Baselines for higher-order cumulants of net-charge distributions from local charge conservation

Based on [arXiv:2002.11398]

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1 Cumulant measurements: experimental challenges

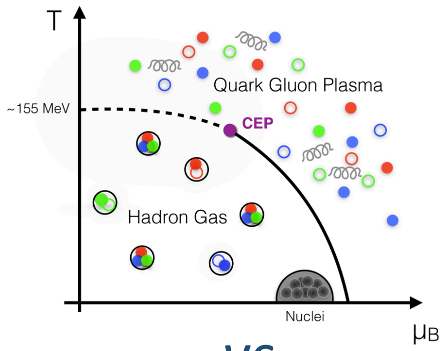
Theory:
lattice calculations are done within Grand Canonical Ensemble.

Reality:
- volume fluctuation
- charge conservation (resonance decays, string fragmentation, jets, …)

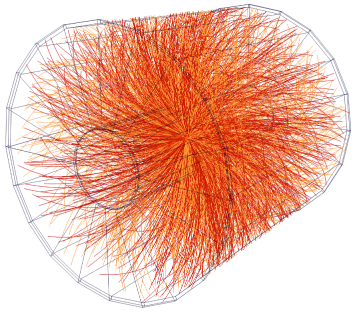
Experiment:
- dependence on centrality selection methods
- finite efficiency of particle registration
- need correction for particle mis-identification
- net-proton as a *proxy* for net-baryon number

} **correction methods are developed**

→ How to interpret experimental values of cumulants and their ratios?



vs



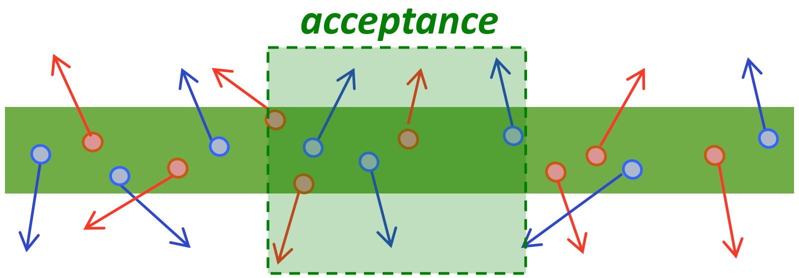
Global *vs* **local** charge conservation and cumulants

To compare net-charge fluctuations with theory (=calculations in GCE),
charge conservation in each event should be taken into account.

Options:

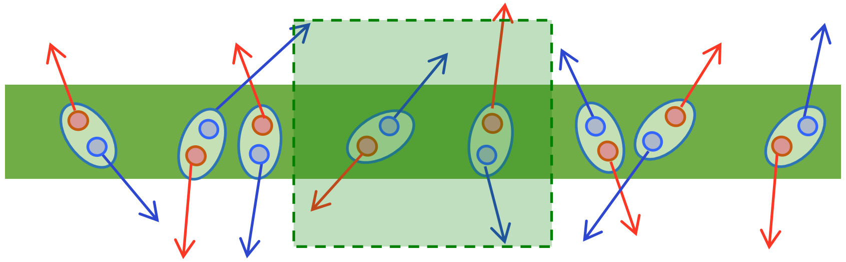
"Global" baryon number conservation:

pairs are produced independently
with y_1^+ and y_2^- within acceptance

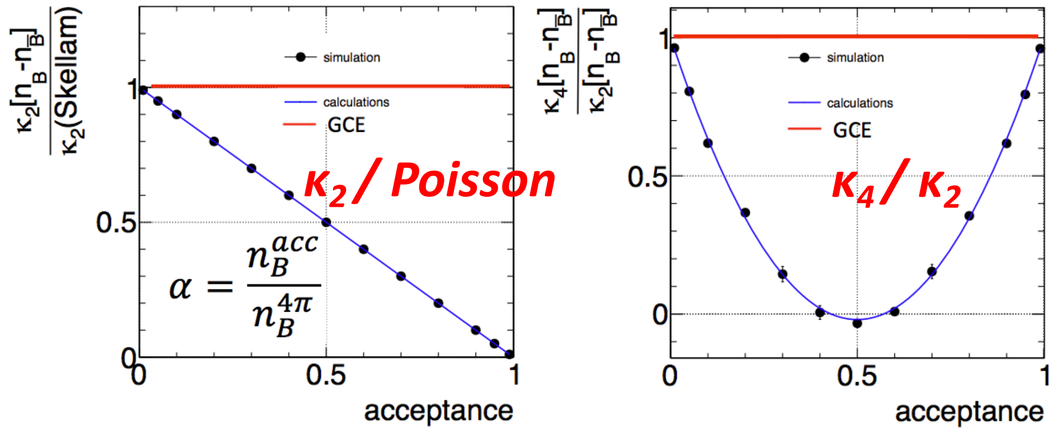


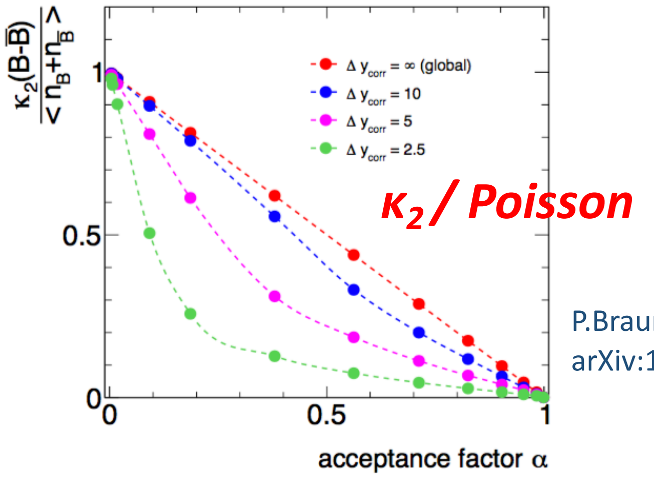
"Local" baryon number conservation:

there is a **finite correlation length** for +,– pairs
(resonance decays, string fragmentation, …)



Example: simple correlation $|y_1^+ - y_2^-| < \Delta y_{corr}/2$:



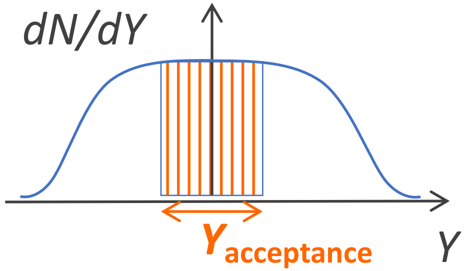


P.Braun-Munzinger et al., arXiv:1907.03032

P. Braun-Munzinger et al., NPA 982 (2019) 307–310; also NPA 1008 (2021) 122141

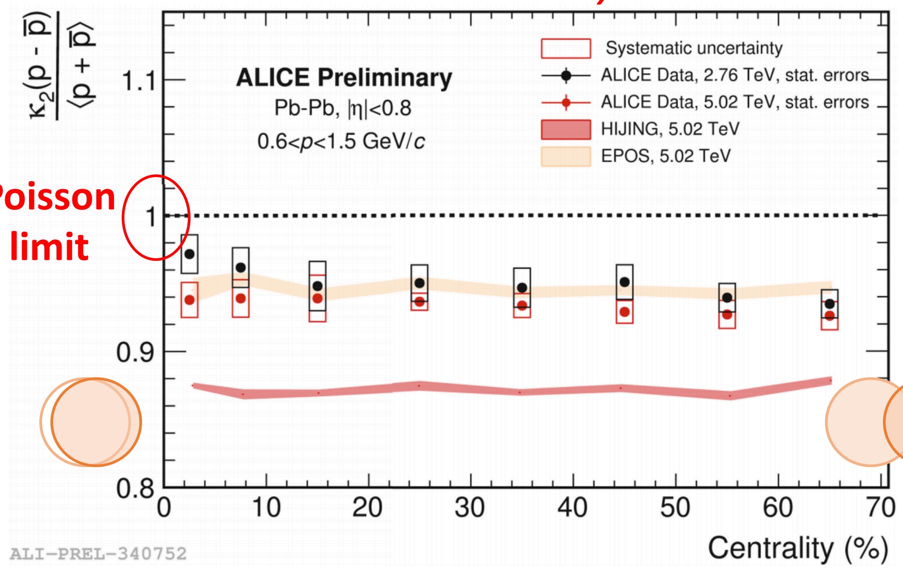
Net-proton fluctuations at LHC (ALICE)

Why measure at LHC: we probe fluctuations at the cross-over region of the Phase Diagram.

