

Quarkonium TMD-Shape functions for electron-proton colliders

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In progress: (S. Fleming, Y. Makris, T. Mehen, and J. Lieffers)

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Quarkonium and TMDs

Quarkonium production is considered as one of the most important processes to access unpolarized and polarized gluon TMDs. Some example of relevant processes are:

$$ep/pp \rightarrow \psi + X/\psi + \psi + X/\psi + \gamma + X/...$$

arXiv:1202.6585 (S. J. Brodsky, F. Fleuret, C. Hadjidakis, J. P. Lansberg)

arXiv:1208.3642 (D. Boer, C. Pisano)

arXiv:1401.7611 (W. J. den Dunnen, J.-P. Lansberg, C. Pisano, M. Schlegel)

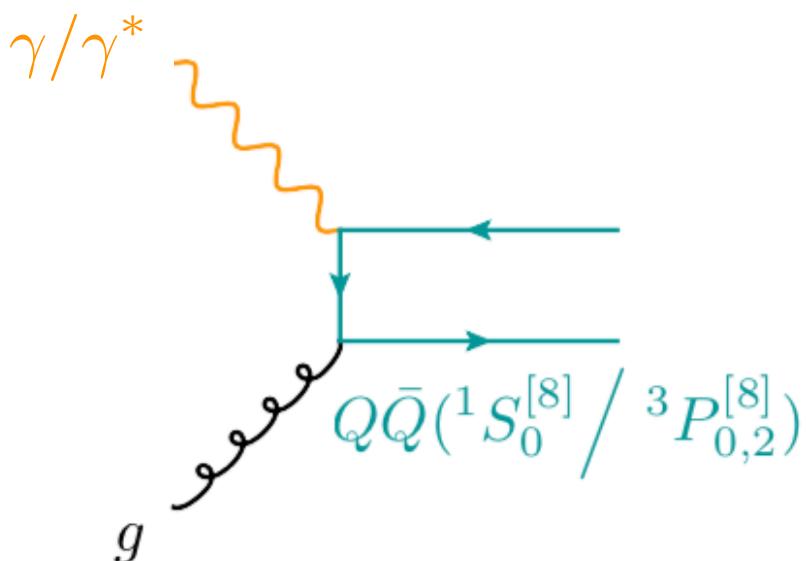
arXiv:1406.5476 (G-P. Zhang)

arXiv:1809.02056 (A. Bacchetta, D. Boer, C. Pisano, P. Taels)

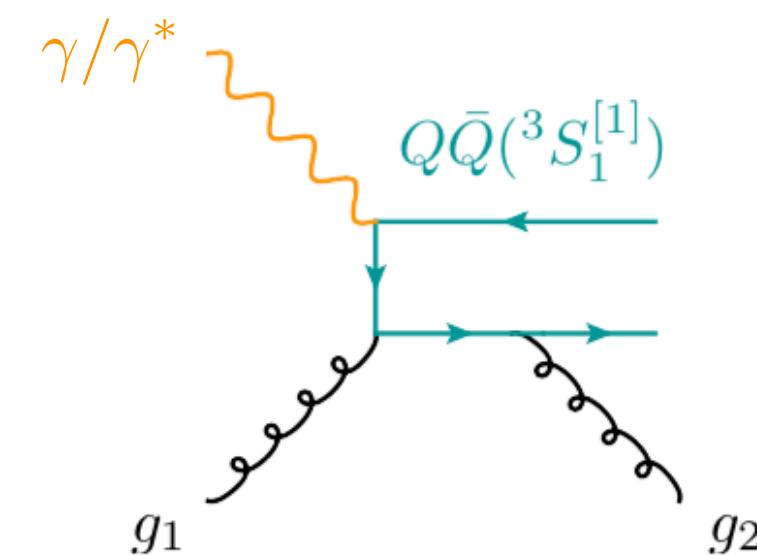
and many others

In this talk ($ep \rightarrow Q + X$)

Color octet



Color singlet



- qT measured w.r.t. photon-proton axis
- Direct photon
- Hard scales: $W \gg m_Q \gg \Lambda_{QCD}$ (photo) $Q > m_Q \gg \Lambda_{QCD}$ (lepto)
- Single light (collinear) direction
- No Glauber vertices included: $\mathcal{L}^{(0)} = \mathcal{L}_{s,c,h} + \mathcal{L}_G$ omitted during this discussion

In this talk ($ep \rightarrow Q + X$)

Quarkonium Shape Funct.

