Status of DPS theory

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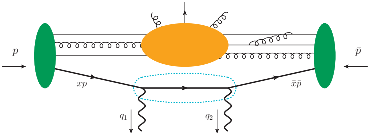
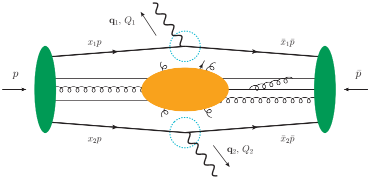
What is double parton scattering (DPS)?

SPS



$$|\mathbf{q}_1^\perp + \mathbf{q}_2^\perp| \sim \Lambda \ll Q$$

DPS



$$|\mathbf{q}_1^\perp|, |\mathbf{q}_2^\perp| \sim \Lambda \ll Q$$

Definition

Double parton scattering (DPS) is a proton-proton scattering process where two partons from each proton undergo **two separate hard interactions**.

- **Here, focus of perturbative QCD description of DPS in pp collisions.**

pioneers: Politzer; Paver, Treleani; Mekhfi;

recent work by: Gaunt, Stirling; Blok, Dokshitzer, Frankfurt; Diehl, Schäfer; Manohar, Waalewijn; Ryskin; Snigierev...

When is DPS important?

Scaling

- differential XS: $\dfrac{\mathrm{d}^2\sigma_{\text{SPS}}}{\mathrm{d}^2q_1\mathrm{d}^2q_2} \sim \dfrac{\mathrm{d}^2\sigma_{\text{DPS}}}{\mathrm{d}^2q_1\mathrm{d}^2q_2} \implies$ same power counting!
- integrated XS: $\dfrac{\sigma_{\text{DPS}}}{\sigma_{\text{SPS}}} \sim \mathcal{O}\left(\dfrac{\Lambda^2}{Q^2}\right) \implies$ phase-space suppressed

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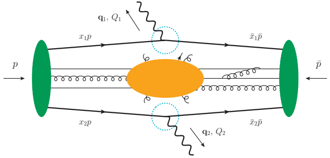
- competitive with SPS in regions of small $|\mathbf{q}_1^\perp|, |\mathbf{q}_2^\perp|$
  $\rightarrow$ e.g. two pairs of back-to-back jets
- DPS relevance increases with collision energy
- enhanced by parton luminosities at small-x, e.g. $F_{gg} \propto (f_g)^2$
  $\rightarrow$ interest for quarkonia production
- DPS dominant contribution for coupling-suppressed processes in SPS
  $\rightarrow$ same-sign WW production at $\mathcal{O}(\alpha_s^2)$ in SPS, but $\mathcal{O}(1)$ in DPS

DPS cross section

For colourless final states, analogous factorized form to the SPS case

- $\hat{\sigma}^{(i)}$ are regular partonic cross sections
- F_{ab} are double parton distributions (DPDs)
- y [GeV^{-1}] is inter-parton transverse separation



Transverse-momentum dependent (TMD) factorization:

$$\frac{d\sigma_{\text{DPS}}}{dq_1^{\perp}\, dq_2^{\perp}} = \frac{1}{C} \sum_{a_1 a_2 b_1 b_2} \hat{\sigma}^{(1)}_{a_1 b_1}\, \hat{\sigma}^{(2)}_{a_2 b_2}$$

$$\times \int d^2 y\, \frac{d^2 z_1}{2\pi^2}\, \frac{d^2 z_2}{2\pi^2}\, e^{-i q_1^{\perp} z_1 - i q_2^{\perp} z_2}\, F_{a_1 a_2}(z_1, z_2, y)\, F_{b_1 b_2}(z_1, z_2, y)$$

