Chromoelectric Distribution Function of Nuclear Matter Probed by Quarkonium

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arXiv: 2009.02408

Virtual Quarkonia As Tools 2021
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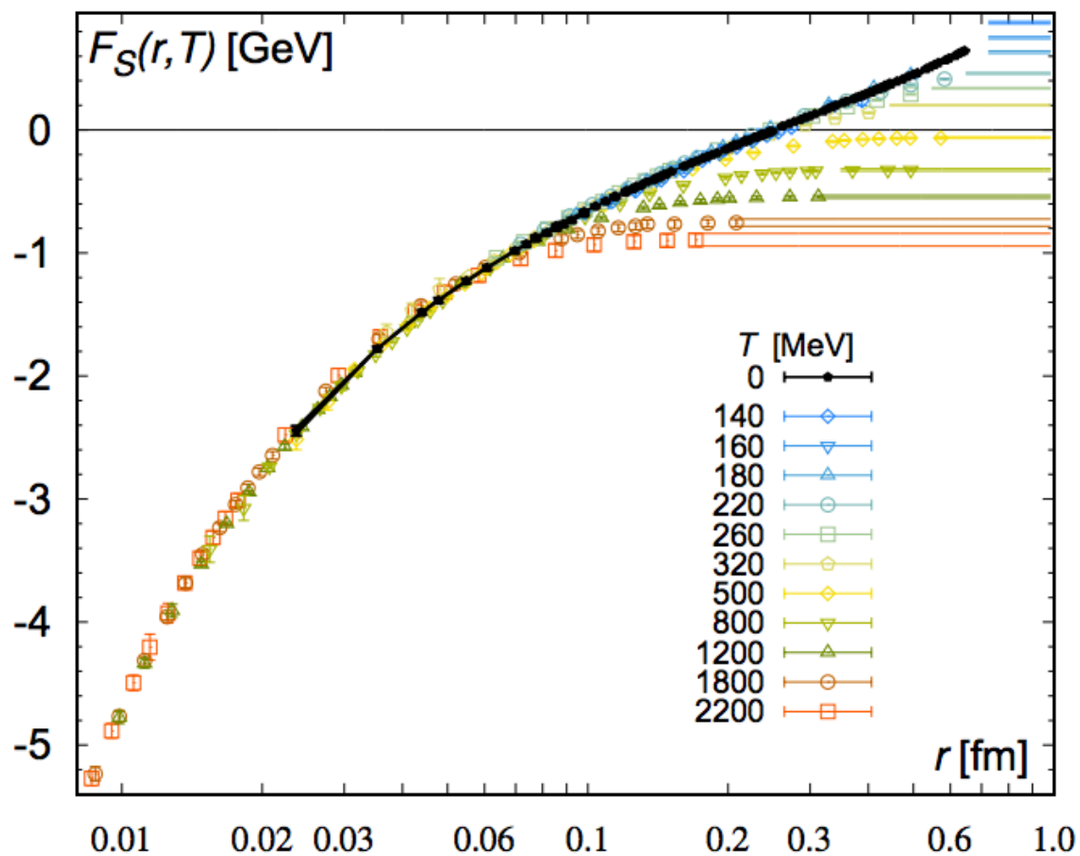
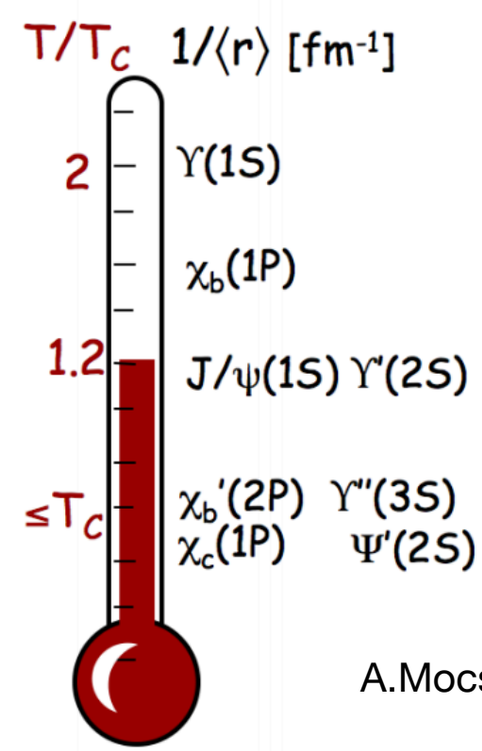
Quarkonium as Probe of Quark-Gluon Plasma

- **Static screening**: suppression of color attraction —> melting at high T —> reduced production —> thermometer

$$T = 0 : V(r) = -\frac{A}{r} + Br \longrightarrow T \neq 0 :$$ Confining part flattened



