

FEYNONIUM: A FRESH LOOK ON AUTOMATIC CALCULATIONS IN NONRELATIVISTIC EFTs

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based on [2006.15451](#) in collaboration with
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Quarkonia as Tools
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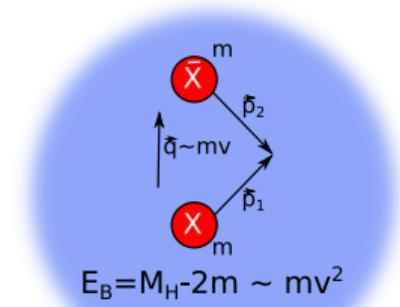


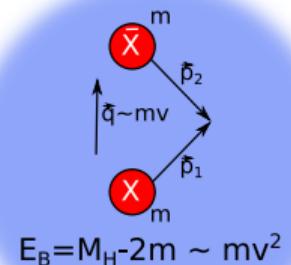
Outline

1 (NR)EFTs and Automation

2 FeynCalc 9.3 and FeynOnium

3 Summary and Outlook

- Nonrelativistic Effective Field Theories (NREFTs) are the key to a systematic description of physical systems with well separated dynamical scales and $v \ll c$
- NRQCD [Caswell & Lepage, 1986; Bodwin et al., 1995] and pNRQCD [Pineda & Soto, 1998a; Brambilla et al., 2000] are especially useful for heavy quarkonia
- Quarkonia are not the only NR bound states one can think of
 - positronium, muonium [Caswell & Lepage, 1986]
 - systems made of NR atoms [Brambilla et al., 2017] and molecules [Brambilla et al., 2018]
 - NR dark matter [Hisano et al., 2003, 2004, 2005; Shepherd et al., 2009; An et al., 2016; Biondini & Laine, 2018; Beneke et al., 2019]
 - heavy neutrinos [Biondini et al., 2013]
- Obtaining predictions within NREFT frameworks requires hard work
- Integrating out scales (e. g. m , mv , ...) leads to a tower of NREFTs
-  Increasing amount of laborious NR calculations



$$E_B = M_H - 2m \sim mv^2$$

- The practical usage of NREFTs is thorny (even at tree-level)

- Explicit construction of higher order operators
- Derivation of Feynman rules for the new operators
- Emergence of unusual propagators
- Loss of manifest Lorentz covariance
- Nonrelativistic expansion of high-energy amplitudes
- Calculation of NR integrals



- Existing approaches to address NREFTs in automatic frameworks focus on covariant projector techniques [Bodwin & Petrelli, 2002]
- If there is no way to avoid genuine NR calculations, you are left with pen and paper or self-written codes

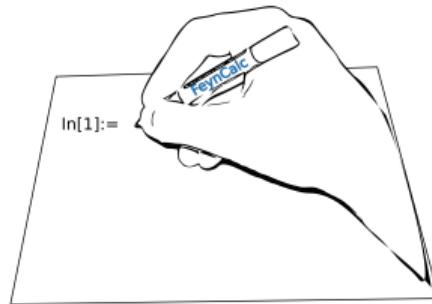
- Many tools for streamlining different aspects of EFT calculations are readily available
- Some EFT-specific codes are **ROSETTA** [Falkowski et al., 2015], **SMEFTSIM** [Brivio et al., 2017], **MATCHINGTOOLS** [Criado, 2018], **CoDEX** [Das Bakshi et al., 2019], **WILSON** [Aebischer et al., 2018], **DEFT** [Gripaios & Sutherland, 2019] **SMEFTFR** [Dedes et al., 2020], **BASISGEN** [Criado, 2019], **SYM2INT** [Fonseca, 2017], **ECO** [Marinissen et al., 2020], , **GRIP** [Banerjee et al., 2020], **WCXF** [Aebischer et al., 2018], **DIRECTDM** [Bishara et al., 2017], **GRINDER** [Grozin, 2000], **SOFTSERVE** [Bell et al., 2019], **MADONIA** [Artoisenet et al., 2008] , **HELAC-ONIA** [Shao, 2013, 2016], **FDC** [Wang, 2004], **FDCHQHP** [Wan & Wang, 2014] ...
- There are even more general purpose tools that can be used for EFTs:
MADGRAPH5_AMC@NLO [Alwall et al., 2014], **GoSam** [Cullen et al., 2012, 2014], **HERWIG++** [Bahr et al., 2008], **SHERPA** [Gleisberg et al., 2009], **WHIZARD** [Moretti et al., 2001; Kilian et al., 2011], **CALCHEP** [Belyaev et al., 2013], **COMPHEP** [Boos et al., 2004], **FENYARTS** [Hahn, 2001], **FENYRULES** [Christensen & Duhr, 2009; Alloul et al., 2014], **NLOCT** [Degrande, 2015], **QGRAF** [Nogueira, 1993], **FIRE** [Smirnov, 2015], **PACKAGE-X** [Patel, 2015, 2017], ...
- Yet the manifest Lorentz covariance is usually taken for granted ...

