Phenomenological assessment of proton mechanical properties

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Overview

1. Physical picture of the energy momentum tensor
2. Relation to generalised parton distributions
3. Extraction from a global DVCS analysis
4. Results
5. Conclusion

Physical picture of the energy momentum tensor

- The proton matrix element of the **energy-momentum tensor (EMT)** can be parametrised in terms of five **gravitational form factors (GFFs)** - for parton of type a

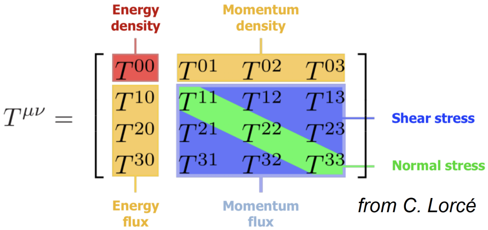
Gravitational form factors

$$\langle p', s' | T_a^{\mu\nu} | p, s \rangle = \bar{u}(p', s') \Bigg\{ \frac{P^\mu P^\nu}{M} A_a(t, \mu^2) + \frac{\Delta^\mu \Delta^\nu - \eta^{\mu\nu} \Delta^2}{M} C_a(t, \mu^2) + M \eta^{\mu\nu} \bar{C}_a(t, \mu^2) + \frac{P^{\{\mu} i\sigma^{\nu\}\rho} \Delta_\rho}{4M} \left[A_a(t, \mu^2) + B_a(t, \mu^2) \right] + \frac{P^{[\mu} i\sigma^{\nu]\rho} \Delta_\rho}{4M} D_a(t, \mu^2) \Bigg\} u(p, s) \quad (1)$$

where

$$\Delta = p' - p, \;\; t = \Delta^2, \;\; P = \frac{p + p'}{2} \quad (2)$$

Physical picture of the energy momentum tensor



from C. Lorcé

In the Breit frame ($\vec{P} = 0,\ t = -\vec{\Delta}^2$), radial distributions of energy and momentum in the proton are described by Fourier transforms of the GFFs w.r.t. variable $\vec{\Delta}$.

- Simplest such distribution: radial pressure anisotropy profile

$$s_a(r, \mu^2) = -\frac{4M}{r^2} \int \frac{\mathrm{d}^3\vec{\Delta}}{(2\pi)^3} e^{-i\vec{\Delta}\cdot\vec{r}} \frac{t^{-1/2}}{M^2} \frac{2}{t^2} \left[t^{5/2}\, C_a(t, \mu^2) \right] \tag{3}$$

- Energy radius of the proton

$$\langle r^2 \rangle_E = 6 \sum_{\text{parton type } a} \left[\frac{\mathrm{d}A_a}{\mathrm{d}t}(0, \mu^2) - \frac{C_a(0, \mu^2)}{M^2} \right] \tag{4}$$

Relation to generalised parton distributions

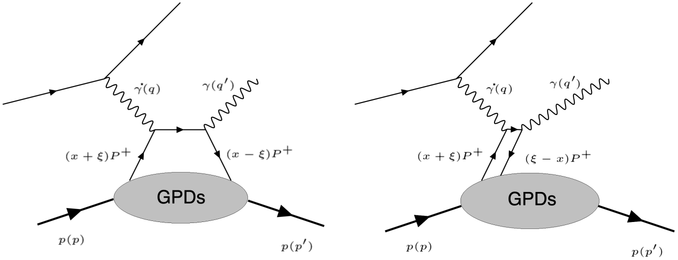
- Remarkably, GFFs can be accessed through **generalised parton distributions (GPDs)** – non-perturbative objects encompassing usual **parton distribution functions (PDFs)** and **elastic form factors (EFFs)** → **multi-dimensional information on hadron structure**
- There are four leading-twist chiral-even GPDs for each parton type inside the proton, noted H^a, E^a, $\widetilde{H}^a$ and $\widetilde{E}^a$, which depend on three variables (x, ξ, t) and a scale μ^2
- Four GFFs can be extracted from those GPDs through, *e.g.* for quarks

