

# Medium evolution of quarkonium within the EFT framework

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UNIÓN EUROPEA

FONDO EUROPEO DE DESENVOLVEMENTO REXIONAL  
"Unha maneira de facer Europa"



EXCELENCIA  
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Xacobeo 2021

# Talk based on the following works

- ① N. Brambilla, M. A. Escobedo, J. Soto and A. Vairo, Phys. Rev. D **96** (2017) no.3, 034021 [arXiv:1612.07248 [hep-ph]].
- ② N. Brambilla, M. A. Escobedo, J. Soto and A. Vairo, Phys. Rev. D **97** (2018) no.7, 074009 [arXiv:1711.04515 [hep-ph]].
- ③ N. Brambilla, M. A. Escobedo, A. Vairo and P. Vander Griend, Phys. Rev. D **100** (2019) no.5, 054025 [arXiv:1903.08063 [hep-ph]].
- ④ N. Brambilla, M. A. Escobedo, M. Strickland, A. Vairo, P. Vander Griend and J. H. Weber, [arXiv:2012.01240 [hep-ph]].

# Outline

- 1 Introduction
- 2 Quarkonium suppression in pNRQCD
- 3 Recent developments
- 4 Conclusions and outlook

# Heavy quarkonium in heavy-ion collisions

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# Heavy quarkonium in heavy-ion collisions

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- Heavy quarks can only be created at the beginning of the collision. It is a hard process.
- However, the existence of a medium changes the probability that a bound state is formed and its lifetime.
- Measuring  $R_{AA}$ , the ratio of quarkonium states measured in heavy-ion collisions divided by the naive extrapolation of pp data, we can extract information about the medium.

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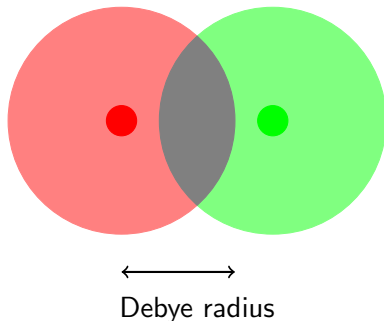
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$$V(r) = -\alpha_s \frac{e^{-m_D r}}{r}$$

At finite temperature



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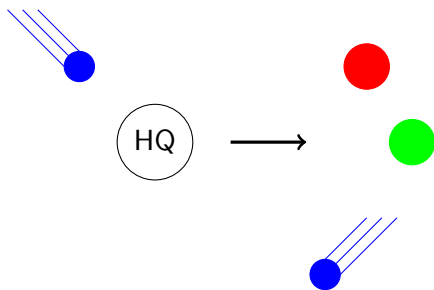
## Inelastic scattering with partons in the medium

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Related to singlet to octet transitions (Brambilla et al. (2008)).

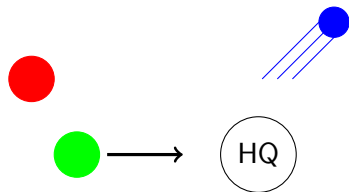
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# Recombination



Two heavy quarks coming from different origin may recombine to form a new quarkonium state.

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- In some cases, decays and recombination can be described with rate or Boltzmann equation in the semi-classical approximation. However, this is not always the case.
- When thermal effects are important, we need to describe all three effects taking into account quantum effects.
- The OQS framework has been discussed in the previous talk. In this talk, I focus on the EFT point of view. See also X. Yao's talk at the end of the session for EFT applications combined with Boltzmann equation.

# Integrating out the heavy quark mass

- Integrating out the scale  $m$  can be useful both to study heavy quark diffusion and quarkonium suppression.
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## Classification of gluons

- Hard gluons, with energy and momentum of order  $m$ .
- Soft gluons, with energy and momentum of order  $mv$ .
- Potential gluons, with energy of order  $mv^2$  and momentum of order  $mv$ .
- ultrasoft gluons, with energy and momentum of order  $mv^2$ .