- Proliferation of new quantities once the manifest Lorentz covariance is lost

$$\begin{aligned} &g^{\mu\nu}, \quad g^{\mu 0}, \quad g^{\mu i}, \quad g^{00}, \quad g^{ij}, \quad \delta^{ij} \\ &\epsilon^{0\mu\nu\rho}, \quad \epsilon^{\mu\nu i}, \quad \epsilon^{\mu i j}, \quad \gamma^0, \quad \gamma^i, \quad \gamma \cdot p, \\ &p^0, \quad p^i, \quad |p|, \quad \sigma^j, \quad \sigma \cdot q \dots \end{aligned}$$

- Even tree-level calculations can quickly become tedious e. g. when expanding an amplitude in relative 3-momenta to a sufficiently high order
- Tedious to automatize if you are to start from scratch
- This is where **FENONIUM** comes to the rescue!
- **FENONIUM**: our project to improve the automation of NREFT calculations

- Open source tool for NREFT calculations at tree- and 1-loop level
- Built upon an existing framework (**FEYNCALC**)
- Sufficiently generic to cover a wide range of NR processes
- Fully worked out examples reproducing selected NREFT results from the literature
- At the moment 2 main deliverables
 - Better handling of nonrelativistic quantities in **FEYNCALC** itself
 - A **FEYNCALC** add-on called **FEYNONIUM** with tools specific to particular NREFTs (currently pNRQCD and NRQCD)



1991	• FEYNCALC 1.0 [Mertig et al., 1991]
1997	• TARCER [Mertig & Scharf, 1998]
2012	• FEYNCALCFORMLINK [Feng & Mertig, 2012]
2016	• FEYNCALC 9.0 [<u>VS</u> et al., 2016]
2017	• FEYNHELPERS [<u>VS</u> , 2016]
2020	• FEYNCALC 9.3 [<u>VS</u> et al., 2020]
2020	• FEYNONIUM [Brambilla, Chung, <u>VS</u> , Vairo, 2020]

"Old" FEYNCALC

Shortcut in FEYNCALC	Meaning
MT $[\mu, \nu]$, MTD $[\mu, \nu]$ MTE $[\mu, \nu]$	$\bar{g}^{\mu\nu}, g^{\mu\nu}, \hat{g}^{\mu\nu}$
FV $[p, \mu]$, FVD $[p, \mu]$, FVE $[p, \mu]$	$\bar{p}^\mu, p^\mu, \hat{p}^\mu$
SP $[p, q]$, SPD $[p, q]$, SPE $[p, q]$	$\bar{p} \cdot \bar{q}, p \cdot q, \hat{p} \cdot \hat{q}$
GA $[\mu]$, GAD $[\mu]$, GAE $[\mu]$	$\bar{\gamma}^\mu, \gamma^\mu, \hat{\gamma}^\mu$
GS $[p]$, GSD $[p]$, GSE $[p]$	$\bar{\gamma} \cdot \bar{p}, \gamma \cdot p, \hat{\gamma} \cdot \hat{p}$
LC $[\mu, \nu, \rho, \sigma]$, LC $[\mu, \nu]$ $[p, q]$	$\bar{\epsilon}^{\mu\nu\rho\sigma}, \bar{\epsilon}^{\mu\nu\rho\sigma} p_\rho q_\sigma$
LCD $[\mu, \nu, \rho, \sigma]$, LCD $[\mu, \nu]$ $[p, q]$	$\epsilon^{\mu\nu\rho\sigma}, \epsilon^{\mu\nu\rho\sigma} \hat{p}_\rho \hat{q}_\sigma$

