A non-critical baseline for fluctuation measurements

Krzysztof Redlich Uni. Wroclaw, Poland

- Modelling equation of state of hadronic phase

  From S-matrix => Hadron Resonance Gas

  => LQCD EoS => Particle Yields in HIC

- Exact charge conservation and hadron production yields in high energy collisions and their scaling: link to ALICE data

- Baryon number conservation and its impact on higher order fluctuation observables in experimental acceptance: comparison to STAR & ALICE cumulants data

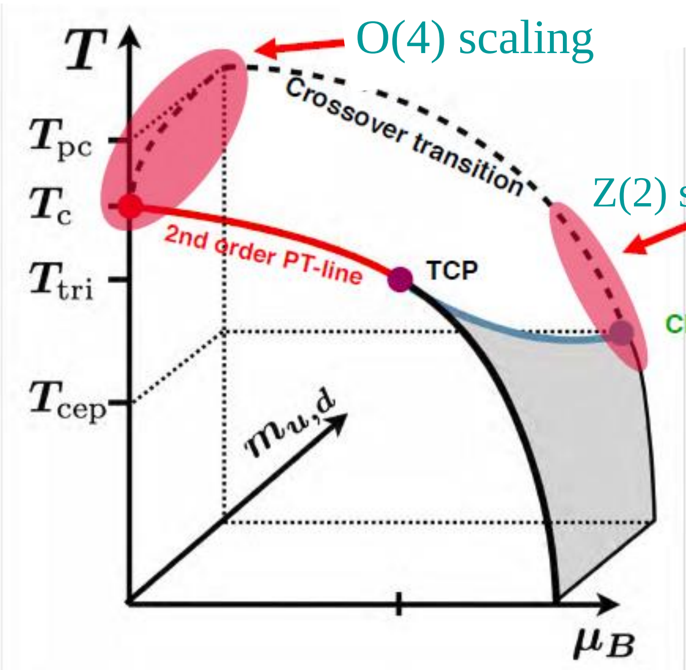
P. Braun-Munzinger, B. Friman, A. Rustamov, J. Stachel & K.R. Nucl. Phys. A 1008 (2021)

J. Cleymans, Pok Man Lo, N. Sharma & K.R. Phys. Rev. C103 (2021)

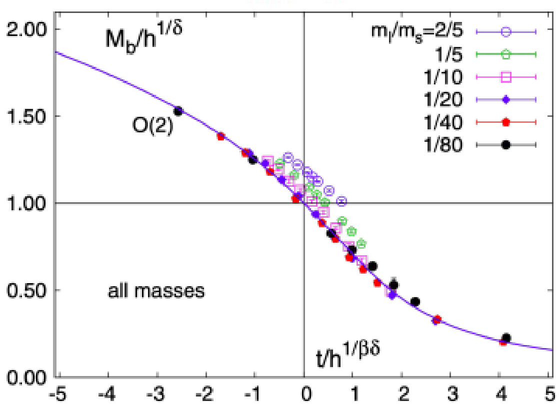
A. Andronic, P. Braun-Munzinger, Pok Man Lo, B. Friman, J. Stachel & K.R Phys. Lett. B 792 (2019)

A. Andronic, P. Braun-Munzinger, J. Stachel & K.R., Nature 561, 302 (2018)

Non-critical baseline for net-charge cumulants



F. Karsch



S. Ejiri *et al.*, PRD 80, 094505 (2009)

- Due to expected O(4) scaling of QCD free energy

$$F = F_R(T,\mu_q,\mu_I) + b^{-1} F_S(b^{(2-\alpha)^{-1}} t(\mu), b^{\beta\delta/\nu} h)$$

- Direct delineation of chiral symmetry restoration via higher order cumulants

$$\chi_S^{(n)}\big|_{\mu=0} \approx h^{(2-\alpha-n/2)/\beta\delta}$$

$$\chi_B^{(n)} = \frac{\partial^n (P/T^4)}{\partial(\mu_B/T)^n} = \chi_R^{(n)} + \chi_S^{(n)}$$

$$\chi_S^{(n)}\big|_{\mu\neq 0} \approx h^{(2-\alpha-n)/\beta\delta}$$

At $\mu = 0$, $\chi_B^{(n\geq 6)}$ are singular at $h \to 0$

At $\mu \neq 0$, $\chi_B^{(n\geq 3)}$ are singular at $h \to 0$

M. A. Stephanov, K. Rajagopal, E. V. Shuryak
Phys.Rev.Lett. 81 (1998) 4816, Phys.Rev.D 60 (1999) 114028

- **CP: 2nd order point - critical fluctuations of net protons.**
  (Hatta, Stephanov PRL 91, 102003 (2003))
- **Crossover: exhibits critical fluctuations in scaling regime**
  (Ejiri, Karsch, Redlich, PLB 633, 275 (2006))