# NRQCD

Caswell and Lepage (1986), Bodwin, Braaten and Lepage (1994)

$$\mathcal{L}_{NRQCD} = \mathcal{L}_g + \mathcal{L}_q + \mathcal{L}_\psi + \mathcal{L}_\chi + \mathcal{L}_{\psi\chi}$$

$$\mathcal{L}_g = -\frac{1}{4}F_{\mu\nu}^a F^{\mu\nu a} + \frac{d_2}{m_Q^2}F_{\mu\nu}^a D^2 F^{\mu\nu a} + d_g^3 \frac{1}{m_Q^2} g f_{abc} F_{\mu\nu}^a F_{\alpha}^{\mu b} F^{\nu\alpha c}$$

$$\mathcal{L}_\psi = \psi^\dagger \left( iD_0 + c_2 \frac{D^2}{2m_Q} + c_4 \frac{D^4}{8m_Q^3} + c_F g \frac{\sigma \mathbf{B}}{2m_Q} + c_D g \frac{\mathbf{D} \times \mathbf{E} - \mathbf{E} \times \mathbf{D}}{8m_Q^2} + i c_S g \frac{\sigma(\mathbf{D} \times \mathbf{E} - \mathbf{E} \times \mathbf{D})}{8m_Q^2} \right) \psi$$

$$\mathcal{L}_\chi = c.c \text{ of } \mathcal{L}_\psi$$

$$\mathcal{L}_{\psi\chi} = \frac{f_1(^1S_0)}{m_Q^2} \psi^\dagger \chi \chi^\dagger \psi + \frac{f_1(^3S_1)}{m_Q^2} \psi^\dagger \sigma \chi \chi^\dagger \sigma \psi + \frac{f_8(^1S_0)}{m_Q^2} \psi^\dagger T^a \chi \chi^\dagger T^a \psi + \frac{f_8(^3S_1)}{m_Q^2} \psi^\dagger T^a \sigma \chi \chi^\dagger T^a \sigma \psi$$

# potential NRQCD Lagrangian at $T=0$

Brambilla, Pineda, Soto and Vairo, NPB566 (2000) 275

Starting from NRQCD and integrating out the scale  $\frac{1}{r}$ .

$$\begin{aligned}\mathcal{L}_{pNRQCD} = & \int d^3r \text{Tr} \left[ S^\dagger (i\partial_0 - h_s) S \right. \\ & \left. + O^\dagger (iD_0 - h_o) O \right] + V_A(r) \text{Tr}(O^\dagger r g E S + S^\dagger r g E O) \\ & + \frac{V_B(r)}{2} \text{Tr}(O^\dagger r g E O + O^\dagger O r g E) + \mathcal{L}_g + \mathcal{L}_q\end{aligned}$$

- Degrees of freedom are singlet and octets.
- Allows to obtain manifestly gauge-invariant results. Simplifies the connection with Lattice QCD.
- If  $1/r \gg T$  we can use this Lagrangian as starting point. In other cases the matching between NRQCD and pNRQCD will be modified.



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# How to compute what we measure?

Experimentally, the most common way to detect quarkonium is through its decay into leptons. What is the pNRQCD operator related with this observable?

$$\text{Tr}(J_{el}^\mu(t, 0) J_{el, \mu}(t, 0) \rho) \propto \text{Tr}(S^\dagger(t, 0) S(t, 0) \rho)$$

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## Reinterpretation

We can understand  $\text{Tr}(S^\dagger(t, x) S(t, x') \rho)$  as the projection of the density matrix to the subspace in which we have a singlet. Quarkonium is an open quantum system interacting with a bath.

# The $\frac{1}{r} \gg T, m_D \gg E$ regime

Brambilla, M.A.E., Soto and Vairo (2017-2018)

Because all the thermal scales are smaller than  $\frac{1}{r}$  but bigger than  $E$  the evolution equation is of the Lindblad form.<sup>1</sup>

$$\partial_t \rho = -i[H(\gamma), \rho] + \sum_k (C_k(\kappa) \rho C_k^\dagger(\kappa) - \frac{1}{2} \{C_k^\dagger(\kappa) C_k(\kappa), \rho\})$$

$$\kappa = \frac{g^2}{6 N_c} \text{Re} \int_{-\infty}^{+\infty} ds \langle T E^{a,i}(s, 0) E^{a,i}(0, 0) \rangle$$

$$\gamma = \frac{g^2}{6 N_c} \text{Im} \int_{-\infty}^{+\infty} ds \langle T E^{a,i}(s, 0) E^{a,i}(0, 0) \rangle$$

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$\kappa$  coincides with the heavy quark diffusion coefficient.