$$\int_{-1}^{1} \mathrm{d}x \, x \, H^q(x, \xi, t, \mu^2) = A_q(t, \mu^2) + 4\xi^2 C_q(t, \mu^2) \tag{5}$$

$$\int_{-1}^{1} \mathrm{d}x \, x \, E^q(x, \xi, t, \mu^2) = B_q(t, \mu^2) - 4\xi^2 C_q(t, \mu^2) \tag{6}$$

$$\sum_q \int_{-1}^{1} \mathrm{d}x \, \widetilde{H}^q(x, \xi, t, \mu^2) = -\sum_q D_q(t, \mu^2) \tag{7}$$

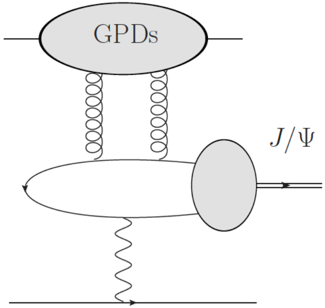
- The last GFF $\bar{C}_a(t, \mu^2)$ can be linked to higher-twist GPDs, and can be studied by **heavy quarkonium production at threshhold** [*e.g.* Joosten, Meziani, 2018]
- Leading-twist GPDs are conveniently accessed through several exclusive processes, like **deeply virtual Compton scattering (DVCS)**: production of a real photon in the scattering of a deeply virtual photon on a hadron target $\rightarrow$ direct sensitivity to quark GPDs at LO



Tree-level depiction of DVCS for $x > |\xi|$ (left) and $\xi > |x|$ (right)

Relation to generalised parton distributions

- Direct sensitivity to gluon GPDs at LO can be provided by **quarkonium production in ultra-peripheral collisions (UPC)**, especially interesting in the very-low ξ (low x_B region) at the LHC [Chapon *et. al.*, 2020] $\rightarrow$ unique possibility to access directly E^g



J/ψ photoproduction in UPC (from C. Mezrag)

- Analysis of different processes (DVCS, quarkonium production, TCS, DVMP, ...) and different observables sensitive to different GPDs is the way to a precise extraction of proton mechanical properties.

Extraction from a global DVCS analysis

- DVCS observables can be parametrised in terms of **Compton form factors** (CFFs) $\mathcal{F}$, which write as convolutions of perturbative **coefficient functions** T_F^a and the **GPDs** F^a:

CFF convolution (leading twist)

$$\mathcal{F}(\xi, t, Q^2) = \sum_{\text{parton type } a} \int_{-1}^{1} \frac{\mathrm{d}x}{\xi} T_F^a\left(\frac{x}{\xi}, \frac{Q^2}{\mu^2}, \alpha_s(\mu^2)\right) F^a(x, \xi, t, \mu^2)^a \tag{8}$$

[a] $F^g(x, \xi, t, \mu^2)/x$ for the usual definition of gluon GPD

- Extracting GPDs from the convolution of eq. (8) is a tedious issue, but we can access information via the **dispersion relations**

LO dispersion relations

$$\mathcal{C}_H(t, Q^2) = \mathrm{Re}\,\mathcal{H}(\xi, t, Q^2) - \frac{1}{\pi}\int_0^1 \mathrm{d}\xi' \, \mathrm{Im}\,\mathcal{H}(\xi', t, Q^2)\left(\frac{1}{\xi - \xi'} - \frac{1}{\xi + \xi'}\right) \tag{9}$$