Collinear factorization:

$$d\sigma_{\text{DPS}} = \frac{1}{C} \sum_{a_1 a_2 b_1 b_2} \hat{\sigma}^{(1)}_{a_1 b_1} \otimes \hat{\sigma}^{(2)}_{a_2 b_2} \otimes \int d^2 y\, F_{a_1 a_2}(y) \otimes F_{b_1 b_2}(y)$$

here I am neglecting $x_i, \bar{x}_i$ in the arguments

Status of factorization

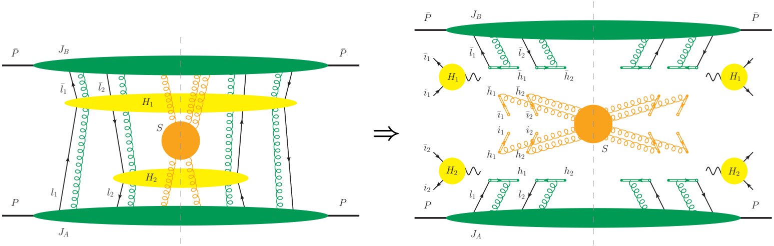
Proof of factorized formulae has been completed: current status at the same level as SPS counterpart.

Diehl, Ostermeier, Schäfer 2011; Diehl, Gaunt, Ostermeier, Plößl, Schäfer 2015

Vladimirov 2016, 2017; Buffing, Diehl, Kasemets 2017; Diehl, RN 2018

Main steps:

- collinear gluons $\longrightarrow$ Wilson lines
- Glauber gluon cancellation
- soft gluons $\longrightarrow$ soft factor
- soft factor re-absorbed into DPDs
  $\hookrightarrow$ rapidity dependence, Collins-Soper equations



Double parton distributions

Generic TMD DPDs

Factorization formula implies definition of "bare" DPDs similar to PDFs

$$F^{(0)}_{a_1 a_2}(x_1, x_2, z_1, z_2, y) \propto \langle p|\, \mathcal{O}_{a_1}(y, z_1)\, \mathcal{O}_{a_2}(0, z_2)\, |p\rangle \,\big|_{z_i^+ = y_i^+ = 0}$$

in terms of operators $\mathcal{O}(y, z) \sim \bar{\psi}(y - \frac{1}{2}z)\, \Gamma\, \psi(y + \frac{1}{2}z)$.

- integrated collinear DPDs $F^{(0)}_{a_1 a_2}(x_1, x_2, y)$ by letting $z_1, z_2 \to 0$
  $\hookrightarrow$ used in integrated cross section calculations
- momentum-space DPDs $F^{(0)}_{a_1 a_2}(x_1, x_2, \Delta)$ by Fourier transform
  $\hookrightarrow$ used in DPD sum rules

Polarization

Note that in an unpolarized proton, two partons can be both polarized.
In DPS, polarized distributions are in principle as relevant as unpolarized ones.

DGLAP evolution for DPDs

Evolution in position space

Renormalizing the bare DPDs adds a scale dependence for each parton:

$$\frac{\mathrm{d}F_{a_1a_2}(x_i, y;\, \mu_1, \mu_2)}{\mathrm{d}\log \mu_1} = 2\left[P_{a_1c}(\mu_1) \underset{1}{\otimes} F_{cb_1}(y;\, \mu_1, \mu_2)\right](x_i)$$

$$\frac{\mathrm{d}F_{a_1a_2}(x_i, y;\, \mu_1, \mu_2)}{\mathrm{d}\log \mu_2} = 2\left[P_{a_2c}(\mu_2) \underset{2}{\otimes} F_{a_1c}(y;\, \mu_1, \mu_2)\right](x_i)$$

Evolution in momentum space

Momentum-space dependent DPDs obey inhomogeneous evolution equations:

$$\begin{aligned}\frac{\mathrm{d}F_{a_1a_2}(x_i, \Delta;\, \mu, \mu)}{\mathrm{d}\log \mu} &\\ = 2\Big[P_{a_1c}(\mu) \underset{1}{\otimes} F_{cb_1}(\Delta;\, \mu, \mu) &+ P_{a_1c}(\mu) \underset{1}{\otimes} F_{cb_1}(\Delta;\, \mu, \mu)\\ + P_{s,\, a_1a_2, a_0}(\mu) \underset{12}{\otimes} f_{a_0}(\mu)\Big](x_i)&\end{aligned}$$

where P_s is the $1 \to 2$ splitting function.

DGLAP evolution for DPDs: numerics

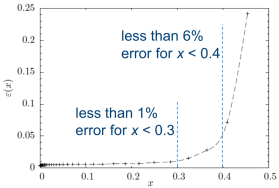
Double DGLAP evolution is a non-trivial numerical task, but it is also the main ingredient for DPS phenomenological studies.