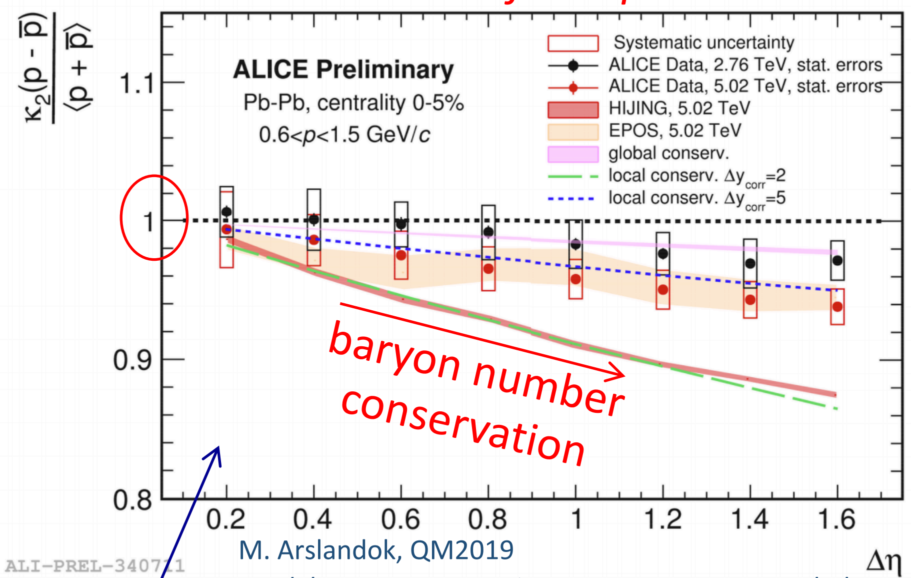


$\kappa_2/\langle p+\bar{p}\rangle$ *vs centrality*

Poisson limit

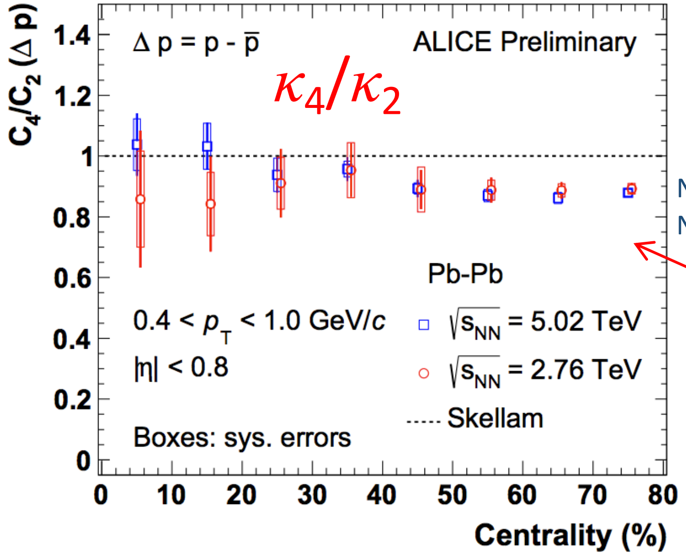


vs width of acceptance



M. Arslandok, QM2019

Models: P. Braun-Munzinger, A.Rustamov, J.Stachel, Nucl.Phys.A 960 (2017) 114; arXiv:1907.03032



N.Behera, QM2018
NPA 982 851-854 (2019)

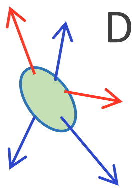
Deviation from 1 (= from Poisson) is treated to be due to **baryon number conservation** (*i.e.* no evidence for dynamical fluctuations so far).

Higher-order ratios: deviation from Skellam. How to interpret?..

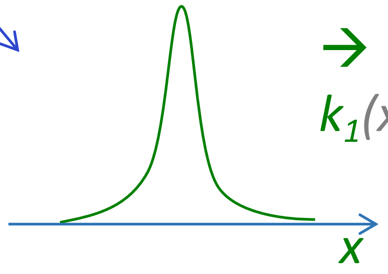
There are **volume fluctuations** and **charge conservation** that “spoil” the signal...

Can we use experimentally available information and find *a better baseline for higher-order cumulant ratios*, rather than Skellam?

2 Cumulants of a system of independent sources

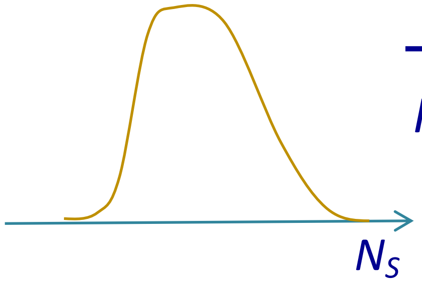


Distribution of <u>some quantity *x*</u> for a single source:

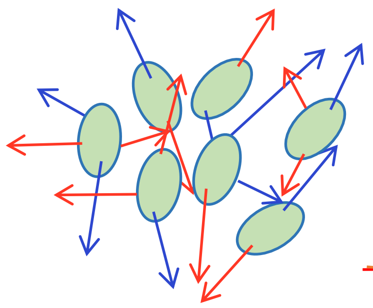


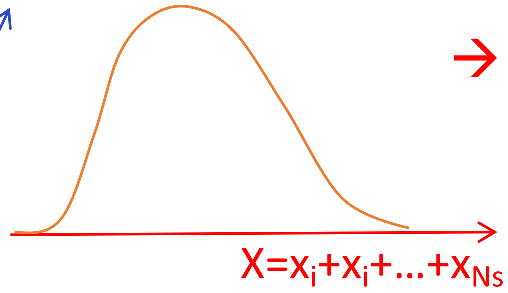
→ *cumulants:*
$k_1(x), k_2, k_3, k_4, \dots$

Distribution of number of sources N_S:



→ *cumulants:*
$K_1(N_S), K_2, K_3, K_4, \dots$





→ *cumulants* ***κ***$\boldsymbol{\kappa_n}$ *of* ***total event-wise distribution*** *of X can be calculated via derivatives of MGF:*

$$M_X(t) = [M_x(t)]^{N_S}$$

Rustamov et al., Nucl.Phys.A 960 (2017) 114

$$\kappa_1 = k_1 K_1,$$
$$\kappa_2 = k_1{}^2 K_2 + k_2 K_1,$$
$$\kappa_3 = k_1{}^3 K_3 + 3k_2 k_1 K_2 + k_3 K_1,$$
$$\kappa_4 = k_1{}^4 K_4 + 6k_2 k_1{}^2 K_3 + k_4 K_1 + \left(3k_2{}^2 + 4k_1 k_3\right) K_2$$

In this paper, *sources* were treated as *"wounded nucleons"*.