1804.10600, A.Bazavov, N.Brambilla
P.Petreczky, A.Vairo, J.H.Weber



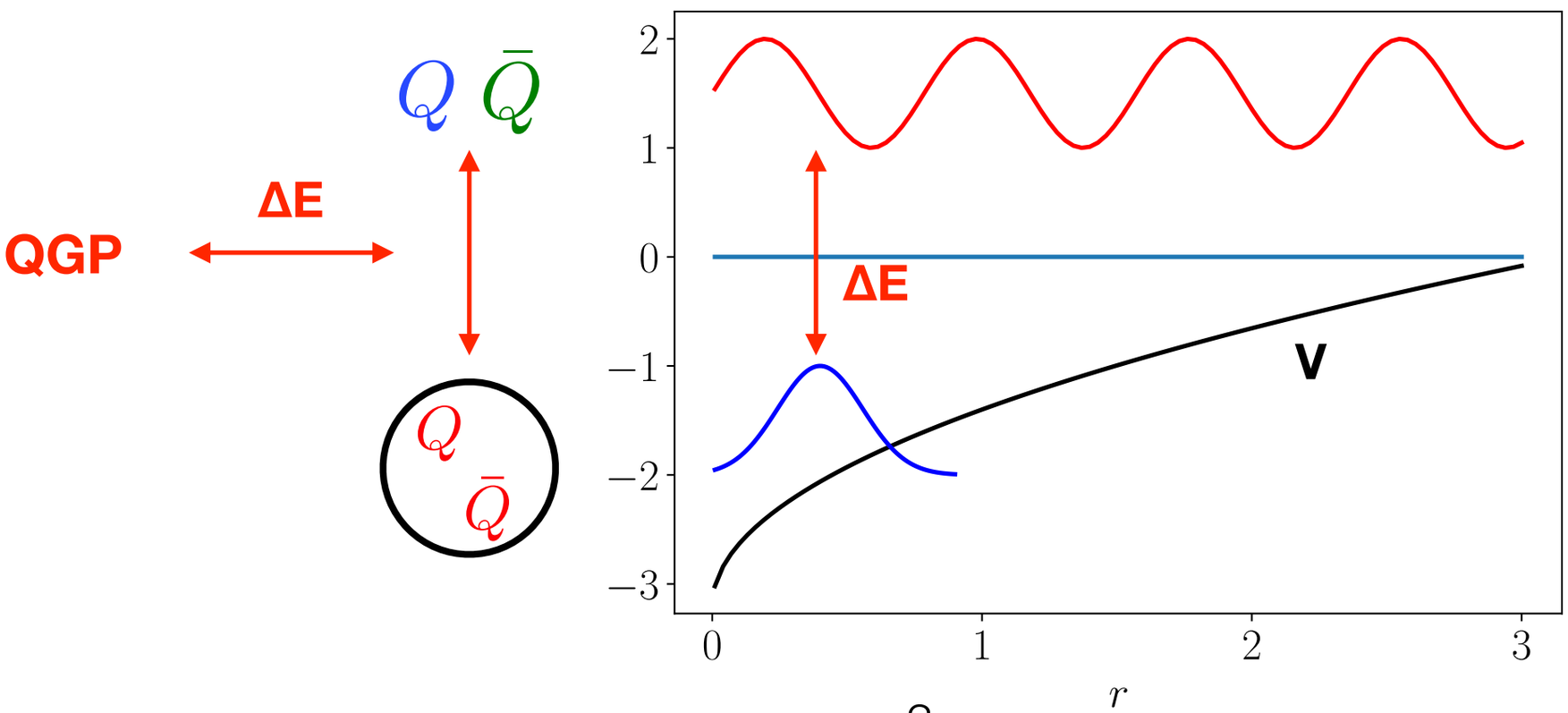
A.Mocsy, 0811.0337

Quarkonium as Probe of Quark-Gluon Plasma

- Static screening: suppression of color attraction —> melting at high T —> reduced production —> thermometer

- **Dynamical screening**: related to imaginary potential, **dissociation** induced by dynamical process, lead to suppression even when T(QGP) < melting T

- **Recombination**: unbound heavy quark pair forms quarkonium, can happen below melting T, crucial for phenomenology and theory consistency



Simple physics picture of thermometer does not work

What QGP properties are we probing by measuring quarkonium?

This talk:

In certain limit, we are probing chromoelectric distribution functions of QGP/nuclear medium

Leading-power, all-order construction, gauge invariant

Two tools: open quantum systems, effective field theory

Open Quantum System

Total system = subsystem + environment: $H = H_S + H_E + H_I$

$\rho(t=0) = \rho_S \otimes \rho_E$

Subsystem & environment

(Heavy quark pairs & QGP)