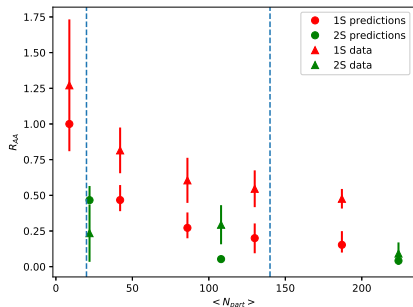
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## From $\kappa$ and $\gamma$ to phenomenological predictions

We can take values of  $\kappa$  (in this case we use lattice QCD results of Francis, Kaczmarek, Laine, Neuhaus and Ohno (2015)) and  $\gamma$  (we use  $\gamma = 0$ ).

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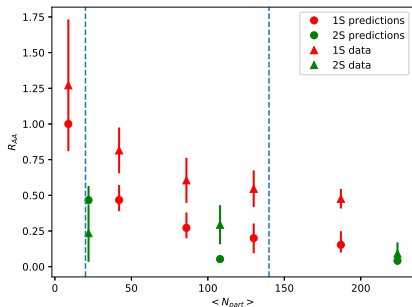


Comparison to CMS data at  $\sqrt{s} = 2.76$  TeV (Phys.Lett. B770 (2017) 357-379), computation done in Brambilla, M.A.E., Soto and Vairo (2017-2018).

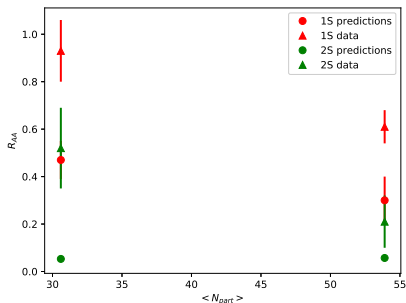


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Comparison to CMS data at  $\sqrt{s} = 5.02$  TeV (Phys. Lett. B 790, 270-293 (2019)), computation shown in Hard Probes 2018.

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# Summary

- Recently, a more precise determinations of  $\kappa$  became available. We have more precise information on the dependence of  $\kappa$  with the temperature.

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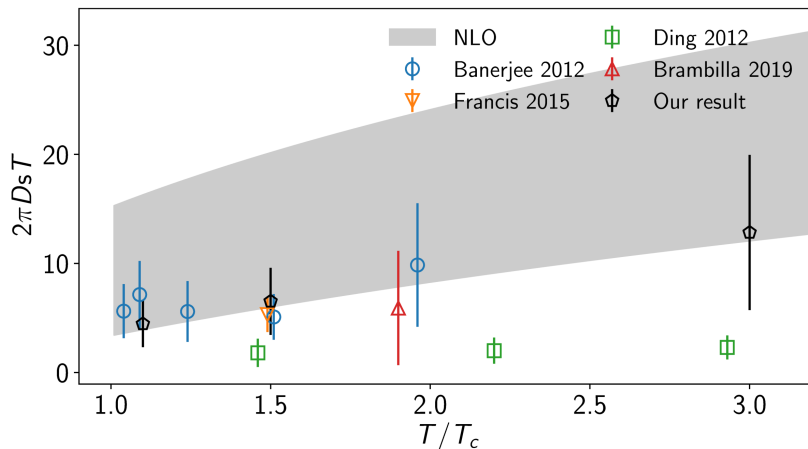
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# Summary

- Recently, a more precise determinations of  $\kappa$  became available. We have more precise information on the dependence of  $\kappa$  with the temperature.
- $\gamma$  can be obtained from Lattice QCD information on the thermal mass shift.
- To solve the Lindblad equation numerically is hard. In the past, we approximated medium evolution with a Bjorken expansion to reduce computational cost. I will discuss how we recently managed to reduce this cost.

# New determination of $\kappa$ . ( $\kappa = 2\pi D_s T$ )

Picture taken from N. Brambilla, V. Leino, P. Petreczky and A. Vairo, Phys. Rev. D **102** (2020) no.7, 074503 [arXiv:2007.10078 [hep-lat]].



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- In the case of  $\kappa$ , we have a determination which is based on an independent set of assumptions which can be compared with what is found studying heavy quark diffusion.
- In the case of  $\gamma$ , it is the first non-perturbative determination.
- The main assumption is that we are in the regime  $\frac{1}{r} \gg T, m_D \gg E$  and that the bound states are Coulombic.