M. Asakawa, K. Yazaki Nucl.Phys.A 504 (1989) 668

Modelling QCD thermodynamic potential in hadronic phase

Pressure of an interacting, a+b ⇔ a+b, hadron gas in equilibrium

$$P(T) \approx P_a^{id} + P_b^{id} + P_{ab}^{\text{int}}$$

The leading order interactions, determined by the two-body scattering phase shift, which is equivalent to the second virial coefficient

$$P^{\text{int}} = \sum_{I,j} \int_{m_{th}}^{\infty} dM \; B_j^I(M) P^{id}(T,M)$$

$$B_j^I(M) = \frac{1}{\pi}\frac{d}{dM}\delta_j^I(M)$$

R. Dashen, S. K. Ma and H. J. Bernstein, Phys. Rev. 187, 345 (1969)
R.Venugopalan, and M. Prakash, Nucl. Phys. A 546 (1992) 718.
W. Weinhold,, and B. Friman, Phys. Lett. B 433, 236 (1998).
Pok Man Lo, Eur. Phys.J. C77 (2017) no.8, 533

Effective weight function — *Scattering phase shift*

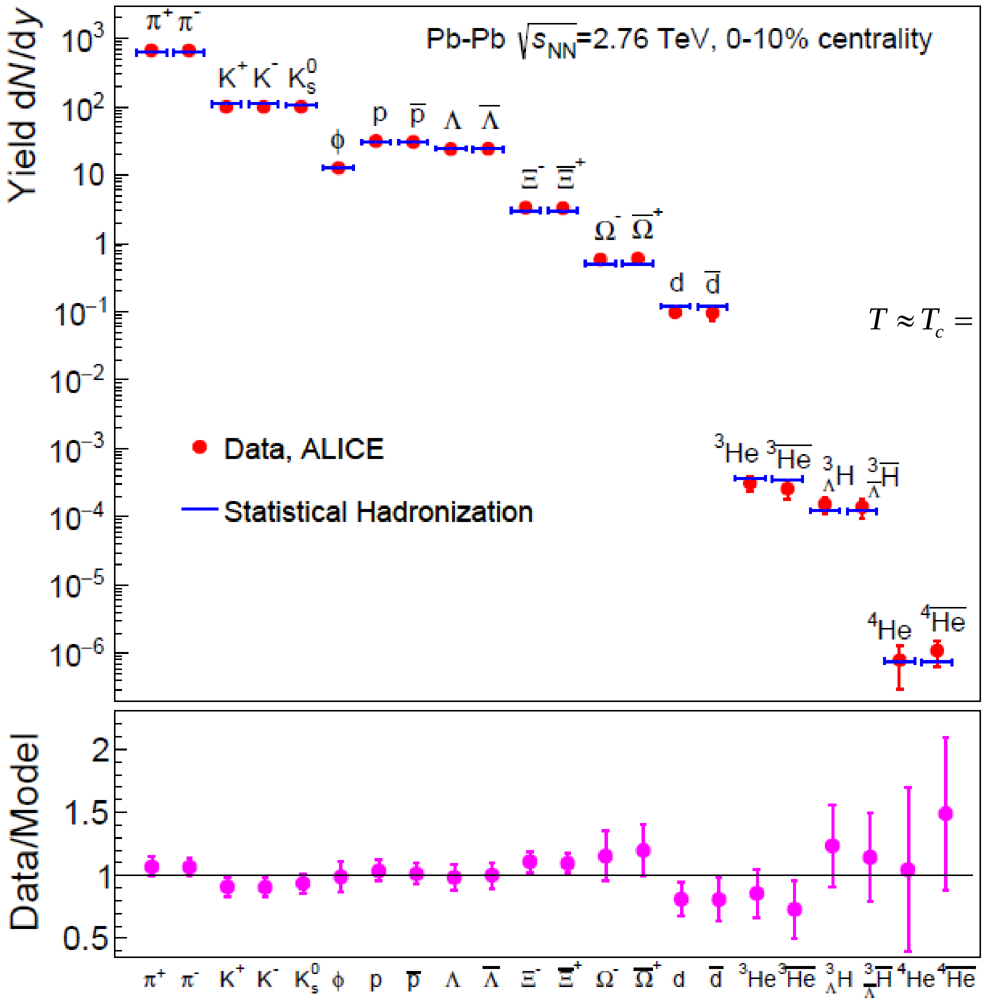
- Interactions driven by narrow resonance of mass M_R

$$B(M) = \delta(M^2 - M_R^2) \Rightarrow P^{\text{int}} = P^{id}(T, M_R) \Rightarrow HRG$$

For finite and small width of resonance, B(M) => Breit-Wigner form

- For non-resonance interactions or for broad resonances $P_{ab}^{\text{int}}(T)$ should be linked to the phase shifts

S-matrix HRG and particle yields in Pb-Pb collisions at the LHC



$$P^{regular}(T,\vec{\mu}) \approx \sum_H P_H^{id} + \sum_R P_R^i$$

The S-matrix HRG model formulated in GC ensemble that incudes empirical information on pion-nucleon interactions provides a very good description of LHC yields data