2 Cumulants of a system of independent sources

Event-wise cumulants:

Rustamov et al., Nucl.Phys.A 960 (2017) 114

$$\kappa_1 = k_1 K_1,$$
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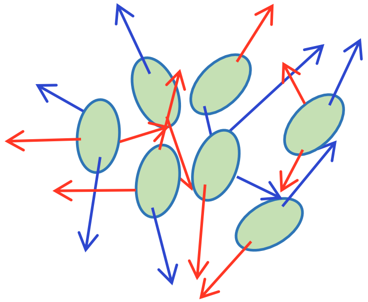
$$\kappa_4 = {k_1}^4 K_4 + 6k_2 {k_1}^2 K_3 + k_4 K_1 + \left(3{k_2}^2 + 4k_1 k_3\right) K_2$$



Cumulants:
k – each source
K – num. of sources
κ – whole system

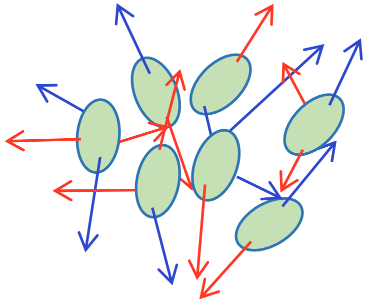
this work,
arXiv:2002.11398

$$\kappa_5 = k_5 K_1 + 5(2k_2 k_3 + k_1 k_4) K_2 + {k_1}^5 K_5 + 10 k_2 {k_1}^3 K_4 + 5\left(3{k_2}^2 k_1 + 2k_1^2 k_3\right) K_3,$$
$$\kappa_6 = {k_1}^6 K_6 + 15k_2 {k_1}^4 K_5 + 20 k_3 {k_1}^3 K_4 + 15 k_4 {k_1}^2 K_3 + 45 {k_2}^2 {k_1}^2 K_4 + 60 k_2 k_3 k_1 K_3 + k_6 K_1 + \left(10{k_3}^2 + 15 k_2 k_4 + 6 k_1 k_5\right) K_2 + 15 {k_2}^3 K_3,$$
$$\kappa_7 = {k_1}^7 K_7 + 21 k_2 {k_1}^5 K_6 + 35 k_3 {k_1}^4 K_5 + 35 k_4 {k_1}^3 K_4 + 105 {k_2}^2 {k_1}^3 K_5 + 21 k_5 {k_1}^2 K_3 + 210 k_2 k_3 {k_1}^2 K_4 + 70 {k_3}^2 k_1 K_3 + 105 k_2 k_4 k_1 K_3 + 105 {k_2}^3 k_1 K_4 + k_7 K_1 + 7(5k_3 k_4 + 3 k_2 k_5 + k_1 k_6) K_2 + 105 {k_2}^2 k_3 K_3,$$
$$\kappa_8 = {k_1}^8 K_8 + 28 k_2 {k_1}^6 K_7 + 56 k_3 {k_1}^5 K_6 + 70 k_4 {k_1}^4 K_5 + 210 {k_2}^2 {k_1}^4 K_6 + 56 k_5 {k_1}^3 K_4 + 560 k_2 k_3 {k_1}^3 K_5 + 28 k_6 {k_1}^2 K_3 + 280 {k_3}^2 {k_1}^2 K_4 + 420 k_2 k_4 {k_1}^2 K_4 + 420 {k_2}^3 {k_1}^2 K_5 + 280 k_3 k_4 k_1 K_3 + 168 k_2 k_5 k_1 K_3 + 840 {k_2}^2 k_3 k_1 K_4 + k_8 K_1 + \left(35 {k_4}^2 + 56 k_3 k_5 + 28 k_2 k_6 + 8 k_1 k_7\right) K_2 + 280 k_2 {k_3}^2 K_3 + 210 {k_2}^2 k_4 K_3 + 105 {k_2}^4 K_4$$

… Fortunately, **at the LHC** we are interested in the case when

$k_1(x) = \langle \Delta n \rangle = \langle n^+ - n^- \rangle = 0$

2 Cumulants of a system of independent sources



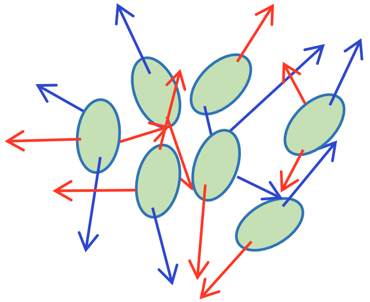
Cumulants:
k – each source
K – num. of sources
κ – whole system

Event-wise cumulants ***κ_1 … κ_8 at the LHC*** *when* **<Δn>=0***:*

$$\kappa_1 = 0,$$
$$\kappa_2 = k_2 K_1,$$
$$\kappa_3 = k_3 K_1,$$
$$\kappa_4 = 3{k_2}^2 K_2 + k_4 K_1,$$

$$\kappa_5 = k_5 K_1 + 10 k_2 k_3 K_2,$$
$$\kappa_6 = 15{k_2}^3 K_3 + k_6 K_1 + \left(10{k_3}^2 + 15 k_2 k_4\right) K_2,$$
$$\kappa_7 = 105 k_3 {k_2}^2 K_3 + k_7 K_1 + 7(5 k_3 k_4 + 3 k_2 k_5) K_2,$$
$$\kappa_8 = 105{k_2}^4 K_4 + 210 k_4 {k_2}^2 K_3 + 280 {k_3}^2 k_2 K_3 + k_8 K_1 + \left(35{k_4}^2 + 56 k_3 k_5 + 28 k_2 k_6\right) K_2.$$

2 Focus on κ_4/κ_2 and κ_6/κ_2 for a superposition of sources



Cumulants:
k – each source
K – num. of sources
κ – whole system

$$\langle \Delta n \rangle = 0$$

$$\begin{cases} \kappa_2(\Delta N) = k_2(\Delta n)\langle N_S \rangle \\ \kappa_4(\Delta N) = k_4(\Delta n)\langle N_S \rangle + 3k_2^2(\Delta n)K_2(N_S) \\ \kappa_6(\Delta N) = k_6\langle N_S \rangle + \left(10k_3^2 + 15k_2k_4\right) K_2(N_S) + 15k_2^3 K_3(N_S) \end{cases}$$

Cumulant ratios:

$$\frac{\kappa_4}{\kappa_2}(\Delta N) = \frac{k_4}{k_2} + 3k_2\frac{K_2(N_S)}{\langle N_S \rangle}$$

$$\frac{\kappa_6}{\kappa_2}(\Delta N) = \frac{k_6}{k_2} + \left(10\frac{k_3^2}{k_2} + 15k_4\right)\frac{K_2(N_S)}{\langle N_S \rangle} + 15k_2^2\frac{K_3(N_S)}{\langle N_S \rangle}$$

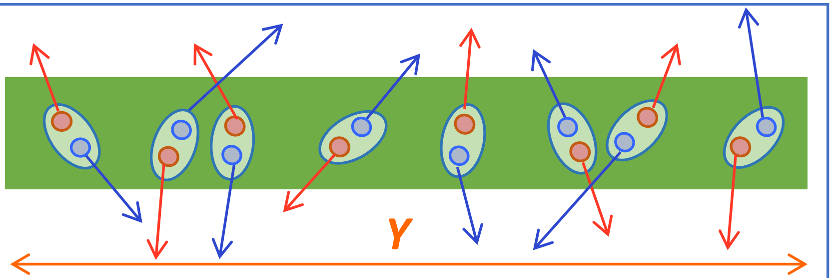
fluctuations in a number of sources

Now we should assign some definite meaning for the sources…

(e.g. in Nucl.Phys.A 960 (2017) 114 *sources* were treated as *"wounded nucleons"*)

Model with particle-antiparticle sources

Assumptions about the sources:



- each source emits **a particle-antiparticle pair**:

$$k_4(\Delta n) = k_2 - 3k_2^2$$

$$k_6(\Delta n) = k_2\left(1 - 15k_2 + 30k_2^2\right)$$

- rapidities of different sources are uncorrelated: $K_r(N_S) \rightarrow K_r(N^-)$ *, r = 1,2,…*

  (a proxy for N_S)

- particles produced from one source do not interact with particles from other sources.

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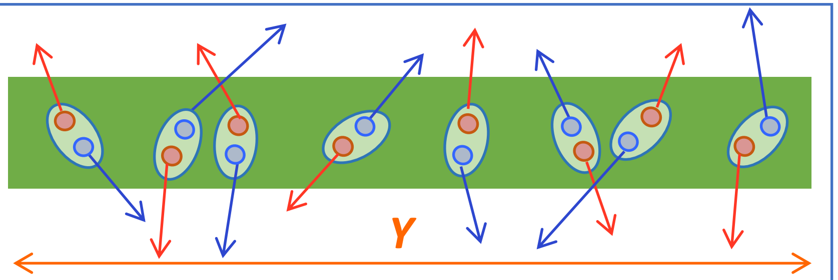


- rapidities of different sources are uncorrelated: $K_r(N_S) \to K_r(N^-)$, *r = 1,2,…* (*a proxy for* N_S)
- particles produced from one source do not interact with particles from other sources.