Unitary evolution

Time reversible

$U(t,0)(\rho_S \otimes \rho_E)U^\dagger(t,0)$

Subsystem & environment

Trace out (integrate out) environment

$\rho_S(t=0)$

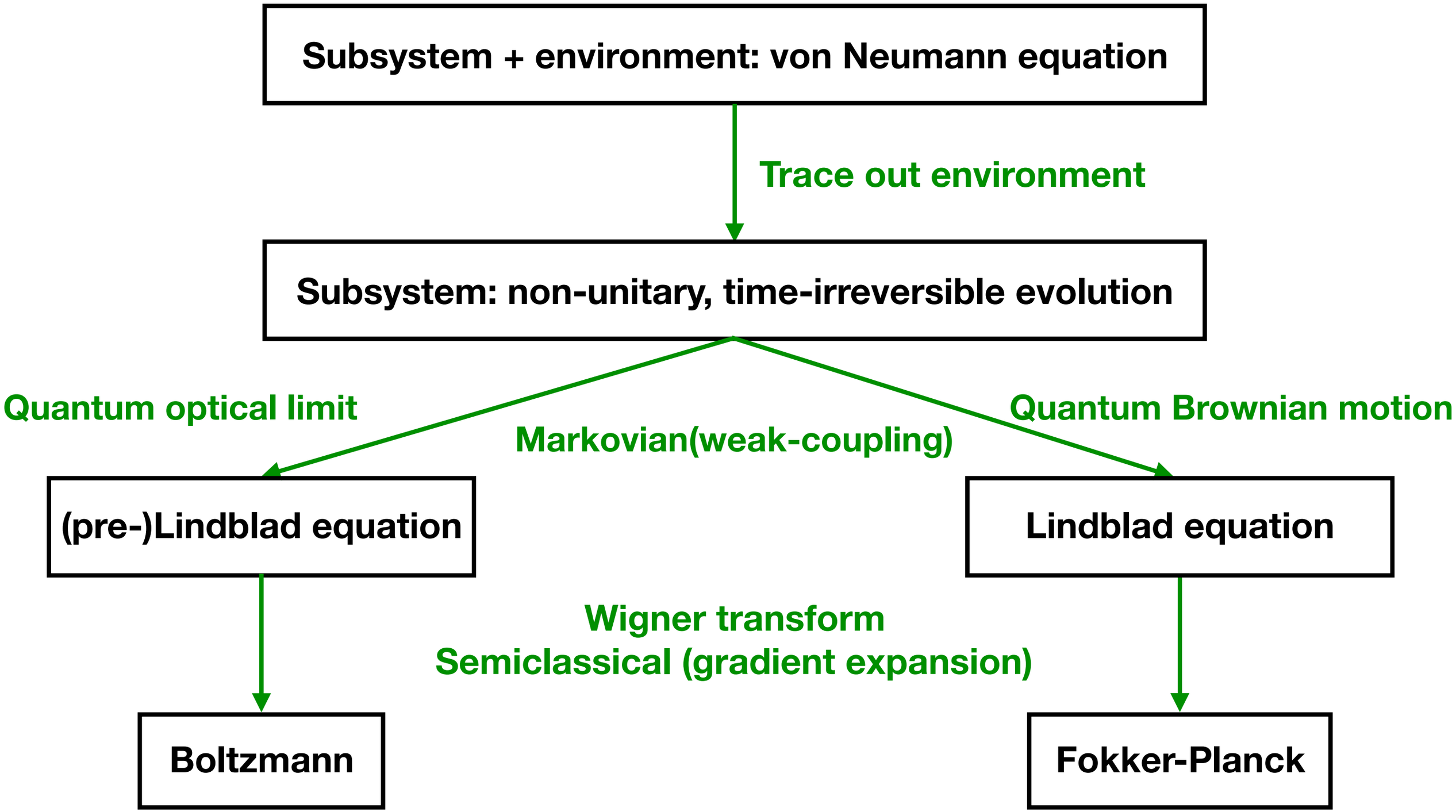
Subsystem

Non-unitary

Time irreversible

$\mathrm{Tr}_E\left[U(t,0)(\rho_S \otimes \rho_E)U^\dagger(t,0)\right]$

From Open Quantum System to Semiclassical Transport



Wigner transform $$f_{nl}(\boldsymbol{x},\boldsymbol{k},t) \equiv \int \frac{d^3k'}{(2\pi)^3} e^{i\boldsymbol{k}'\cdot\boldsymbol{x}} \left\langle \boldsymbol{k}+\frac{\boldsymbol{k}'}{2}, nl, 1 \middle| \rho_S(t) \middle| \boldsymbol{k}-\frac{\boldsymbol{k}'}{2}, nl, 1 \right\rangle$$