- Measured yields reproduced at

  $T = 156.6 \pm 1.7 \text{MeV}$

  $\mu = 0.7 \pm 3.8$ MeV

  $\chi^2 / dof = 16.7/19$

  $V_{\Delta y=1} = 4175 \pm 380$ fm^3

- A fireball in central Pb-Pb collisions is matter at the QCD phase boundary

A. Andronic, P. Braun-Munzinger, Pok Man Lo, B. Friman, J. Stachel & K.R Phys. Lett. B 792, 304 (2019)

A. Andronic, P. Braun-Munzinger, J. Stachel & K.R., Nature 561, 302 (2018)

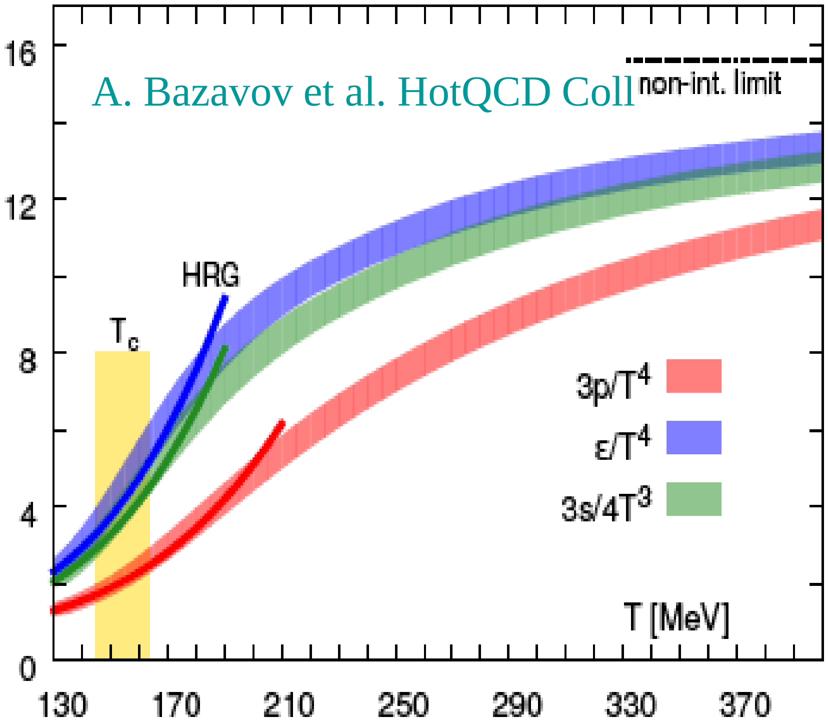
Quark-Hadron duality near the QCD phase boundary

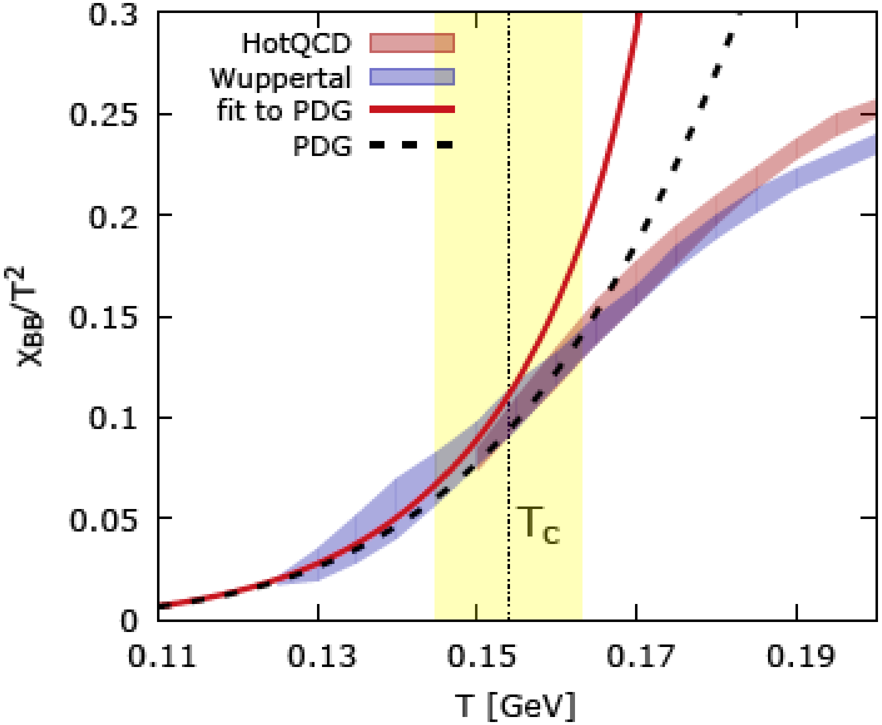
$$P(T,\vec{\mu}) \approx \sum_H P_H^{id} + \sum_R P_R^i$$

see also: J. Goswami, et al., 2011.02812 [hep-lat]
R. Bellwied, et al. 2102.06625 [hep-lat]
S. Borsányi, et. al 2102.06660 [hep-lat]

$$P_R^i = \pm \frac{T g_i}{2\pi^2} \int p^2 dp \int dM \ln(1 \pm e^{-\beta(E_i - \vec{q}_i \vec{\mu}_i)}) F_R^{BW}(M)$$

Pok Man Lo, M. Marczenko et. al.
Eur.Phys.J.A 52 (2016) 8, 235





- SM Hadron Resonace Gas thermodynamic potential provides good approximation of the QCD equation of states in confined phase
- Good description of net- baryon number fluctuations and in further sectors of hadronic quantum number on correlations and fluctuations

