
- Hermiticity [Lipatov, 1997; Bondarenko, Zubkov, 2018]
- $R_\pm \to g$ transition is given by "non-sense" polarization $n^\mu_\mp$.

$$\Rightarrow \mathcal{T}_\pm[x, A_\mu] = \frac{i}{g_s}\partial_\perp^2 \left(W_\infty[x_\pm, \mathbf{x}_T, A_\mu] - W^\dagger_\infty[x_\pm, \mathbf{x}_T, A_\mu]\right),$$

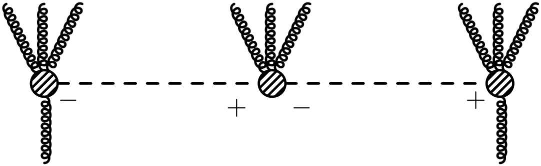
Where:

$$\begin{aligned} W_{x_\mp}[x_\pm, \mathbf{x}_T, A_\pm] &= P\exp\left[\frac{-ig_s}{2}\int\limits_{-\infty}^{x_\mp} dx'_\mp A_\pm(x_\pm, x'_\mp, \mathbf{x}_T)\right] \\ &= \left(1 + ig_s\partial_\pm^{-1}A_\pm\right)^{-1}, \end{aligned}$$

and $\partial_\pm^{-1} \to -i/(k^\pm + i\varepsilon)$ in the Feynman rules.
After IBP trick:

$$\boxed{S_{\text{int.}} = \int dx \frac{i}{g_s}\,\text{tr}\left[R_+(x)\partial_\perp^2\partial_-\left(W_{x_+}[A_-] - W^\dagger_{x_+}[A_-]\right) + (+\leftrightarrow -)\right]},$$

Structure of Induced interactions



Induced interactions of particles and Reggeons:

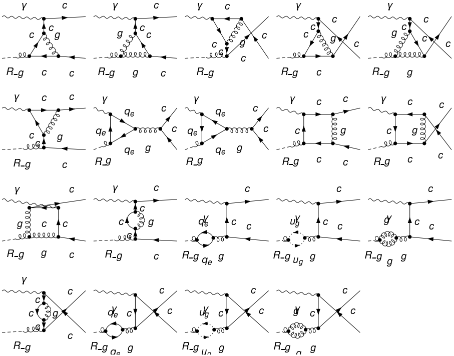
$$\frac{i}{g_s}\,\mathrm{tr}\left[R_+\partial_\perp^2\partial_- W\left[A_-\right] + R_-\partial_\perp^2\partial_+ W\left[A_+\right]\right],$$

expansion of P-exponent generaties induced vertices:

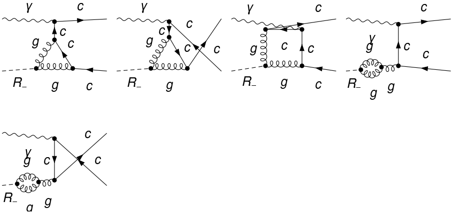
$$\begin{aligned}
&\mathrm{tr}\Big[\big(R_+\partial_\perp^2 A_- + R_-\partial_\perp^2 A_+\big) + \\
&(-ig_s)(\partial_\perp^2 R_+)(A_-\partial_-^{-1}A_-) + (-ig_s)^2(\partial_\perp^2 R_+)(A_-\partial_-^{-1}A_-\partial_-^{-1}A_-) + \\
&(-ig_s)(\partial_\perp^2 R_-)(A_+\partial_+^{-1}A_+) + (-ig_s)^2(\partial_\perp^2 R_-)(A_+\partial_+^{-1}A_+\partial_+^{-1}A_+) \\
&+O(g_s^3)\Big]\,.
\end{aligned}$$

$R\gamma \to c\bar{c}\left[{}^1S_0^{(8)}\right]$ @ 1 loop

Rg-coupling diagrams:

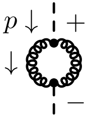


Induced coupling diagrams:



- Expressions for all diagrams had been generated using custom `FeynArts` model-file, scalar products converted to denominators
- Since heavy-quark momenta $= \pm p/2$, not all (quadratic) denominators are linearly-independent. Linear-dependence had been resolved before IBP
- IBP reduction to master integrals has been performed using `LiteRed`.