Scaled factorial moments: $R_2(N_S) = \frac{\langle N_S(N_S-1)\rangle}{\langle N_S\rangle^2} - 1$ $R_3(N_S) = \frac{\langle N_S(N_S-1)(N_S-2)\rangle}{\langle N_S\rangle^3} - 1$

using eqs. from prev. slide

$$\frac{\kappa_4}{\kappa_2}(\Delta N) = 1 + 3\kappa_2(\Delta N) R_2(N^-)$$

$$\frac{\kappa_6}{\kappa_2}(\Delta N) = 1 + 15\kappa_2(\Delta N)\left[(1 - 3\kappa_2(\Delta N)) R_2(N^-) + \kappa_2(\Delta N) R_3(N^-)\right]$$

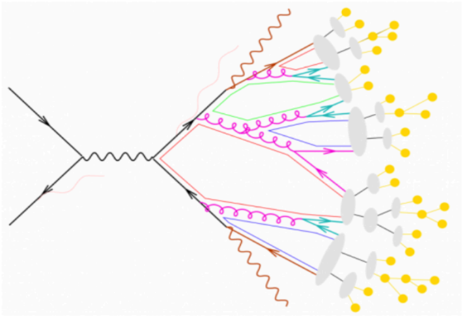
$$k_2(\Delta n) \overset{(*)}{=} \frac{1}{\langle N^-\rangle}\kappa_2(\Delta N)$$

– the baselines: higher-order genuine correlations should lead to deviation from these baselines

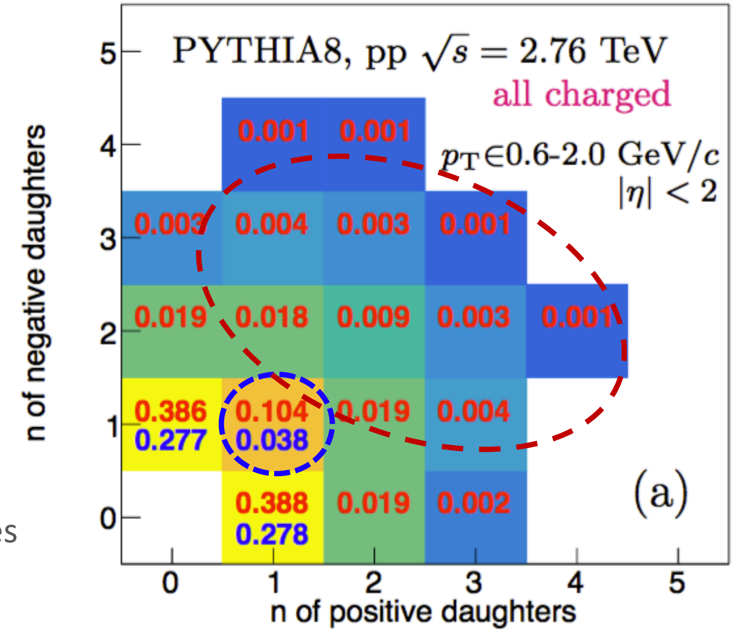
So, what is needed from an experiment?
1) net-particle κ_2 (direct calc. or via the BF integral)
2) moments of the number of sources

(*) there could be a mixture of sources of different nature (e.g. resonances of several types) – in this case it's enough to consider an "averaged" source, which is characterized by the BF, I.A., Acta Phys. Polon. B50, 981 (2019)

3 Is it justified? Look at positive and negative particles from "sources" in PYTHIA



all charged daughters per source (*):

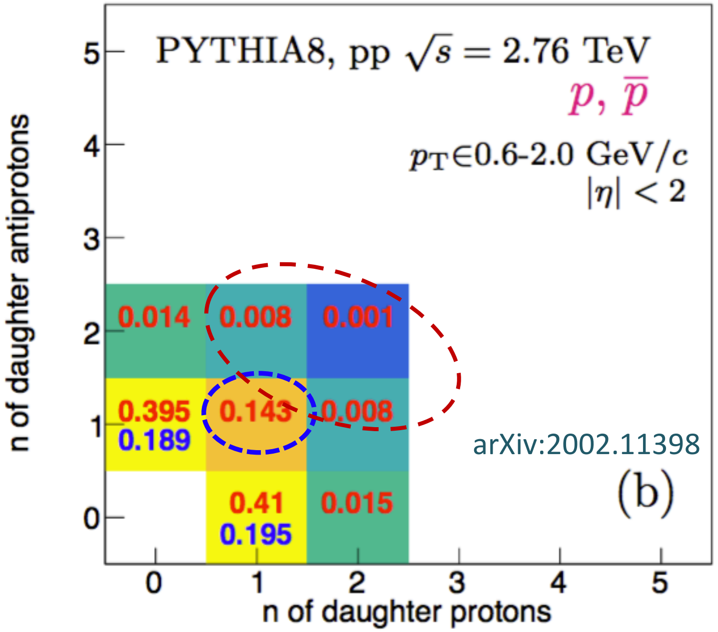


red – any mother
blue – from resonances

77% of sources: only 1 daughter
10% of sources give a (+ –) pair
9% of sources give several unlike-sign pairs
(→ multiparticle correlations from a source)

(*) sources=mothers in PYTHIA: resonances, quarks, gluons

***protons & antiprotons** per source:*



81% of sources: only 1 daughter
14% give a (+ –) pair
only **1.7%** of sources give several unlike-sign pairs

→ For the net-protons at the LHC, the (+ –) **pair production** is the most relevant (*string fragmentation?..*)

3 What about scaled factorial moments of $N_{sources}$?

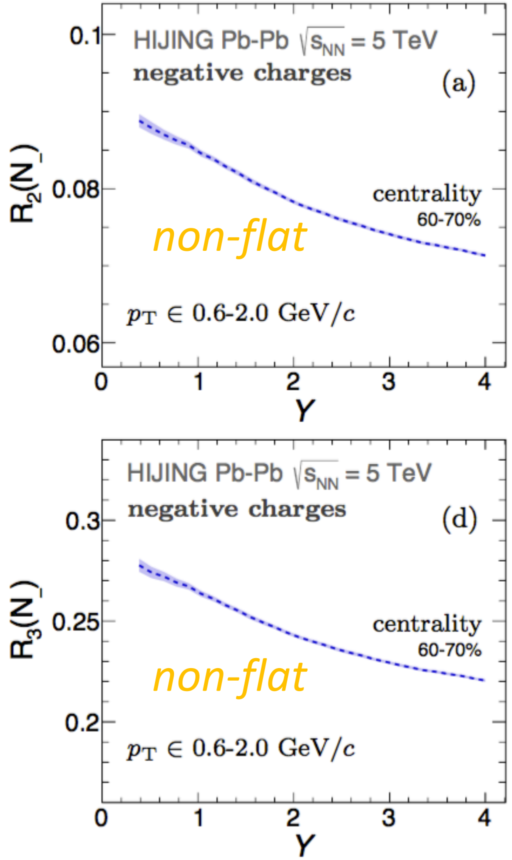
A property of the scaled f.m. R_r: if sources are uncorrelated in rapidity, R_r are independent of an acceptance width *Y*.

→ *Check this in HIJING:*

$$R_2(N_S) = \frac{\langle N_S(N_S-1)\rangle}{\langle N_S\rangle^2} - 1$$

$$R_3(N_S) = \frac{\langle N_S(N_S-1)(N_S-2)\rangle}{\langle N_S\rangle^3} - 1$$

all negative particles:



change with *Y*
(particle rapidities are correlated)

What about scaled factorial moments of $N_{sources}$?