Net-baryon fluctuations in the SM-HRG: GC ensemble

- Thermodynamic potential in SM-HRG in baryonic sector

$$\ln Z^{GC} = z_B e^{\mu_B/T} + z_{\bar{B}} e^{-\mu_B/T}$$

$z_B, z_{\bar{B}}$: include interactions in S-matrix

- Fluctuations of net baryon number

Susceptibilities => cumulants

$$\chi_n = \frac{\partial^n (P/T^4)}{\partial(\mu_B/T)^n} \quad => \quad \kappa_n = VT^3 \chi_n$$

- Cumulants in SM-HRG

$$\kappa_{2n} = \langle N_B \rangle + \langle N_{\bar{B}} \rangle \qquad \kappa_{2n+1} = \langle N_B \rangle - \langle N_{\bar{B}} \rangle$$

- Cumulants ratio

$$\frac{\kappa_{2n}}{\kappa_{2k}} = \frac{\kappa_{2n+1}}{\kappa_{2k+1}} = 1 \quad , \frac{\kappa_{2n+1}}{\kappa_{2k}} = \frac{\langle N_B \rangle - \langle N_{\bar{B}} \rangle}{\langle N_B \rangle + \langle N_{\bar{B}} \rangle} = \mathrm{th}(\frac{\mu_B}{T})$$

- Baryon and antibaryon Poisson distributed

$$P(N_B) = \frac{\langle N_B \rangle^{N_B}}{N_B!} e^{-\langle N_B \rangle} \qquad P(N_{\bar{B}}) = \frac{\langle N_{\bar{B}} \rangle^{N_{\bar{B}}}}{N_{\bar{B}}!} e^{-\langle N_{\bar{B}} \rangle}$$

- Probability of net-baryon number $B = N_B - N_{\bar{B}}$ is the Skellam distribution

$$\mathrm{P}(B) = \left(\frac{< N_B >}{< N_{\bar{B}} >} \right)^{B/2} I_B(2\sqrt{\langle N_B \rangle \langle N_{\bar{B}} \rangle})\, \mathrm{e}^{-(<N_B>+<N_{\bar{B}}>)}$$

- Moments of net-baryon number

$$\mu_n = \langle (N_B - N_{\bar{B}})^n \rangle = \langle B^n \rangle = \sum_B B^n P(B)$$

- Cumulants: partial Bell polynomial

$$\kappa_n = \sum_{k=1}^{n} (-1)^{k-1} (k-1)! B_{n,k}(\mu_{1,..,}\mu_{n-k+1})$$

$$\kappa_1 = \mu_1$$

$$\kappa_2 = \mu_2 - (\mu_1)^2$$

Particle yields linked to $dN_{ch}/d\eta$: from pp, pA to AA

- Increase of strangeness production with increasing multiplicity until saturation,

  as well as,

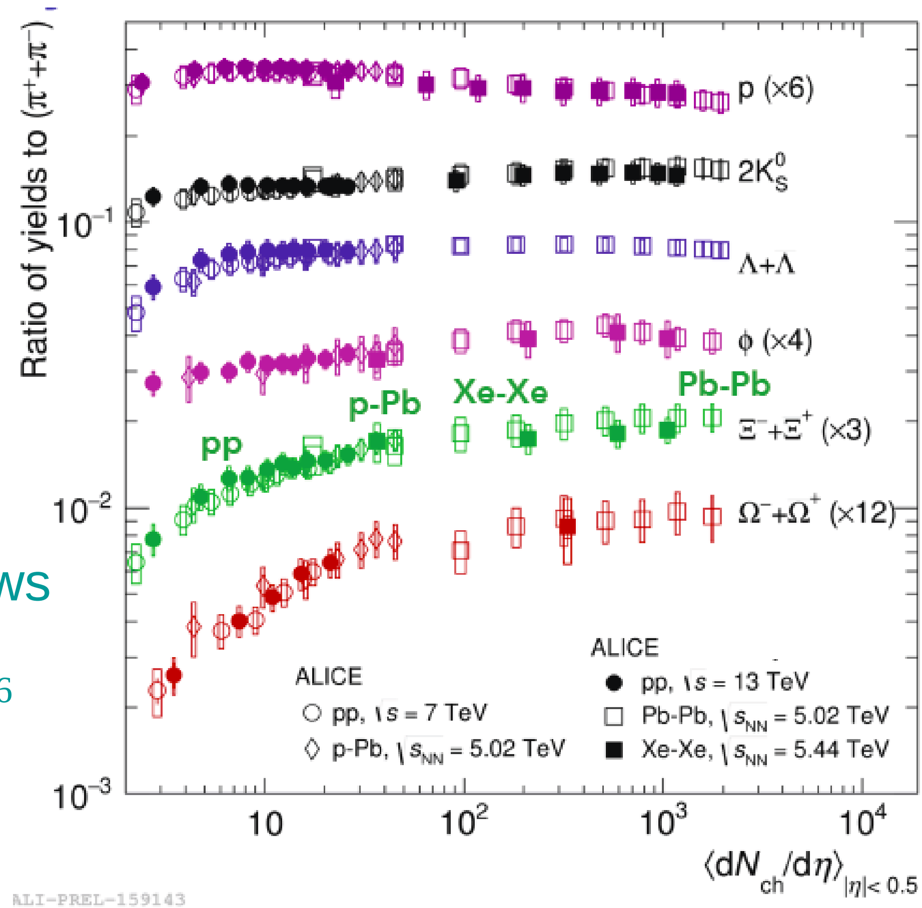
  its dependence on strange quantum number of hadrons can be

  linked to "canonical suppression effect"

  i.e.