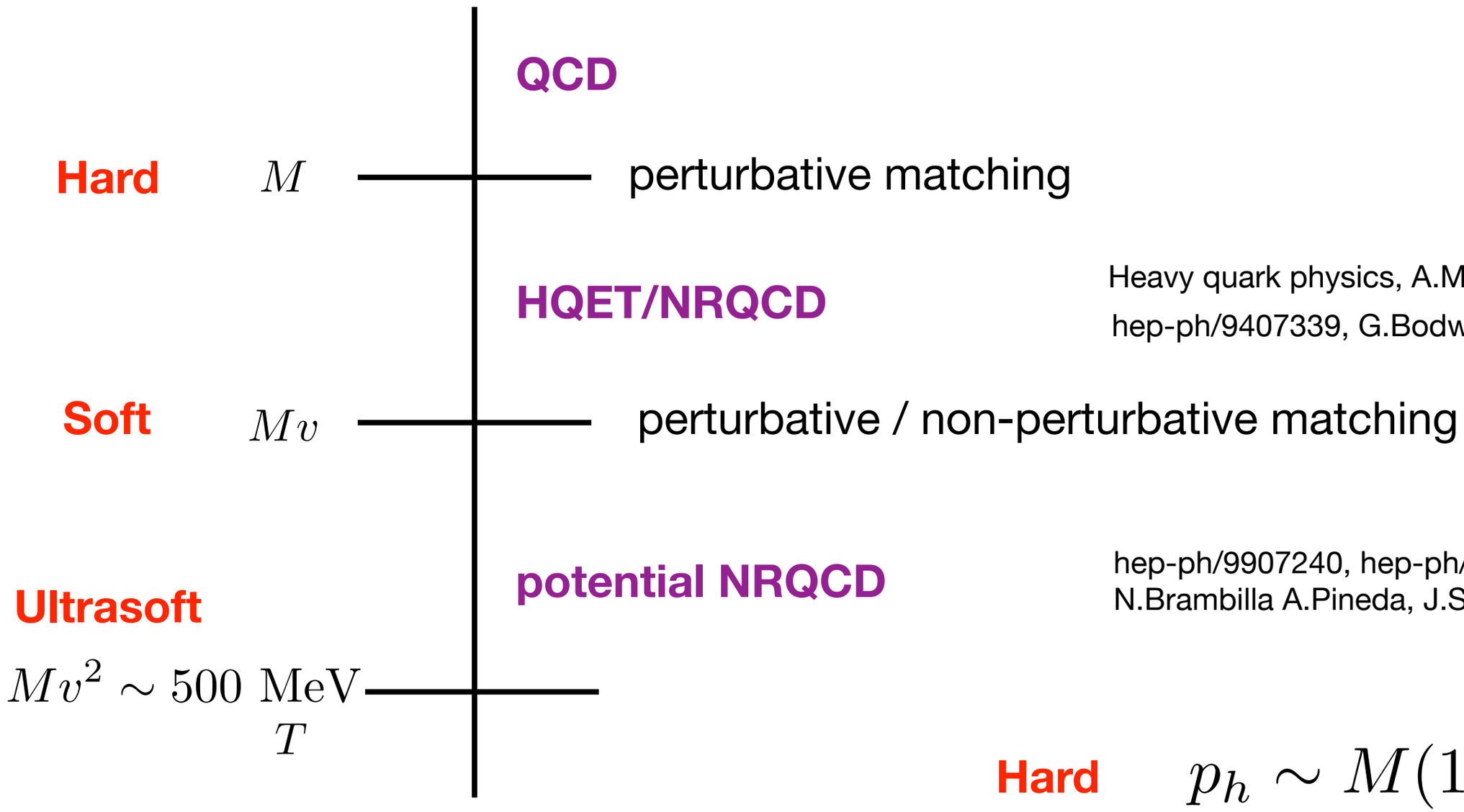
Separation of Scales and pNRQCD

Separation of scales

$$M \gg Mv \gg Mv^2,\ T,\ \Lambda_{QCD}$$

$v^2 \sim 0.3$ charmonium

$v^2 \sim 0.1$ bottomonium



Heavy quark physics, A.Manohar, M.Wise

hep-ph/9407339, G.Bodwin, E.Braaten, G.Lepage

hep-ph/9907240, hep-ph/0410047,
N.Brambilla A.Pineda, J.Soto, A.Vairo

Hard $p_h \sim M(1,1,1,1)$

Soft $p_s \sim Mv(1,1,1,1)$

Ultrasoft $p_{us} \sim Mv^2(1,1,1,1)$

Leading Power

- **Nonrelativistic & multipole expansions: v & r**

$$\mathcal{L}_{\rm pNRQCD} = \int d^3r\, {\rm Tr}\Big({\rm S}^\dagger (i\partial_0 - H_s){\rm S} + {\rm O}^\dagger (iD_0 - H_o){\rm O} + \boxed{V_A({\rm O}^\dagger \boldsymbol{r}\cdot g\boldsymbol{E}{\rm S} + {\rm h.c.})} + \frac{V_B}{2}{\rm O}^\dagger\{\boldsymbol{r}\cdot g\boldsymbol{E}, {\rm O}\} + \cdots \Big)$$

Dipole interaction

- **Boltzmann equation at leading-power in v & r, leading-order in g**

Dissociation and recombination rates depend on QGP via

XY, T.Mehen 1811.07027

$$\mathrm{Tr}_E\big(\rho_E E_i(t_1,\boldsymbol{x}_1)E_i(t_2,\boldsymbol{x}_2)\big) = \langle E_i(t_1,\boldsymbol{x}_1)E_i(t_2,\boldsymbol{x}_2)\rangle_T$$

Not gauge invariant !

All-Order Construction: Sum A0 Interactions

$$\mathcal{L}_{\rm pNRQCD} = \int d^3r\, {\rm Tr}\Big({\rm S}^\dagger(i\partial_0 - H_s){\rm S} + \boxed{{\rm O}^\dagger(iD_0 - H_o){\rm O}} + V_A({\rm O}^\dagger \boldsymbol{r}\cdot g\boldsymbol{E}{\rm S} + {\rm h.c.}) + \frac{V_B}{2}{\rm O}^\dagger\{\boldsymbol{r}\cdot g\boldsymbol{E}, {\rm O}\} + \cdots\Big)$$

Octet—A0 interaction not suppressed by v or r

Need sum A0 to all orders at leading power

Field redefinition:

$$\begin{aligned}
{\rm O}(\boldsymbol{R},\boldsymbol{r},t) &= \mathcal{W}_{[(\boldsymbol{R},t),(\boldsymbol{R},t_0)]}\widetilde{\rm O}(\boldsymbol{R},\boldsymbol{r},t)\\
\widetilde{E}_i(\boldsymbol{R},t) &= \mathcal{W}_{[(\boldsymbol{R},t_0),(\boldsymbol{R},t)]}E_i(\boldsymbol{R},t)\\
\mathcal{W}_{[(\boldsymbol{R},t_f),(\boldsymbol{R},t_i)]} &= \mathcal{P}\exp\left(ig\int_{t_i}^{t_f} ds\mathcal{A}_0(\boldsymbol{R},s)\right)
\end{aligned}$$

New form of dipole interaction:

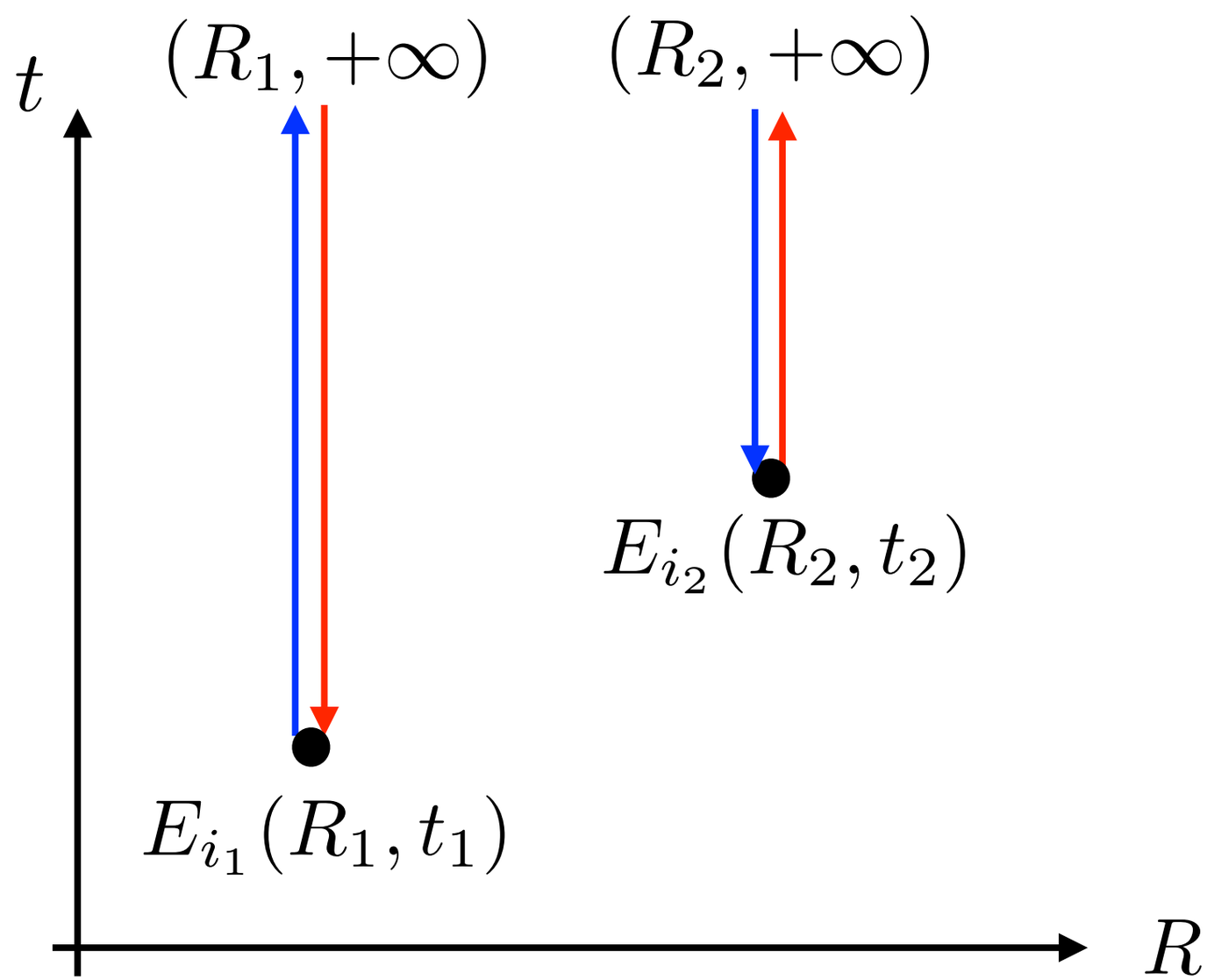
$$g\int d^3r {\rm Tr}\Big(\widetilde{\rm O}^\dagger r_i\widetilde{E}_i{\rm S} + {\rm S}^\dagger r_i\widetilde{E}_i^\dagger\widetilde{\rm O}\Big)$$

Chromoelectric Distribution Function of QGP

$$g^{E++}_{i_1 i_2}(t_1, t_2, \boldsymbol{R}_1, \boldsymbol{R}_2) = \left\langle E_{i_1}(\boldsymbol{R}_1, t_1)\mathcal{W}_{[(\boldsymbol{R}_1,t_1),(\boldsymbol{R}_1,+\infty)]}\mathcal{W}_{[(\boldsymbol{R}_2,+\infty),(\boldsymbol{R}_2,t_2)]}E_{i_2}(\boldsymbol{R}_2, t_2)\right\rangle_T$$

Wilsons not connected at infinite time!

For gauge invariance, need spatial gauge link



Wilson Lines at Infinite Time

$$\mathcal{L}_{\rm pNRQCD} = \int d^3r\, {\rm Tr}\Big({\rm S}^\dagger (i\partial_0 - H_s){\rm S} + {\rm O}^\dagger (iD_0 - H_o){\rm O} + V_A({\rm O}^\dagger \boldsymbol{r}\cdot g\boldsymbol{E}{\rm S} + {\rm h.c.}) + \frac{V_B}{2}{\rm O}^\dagger\{\boldsymbol{r}\cdot g\boldsymbol{E}, {\rm O}\} + \cdots \Big)$$

Coulomb interaction between octet heavy quark pair included in potential

But Coulomb between octet center-of-mass motion and medium not considered

For Coulomb modes $p_c^\mu \sim A_c^\mu \sim M(v^2, v, v, v)$

$$\int {\rm d}^3r\, {\rm Tr}\Big({\rm O}^\dagger(\boldsymbol{R},\boldsymbol{r},t)\Big(iD_0 + \boxed{\frac{\boldsymbol{D}_{\boldsymbol{R}}^2}{4M}} + \frac{\nabla_{\boldsymbol{r}}^2}{M} - V_o(\boldsymbol{r}) + \cdots\Big){\rm O}(\boldsymbol{R},\boldsymbol{r},t)\Big)$$