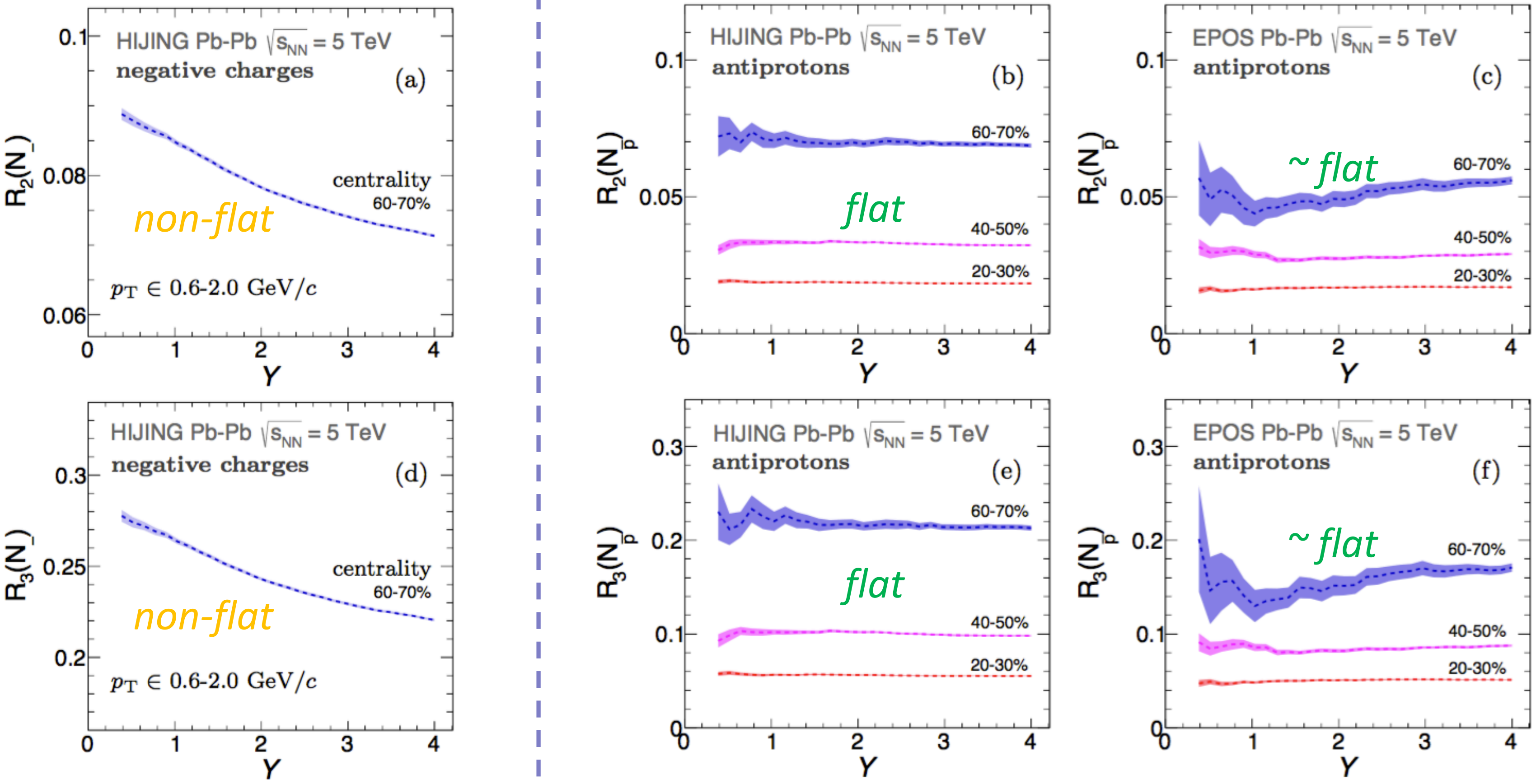
A property of the scaled f.m. R_r: if sources are uncorrelated in rapidity, R_r are independent of an acceptance width Y.

→ *Check this in HIJING:*

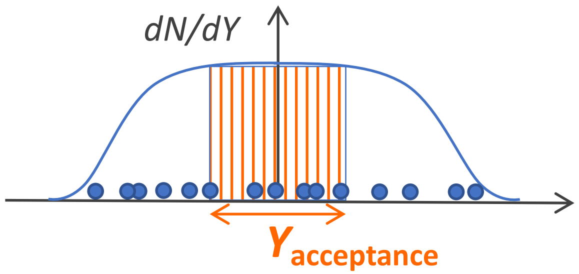
$$R_2(N_S) = \frac{\langle N_S(N_S-1)\rangle}{\langle N_S\rangle^2} - 1$$

$$R_3(N_S) = \frac{\langle N_S(N_S-1)(N_S-2)\rangle}{\langle N_S\rangle^3} - 1$$

all negative particles: *antiprotons:* *... also in EPOS:*



"binomial sampling" from rapidity distribution:



centrality: by multiplicity in $3<|\eta|<5$

change with Y
(particle rapidities are correlated)

R_r – stable with Y
→ Independent production, can be used as proxy for N_S!

(note about plots: there are point-by-point correlations)

3 Check baselines with HIJING and EPOS: acceptance dependence

Consider a very wide 60-90% centrality class:
(to have enough statistics)

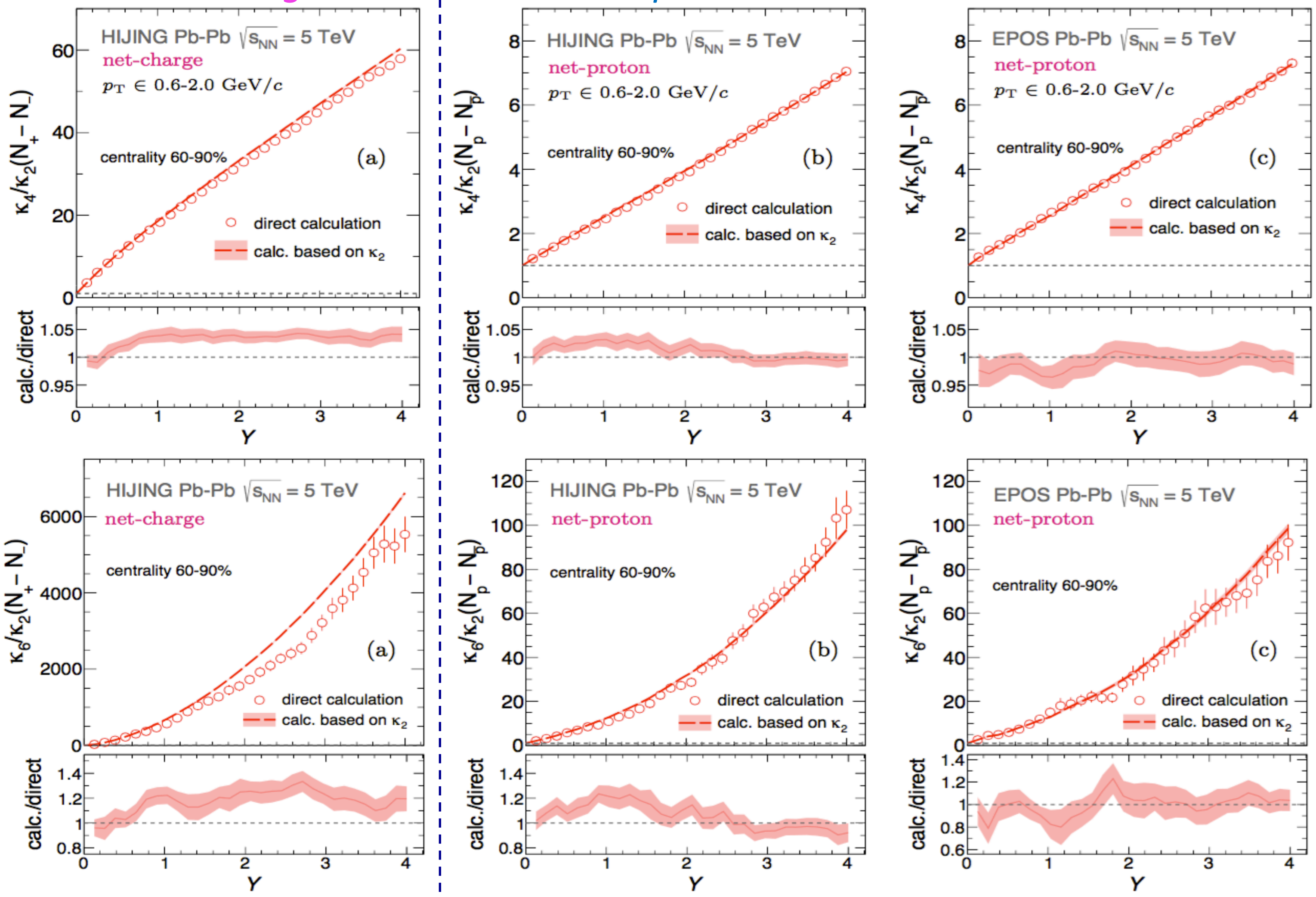
$$\frac{\kappa_4}{\kappa_2}(\Delta N) = 1 + 3\kappa_2(\Delta N) R_2(N^-)$$