Gaunt-Stirling DPD code (private) [Gaunt, Stirling '09]

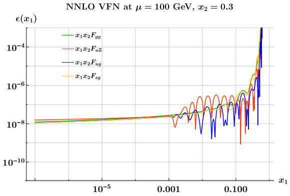
- LO DGLAP (both $\boldsymbol{y}$- and $\boldsymbol{\Delta}$-dependent)

Only publicly available set: `GS09` [gsdpdf.hepforge.org]

- based on products of `MSTW2008` PDFs
- $\boldsymbol{y}$-integrated DPDs



[J. Gaunt's talk @ MPI10]



ChiliPDF project [Diehl, RN, Tackmann, Plößl]

- unequal-scale evolution ($\boldsymbol{\mu_1 \neq \mu_2}$)
- NNLO DGLAP, with NNLO flavor matching
- polarized DGLAP
- flexible input (numerical, analytical, ...)

To the best of my knowledge, private DPD evolution codes have been developed by other groups (e.g. in [Elias, Golec-Biernat, Staśto '18])

DPD sum rules

- integrated DPDs (i.e. momentum-space DPDs at $\Delta = 0$) obey **sum rules** analogous to the PDF ones, and expressed in terms of PDFs
- these are used to constrain DPD models

Momentum sum rule

$$\sum_{a_2} \int_0^{1-x_1} dx_2\, x_2\, F_{a_1 a_2}(x_1, x_2, 0;\, \mu) = (1 - x_1)\, f_{a_1}(x_1;\, \mu)$$

Number sum rule

$$\int_0^{1-x_1} dx_2\, \left[F_{a_1 a_2}(x_1, x_2, 0;\, \mu) - F_{a_1 \bar{a}_2}(x_1, x_2, 0;\, \mu)\right]$$
$$= \left(N_{a_2,\mathrm{v}} + \delta_{a_1 \bar{a}_2} - \delta_{a_1 a_2}\right)\, f_{a_1}(x;\, \mu)$$

where $N_{a,\mathrm{v}}$ is the number of valence partons of type a

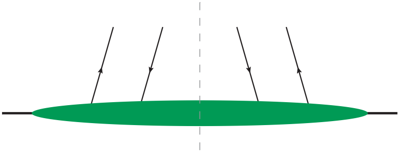
DPDs from perturbative splitting

A class of DPD Ansätze at small y

When $y \to 0$ DPDs are sum of "**intrinsic**" and "**splitting**" piece

$$F(y)|_{y\to 0} = F_{\text{int}}(y) + F_{\text{spl}}(y)$$

At larger y, DPDs can be modeled so that $F(y) \to 0$ as $y \to \infty$.



intrinsic (F_{int} or "$\mathbf{2}$")

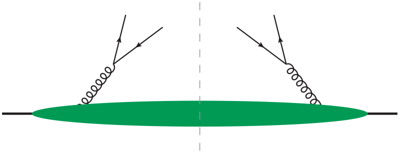
twist-4 distribution at small y, nonperturbative

Perturbative splitting

- $F_{\text{spl}}(y) \propto \dfrac{1}{y^2}$
- UV divergence in cross-section

$$\int d^2y\, F_1\, F_2 \sim \int \frac{d^2y}{y^4}$$

- comes from region of overlap between SPS and DPS



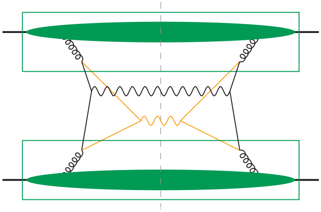
perturbative splitting (F_{spl} or "$\mathbf{1}$")

- LO: $F_{ab} \propto P_{a_0 \to ab} \cdot f_{a_0}$ [Diehl et al. 2011]
- NLO: calculated [Diehl et al. 2019]

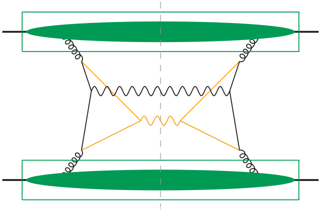
Double-counting between SPS and DPS

The UV divergence in y is associated to the double counting of SPS and DPS contributions in the region where $y \to 0$:

DPS interpretation (1v1)



SPS interpretation



Solution: DGS scheme

The DGS subtraction scheme cancels the UV divergence at all orders:

$$\boxed{\sigma = \sigma_{\text{SPS}} + \sigma_{\text{DPS}} - \sigma_{\text{sub}}}\,, \qquad \sigma_{\text{sub}} = \sigma_{\text{DPS}} \text{ with } F_{1,2} = F_{\text{spl}}$$

where the DPS cross section is regularized introducing a cutoff $\nu \sim Q$

$$\sigma_{\text{DPS}} \propto \int \mathrm{d}^2 y\, F_1(y)\, F_2(y) \to \int \mathrm{d}^2 y\, \Phi^2(y\nu)\, F_1(y)\, F_2(y)$$

Simple cutoff regulator $\Phi(y\nu) = \Theta(y\nu - 2e^{-\gamma_E})$. Diehl, Gaunt, Schönwald 2015

Intrinsic DPDs: simple models

Product Ansatz

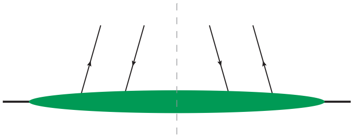
At a scale μ_0 the **intrinsic** DPDs are given by the product of PDFs and a **geometric factor**:

$$F_{ab}^{\text{int}}(x_1, x_2, y;\, \mu_0) = f_a(x_1;\, \mu_0)\, f_b(x_2;\, \mu_0)\, G(x_1, x_2, y)$$

Parton correlations from G and DGLAP evolution.

$\hookrightarrow$ iterated improvements to fulfil sumrules

Diehl, Gaunt, Lang, Plößl, Schäfer 2020



Pocket formula

Simplify assumption if DPDs are product of PDFs **at all scales**, and G independent of x_i:

$$F_{ab}^{\text{int}}(x_1, x_2, y;\, \mu_1, \mu_2) = f_a(x_1;\, \mu_1)\, f_b(x_2;\, \mu_2)\, G(y)$$

- neglect all parton correlations
- violates momentum conservation for $x_1 + x_2 > 1$
- violates double DGLAP equations
- ... but: leads to very convenient XS formula (the **DPS pocket formula**)

$$\boxed{\sigma_{\text{DPS}} = \frac{1}{C}\,\frac{\sigma_1^{\text{SPS}}\sigma_2^{\text{SPS}}}{\sigma_{\text{eff}}}}, \qquad \text{with } \sigma_{\text{eff}} = \frac{1}{\int d^2y\, G^2(y)}$$

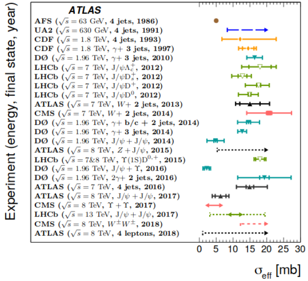
Estimates of σ_{eff}

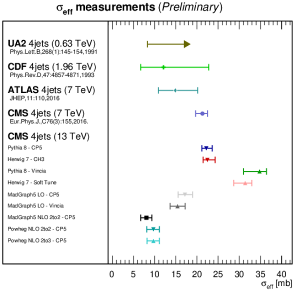
Experimental measurements of $\boldsymbol{\sigma}_{\text{eff}}$ have given **very different results** among processes and experiments, spanning a range from $\sim \mathbf{1}$ to $\sim \mathbf{35}$ mb.

latest measurements (4-jets CMS)

[CMS-PAS-SMP-20-007]

$\boldsymbol{\sigma}_{\text{eff}}$ measurements [CERN-EP-2018-274]





For **quarkonia production**, the extracted values of $\boldsymbol{\sigma}_{\text{eff}}$ are usually on the lower side, pointing to a larger effect of inter-partonic correlations.