- A brief review
- Diagrammatic analysis for ep

Color octet mechanisms

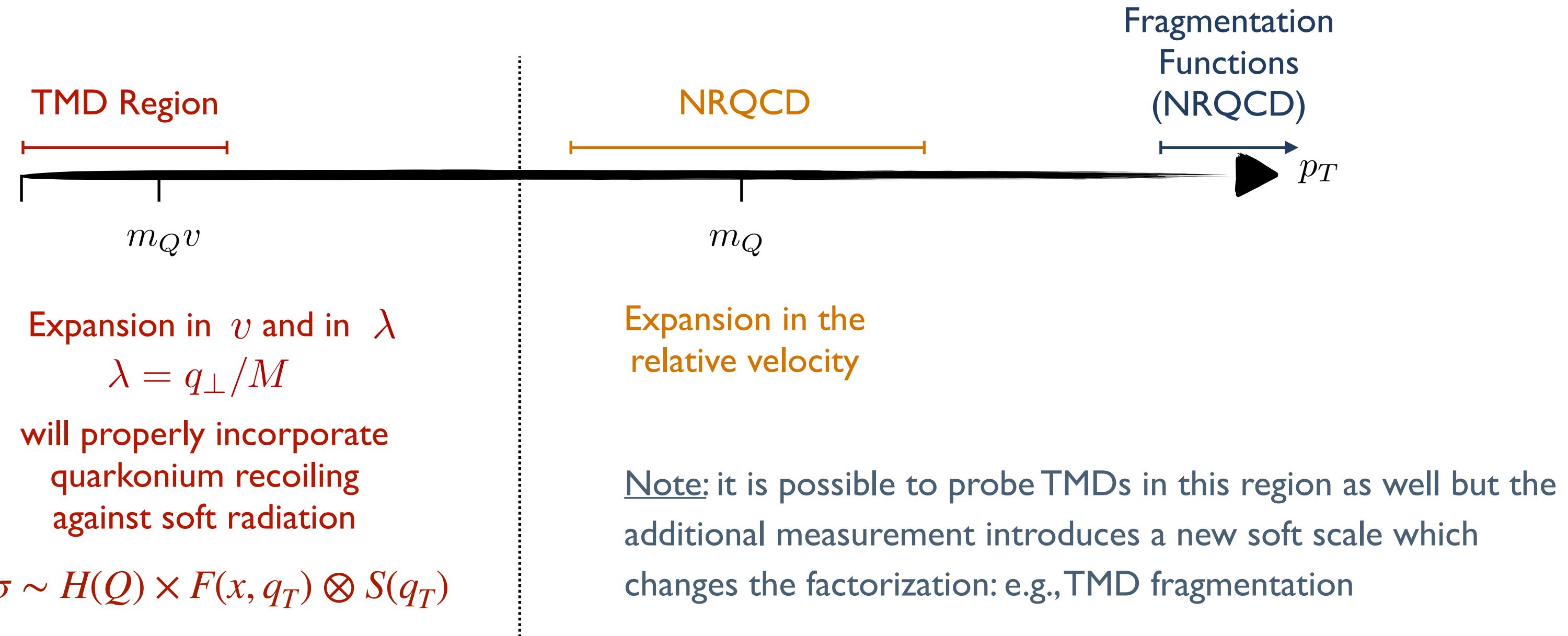
- The operators and matching
- The factorization and shape funct.
- Renormalization Group and scales

Color singlet mechanism

- Sub-leading factorization
- The operators and matching
- Factorization and Shape funct.

NRQCD regimes

Quarkonium spectrum vs EFT regions

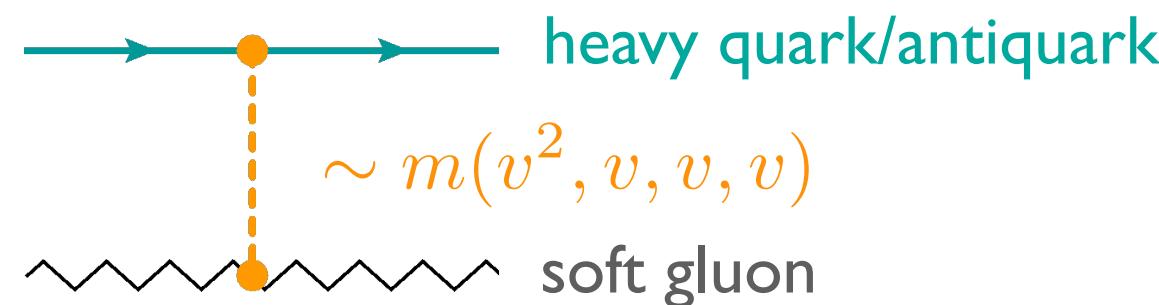


Quarkonium TMD Shape functions

NRQCD matrix elements with TMD structure:

$$S_{Q\bar{Q}[n]\rightarrow Q}(\vec{q}_T) = \langle \psi^\dagger \mathcal{K}[n] \chi \underbrace{S_v \dots}_{\text{Quarks}} \underbrace{\delta^{(2)}(\vec{q}_T - \mathcal{P}_\perp)}_{\text{TMD}} \underbrace{\mathcal{P}_Q \dots}_{\text{Bound state}} \rangle$$

Quarkonium TMD-shape functions* encode both soft and non-perturbative quarkonium related effects. Further factorization is not possible due to Coulomb-like interactions:



arXiv:1907.06494 (M. Echevarria)

arXiv:1910.03586 (S. Fleming, Y. Makris, and T. Mehen)

Quarkonium TMD Shape functions

NRQCD matrix elements with TMD structure:

$$S_{Q\bar{Q}[n]\rightarrow Q}(\vec{q}_T) = \langle \underbrace{\psi^\dagger \mathcal{K}[n] \chi}_{\text{Soft gluons}} \underbrace{S_v \dots}_{\text{Bound state}} \underbrace{\delta^{(2)}(\vec{q}_T - \mathcal{P}_\perp)}_{\text{Bound state}} \underbrace{\mathcal{P}_Q \dots}_{\text{Bound state}} \rangle$$

Quarkonium at the kinematic end-point:

arXiv:hep-ph/9705286 (M. Beneke, I. Z. Rothstein, and M. B. Wise)

arXiv:hep-ph/0211303 (S. Fleming and A. K. Leibovich)