  constraints imposed on thermal particle yields due to exact strangeness conservation. This requires canonical ensemble formulation of conservation laws

Smooth evolution of particle yields as function of charged particle multiplicity, and strangeness suppression



J. Cleymans, E. Suhonen & K.R. Z.Phys.C 51 (1991) 137Z.Phys.C 76 (1997) 269

S. Hamieh, A. Tounsi & K.R. Phys. Lett. B486 (2000), Eur.Phys.J. C24 (2002) ,

J. Cleymans, H. Oeschler & K.R. Phys. Rev. C59 (1999) 1663

Strangeness canonical suppression with yields of charged particles

- Strangeness conservation must be exact

$$Z^{GC}(\mu) = Tr[e^{-\beta(H-\mu S)}] \Rightarrow Z_S^C = Tr[e^{-\beta H}\delta_S]$$

$$Z^{GC}(\lambda) = \sum_{S=-\infty}^{\infty} \lambda^S Z_S^C \quad \Rightarrow Z_S^C \simeq \int_{-\pi}^{\pi} d\varphi e^{i\phi S} e^{\ln(Z^{GC}(\mu \to i\varphi))}$$

$$\ln Z^{GC}(\mu,T,V) = \sum_{s=-3}^{3} z_s e^{s\mu/T}$$

Interactions in z_s included: S-matrix

J. Cleymans, Pok Man Lo, N. Sharma & K.R. Phys. Rev. C103 014904 (2021)

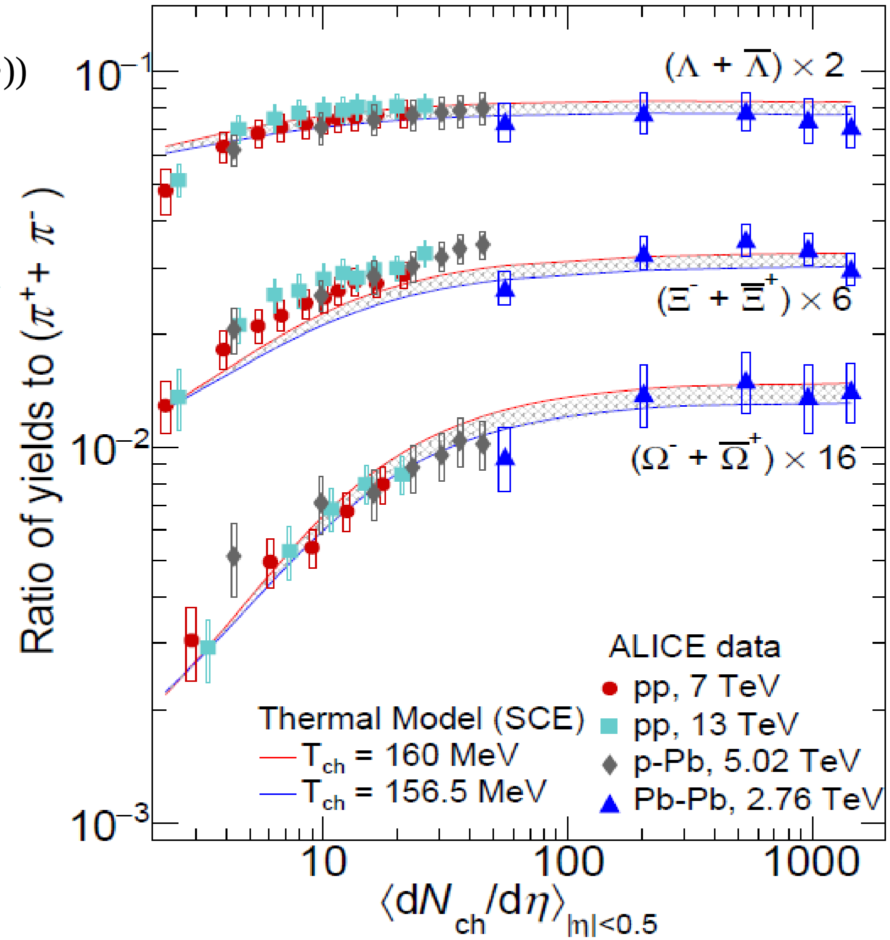
- This implies strangeness suppression effect

$$< N_s >_A^C \approx V_A n^{GC} \bullet \frac{I_s(2V_C n_{s=1}^{th}(T))}{I_0(2V_C n_{s=1}^{th}(T))}$$

where volume parameters $V_{A(C)} \sim dN_{ch}/d\eta$

V_C - full phase-space volume where S is exactly conserved

V_A - effective fireball volume in the acceptance



The suppression factor $I_S(x)/I_0(x) \leq 1$ decreases with decreasing x, and increasing strange s-quantum number of hadron.