Rapidity divergences and regularization

$$\Pi^{(1)}_{ab} = q \downarrow$$


$$= g_s^2 C_A \delta_{ab} \int \frac{d^d q}{(2\pi)^D} \frac{\left(\mathbf{p}_T^2 (n_+ n_-)\right)^2}{q^2 (p-q)^2 q^+ q^-}$$

Cutoff in rapidity [Lipatov, 1995] ($q^{\pm} = \sqrt{q^2 + \mathbf{q}_T^2} e^{\pm y}$, $p^+ = p^- = 0$):

$$\int \frac{dq^+ dq^-}{q^+ q^-} = \int\limits_{y_1}^{y_2} dy \int \frac{dq^2}{q^2 + \mathbf{q}_T^2},$$

$$\Pi^{(1)}_{ab} \sim \delta_{ab} \mathbf{p}_T^2 \times \underbrace{C_A g_s^2 \int \frac{\mathbf{p}_T^2 d^{D-2}\mathbf{q}_T}{\mathbf{q}_T^2 (\mathbf{p}_T - \mathbf{q}_T)^2}}_{\omega_g(\mathbf{p}_T^2)} \times (y_2 - y_1) + \text{finite terms}$$

Tilted Wilson-line regularization [Hentschinski, Sabio Vera, Chachamis *et. al.*, 2012-2013]:

$$\boxed{\tilde{n}^{\pm} = n^{\pm} + r \cdot n^{\mp},\ \tilde{k}^{\pm} = k^{\pm} + r \cdot k^{\mp},\ r \to 0,}$$

+ modified kinematics [M.N.,2019]: $\boxed{\tilde{\partial}_+ R_- = \tilde{\partial}_- R_+ = 0.}$

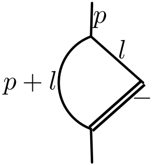
“Tadpoles” and “Bubbles”.

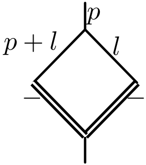
“Tadpoles” (one quadratic propagator):

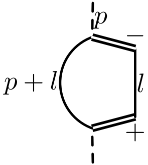
$$A_{[-]}(p) = \int \frac{[d^d l]}{(p+l)^2[\tilde{l}^-]},\ A_{[--]}(p) = \int \frac{[d^d l]}{l^2[\tilde{l}^-][\tilde{l}^- - \tilde{p}^-]}$$

where $[d^D l] = \frac{(\mu^2)^\epsilon d^d l}{i\pi^{D/2} r_\Gamma}$, $r_\Gamma = \Gamma^2(1-\epsilon)\Gamma(1+\epsilon)/\Gamma(1-2\epsilon)$,
$1/[\tilde{l}^-] = \left(1/(\tilde{l}^- + i\varepsilon) + 1/(\tilde{l}^- - i\varepsilon)\right)/2$

“Bubbles” (two quadratic propagators):







$$B_{[-]}(p) = \int \frac{[d^d l]}{l^2(p+l)^2[\tilde{l}^-]},$$

$$B_{[--]}(p) = \int \frac{[d^d l]}{l^2(p+l)^2[\tilde{l}^-][\tilde{l}^- + \tilde{p}^-]}$$

$$B_{[+-]}(p) = \int \frac{[d^d l]}{l^2(p+l)^2[\tilde{l}^+][\tilde{l}^-]},$$

where $p^+ = p^- = 0$ for the last integral.