Extraction from a global DVCS analysis

- At LO, the subtraction constant $\mathcal{C}_H(t, Q^2)$ can be linked to the so-called **D-term** by

$$\mathcal{C}_H(t, Q^2) = 2\sum_q e_q^2 \int_{-1}^{1} \mathrm{d}z \, \frac{D^q_{\mathrm{term}}(z, t, Q^2)}{1 - z} \tag{10}$$

and from

$$\int_{-1}^{1} \mathrm{d}x \, x \, H^q(x, \xi, t, \mu^2) = A_q(t, \mu^2) + 4\xi^2 C_q(t, \mu^2) \tag{11}$$

we find that the GFF $C_q(t, \mu^2)$ is linked to this D-term by

$$\int_{-1}^{1} \mathrm{d}z \, z D^q_{\mathrm{term}}(z, t, \mu^2) = 4 C_q(t, \mu^2) \tag{12}$$

GFF C_a extraction

Experiment $\rightarrow$ CFF $\mathcal{H}/\mathcal{E}$ $\rightarrow$ subtraction constant $\rightarrow$ integral over D-term $\rightarrow$ GFF C_a

Extraction from a global DVCS analysis

- How to get from

$$\int_{-1}^{1} \mathrm{d}z \, \frac{D^q_{\text{term}}(z,t,\mu^2)}{1-z} \quad \text{to} \quad \int_{-1}^{1} \mathrm{d}z \, z D^q_{\text{term}}(z,t,\mu^2) \ ? \tag{13}$$

Through the known evolution of the D-term with scale! Let's expand the D-term in the Gegenbauer polynomial basis

$$D^q_{\text{term}}(z,t,\mu^2) = (1-z^2) \sum_{\text{odd } n} d^q_n(t,\mu^2) \, C^{3/2}_n(z) \tag{14}$$

Then

GFF C_a extraction

$$\int_{-1}^{1} \mathrm{d}z \, \frac{D^q_{\text{term}}(z,t,\mu^2)}{1-z} = 2 \sum_{\text{odd } n} d^q_n(t,\mu^2) \quad \text{and} \quad \int_{-1}^{1} \mathrm{d}z \, z D^q_{\text{term}}(z,t,\mu^2) = \frac{4}{5} \, d_1(t,\mu^2) \tag{15}$$

Extraction from a global DVCS analysis

GFF C_a extraction

$$\int_{-1}^{1} \mathrm{d}z \, \frac{D^q_{\mathrm{term}}(z, t, \mu^2)}{1 - z} = 2 \sum_{\text{odd } n} d^q_n(t, \mu^2) \quad \text{and} \quad \int_{-1}^{1} \mathrm{d}z \, z D^q_{\mathrm{term}}(z, t, \mu^2) = \frac{4}{5} d_1(t, \mu^2) \tag{16}$$

- Each term $d^q_n(t, \mu^2)$ evolves with μ^2 in an independent way, which allows to separate the different terms of the sum on the left, if the subtraction constant is known precisely on a large enough lever-arm in Q^2.
- In practice, we have to rely on various degrees of modelling to perform this separation.

Results

1. **CFF extraction** from a global analysis of world DVCS data is performed in [Moutarde, Sznajder, Wagner, 2019] thanks to a **neural network (NN) parametrization** of CFFs. Replicas of the NN are freely accessible on PARTONS (http://partons.cea.fr/)

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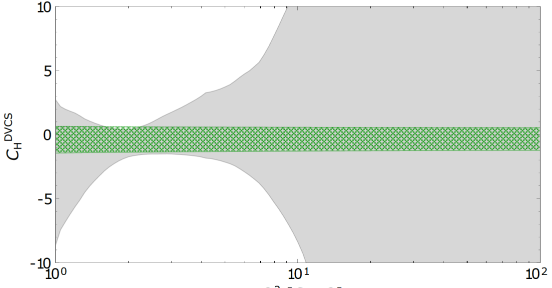