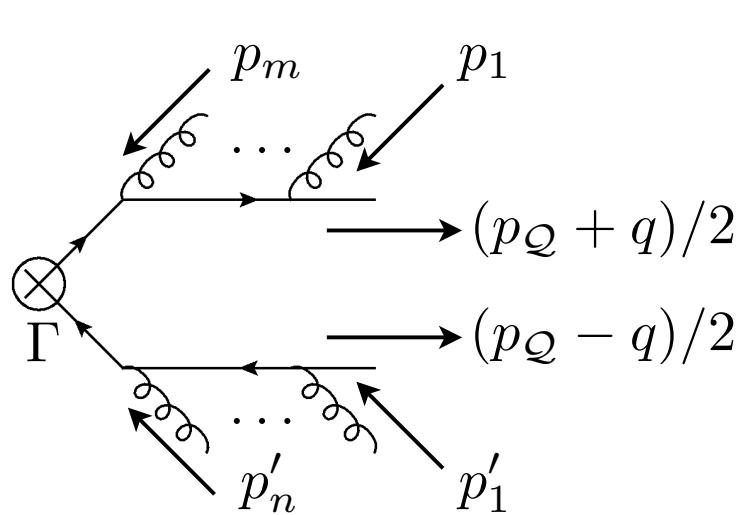
arXiv:hep-ph/010631 (C.W. Bauer, C-W Chiang, S. Fleming, A. K. Leibovich, and I. Low)

arXiv:hep-ph/0306139 (S. Fleming, A. K. Leibovich, and T. Mehen)

arXiv:hep-ph/0607121 (S. Fleming, A. K. Leibovich, and T. Mehen)

arXiv:0705.3230 (A. K. Leibovich and X. Liu)

Diagrammatic analysis at tree level



$$\begin{aligned}
 &= \text{Diagram with } \mathbf{S} \text{ (labeled S)} \quad (1 + \mathcal{O}(\lambda)) \quad + \\
 &\quad \text{Diagram with } \mathbf{P} \text{ (labeled P)} \quad (1 + \mathcal{O}(\lambda)) \quad + \dots
 \end{aligned}$$

$$d_{\Gamma}(m, n) = d_{\Gamma}^{(0)}(m, n) \ (1 + \mathcal{O}(\lambda)) + d_{\Gamma}^{(1)}(m, n) \ (1 + \mathcal{O}(\lambda)) + \dots$$

S-wave

$$d_{\Gamma}^{(0)} = \boxed{\left(u^{(0)}\right)^{\dagger} S_v^{\dagger} \Gamma^{(0)} S_v v^{(0)}}$$

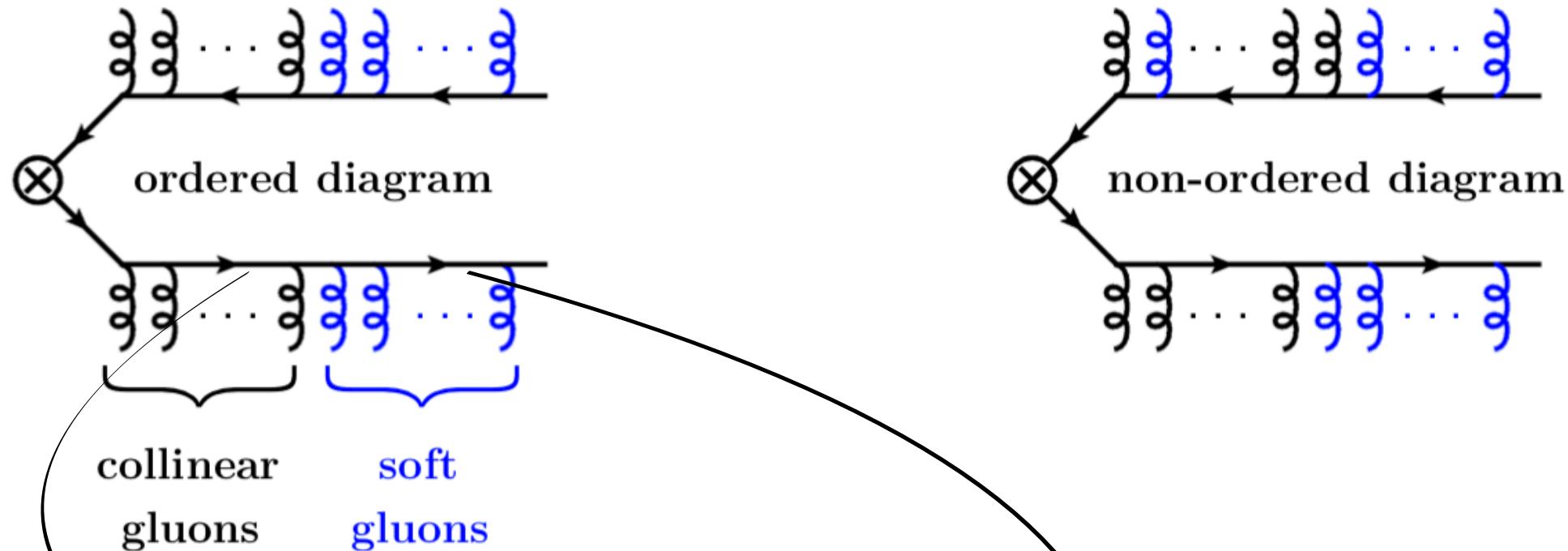
$$S_v(x, -\infty) = \text{P} \left[\exp \left(-ig \int_{-\infty}^0 d\tau v \cdot A_{soft}(x^\mu + v^\mu \tau) \right) \right]$$

P-wave

$$d_{\Gamma}^{(1)} = \boxed{\frac{g}{2m} \left(u^{(0)}\right)^{\dagger} \left\{ S_v^{\dagger} \Gamma^{(0)} S_v, \left[\frac{1}{v \cdot \mathcal{P}} \mathbf{q} \cdot \mathbf{B}_s \right] \right\} v^{(0)}} + \boxed{\left(u^{(0)}\right)^{\dagger} S_v^{\dagger} \mathbf{q} \cdot (\Gamma^{(1)} - \frac{1}{4m} \{\Gamma^{(0)}, \gamma\}) S_v v^{(0)}}$$