Rapidity divergences at one loop

Only log-divergence $\sim \ln r$ (Blue cells in the table) is related with Reggeization of particles in t-channel.
Integrals which **do not** have log-divergence may still contain the power-dependence on r:

- $r^{-\epsilon} \to 0$ for $r \to 0$ and $\epsilon < 0$.
- $r^{+\epsilon} \to \infty$ for $r \to 0$ and $\epsilon < 0$ – **weak-power divergence** (Pink cells in the table)
- $r^{-1+\epsilon} \to \infty$ – **power divergence.** (Red)

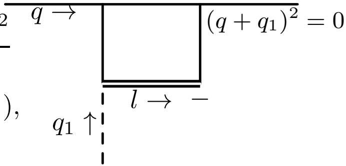
(# LC prop.) \ (# quadr. prop.)	1	2	3	4
1	$A_{[-]}$	$B_{[-]}$	$C_{[-]}$	…
2	$A_{[+-]}$	$B_{[+-]}$	$C_{[+-]}$	…
3	…	…	…	…

The **weak-power** and **power-divergences** cancel between Feynman diagrams describing one region in rapidity, so only log-divergences are left.

Triangle integrals, logarithmic RD

Result for $Q^2 = 0$:

$$C_{[-]}(t_1, 0, q^-) = \frac{1}{q^- t_1}\left(\frac{\mu^2}{t_1}\right)^{\epsilon} \frac{1}{\epsilon}\left[\ln r + i\pi - \ln\frac{|q_-|^2}{t_1}\right.$$
$$\left. -\psi(1+\epsilon) - \psi(1) + 2\psi(-\epsilon)\right] + O(r^{1/2}),$$



coincides with the result of [G. Chachamis, *et. al.*, 2012].

Result for $Q^2 \neq 0$ [M.N., 2019]:

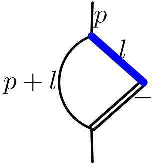
$$C_{[-]}(t_1, Q^2, q_-) = C_{[-]}(t_1, 0, q_-) + \left(\frac{\mu^2}{t_1}\right)^{\epsilon} \frac{I(Q^2/t_1)}{q_- t_1} - \frac{1}{t_1}\underbrace{\Delta B_{[-]}(Q^2, q_-)}_{\propto r^{-\epsilon}},$$

where

$$\begin{aligned} I(X) &= -\frac{2X^{-\epsilon}}{\epsilon^2} - \frac{2}{\epsilon}\int\limits_0^X \frac{(1-x^{-\epsilon})dx}{1-x} \\ &= -\frac{2X^{-\epsilon}}{\epsilon^2} + 2\left[-\mathrm{Li}_2(1-X) + \frac{\pi^2}{6}\right] + O(\epsilon). \end{aligned}$$

New mass-dependent master integrals

Example: "bubble-type" integral with massive propagator arises during IBP-reduction:

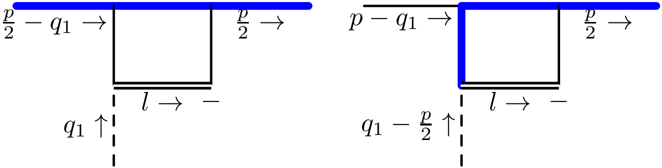


$$= \frac{1}{2\tilde{p}^-}\frac{\mu^{2\epsilon}}{\epsilon^2}\left\{\frac{1}{\cos(\pi\epsilon)}\left[\left(\frac{m^2}{M_T^4}\right)^{\epsilon} - (\tilde{p}_-)^{-2\epsilon} r^{\epsilon}\right] + \frac{\Gamma(1-2\epsilon)\Gamma^2(1+\epsilon)}{\pi^2}\left[\frac{m^{-2\epsilon}\sin(\pi\epsilon)^2}{\epsilon^2} - \left(\frac{\tilde{p}_-^2}{M_T^4}\right)^{\epsilon}\frac{\pi r^{-\epsilon}\,\mathrm{tg}(\pi\epsilon)}{\epsilon}\right]\right\} + O(r),$$

where $M_T^2 = m^2 + \mathbf{p}_T^2$. New kinds of mass-dependent $r^{\pm\epsilon}$-terms appear, **which should cancel between diagrams.**

New mass-dependent master integrals

New mass-dependent “triangle-type” integrals arise as result of IBP-reduction:



Their calculation is in progress.