Implemented for FEYNONIUM

Shortcut in FEYNCALC	Meaning
KD $[i, j]$, KDD $[i, j]$, KDE $[i, j]$	$\bar{\delta}^{ij}, \delta^{ij}, \hat{\delta}^{ij}$
CV $[p, i]$, CVD $[p, i]$, CVE $[p, i]$	$\bar{p}^i, p^i, \hat{p}^i$
CSP $[p, q]$, CSPD $[p, q]$, CSPE $[p, q]$	$\bar{p} \cdot \bar{q}, p \cdot q, \hat{p} \cdot \hat{q}$
TGA $[]$	$\bar{\gamma}^0$
CGA $[i]$, CGAD $[i]$, CGAE $[i]$	$\bar{\gamma}^i, \gamma^i, \hat{\gamma}^i$
CGS $[p]$, CGSD $[p]$, CGSE $[p]$	$\bar{\gamma} \cdot \bar{p}, \gamma \cdot p, \hat{\gamma} \cdot \hat{p}$
CLC $[i, j, k]$, CLC $[i, j]$ $[p]$	$\bar{\epsilon}^{ijk}, \bar{\epsilon}^{ijk} \bar{p}^k$
CLCD $[i, j, k]$, CLCD $[i, j]$ $[p]$	$\epsilon^{ijk}, \epsilon^{ijk} p^k$
SI $[\mu]$, SID $[\mu]$, SIE $[\mu]$	$\bar{\sigma}^\mu, \sigma^\mu, \hat{\sigma}^\mu$
SIS $[p]$, SISD $[p]$, SISE $[p]$	$\bar{\sigma} \cdot \bar{p}, \sigma \cdot p, \hat{\sigma} \cdot \hat{p}$
CSI $[i]$, CSID $[i]$, CSIE $[i]$	$\bar{\sigma}^i, \sigma^i, \hat{\sigma}^i$
CSIS $[p]$, CSISD $[p]$, CSISE $[p]$	$\bar{\sigma} \cdot \bar{p}, \sigma \cdot p, \hat{\sigma} \cdot \hat{p}$

- New symbols to represent various nonstandard propagators

Shortcut in FEYNCALC	Meaning
<code>FAD[{k - p₁ - ..., m, n}]</code>	$\left[\frac{1}{(k-p_1-\dots)^2 - m^2 + i\eta} \right]^n$
<code>SFAD[{ {k - p₁ - ..., ±k.(q₁ + ...)}, {±m², ±1}, n}]</code>	$\left[\frac{1}{(k-p_1-\dots)^2 \pm k.(q_1+\dots) \mp m^2 \pm i\eta} \right]^n$
<code>CFAD[{ {k - p₁ - ..., ±k.(q₁ + ...)}, {±m², ±1}, n}]</code>	$\left[\frac{1}{(k-p_1-\dots)^2 \pm k.(q_1+\dots) \pm m^2 \pm i\eta} \right]^n$
<code>GFAD[{ {x, ±1}, n}]</code>	$\left[\frac{1}{x \pm i\eta} \right]^n$

- FAD: original symbol for covariant quadratic propagators
- SFAD: new symbol for covariant quadratic or eikonal propagators
- CFAD: new symbol for Cartesian quadratic or eikonal propagators
- GFAD: new symbol for generic propagators