$$B_s^\mu = -\frac{1}{g} S_v^{\dagger} [(\mathcal{P}^\mu - g A^\mu) S_v]$$

Diagrammatic analysis for $(e)g_n \rightarrow Q\bar{Q} + X_n + X_s$



$$\Pi_n \sim \frac{1}{E_n}$$

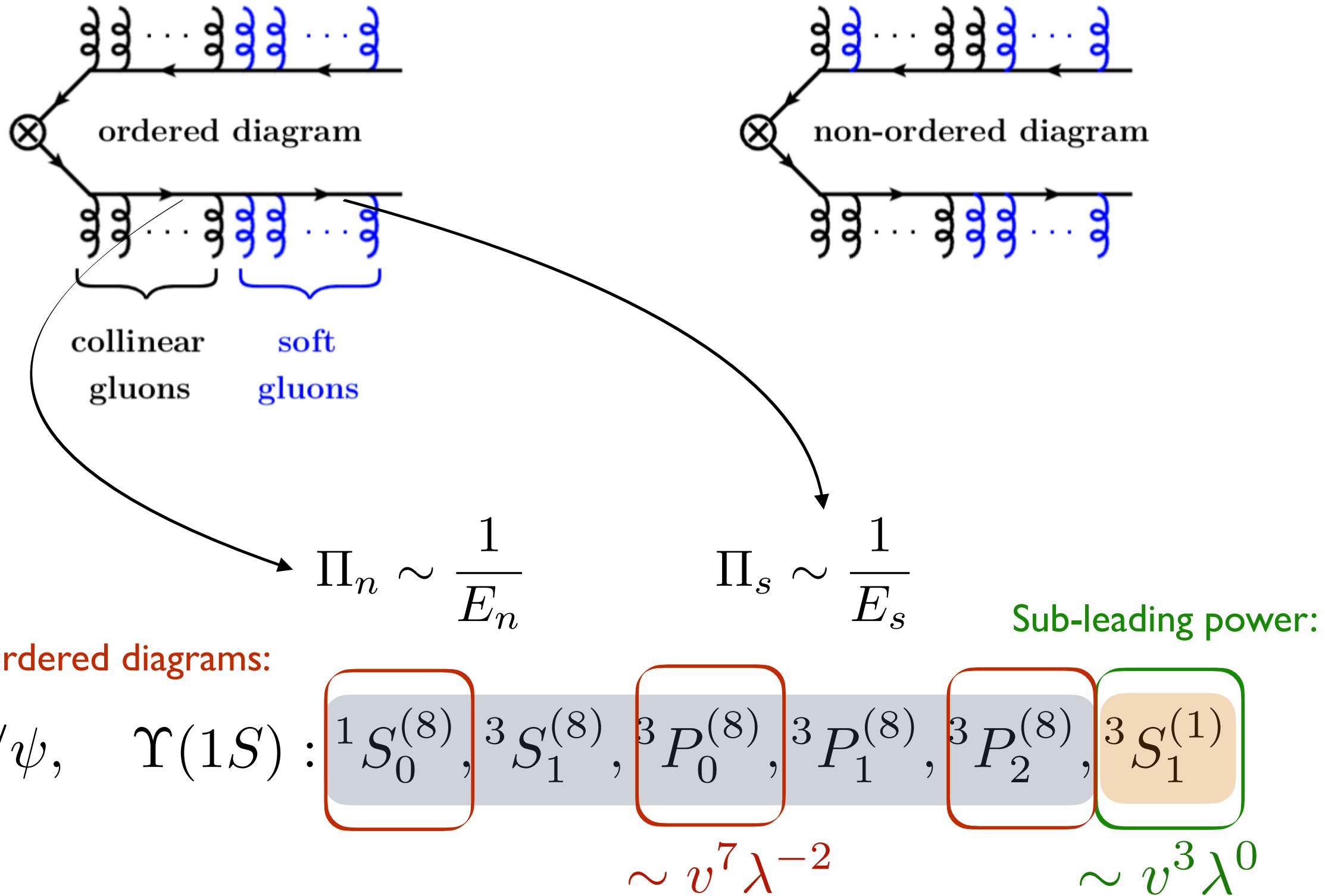
$$\Pi_s \sim \frac{1}{E_s}$$

$J/\psi, \Upsilon(1S) : {}^1S_0^{(8)}, {}^3S_1^{(8)}, {}^3P_0^{(8)}, {}^3P_1^{(8)}, {}^3P_2^{(8)}, {}^3S_1^{(1)}$

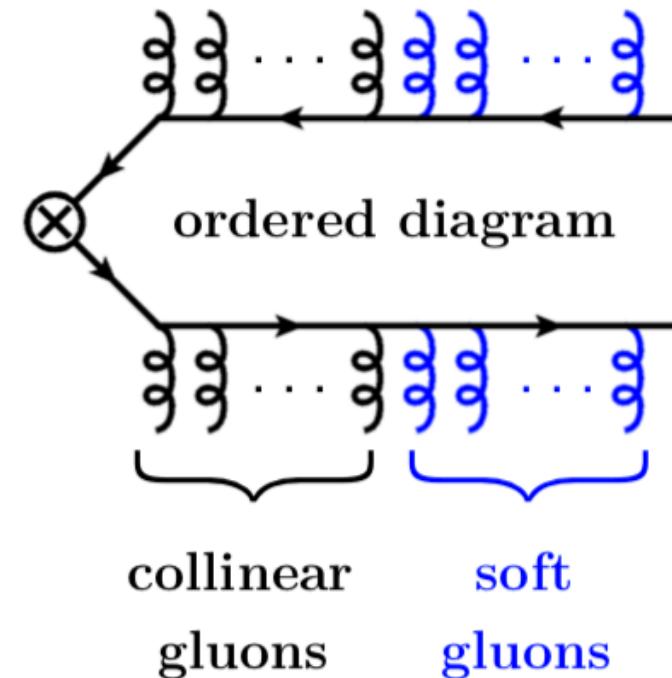
$$v^7$$

$$v^3$$

Diagrammatic analysis for $(e)g_n \rightarrow Q\bar{Q} + X_n + X_s$



Color octet: Matching operators



$$L_\mu \left\langle J_{Q\bar{Q}}^\mu(0) \Big| Q\bar{Q}[n] + X \right\rangle = \mathbf{L}_\perp^i \int d\omega C^i(\omega, \mu; [n]) \times \left\langle O(\omega; [n]) \Big| Q\bar{Q}[n] + X \right\rangle$$