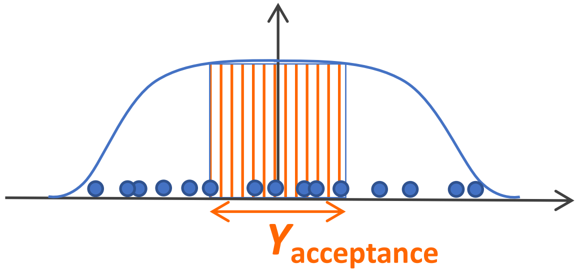
$$\frac{\kappa_6}{\kappa_2}(\Delta N) = 1 + 15\kappa_2(\Delta N)\left[(1 - 3\kappa_2(\Delta N)) R_2(N^-) + \kappa_2(\Delta N) R_3(N^-)\right]$$

net-charge: *net-protons:* *... also in EPOS:*

κ_4/κ_2

κ_6/κ_2





Net-charge: deviations from the model

Good description for the net-protons (!)

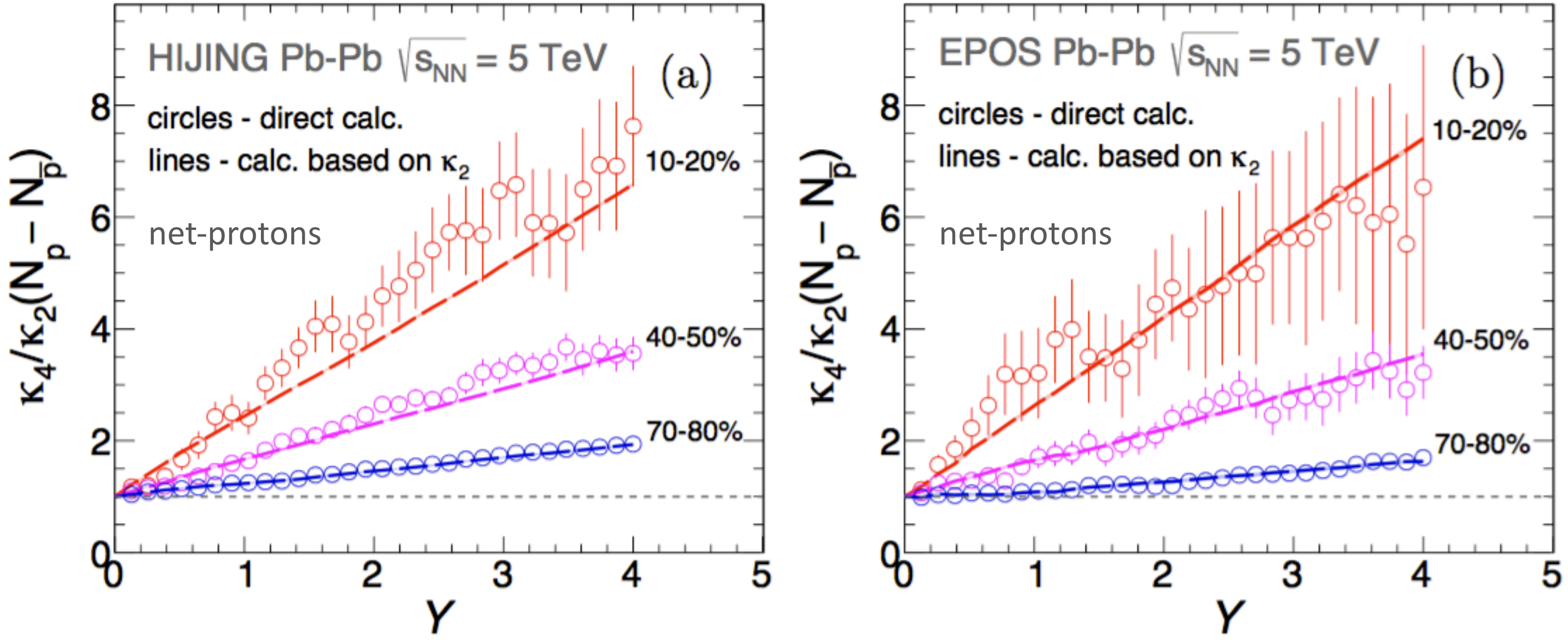
(note about plots: there are point-by-point correlations)

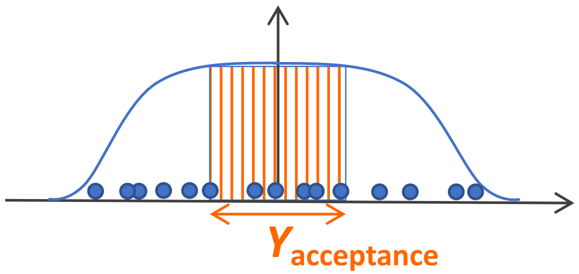
Net-proton κ_4/κ_2 in HIJING and EPOS: acceptance dependence

$$\frac{\kappa_4}{\kappa_2}(\Delta N) = 1 + 3\kappa_2(\Delta N) R_2(N^-)$$

Narrower centrality classes (10% width) for net-proton κ_4/κ_2:

HIJING **EPOS**





(note about plots: there are point-by-point correlations)

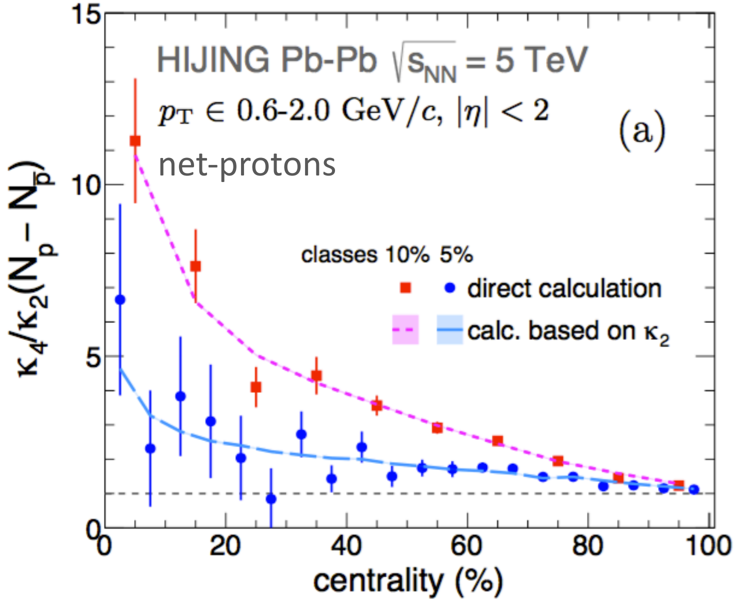
- A reasonable agreement between the model and direct calculations is observed.