Outlook

- HEF rigorously resums corrections enhanced by "rapidity" logs: $Y_{\pm} = \ln(\mu_Y/|\mathbf{q}_T|) + \ln(1/z_{\pm})$ to LLA and NLLA. Obtaining the complete solution for LLA UPDF is hard, therefore approximations had been used. Situation at $\mathrm{N}^{k>2}$LLA in QCD is more complicated.
- In the region $z_{\pm} \ll 1$, emissions with $|\mathbf{q}_T| \sim \mu_Y \ll \sqrt{S}$ are allowed, so HEF is **not** limited to $|\mathbf{q}_T| \ll \mu_Y$
- There is an overlap between HEF and TMD-factorization (or CSS, or SCET) w.r.t. resummation of $\ln(\mu_Y/|\mathbf{q}_T|)$
- KMRW and Blümlein UPDFs can be considered as two "opposite" approximations to exact LLA UPDF in HEF: resummation of $(\alpha_s \ln^2(\mu_Y/|\mathbf{q}_T|))^n$ and $(\alpha_s \ln(\mu_F/|\mathbf{q}_T|)\ln(1/z))^n$ respectively.
- Calculation of $\gamma R \to c\bar{c}\left[{}^{2S+1}L_J\right]$ contributions at one loop is a good starting point for the full NLO calculation for heavy quarkonium production in HEF. All master integrals needed at NLO are contained in this calculation. At NLO for H, the scale-dependence should be reduced even with LLA UPDF.
- Cancellation of $r^{\pm\epsilon}$ terms will serve as a strong cross-check of one-loop result. Only $\ln r$-divergence should be left.

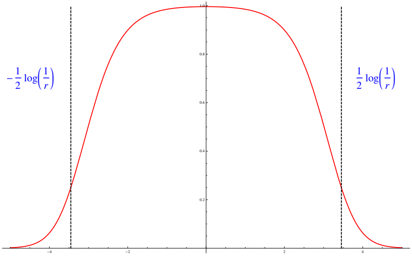
Thank you for your attention!

Backup: rapidity divergences in real corrections

Constraint $\tilde{\partial}_+ R_- = \tilde{\partial}_- R_+ = 0$. Lipatov's vertex ($k = q_1 - q_2$, $k^2 = 0$):

$$\Gamma_{+\mu-} = -(\tilde{n}_+ \tilde{n}_-) \left((q_1 + q_2)_\mu + q_1^2 \frac{\tilde{n}^-_\mu}{\tilde{q}_2^-} + q_2^2 \frac{\tilde{n}^+_\mu}{\tilde{q}_1^+} \right) + 2 \left(\tilde{q}_1^+ \tilde{n}^-_\mu + \tilde{q}_2^- \tilde{n}^+_\mu \right),$$

without modified constraint, the Slavnov-Taylor identity $\boxed{k^\mu \Gamma_{+\mu-} = 0}$ is violated by terms $O(r)$.



The square of regularized LV:

$$\Gamma_{+\mu-}\Gamma_{+\nu-}P^{\mu\nu} = \frac{16\mathbf{q}_{T1}^2\mathbf{q}_{T2}^2}{\mathbf{k}_T^2} f(y),$$

$$\longleftarrow f(y) = \frac{1}{(re^{-y} + e^y)^2 (re^y + e^{-y})^2},$$

$$\int\limits_{-\infty}^{+\infty} dy\ f(y) = -1 - \ln r + O(r)$$

Backup: On the energy dependence of p_T-integrated $pp \to \mathcal{H} + X$ cross-section

p_T-integrated cross-section with DL UPDF can be put in the form:

$$\left.\frac{d\sigma}{dy}\right|_{y=0} = \int \frac{dN_1 dN_2}{(2\pi i)^2} \frac{\Gamma(1-\gamma_{N_1})\Gamma(1-\gamma_{N_2})e^{-2\gamma_E(\gamma_{N_1}+\gamma_{N_2})}}{\Gamma(\gamma_{N_1})\Gamma(\gamma_{N_2})} f_{N_1}(\mu_F) f_{N_2}(\mu_F)$$
$$\times \int \frac{d^2\mathbf{q}_{T1} d^2\mathbf{q}_{T2}}{\pi \mathbf{q}_{T1}^2 \mathbf{q}_{T2}^2} \left(\frac{\mathbf{q}_{T1}^2}{\mu_F^2}\right)^{\gamma_{N_1}} \left(\frac{\mathbf{q}_{T2}^2}{\mu_F^2}\right)^{\gamma_{N_2}} \left(\frac{M_T}{\sqrt{S}}\right)^{-N_1-N_2} \frac{\overline{|\mathcal{M}(\mathbf{q}_{T1},\mathbf{q}_{T2})|^2}}{M_T^4},$$

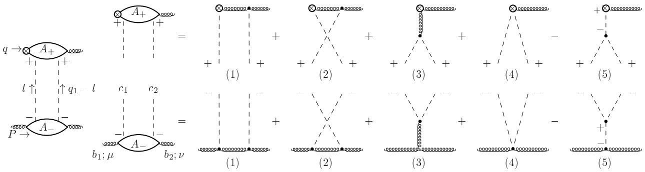
where $M_T^2 = M^2 + (\mathbf{q}_{T1}+\mathbf{q}_{T2})^2$, $\gamma_N = \frac{\alpha_s C_A}{\pi N}$ and e.g. for ${}^1S_0$-states:

$$\overline{|\mathcal{M}(\mathbf{q}_{T1},\mathbf{q}_{T2})|^2} \propto \frac{2M^2\left(M^2 + (\mathbf{q}_{T1}+\mathbf{q}_{T2})^2\right)^2}{\left(M^2 + \mathbf{q}_{T1}^2 + \mathbf{q}_{T2}^2\right)^2} \sin^2\phi_{12}.$$

It is unlikely, that some fixed μ_F scale-choice can cancel all $\alpha_s/N_{1,2}$-corrections.

Backup: on Multi-Reggeon exchanges

Example EFT calculation of the 2-Reggeon exchange [M.N., 2019]:



$$\mathcal{M}^{(RR),b_1 b_2 b_3}_{\mu\nu\sigma} = \frac{1}{8}\int \frac{d^{d-2}\mathbf{l}_T}{(2\pi)^d} \frac{A^{+,c_2 c_1 b_3}_{\sigma}(l_T) A^{-,b_1 c_1 c_2 b_2}_{\mu\nu}(l_T)}{\mathbf{l}_T^2(\mathbf{q}_{T1}-\mathbf{l}_T)^2},$$

$$A^{(-,1+2+4),b_1 c_1 c_2 b_2}_{\mu\nu} = (-ig_s^2) v_{-\mu\nu}(P, P-q_1) \int\limits_{-\infty}^{+\infty} \frac{dl_-}{\sqrt{2}} \times$$

$$\left\{ \frac{f^{b_1\{c_1 d}f^{dc_2\}b_2}}{2} \left[\frac{(-2P_+)}{-P_+l_- - \mathbf{l}_T^2 + i\varepsilon} + \frac{(-2P_+)}{P_+l_- + 2\mathbf{l}_T\mathbf{q}_{T1} - \mathbf{l}_T^2 + i\varepsilon} \right] \right.$$

$$\left. + \quad \frac{f^{b_1[c_1 d}f^{dc_2]b_2}}{2} \left[\frac{(-2P_+)}{-P_+l_- - \mathbf{l}_T^2 + i\varepsilon} - \frac{(-2P_+)}{P_+l_- + 2\mathbf{l}_T\mathbf{q}_{T1} - \mathbf{l}_T^2 + i\varepsilon} \right] \right\},$$

Integral in front of $f^{b_1\{c_1 d}f^{dc_2\}b_2}$ gives $i\pi\sqrt{2}$, integral in front of $f^{b_1[c_1 d}f^{dc_2]b_2}$ vanishes after subtractions.