S-wave:

$$C^{ij}(\omega, \mu; {}^1S_0^{[8]}) = -\frac{e_Q}{m_Q} \epsilon_\perp^{ij}$$

$$O^i(\omega; {}^1S_0^{[8]}) = -g \mathcal{S}_v^{ac} \mathcal{S}_{\bar{n}}^{ab} \left[\delta(\omega - n \cdot \mathcal{P}) \mathbf{B}_{\bar{n}\perp}^{i,b} \right] (\psi_{\mathbf{p}}^\dagger T^c \chi_{-\mathbf{p}})$$

P-wave:

$$C^{ijkl}(\omega, \mu; {}^3P_J^{[8]}) = i \frac{e_Q}{m_Q^2} (\delta_\perp^{ij} \delta_\perp^{kl} + \delta_\perp^{kj} \delta_\perp^{il} + \delta_\perp^{ik} n^j n^\ell)$$

$$O^{jkl}(\omega; {}^3P_J^{[8]}) = g \mathcal{S}_v^{ac} \mathcal{S}_{\bar{n}}^{ab} \left[\delta(\omega - n \cdot \mathcal{P}) \mathbf{B}_{\bar{n}\perp}^{k,b} \right] \left[\psi_{\mathbf{p}}^\dagger \frac{\vec{\mathcal{P}}^j}{2} \boldsymbol{\sigma}^\ell T^c \chi_{-\mathbf{p}} \right]$$

Factorization

$$2E_{\mathcal{Q}} \frac{d\sigma}{d^3 p_{\mathcal{Q}}} = \frac{1}{32\pi^3 s} \sum_X \frac{1}{2} \sum_{s_P} \left| \langle P | J_{Q\bar{Q}}^\mu(0) | \mathcal{Q} + X \rangle \right|^2 (2\pi)^4 \delta^4(p_\gamma + p_P - p_{\mathcal{Q}} - p_X)$$

↓
Apply matching for SCET-like factorization procedure

$$\frac{d\sigma}{d^2 \mathbf{q}_T} = \sigma_0({}^1S_0^{[8]}) H(2m_Q, \mu; {}^1S_0^{[8]}) \int d^2 \mathbf{k} \mathcal{B}_{g/P}(x, \mathbf{k}) S(\mathbf{q} - \mathbf{k}; {}^1S_0^{[8]})$$

Shape functions

Color octet contributions

The shape functions

$$S(\mathbf{q}; {}^1S_0^{[8]}) = \frac{1}{4m_Q \langle {}^1S_0^{[8]} \rangle} \langle \mathcal{S}_v^{ac} \mathcal{S}_{\bar{n}}^{ab} [\delta^{(2)}(\mathbf{q}_s - \mathcal{P}_\perp) \psi_p^\dagger T^c \chi_{-\mathbf{p}}] a_\mathcal{Q}^\dagger a_\mathcal{Q} (\psi_p^\dagger T^f \chi_{-\mathbf{p}})^\dagger \mathcal{S}_v^{\dagger df} \mathcal{S}_{\bar{n}}^{\dagger de} \rangle$$

$$S(\mathbf{k}; {}^3P_J^{[8]}) = \frac{2J+1}{4m_Q \langle {}^3P_0^{[8]} \rangle} \frac{1}{3} \langle \mathcal{S}_v^{ac} \mathcal{S}_{\bar{n}}^{ab} [\delta^{(2)}(\mathbf{q}_s - \mathcal{P}_\perp) [\psi_p^\dagger \frac{\vec{\mathcal{P}} \cdot \boldsymbol{\sigma}}{2} T^c \chi_{-\mathbf{p}}]] a_\mathcal{Q}^\dagger a_\mathcal{Q} [\psi_p^\dagger \frac{\vec{\mathcal{P}} \cdot \boldsymbol{\sigma}}{2} T^c \chi_{-\mathbf{p}}]^\dagger \mathcal{S}_v^{\dagger df} \mathcal{S}_{\bar{n}}^{\dagger de} \rangle$$