Centrality dependence of net-proton κ_4/κ_2

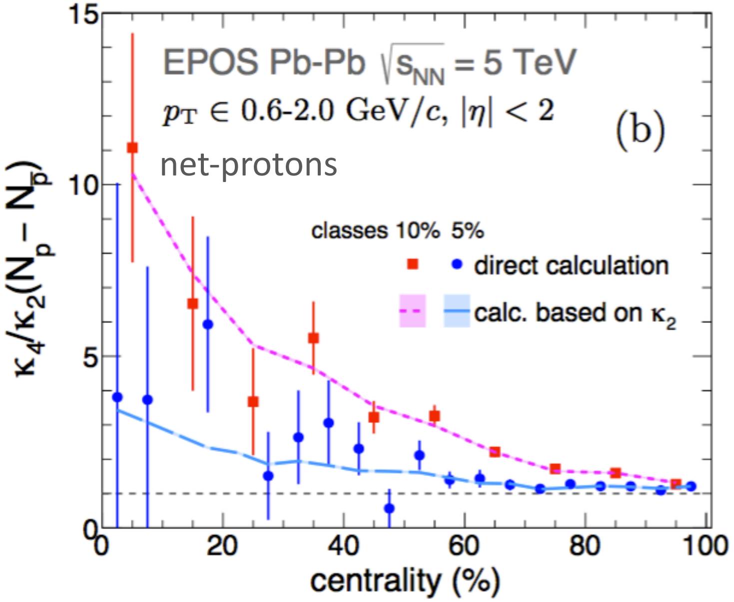
Within p_T **0.6-2.0 GeV/c** and **$|\eta| < 2$**:

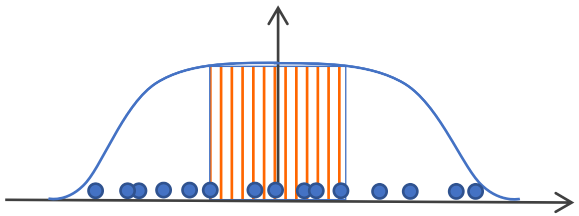
$$\frac{\kappa_4}{\kappa_2}(\Delta N) = 1 + 3\kappa_2(\Delta N)R_2(N^-)$$

HIJING



EPOS





- [again] A reasonable agreement is observed.
- A decrease with class width

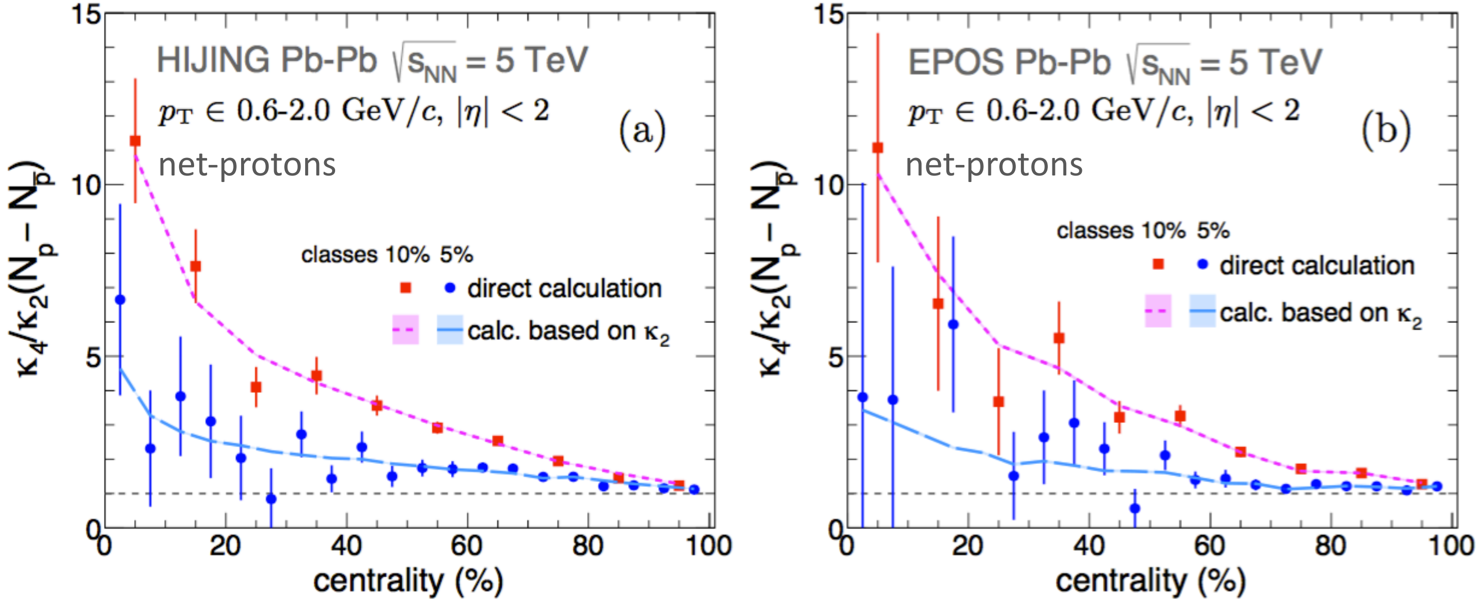
Centrality dependence of net-proton κ_4/κ_2

3

Within p_T **0.6-2.0 GeV/c** and **$|\eta| < 2$**:

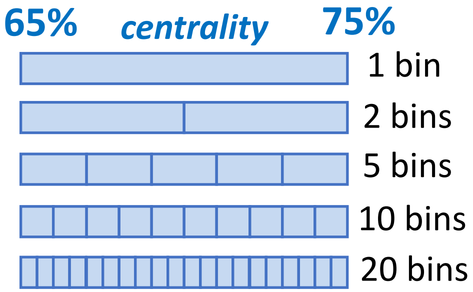
$$\frac{\kappa_4}{\kappa_2}(\Delta N) = 1 + 3\kappa_2(\Delta N) R_2(N^-)$$

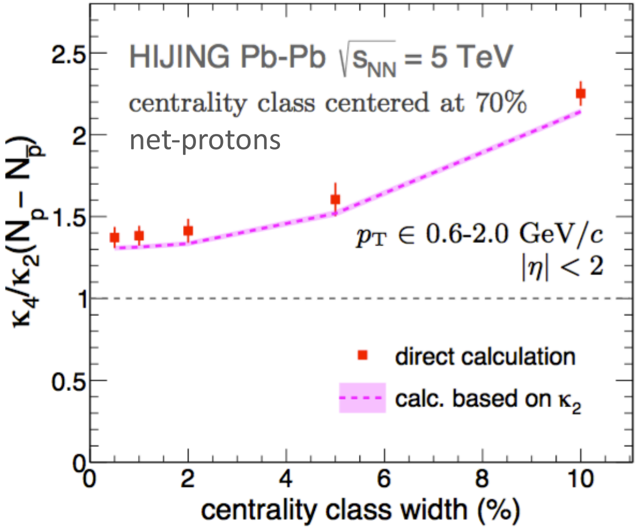
HIJING **EPOS**



Try "centrality bin-width correction":

Luo, et al., J. Phys. G 40, 105104 (2013)





→ In narrow classes, the ratio "converges" to a (non-trivial) value around 1.3-1.4.

- A reasonable agreement between the model and direct calculations is observed in HIJING.

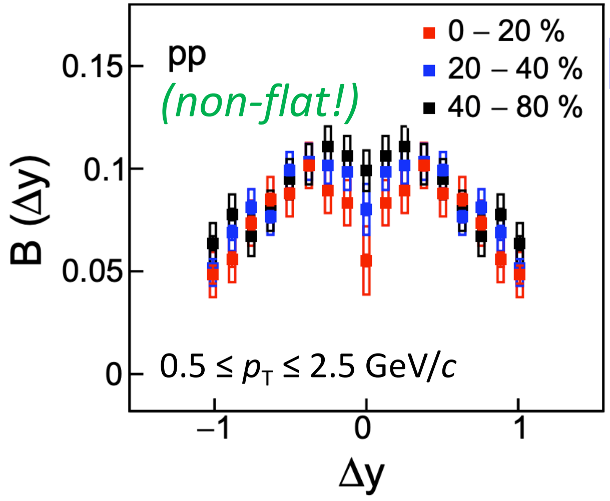
Summing up: baseline for κ_4/κ_2 from R_2 and balance function

$$\frac{\kappa_4}{\kappa_2}(\Delta N) = 1 + 3\kappa_2(\Delta N) R_2(N^-) = 1 + 6\langle N_p\rangle(1 - \tilde{I}_{BF}) R_2(N^-)$$

$$R_2(N_S) = \frac{\langle N_S(N_S-1)\rangle}{\langle N_S\rangle^2} - 1$$

Balance function for protons:

Jinjin Pan, PhD thesis, arXiv:1911.02234

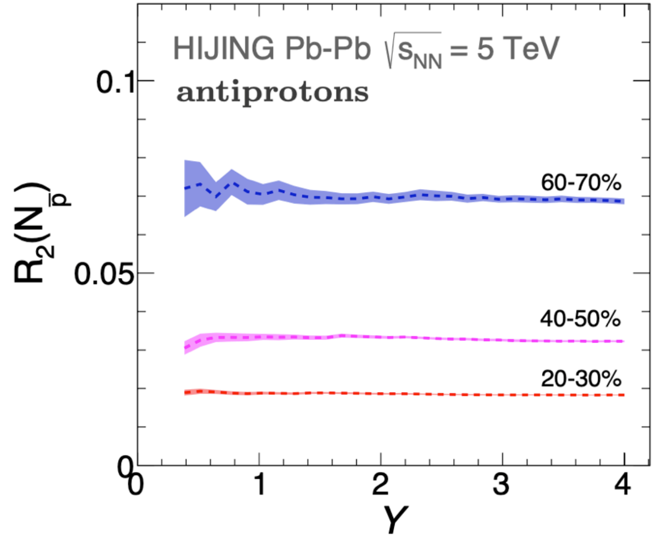


BF integral:

$$\tilde{I}_{BF} = \int_{-Y}^{Y} B(\Delta y)\left(1 - \frac{|\Delta y|}{Y}\right) d\Delta y$$

$$\kappa_2(\Delta N) = 2\langle N_p\rangle(1 - \tilde{I}_{BF})$$ arXiv:2002.11398

criteria to check: is R_r flat?..



Sensitivity to various effects:

	$R_{2,3,\dots}$	balance function
radial flow	no	yes
final-state rescatterings	no	yes
volume fluctuations	yes	no

→ Look for deviations of κ_4/κ_2 from this baseline
→ Similarly for higher orders

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Finally: net-proton *factorial cumulants*

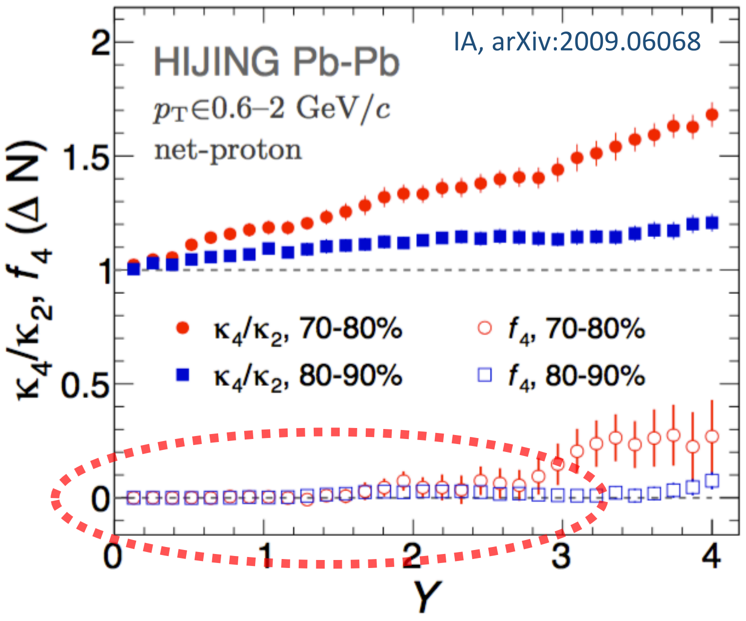
Factorial cumulants of order ***k*** remove correlations of lower orders ***r < k***.

Kitazawa, Luo, PRC 96, 024910 (2017);
Ollitrault et al., Phys. Rev. C 99, 034902 (2019)

$$f_4 = \kappa_4 - 6\big(\langle NQ^2\rangle - \langle N\rangle\langle Q^2\rangle - 2\langle NQ\rangle\langle Q\rangle + 2\langle N\rangle\langle Q\rangle^2\big) + 8\big(\langle Q^2\rangle - \langle Q\rangle^2\big) + 3\big(\langle N^2\rangle - \langle N\rangle^2\big) - 6\langle N\rangle$$

$N = N^+ + N^-$
ΔN is denoted as Q

Calculation in HIJING:



Net-proton f_4 are suppressed in HIJING (no multi-particle correlations).

Notes:

- in distinction from ordinary cumulants, factorial cumulants cannot be directly compared with the LQCD
- however, they should be *non-zero* in the presence of critical behavior (multi-particle correlations)
- it is important to calculate them in real data.

Summary

- A combination of the charge conservation and the V.F. can produce non-trivial values of the higher-order cumulants without any criticality in the system.
- Expressions for cumulants and their ratios were derived under the assumption that particle-antiparticle pairs are produced from sources that are nearly uncorrelated in rapidity. Experimentally, it's enough to measure the $\kappa_2(\Delta N)$ (connected to BF) and lower-order cumulants of number of positive (or negative) particles within the experimental acceptance.
- Model calculations show good agreement with the direct analysis in HIJING and EPOS at LHC energies.
- **Provided expressions would give a more natural baseline for cumulant ratios at LHC energies, rather than the Skellam limit or MC simulations.** Any higher-order genuine correlations should lead to deviation from this baseline.
- Alternative / complementary way is to calculate factorial cumulants in real data.

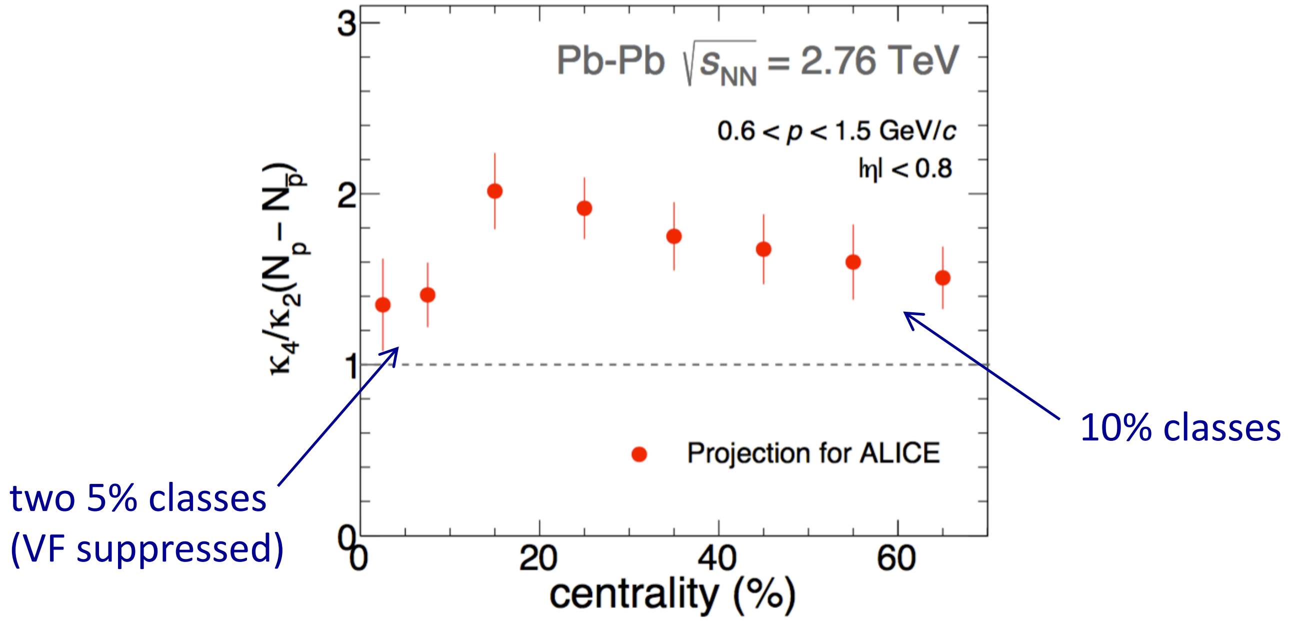
Thank you for your attention!

This work is supported by the Russian Science Foundation, grant 17-72-20045.

κ_4/κ_2: projection for ALICE from the model

$$\frac{\kappa_4}{\kappa_2}(\Delta N) = 1 + 3\kappa_2(\Delta N) R_2(N^-)$$

Using values of κ_2 of net-proton and protons from in *ALICE, PLB 807, 135564 (2020)*:



- These points may be considered as a baseline for direct calculations of the κ_4/κ_2 ratios in data (within the same centrality classes).