What is left

Recent theory developments

- **parton shower** combining SPS and DPS, accounting for the "$\mathbf{1 \to 2}$" splitting and implementing the SPS-DPS double counting (`dShower`)

  Cabouat, Gaunt 2020
- **lattice QCD**: extracted moments of the pion DPD and of the proton DPD

  Bali et al. 2018, Zimmermann (PhD Thesis) 2020

What I did not talk about (but probably more in next talk by Matteo)

- many phenomenological models for DPDs

  $\hookrightarrow$ constituent quark models [Rinaldi, Scopetta, Ceccopieri], "bag" model [Manohar, Waalewijn], valence quark models [Broniowski, Ruiz Arriola], KMR approach [Golec-Biernat, Staśto], . . .
- many phenomenological studies with DPS Blok, Dokshitzer, Frankfurt, Strikman, Maciuła, Szczurek, Kutak, van Hameren, Gaunt, Kom, Kulesza, Stirling, Fedkyevich, Kasemets, Myska, Cotogno, Lansberg, Yamanaka, Zhang, Shao, Ceccopieri, Rinaldi, Scopetta, ...
- DPS in pA collisions and TPS (triple parton scattering) D'Enterria, Snigirev

Summary

- **DPS** contributions can be **comparable or even dominant** w.r.t. SPS in some cases, including quarkonia production
- status of DPS **factorization proofs** is at the same level as for SPS
- **double-counting** of SPS and DPS in small-y region is understood
- double DGLAP evolution and flavor matching are under control with tools developments
- perturbative **splitting** form of DPDs known **up to NLO**
- we have all ingredients to **compute DPS cross section at LO** in full QCD w.o. approximations (replacing pocket formula)
- it would be interesting to study the **colour non-singlet** DPDs
- a lot of progress and a lot of interest from many fields (an entire session of Quarkonia2020 was on DPS!)

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Thank you!

Modeling the y-dependence

Product Ansatz

An example for the geometric factor $G(x_1, x_2, y)$ appearing in the product Ansatz of DPDs:

$$F_{\text{int}\,a_1 a_2}(x_1, x_2, y, \mu_0, \mu_0) = f_{a_1}(x_1, \mu_0)\, f_{a_2}(x_2, \mu_0) \times \frac{\exp\left(-\frac{y^2}{4h_{a_1 a_2}}\right)}{4\pi h_{a_1 a_2}}\, \Theta(1 - x_1 - x_2) \left(\frac{1 - x_1 - x_2}{(1 - x_1)(1 - x_2)}\right)^{n_{a_1 a_2}},$$

LO splitting

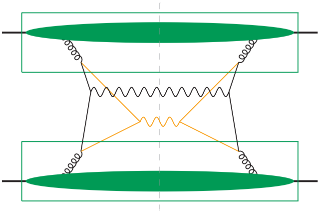
The LO splitting expression:

(distinguish the **geometric factor** and the **dimensional factor**)

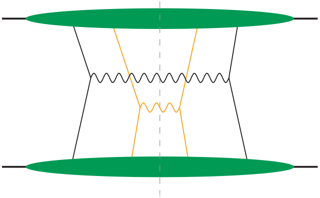
$$F_{\text{spl}\,a_1 a_2}(x_1, x_2, y, \mu_y, \mu_y) = \frac{1}{\pi y^2} \exp\left(-\frac{y^2}{4h_{a_1 a_2}}\right) \frac{\alpha_s(\mu_y)}{2\pi} T_{a_0 \to a_1 a_2}\left(\frac{x_1}{x_1 + x_2}\right) \frac{f_{a_0}(x_1 + x_2, \mu_y)}{x_1 + x_2}$$

The nucleon widths $h_{a_1 a_2}$ can also depend on x_i, and can be taken e.g. from TMD studies.

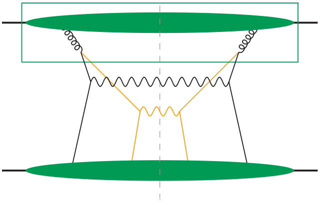
Interplay of splitting and intrinsic contributions



1v1 $\longrightarrow$ divergence is $\dfrac{\mathbf{1}}{\boldsymbol{y^4}}$, subtracted



2v2 $\longrightarrow$ not divergent



2v1 $\longrightarrow$ divergence is $\dfrac{\mathbf{1}}{\boldsymbol{y^2}} \longrightarrow \boldsymbol{\log y}$ terms