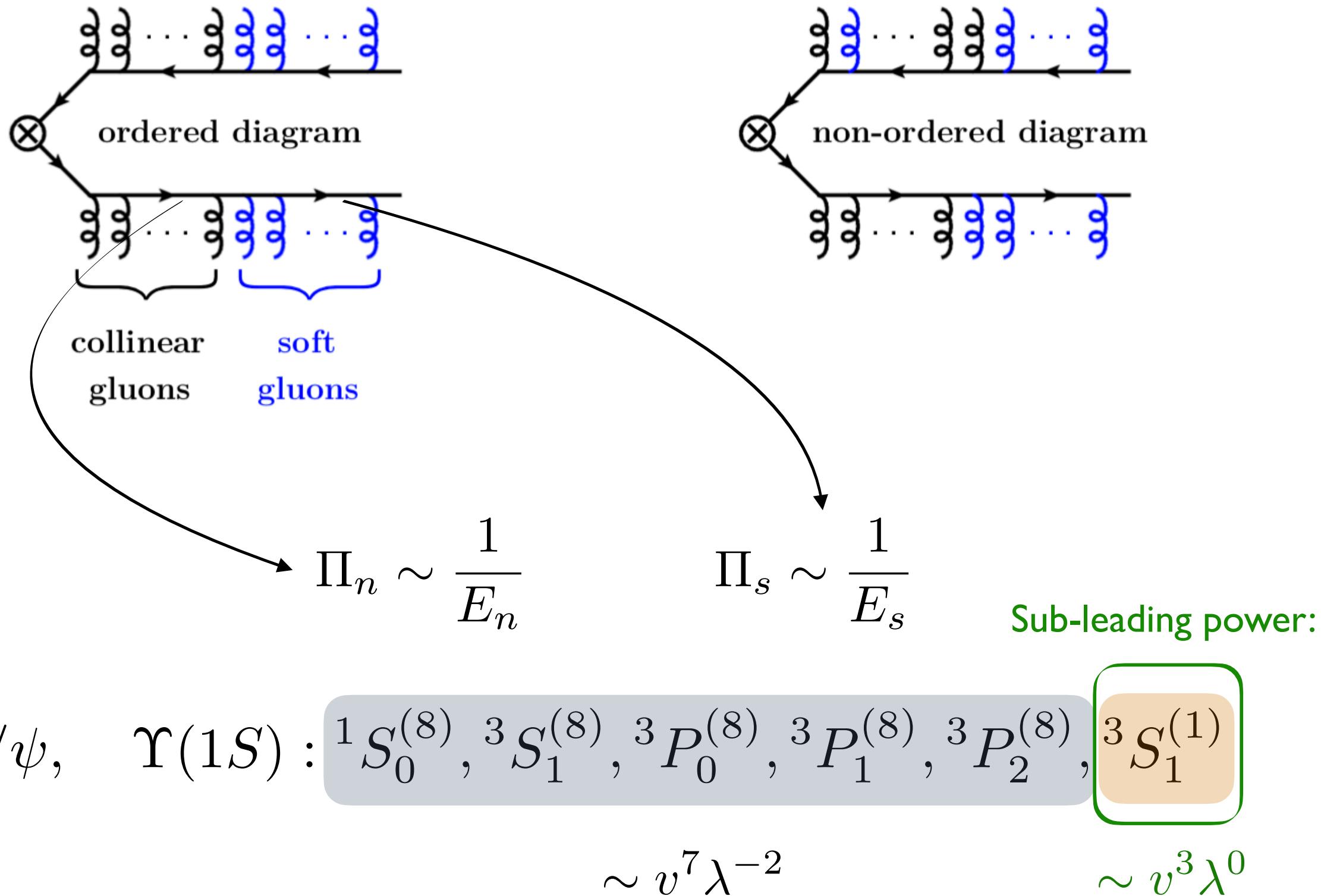
$$= \left(\delta^{(2)}(\mathbf{q}_\perp) + \frac{\alpha_s^2 C_A}{2\pi} \left\{ 4 \ln \left(\frac{\nu}{\mu} \right) \mathcal{L}_0(q_\perp^2, \mu^2) - 2 \mathcal{L}_1(q_\perp^2, \mu^2) - \boxed{2 \mathcal{L}_0(q_\perp^2, \mu^2)} - \frac{\pi}{12} \delta^{(2)}(\mathbf{q}_\perp) \right\} \right) + \mathcal{O}(\alpha_s^2)$$

- Half the rapidity divergences
- Color-octet logarithm
- At NLO the same for S and P wave

$$H_n = 1 + \frac{\alpha_s C_A}{2\pi} \left\{ 2D(n) - \frac{\pi^2}{12} - \boxed{\ln \left(\frac{\mu^2}{s} \right)} - \frac{\beta_0}{2C_A} \ln \left(\frac{\mu^2}{s} \right) - \frac{1}{2} \ln^2 \left(\frac{\mu^2}{s} \right) \right\} + \mathcal{O}(\alpha_s^2)$$

arXiv:hep-ph/9708349 (F. Maltoni, M. L. Mangano, and A. Petrelli)

Diagrammatic analysis at tree level

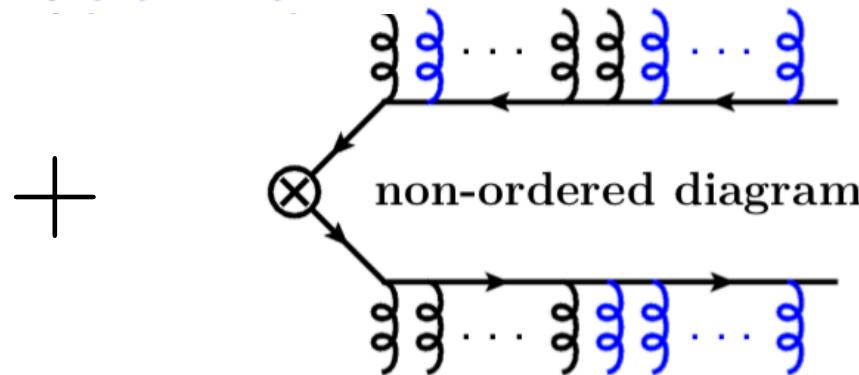
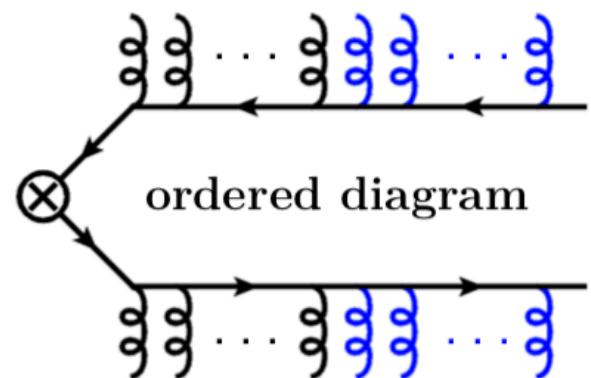