Net-baryon fluctuations in canonical ensemble

- Canonical partition function with exact conservation of net-baryon number,

$$\ln Z^{GC} = z_B e^{\mu_B/T} + z_{\bar{B}} e^{-\mu_B/T} \quad => \quad Z_B^C = \sum_{N_B}\sum_{N_{\bar{B}}} \frac{(z_B)^{N_B}}{N_B!}\frac{(z_{\bar{B}})^{N_{\bar{B}}}}{N_{\bar{B}}!}\delta(N_B - N_{\bar{B}} - B)$$

$$z = \sqrt{z_B z_{\bar{B}}}$$

$$Z_B^C = \left(\frac{z_B}{z_{\bar{B}}}\right)^{B/2} I_B(2z), \qquad \langle N_B \rangle = z\frac{I_{B-1}(2z)}{I_B(2z)}, \qquad \langle N_{\bar{B}} \rangle = z\frac{I_{B+1}(2z)}{I_B(2z)} \quad \text{and} \quad \langle N_B \rangle - \langle N_{\bar{B}} \rangle = B$$

In C-ensemble due to exact charge conservation there is no fluctuations of conserved charges in full phase space, however they are there in a subsystem (V. Koch).

□ In experiments a subsystem is defined by the acceptance that corresponds to cuts in momentum space. In STAR and ALICE net-proton fluctuation analysis the acceptance window corresponds to: $\Delta y = 1 \qquad 0.4 < p_t < 2\ GeV$

A. Bzdak, V. Koch, V. Skokov, PRC87 (2013) 014901

P. Braun-Munzinger, A. Rustamov, J. Stachel, NPA982, (2019), 307

P. Braun-Munzinger, B. Friman, A. Rustamov, J. Stachel & K.R, Nucl. Phys. A (2021), 122141

M. Kitazawa and M. Asakawa, Phys. Rev. C 86, 024904 (2012)

V. Begun, M. Gazdzicki, M. I. Gorenstein and O. Zozulya, Phys. Rev. C 70, 034901 (2004)

J. Cleymans, K. Redlich and L. Turko, Phys. Rev. C 71, 047902 (2005)

V. Vovchenko, R. V. Poberezhnyuk and V. Koch, JHEP 10, 089 (2020)

B_A-fluctuations in acceptance window and exact B-conservation

P. Braun-Munzinger, et al. Nucl. Phys. A (2021)

- Acceptance window in momentum space: Splitting $z_B = z_A + z_R$ and $z_{\bar{B}} = z_{\bar{A}} + z_{\bar{R}}$, then

$$Z_B^C = \sum_{N_B} \sum_{N_{\bar{B}}} \frac{(z_A + z_R)^{N_B}}{N_B!} \frac{(z_{\bar{A}} + z_{\bar{R}})^{N_{\bar{B}}}}{N_{\bar{B}}!} \delta(N_B - N_{\bar{B}} - B) = \left(\frac{z_B}{z_{\bar{B}}}\right)^{B/2} I_B(2z),$$

- Applying binomial theorem to $(z_A + z_R)^{N_B}$ and $(z_{\bar{A}} + z_{\bar{R}})^{N_{\bar{B}}}$, introduce $\alpha_B = \frac{z_A}{z_B}, \quad \alpha_{\bar{B}} = \frac{z_{\bar{A}}}{z_{\bar{B}}}$

  thus also $z_{\bar{R}} = z_{\bar{B}}(1-\alpha_{\bar{B}}),\ z_R = z_B(1-\alpha_B)$

$$Z_B^C = \sum_{B_A} \left(\frac{z_B}{z_{\bar{B}}}\right)^{\frac{B}{2}} \left(\frac{\alpha_B}{\alpha_{\bar{B}}}\right)^{B_A/2} \left(\frac{1-\alpha_B}{1-\alpha_{\bar{B}}}\right)^{(B-B_A)/2} I_{B_A}(2z\sqrt{\alpha_B \alpha_{\bar{B}}}) I_{B-B_A}(2z\sqrt{(1-\alpha_B)(1-\alpha_{\bar{B}})})$$

up to normalization probability to find in the acceptance => $P_A(B_A)$

as by A. Bzdak et. al.

- The phase-space fractions α_B and $\alpha_{\bar{B}}$ are link to the fraction of (anti)baryons in the acceptance

$$\langle N_B \rangle_A = \alpha_B z \frac{I_{B-1}(2z)}{I_B(2z)} = \alpha_B \langle N_B \rangle \quad => \quad \alpha_B = \frac{\langle N_B \rangle_A}{\langle N_B \rangle}$$

$$\langle N_{\bar{B}} \rangle_A = \alpha_{\bar{B}} z \frac{I_{B+1}(2z)}{I_B(2z)} = \alpha_{\bar{B}} \langle N_{\bar{B}} \rangle \quad => \quad \alpha_{\bar{B}} = \frac{\langle N_{\bar{B}} \rangle_A}{\langle N_{\bar{B}} \rangle}$$

for net-proton probability in acceptance

$$\alpha_B => \alpha_p = \frac{\langle N_p \rangle_A}{\langle N_B \rangle}$$

$$\alpha_{\bar{B}} => \alpha_{\bar{p}} \frac{\langle N_{\bar{p}} \rangle_A}{\langle N_{\bar{B}} \rangle}$$