- What is inside the **FEYNONIUM** add-on?
- Expressing Dirac spinor chains in terms of Pauli matrices and Pauli spinors:
`FMSpinorChainExplicit2`
- Special kinematic configurations [Braaten & Chen, 1996] for spinors describing a heavy nonrelativistic system via `FMSpinorChainExplicit`
- Covariant projectors for heavy nonrelativistic systems [Bodwin & Petrelli, 2002]:
`FMInsertCovariantProjector`
- Projections with $J = 0, 1$ and 2 for 3-dimensional Cartesian tensors up to rank 5:
`FMCartesianTensorDecomposition`
- Repetitive application of the 3D Schouten's identity

$$\epsilon^{ijk} \mathbf{p}^l - \epsilon^{jkl} \mathbf{p}^i + \epsilon^{kli} \mathbf{p}^j - \epsilon^{lij} \mathbf{p}^k = 0,$$

via `FMCartesianSchoutenBruteForce`

- Feynman rules for pNRQCD vertices in the weak-coupling regime at order r (cf. figure 5 of [Brambilla, Pineda, et al., 2005])

- Example calculations bundled with FEYNONIUM

- 1-loop level

- Euler-Heisenberg Lagrangian [Heisenberg & Euler, 1936]
- 1-loop correction to the heavy nucleon propagator in baryonic ChPT [Ecker & Mojzis, 1996; Scherer, 2003]
- Dimension six 4-fermion operators in NRQCD (unequal mass case) [Pineda & Soto, 1998b; Brambilla, Vairo, & Rosch, 2005]
- Virtual corrections to inclusive hadronic decays of P -wave quarkonia in NRQCD [Petrelli et al., 1998]
- One-loop running of the chromoelectric dipole interaction in pNRQCD [Brambilla et al., 2000; Pineda & Soto, 2000]

- Tree-level

- $J/\psi \rightarrow 3\gamma$ decay in NRQCD [Ore & Powell, 1949; Bodwin et al., 1995]
- $Q\bar{Q} \rightarrow \gamma\gamma$ decays in NRQCD [Brambilla et al., 2006]
- Relativistic corrections to quarkonium light-cone distribution amplitudes [Brambilla et al., 2019]

Summary

- 💡 **FEYNCALC** 9.3 and **FEYNONIUM** are a huge leap forward in the automation of NREFTs
- 💡 We are not aware of a similar public code for loop calculations able to handle nonrelativistic expressions and a wide range of nonstandard integrals
- 💡 We did not write everything from a scratch but extended an existing software for Lorentz covariant calculations ⇒ Blueprint for other tools?
- ⚙️ Not a fully automatic all-in-one solution, but a handy tool for knowledgeable people
- ⚙️ The current focus is on tree-level and 1-loop calculations
- ✓ Already field-tested in several publications by us [Brambilla, Chen, Jia, VS, Vairo, 2016; Brambilla, Chung, VS, Vairo, 2019;] and other people [Assi & Kniehl, 2020a, 2020b]

Outlook

- 🔍 **FEYNONIUM** is an on-going project, not yet feature-complete ...
- 🔍 Endless possibilities to extend this framework (new algorithms, new NREFTs, new examples, ...)
- 🔍 Looking to hear the community feedback regarding most wanted features
- 🔍 A built-in interface to **QGRAF** [Nogueira, 1993] is already in the testing phase ⇒ automatic generation of NREFT diagrams, visualization using **TIKZ-FEYNMAN** [?, ?] or **GRAPHVIZ**
- 🔍 Some more features being currently tested

Derive Feynman rule for the given NR operator

```
In[76]:= vertex = - $\tau_2 g / (8 M^2)$  QuantumField[QuarkFieldPsiDagger, PauliIndex[di1]].  

AntiCommutator[RightPartialD[CartesianIndex[i]].RightPartialD[CartesianIndex[i]],  

QuantumField[Phi]].QuantumField[QuarkFieldPsi, PauliIndex[di1]]  

FeynRule2[vertex,  

FCVacBra[].FCAnnihilator[QuarkFieldPsi][p2].FCAnnihilator[Phi][p3],  

FCCreator[QuarkFieldPsi][p1].FCVacKet[], FinalSubstitutions → {p3 → p1 - p2}]
```

Out[76]=
$$-\frac{g \tau_2 \psi^{\dagger di1} (\{\vec{\partial}_i, \vec{\partial}_j, \phi\}) \psi^{di1}}{8 M^2}$$

Deriving the Feynman rule for the following matrix element: $\langle 0 | a_\psi(p2).a_\phi(p3). \left(-\frac{g \psi^{\dagger di1} (\{\vec{\partial}_i, \vec{\partial}_j, \phi\}) \psi^{di1} \tau_2}{8 M^2} \right) a_\psi^\dagger(p1). | 0 \rangle$