Color singlet: “Sub-leading” contribution

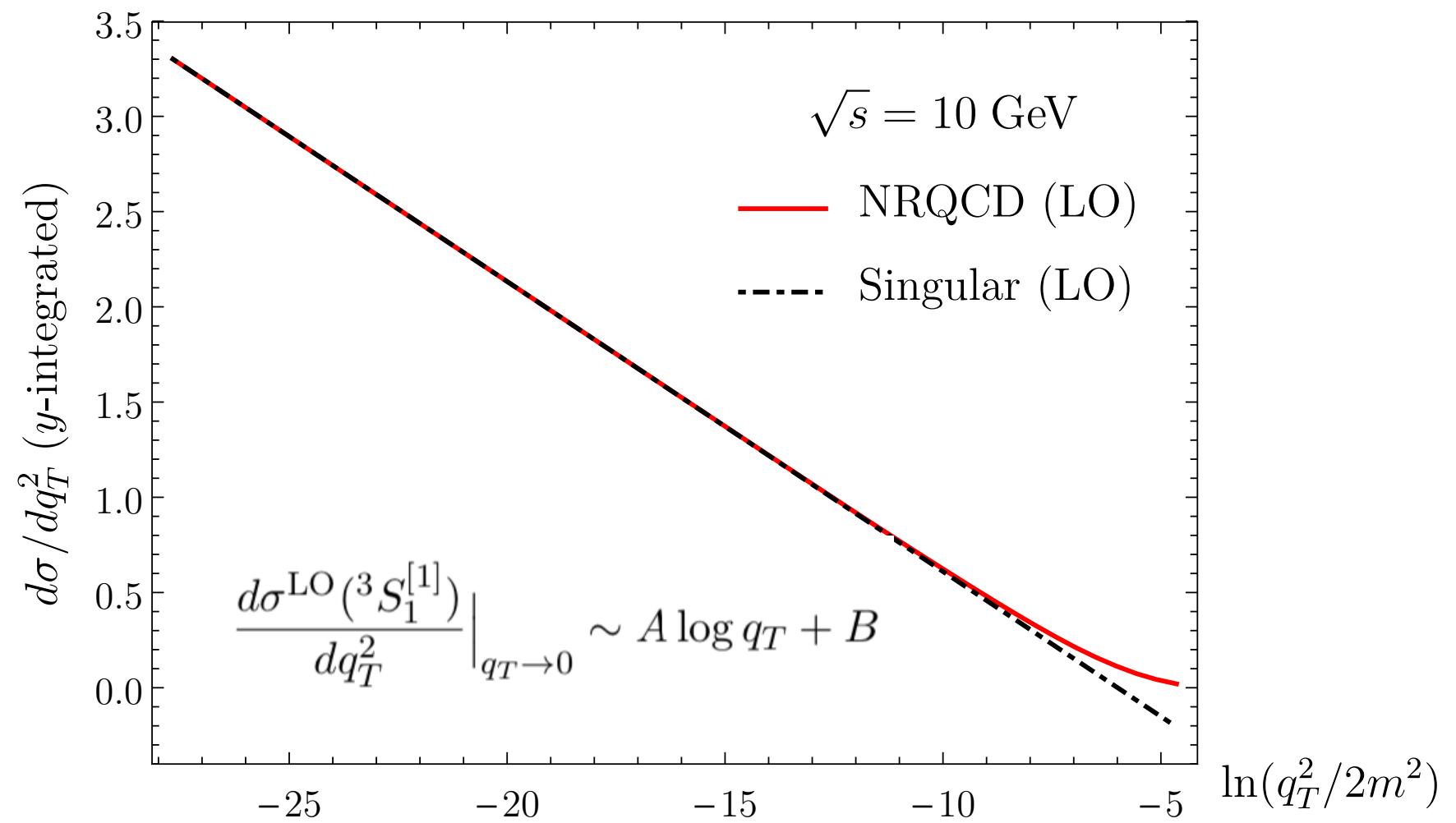
$$\frac{d\sigma}{dq_T} = \boxed{c_{00}\delta^{(2)}(q_T) + c_{01}\frac{1}{q_T^2} + c_{02}\frac{\ln(q_T)}{q_T^2} + \dots} \sim v^7\lambda^{-2}$$

$$\boxed{+c_{11} + c_{02}\ln(q_T) + \dots} \sim v^3\lambda^0$$

+ ...



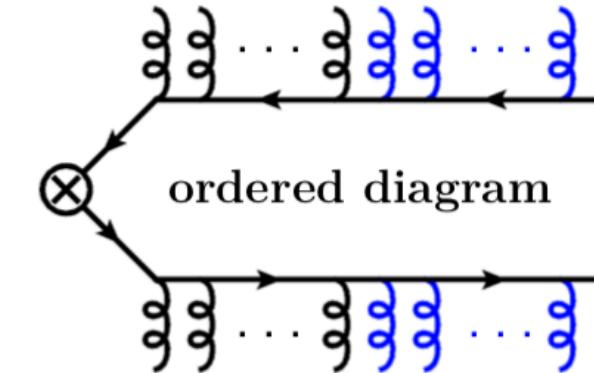
The color singlet operators are suppressed in the λ power counting but enhanced in the relative velocity, v .