# Determining $\kappa$ and $\gamma$ from quarkonium properties

## Equations for $\kappa$ and $\gamma$

$$\Gamma = \kappa \langle r^2 \rangle$$

$$\delta M = \frac{1}{2} \gamma \langle r^2 \rangle$$

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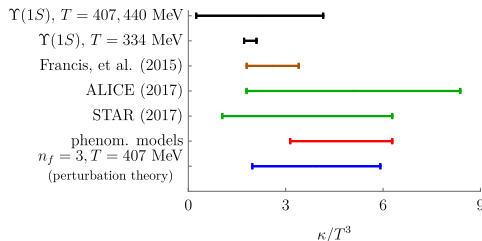
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## Lattice QCD data

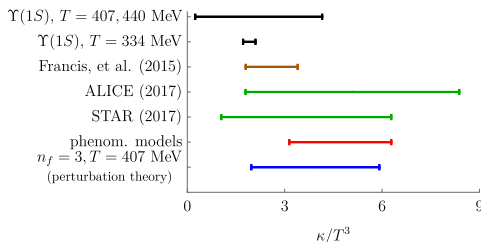
- We use the results of Kim, Petreczky and Rothkopf (2018) for the thermal mass shift and as a lower bound for the decay width.
- We use the results of Aarts, Allton, Kim, Lombardo, Oktay, Ryan, Sinclair and Skullerud (2011) as upper bound for the decay width.
- Data at  $T = 334 \text{ MeV}$ , not used originally in our paper, is taken from Larsen, Meinel, Mukherjee and Petreczky (2019).

# Determination of $\kappa$



Picture taken from Brambilla, M.A.E, Vairo and Vander Griend (2019)

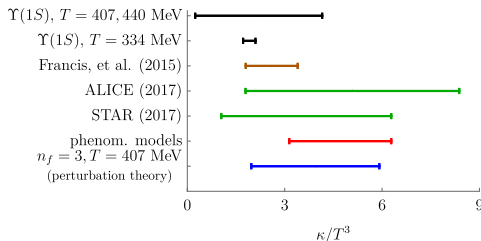
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- We took the value of Kim, Petreczky and Rothkopf (2018) at  $T = 407 \text{ MeV}$  as a lower bound and the value of Aarts, Allton, Kim, Lombardo, Oktay, Ryan, Sinclair and Skullerud (2011) at the highest temperature available as an upper bound.

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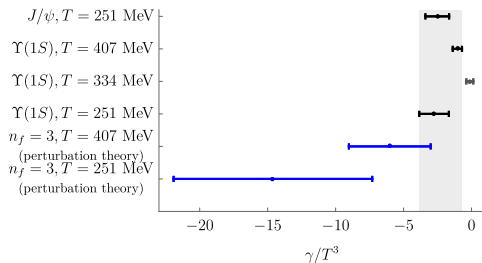


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- Our result compares reasonably well to other determinations.

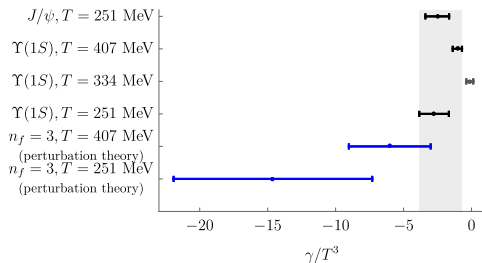


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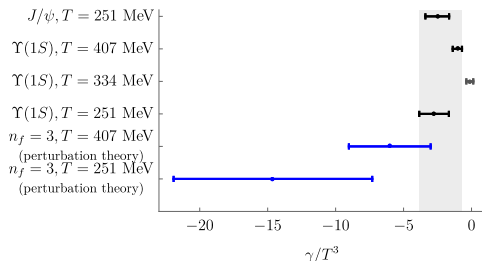
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- Results from different quarkonium state at the same temperature ( $T = 251 \text{ MeV}$ ) are compatible with each other.

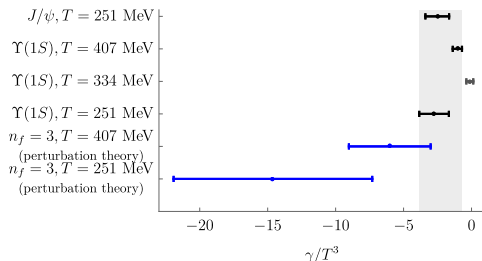
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- Results from different quarkonium state at the same temperature ( $T = 251 \text{ MeV}$ ) are compatible with each other.
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- Lattice extracted results are much smaller than perturbative calculations.

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- In our previous papers, we used a  $N_r$  size lattice to discretize the radial component and we expand in angular momentum, with  $l_{max}$  the higher  $l$  taken into account. We had to compute the evolution of a  $(2N_r \cdot l_{max}) \times (2N_r \cdot l_{max})$  matrix. Doubling the lattice size multiplies the computational cost by four and  $l_{max}$  can not be infinite.