Cumulants in the acceptance window

- With the probability distribution for net-(proton)baryon number $P_A(B_A)$ analytic expressions for cumulants κ_n is obtained for any n
- Furthermore, Python software package is provided for generating κ_n as analytic formula and numerical values
- Example of an explicit form:

$$\kappa_1 = \langle N_B \rangle_A - \langle N_{\bar{B}} \rangle_A = \alpha_B \langle N_B \rangle - \alpha_{\bar{B}} \langle N_{\bar{B}} \rangle$$

$$\kappa_2 = \langle N_B \rangle \alpha_B (1-\alpha_B) + \langle N_{\bar{B}} \rangle \alpha_{\bar{B}} (1-\alpha_{\bar{B}}) + (z^2 - \langle N_B \rangle \langle N_{\bar{B}} \rangle)(\alpha_B - \alpha_{\bar{B}})^2$$

$\alpha_B, \alpha_{\bar{B}} \to 1 \quad \kappa_2 \to 0$

$\alpha_B, \alpha_{\bar{B}} << 1$

$$\kappa_2 = \langle N_B \rangle_A + \langle N_{\bar{B}} \rangle_A$$

- To quantify κ_n for net-proton fluctuations one needs to specify:

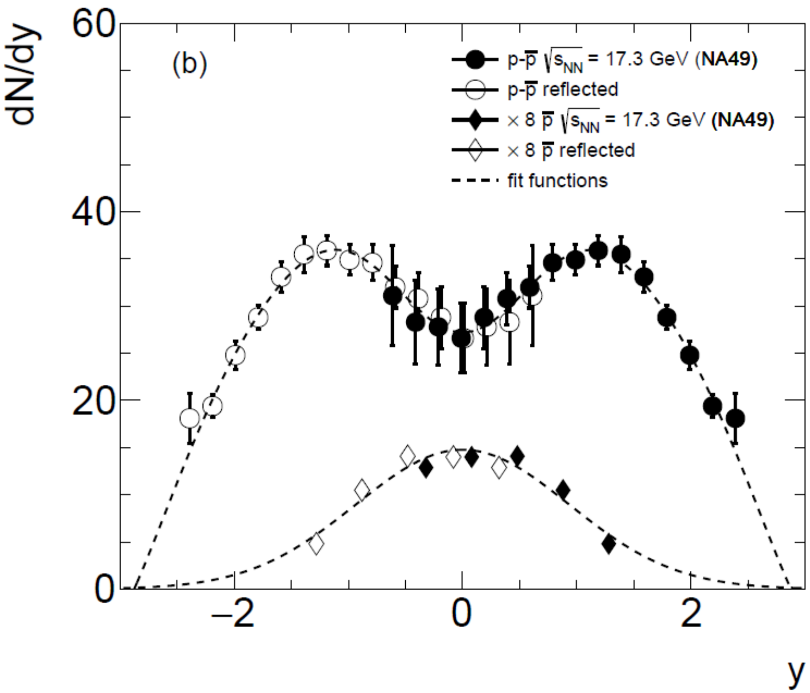
$\alpha_p,\ \alpha_{\bar{p}},\ \langle N_B \rangle,\ \langle N_{\bar{B}} \rangle$

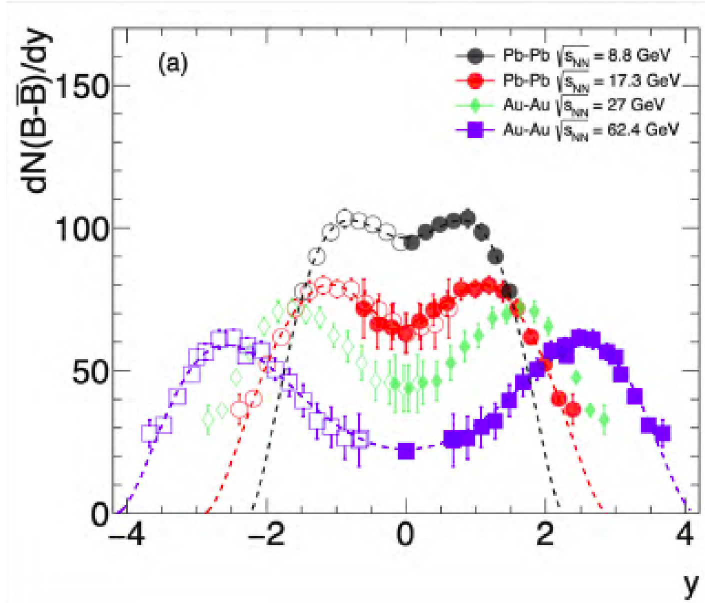
- The value of $z = \sqrt{z_B z_{\bar{B}}}$ is obtained as the solution of:
- $\langle N_B \rangle = z \frac{I_{B-1}(2z)}{I_B(2z)}$, or alternatively can be calculated from SM-HRG model