C.m. kinetic term same order as D0, so leading power in v

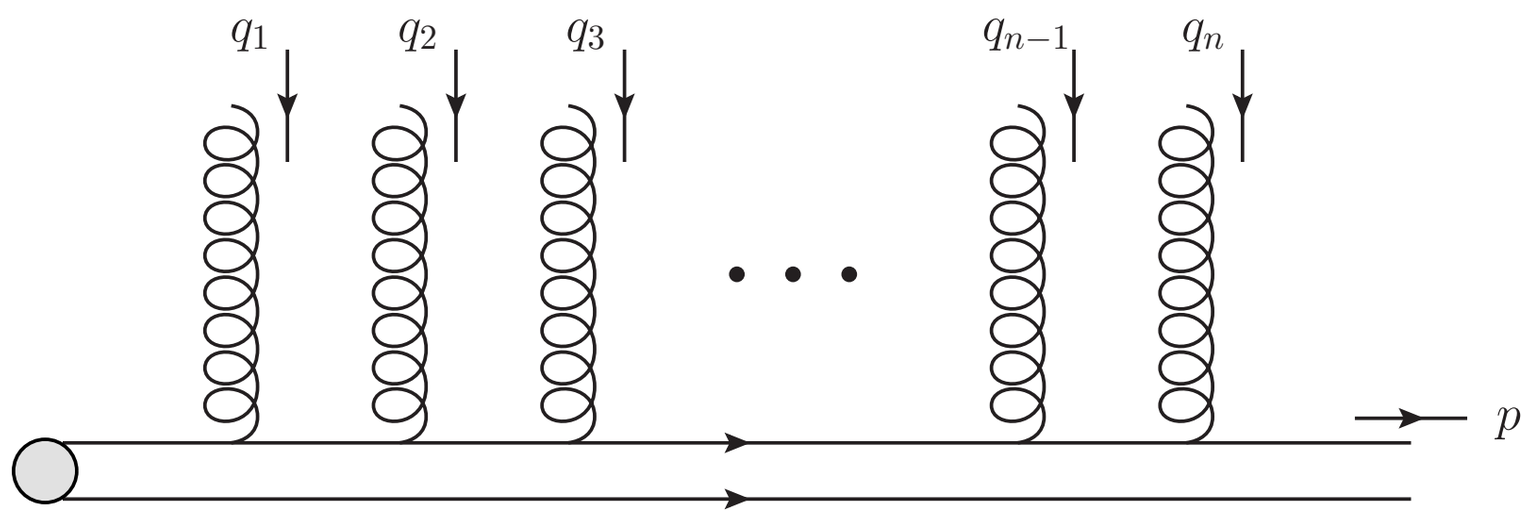
Write out c.m. kinetic term

$$\int {\rm d}^3r\, {\rm Tr}\Big({\rm O}^\dagger(\boldsymbol{R},\boldsymbol{r},t)\frac{\nabla_{\boldsymbol{R}}^2}{4M}{\rm O}(\boldsymbol{R},\boldsymbol{r},t) - \frac{ig}{4M}{\rm O}^\dagger(\boldsymbol{R},\boldsymbol{r},t)\Big(\boldsymbol{A}(\boldsymbol{R},t)\cdot\nabla_{\boldsymbol{R}}$$

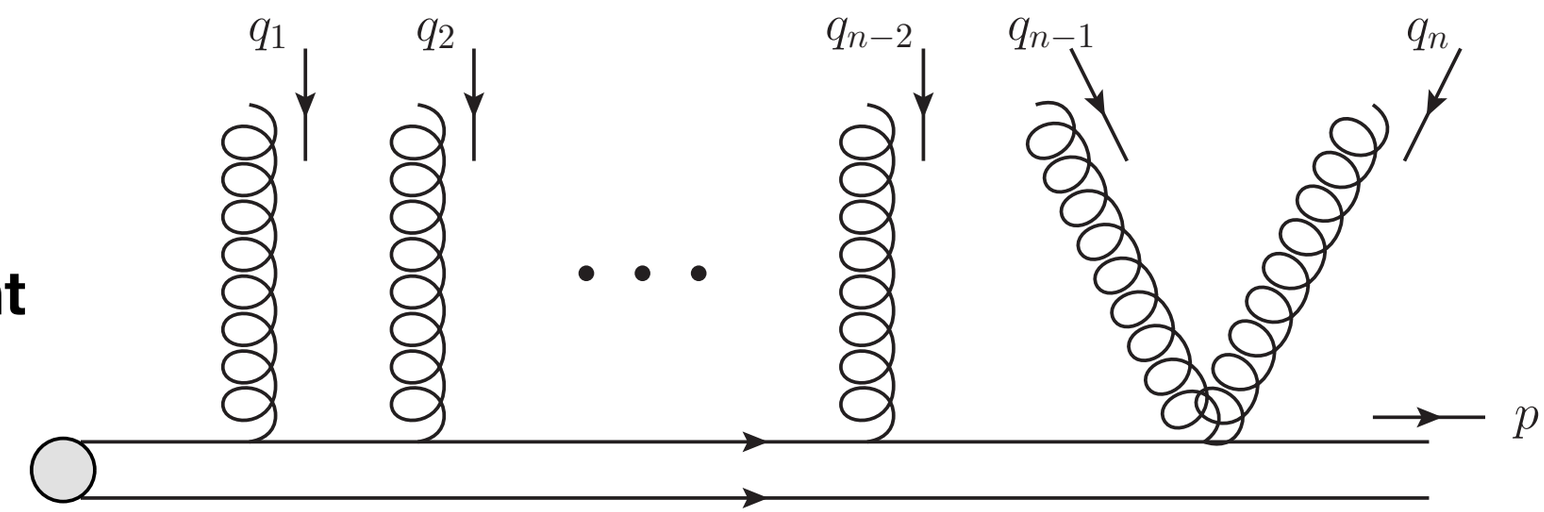
$$+ \nabla_{\boldsymbol{R}}\cdot\boldsymbol{A}(\boldsymbol{R},t)\Big){\rm O}(\boldsymbol{R},\boldsymbol{r},t) - \frac{g^2}{4M}{\rm O}^\dagger(\boldsymbol{R},\boldsymbol{r},t)\boldsymbol{A}^2(\boldsymbol{R},t){\rm O}(\boldsymbol{R},\boldsymbol{r},t)\Big).$$