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- Using MWFM, we need to simulate many times a stochastic evolution. However, the state of the system is represented by a vector of size  $N_r$ , a bit to store the color state and an integer to store the  $l$  quantum number. Doubling the lattice size only doubles the cost and  $l_{max}$  can be  $\infty$ .



# The Monte-Carlo Wave Function method

Take the Lindblad equation

$$\partial_t \rho = -i[H(\gamma), \rho] + \sum_k (C_k(\kappa) \rho C_k^\dagger(\kappa) - \frac{1}{2} \{C_k^\dagger(\kappa) C_k(\kappa), \rho\})$$

Let us define

$$\Gamma_n = C_n^\dagger C_n \quad \Gamma = \sum_n \Gamma_n$$

and

$$H_{\text{eff}} = H - i\Gamma$$

$\rho(t) = \sum_n p_n |\Psi_n(t)\rangle \langle \Psi_n(t)|$ . If we know how to evolve the case  $\rho(t) = |\Psi(t)\rangle \langle \Psi(t)|$ , it is straightforward to generalize.

# The Monte-Carlo Wave Function method

The algorithm to evolve from  $t$  to  $t + dt$

- With probability  $1 - \langle \Psi(t) | \Gamma | \Psi(t) \rangle dt$ .
  - ▶ Evolve the wave-function with  $(1 - iH_{\text{eff}}dt)|\Psi(t)\rangle$ . In our case, this implies solving a 1D Schrödinger equation because  $H_{\text{eff}}$  does not mix states with different color or angular momentum.
- With probability  $\langle \Psi(t) | \Gamma_n | \Psi(t) \rangle dt$ .
  - ▶ Take a quantum jump,  $|\Psi(t)\rangle \rightarrow C_n|\Psi(t)\rangle$ .
  - ▶ Only here transitions between different color and angular momentum are allowed.
- Normalize the resulting wave-function.

The average of this stochastic evolution of the wave-function is equivalent to the Lindblad equation for the density matrix.

# Results

- Thanks to the reduction of the numerical cost, we can substitute the Bjorken by state-of-the-art hydrodynamical evolution (aHydroQP. Alqahtani, Nopoush and Strickland (2015)).

# Results

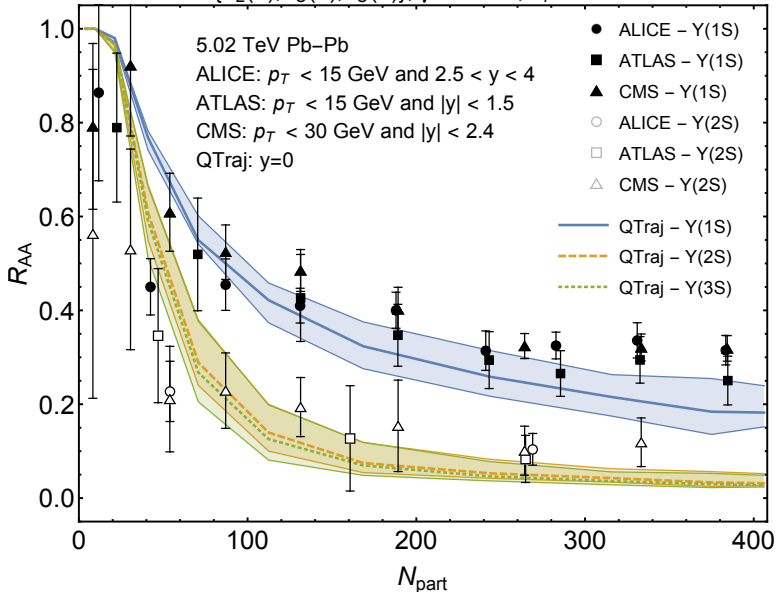
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- We observe that the effect of the quantum jumps is very small. Indeed, evolving with  $H_{\text{eff}}$  is a very good approximation.

# Results

$$\hat{\kappa} \in \{\hat{\kappa}_L(T), \hat{\kappa}_C(T), \hat{\kappa}_U(T)\}, \hat{y} = -1.75, T_f = 250 \text{ MeV}$$



# Plan

- 1 Introduction
- 2 Quarkonium suppression in pNRQCD
- 3 Recent developments
- 4 Conclusions and outlook

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- $\kappa$  is the heavy quark diffusion coefficient. The use of more precise Lattice data leads to a better agreement with experimental data.
- The use of the MCWF method reduces the computational cost. Thanks to this we can use a more realistic hydrodynamical description and obtain more realistic predictions.