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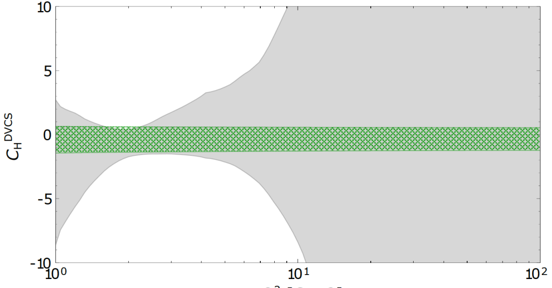

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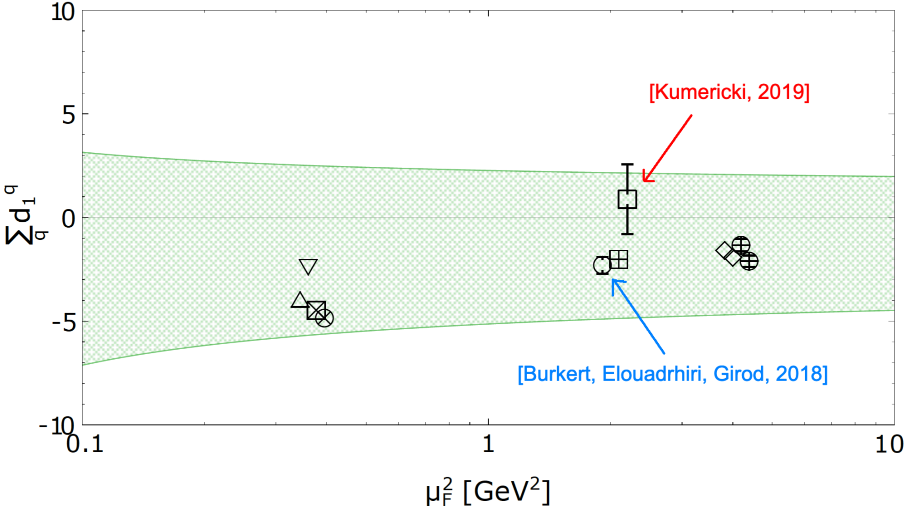
4. Since pressure profiles are obtained by Fourier transform w.r.t. variable $\vec{\Delta}$, and $t = -\vec{\Delta}^2$, assuming a given t-dependence essentially amounts to assuming the general shape of the obtained profile.

Results

Results obtained for $\sum_q d_1^q(t=0,\mu^2) = 5\sum_q C_q(t=0,\mu^2)$



Relaxing assumptions

- If we relax our assumptions, the uncertainty on the extraction of d_1 increases further notably.
- Allowing a simultaneous fit of d_1^q and d_3^q shows a very large correlation of uncertainties $d_1^q \approx -d_3^q$, increasing the uncertainty on d_1 by a factor 20 at $\mu_F^2 = 2$ GeV^2

$$d_1^{uds}(\mu_{\mathrm{F}}^2) \quad -0.5 \pm 1.2 \qquad \longrightarrow \qquad \begin{array}{lc} d_1^{uds}(\mu_{\mathrm{F}}^2) & 11 \pm 25 \\ d_3^{uds}(\mu_{\mathrm{F}}^2) & -11 \pm 26 \end{array}$$

- Indeed, if $d_1^q \approx -d_3^q$ and all other d_n^q are 0 over the range of experimental data, the subtraction constant is numerically very small, but the d_1 term is not.
- The gluon sector has been radiatively generated in this study from a low factorisation scale. When gluons are fitted independently, their uncertainty is 2 orders of magnitude larger than that of quarks.

Conclusion

- With current kinematic spread and precision of experimental data, extracted GFFs are compatible with 0 for flexible CFF parametrizations.
- Relaxing our further modelling assumptions results in a considerable increase of uncertainties. Observables directly sensitive to certains GPDs (especially gluons at LO) and a diversification of experimental processes will be very beneficial.
- An extension of the kinematic domain is required to properly extract d_1 from the other terms in the expansion. This requires future experiments at EIC or EIcC. More precise data from JLab and CERN will also improve the picture.