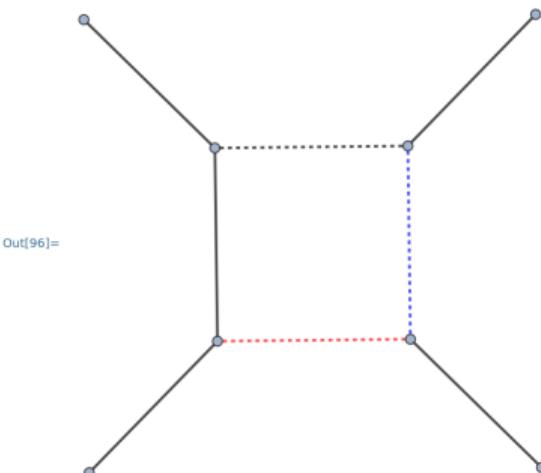
Out[77]=
$$\frac{i g \tau_2 \xi^\dagger \cdot \xi (\bar{p1}^2 + \bar{p2}^2)}{8 M^2}$$

Convert multi-loop integrals from a propagator representation to a graph representation

```
In[94]:= int = FAD[{p, m1^2}, {p - q1, m3^2}, {p - q1 - q2, m2^2}, {p - q1 - q2 - q3, 0}]
Out[94]= 
$$\frac{1}{(p^2 - m1^4) \cdot ((p - q1)^2 - m3^4) \cdot ((p - q1 - q2)^2 - m2^4) \cdot ((p - q1 - q2 - q3)^2)}$$


In[95]:= out = FCLoopIntegralToGraph[int, {p}]
Out[95]= 
$$\left\{ \begin{array}{l} \{-4 \rightarrow 4, -3 \rightarrow 1, -2 \rightarrow 2, -1 \rightarrow 3, 1 \rightarrow 2, 1 \rightarrow 4, 2 \rightarrow 3, 3 \rightarrow 4\}, \\ \{q1 - q2 - q3, q1, q2, q3, \{p - q1 - q2, 1, -m2^4\}, \{p - q1 - q2 - q3, 1, 0\}, \{p - q1, 1, -m3^4\}, \{p, 1, -m1^4\}\}, \\ \{0, 0, 0, 0, \frac{1}{(p^2 - m1^4 + i\eta)}, \frac{1}{((p - q1)^2 - m3^4 + i\eta)}, \frac{1}{((p - q1 - q2)^2 - m2^4 + i\eta)}, \frac{1}{((p - q1 - q2 - q3)^2 + i\eta)}\}, 1 \end{array} \right\}$$


In[96]:= GraphPlot[makeEdgeTaggedGraph @@ out]
```



Derive Feynman parametrization of a multi-loop integral

In[41]:= `int = CVD[p, μ] × CFAD[{p, m₁²}, p + q₁, p + q₂]`

$$\text{Out[41]}= \frac{p^\mu}{(p^2 + m_1^2 - i\eta)((p + q_1)^2 - i\eta)((p + q_2)^2 - i\eta)}$$

In[42]:= `FCFeynmanParametrize[int, {p}, Names → x]`

$$\begin{aligned} \text{Out[42]}= & \left\{ (x(1) + x(2) + x(3))^{2-D} \right. \\ & (m_1^2 x(1)^2 + m_1^2 x(1) x(2) + m_1^2 x(1) x(3) + q_1^2 x(1) x(2) + q_2^2 x(1) x(3) + q_1^2 x(2) x(3) + q_2^2 x(2) x(3) - \\ & \left. 2 x(2) x(3) (q_1 \cdot q_2) \right)^{\frac{D}{2}-3} \left(\frac{1}{2} x(2) \Gamma\left(\frac{1-D}{2} + 3\right) q_1^\mu + \frac{1}{2} x(3) \Gamma\left(\frac{1-D}{2} + 3\right) q_2^\mu \right), 1, \{x(1), x(2), x(3)\} \end{aligned}$$

We have a long way ahead of us