# Outlook

- The MCWF method might be useful at other temperature regimes (maybe  $T \sim \frac{1}{r}$ ). We plan to publish our code soon.

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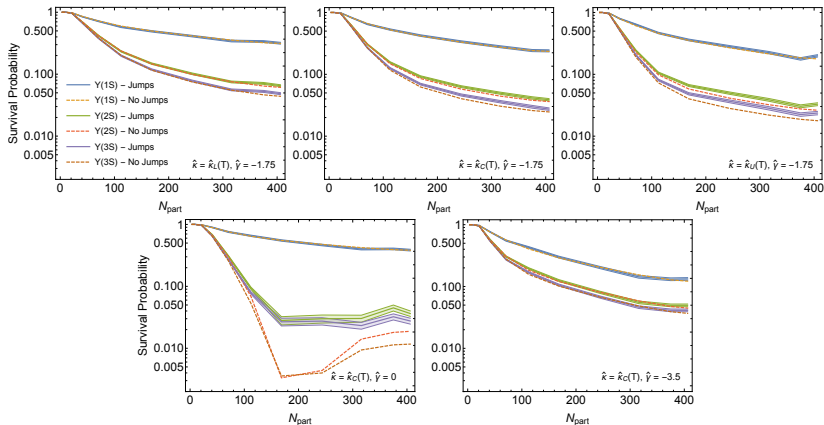
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- We are looking at other observables, like  $v_2$  of bottomonium and  $R_{AA}$  vs  $p_T$ .
- There are things that we can improve:
  - ▶ Extend the formalism to a wider range of temperature regimes. Master equations that are not of the Lindblad form.
  - ▶ Improve the initial conditions.

# Thanks!

# $H_{eff}$ against full evolution





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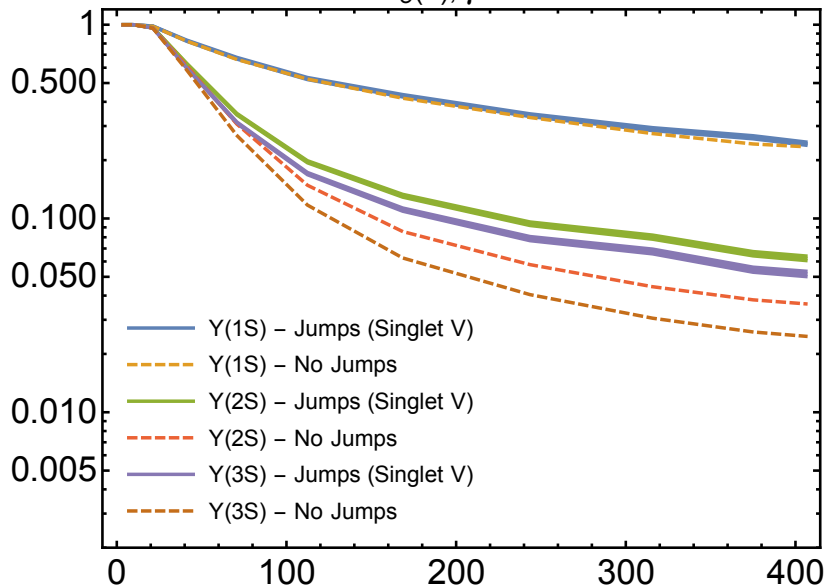
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- The octet has a repulsive potential. The quark and the antiquark separate and, if they do jump back to a singlet state, they are less likely to bound.

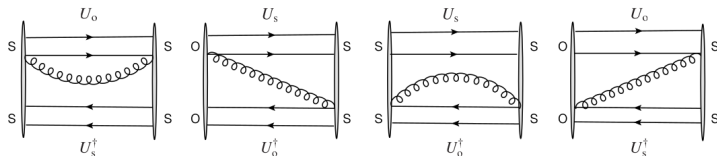
If the octet had an attractive potential...

$$\hat{k} = \hat{k}_C(T), \hat{y} = -1.75$$



# The evolution of the density matrix

4 diagrams that connect any state at time  $t$  with a singlet at time  $t + dt$ .



These diagrams represent the evolution of the density matrix

$$|\psi(t)\rangle \longrightarrow |\psi(t + dt)\rangle$$

$$\langle\phi(t)| \longleftarrow \langle\phi(t + dt)|$$

# The evolution of the density matrix

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