Color singlet: Operators

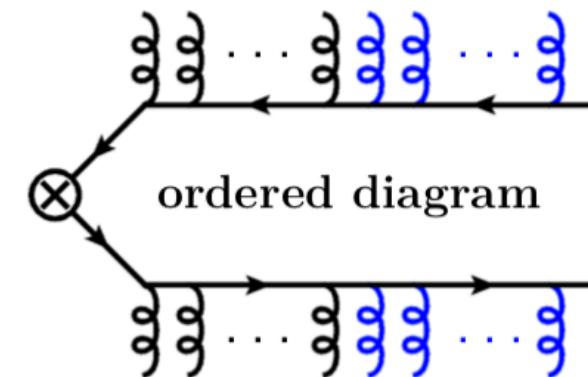
collinear-collinear arXiv:hep-ph/0211303 (S. Fleming and A. K. Leibovich)

$$O_{\text{cc-1}}^{jkl}(\omega, \bar{\omega}; {}^3S_1^{[1]}) = g^2 \left[\delta(\omega - n \cdot \mathcal{P}) \mathbf{B}_{\bar{n}\perp}^{j,a} \delta(\bar{\omega} - \overset{\leftrightarrow}{\mathcal{P}}) \mathbf{B}_{\bar{n}\perp}^{k,a} \right] (\psi \boldsymbol{\sigma}^\ell \chi)$$



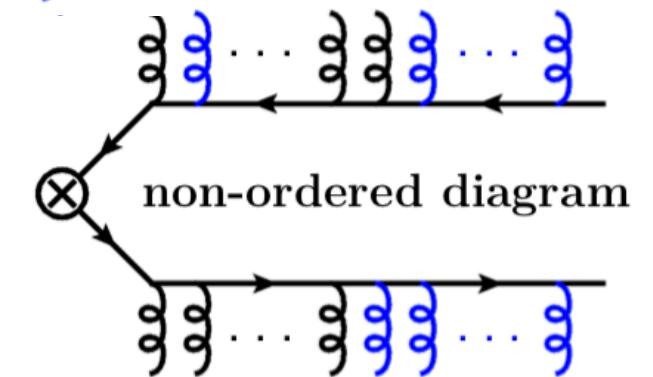
collinear-soft

$$O_{\text{cs-1}}^{jkl}(\omega; {}^3S_1^{[1]}) = g^2 \mathbf{B}_s^{j,a} \left[\delta(\omega - n \cdot \mathcal{P}) \mathbf{B}_{\bar{n}\perp}^{k,a} \right] (\psi \boldsymbol{\sigma}^\ell \chi)$$



$$O_{\text{cs-2}}^{jkl}(\omega; {}^3S_1^{[1]}) = g^2 \left[\frac{(\mathcal{P} \times \mathbf{B}_s^a)^j}{v \cdot \mathcal{P}} \right] \left[\delta(\omega - n \cdot \mathcal{P}) \mathbf{B}_{\bar{n}\perp}^{k,a} \right] (\psi \boldsymbol{\sigma}^\ell \chi)$$

+



Color singlet: Factorization

$$d\sigma(\text{cc-1}) \sim H_{\mu\nu\rho\sigma}(M, \mu) \otimes \mathcal{B}_\perp^{\mu\nu\rho\sigma}(z, \mathbf{b}, M, \mu) \times \langle {}^3S_1^{[1]} \rangle$$

$$d\sigma({}^3S_1^{[1]}) = d\sigma(\text{cc-1}) + d\sigma(\text{cs-1}) + d\sigma(\text{cs-2})$$

$$\mathcal{B}_\perp^{\mu\nu\rho\sigma} \sim \text{Im} \left[\langle P | T[B_{n\perp}^{\mu a} B_{n\perp}^{\nu a}(x^+, 0^-, x^\perp) B_{n\perp}^{\rho a} B_{n\perp}^{\sigma a}(0)] | P \rangle \right]$$

For perturbative values of transverse momentum we can match onto the collinear PDF.

$$\mathcal{B}_\perp(z, M, \mu) = \int_0^{1-z} dy C_\perp^{(1)}(y, z, x) \otimes f_{g/P}(x, \mu)$$

Leading order contributions suffer from rapidity divergences which cancel in the sum of all operators

Note: the factorization for the cc-1 term does not involve any shape or soft function, only the standard quarkonium LDME.

Color singlet: Factorization

$$\frac{d\sigma(\text{cs-1/2})}{d^2\mathbf{q}_T} = \sigma_0(^3S_1^{[1]}) H_{\text{cs-1/2}}^{jj'}(2m_Q, \mu) \int d^2\mathbf{k} \mathcal{B}_{g/P}^\perp(x, \mathbf{k}) S^{jj'}(\mathbf{q} - \mathbf{k}; \text{cs-1/2})$$

$$S^{jj'}(\mathbf{q}_s; \text{cs-1}) = \frac{1}{4m_Q(N_c^2 - 1)} \langle \mathbf{B}_s^{ja} [\delta^{(2)}(\mathbf{q}_s - \mathcal{P}_\perp) \psi \boldsymbol{\sigma}^\ell \chi] a_\mathcal{Q} a_\mathcal{Q}^\dagger (\psi \boldsymbol{\sigma}^{\ell'} \chi) \mathbf{B}_s^{j'a} \rangle$$

$$S^{jj'}(\mathbf{q}_s; \text{cs-2}) = \frac{\varepsilon^{nmj} \varepsilon^{n'm'j'}}{4m_Q(N_c^2 - 1)} \left\langle \frac{\mathcal{P}^n \mathbf{B}_s^{ma}}{v \cdot \mathcal{P}} [\delta^{(2)}(\mathbf{q}_s - \mathcal{P}_\perp) \psi \boldsymbol{\sigma}^\ell \chi] a_\mathcal{Q} a_\mathcal{Q}^\dagger (\psi \boldsymbol{\sigma}^{\ell'} \chi) \frac{\mathcal{P}^{n'} \mathbf{B}_s^{m'a}}{v \cdot \mathcal{P}} \right\rangle$$

These shape function have rapidity divergences at tree level. This complicates the renormalization and resummation procedure.

Summary

- Presence of TMD-shape functions reveals new NP-effects unique to quarkonium production
- New TMD evolution associated with the color-octet channels
- Sub-leading factorization for the color-singlet channel revises non trivial effects for quarkonium TMDs