Wilson Lines at Infinite Time: Resum Coulomb

Single Coulomb attachment



Double Coulomb attachment

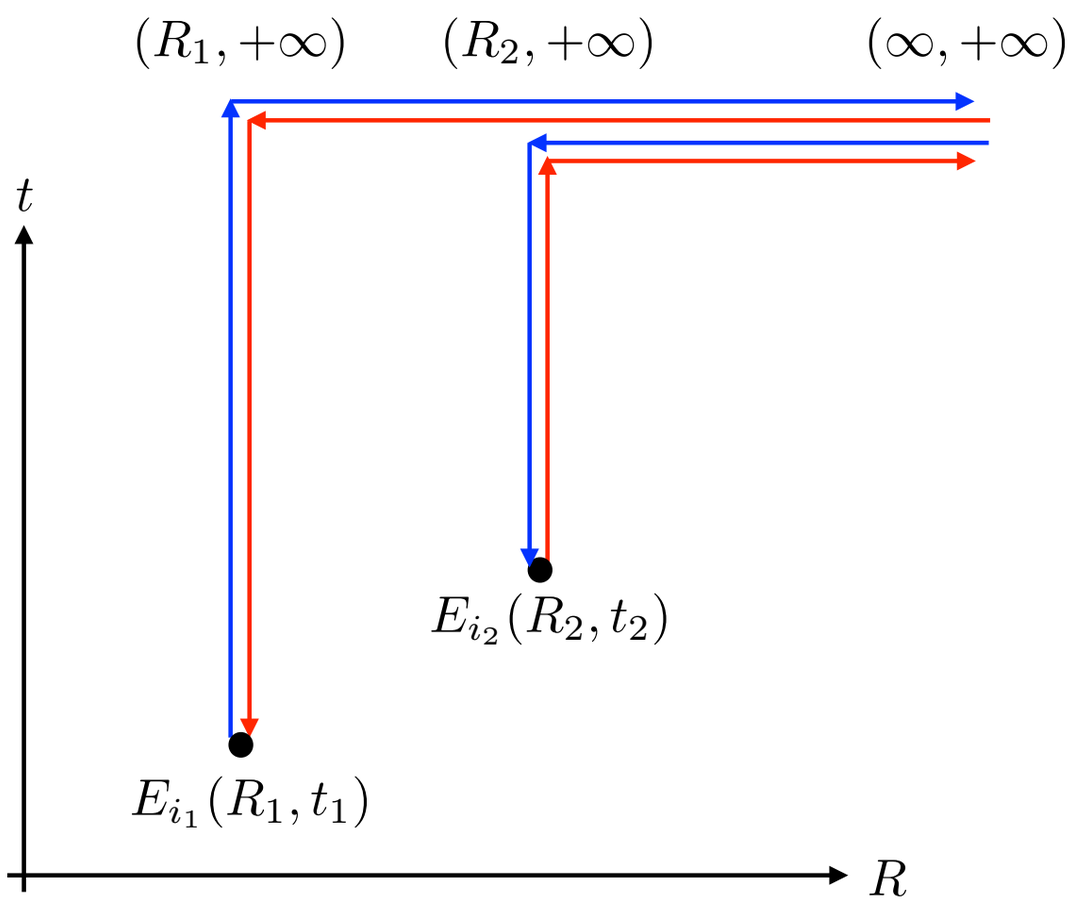


Calculations have some similarity with A.V. Belitsky, X. Ji, F. Yuan hep-ph/0208038

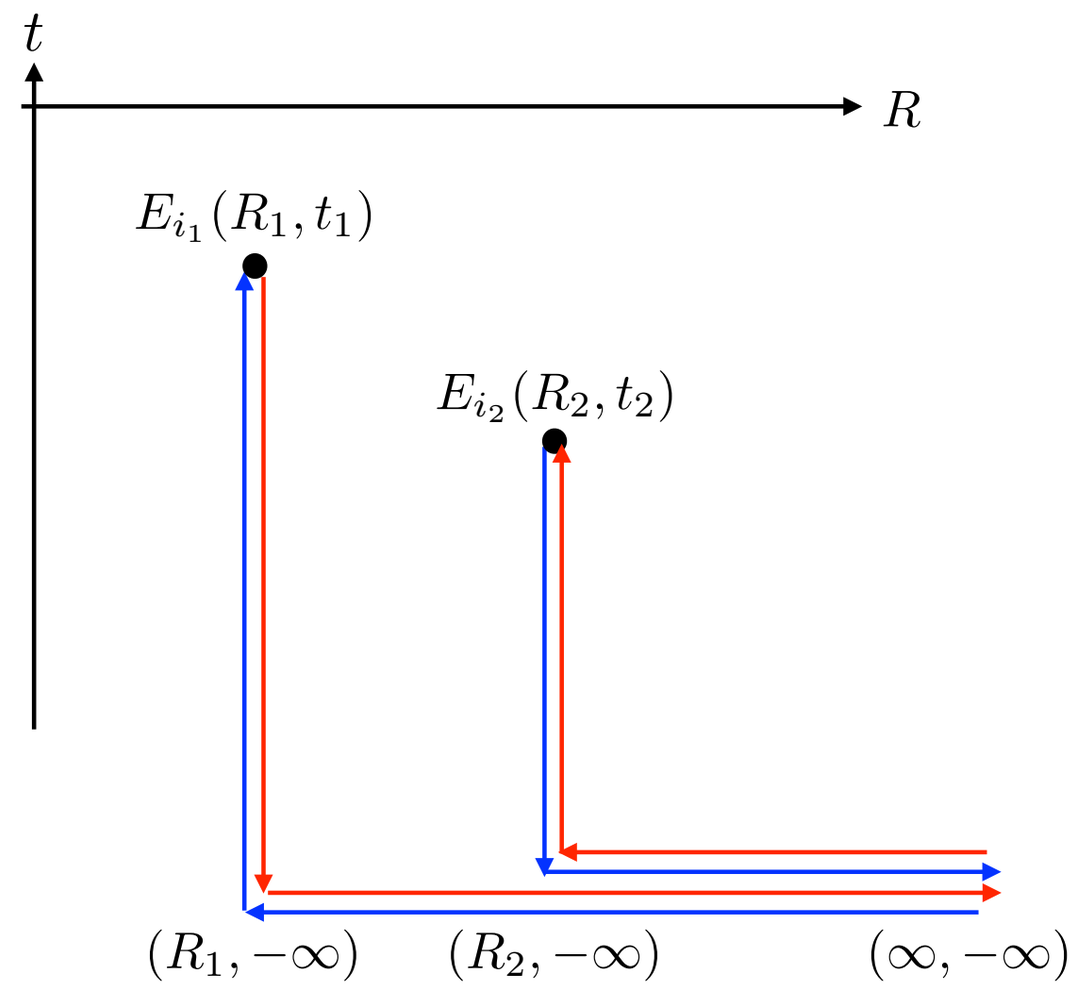
Chromoelectric Distribution Function of QGP

Staple shaped Wilson lines

For dissociation: final-state interaction



For recombination: initial-state interaction



Inclusive v.s. Differential Reaction Rates

- **Take dissociation rate as example**

$$R^-_{nl}(\boldsymbol{x},\boldsymbol{k},t) = \sum_{i_1,i_2}\int \frac{\mathrm{d}^3p_{\mathrm{cm}}}{(2\pi)^3}\frac{\mathrm{d}^3p_{\mathrm{rel}}}{(2\pi)^3}\frac{\mathrm{d}^4q}{(2\pi)^4}(2\pi)^4\delta^3(\boldsymbol{k}-\boldsymbol{p}_{\mathrm{cm}}+\boldsymbol{q})\delta(E_{nl}-E_p+q^0)\, d^{nl}_{i_1,i_2}(\boldsymbol{p}_{\mathrm{rel}})g^{E++}_{i_1i_2}(q^0,\boldsymbol{q})$$

$$d^{nl}_{i_1i_2}(\boldsymbol{p}_{\mathrm{rel}}) \equiv g^2\frac{1}{N_c}\langle\psi_{nl}|r_{i_1}|\Psi_{\boldsymbol{p}_{\mathrm{rel}}}\rangle\langle\Psi_{\boldsymbol{p}_{\mathrm{rel}}}|r_{i_2}|\psi_{nl}\rangle$$

- **Inclusive rate**

$$R^-_{nl} = \int \frac{\mathrm{d}^3p_{\mathrm{rel}}}{(2\pi)^3}\bar{d}_{nl}(\boldsymbol{p}_{\mathrm{rel}})G^{E++}\Big(\frac{(\boldsymbol{p}_{\mathrm{rel}})^2}{M}-E_{nl}\Big)$$

$$G^{E++}(q_0) = \int dt\, e^{-iq_0t}\,\big\langle E_i(t)\mathcal{W}_{[t,0]}E_i(0)\big\rangle$$

Momentum independent distribution has been constructed in

N.Brambilla, M.A.Escobedo J.Soto, A.Vairo 1711.04515

Zero frequency limit = heavy quark diffusion coefficient

- **Differential rate**

$$(2\pi)^3\frac{\mathrm{d}R^-_{nl}}{\mathrm{d}^3p_{\mathrm{cm}}} = \int \frac{\mathrm{d}^3p_{\mathrm{rel}}}{(2\pi)^3}\bar{d}_{nl}(\boldsymbol{p}_{\mathrm{rel}})g^{E++}\Big(\frac{(\boldsymbol{p}_{\mathrm{rel}})^2}{M}-E_{nl},\boldsymbol{p}_{\mathrm{cm}}-\boldsymbol{k}\Big)$$

Momentum dependent distribution

- **Similar to PDF v.s. TMDPDF, though different in time axis**

Summary

- What are we probing by measuring quarkonium?
- Open quantum + EFT: leading-power, all-order construction
- Reaction rates depend on chromoelectric distribution function
  - Inclusive rates depend on momentum independent one, straight-line Wilson line structure
  - Differential rates depend on momentum dependent one, staple-shape Wilson line structure
- Easily generalized to cold nuclear matter by replacing environment density matrix