Empirical inputs: $\langle N_B \rangle, \langle N_{\bar{B}} \rangle, \langle N_p \rangle_A, \langle N_{\bar{p}} \rangle_A \Rightarrow \alpha_p, \alpha_{\bar{p}}, z$

Use data from NA49 and BRAHMS

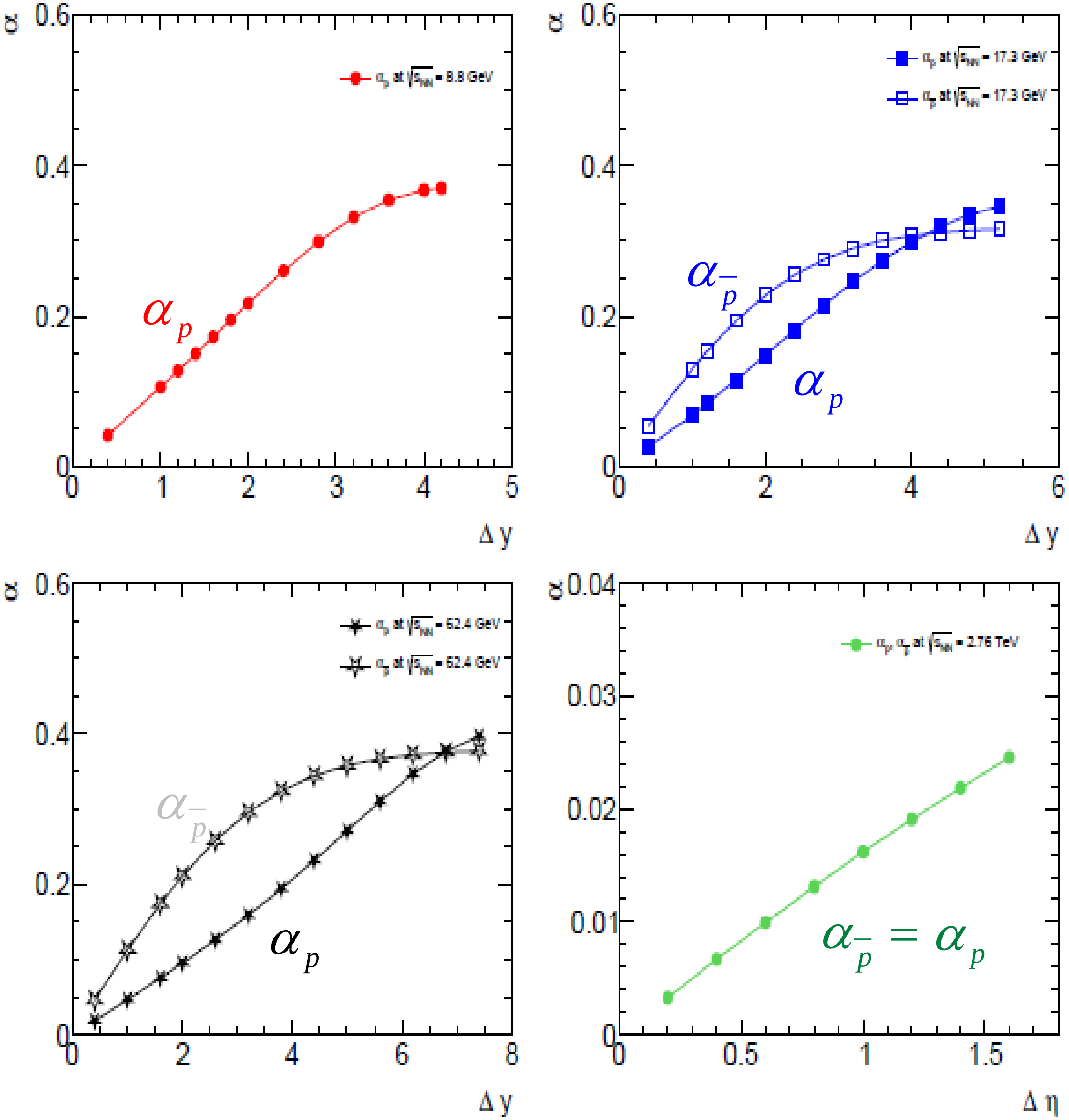
Appelshäuser, PRL 82, 2471 (1999)
Anticic, PRC 83, 014901 (2011)
Arsene, PLB 677, 267 (2009)





$\sqrt{s_{NN}}$ [GeV]	$\langle N_B \rangle$	$\langle N_{\bar{B}} \rangle$	$\langle N_p \rangle$	$\langle N_{\bar{p}} \rangle$	z
8.8	353	2	130	0.51	26.608
17.3	368	16	154.6	4.36	76.833
27	373 (377)	30 (34)	–	–	105.914 (113.354)
62.4	384	70	181.5	33.23	164.132

Note, very different rapidity distribution of protons and antiprotons, thus $\alpha_p \neq \alpha_{\bar{p}}$



Acceptance probability:

The acceptance probability for protons and antiprotons

$$\alpha_p = \frac{< N_p >_A}{< N_B >} \qquad \alpha_{\bar{p}} = \frac{< N_{\bar{p}} >_A}{< N_{\bar{B}} >}$$

Extracted from data at different energies

The STAR acceptance window for protons and antiprotons:

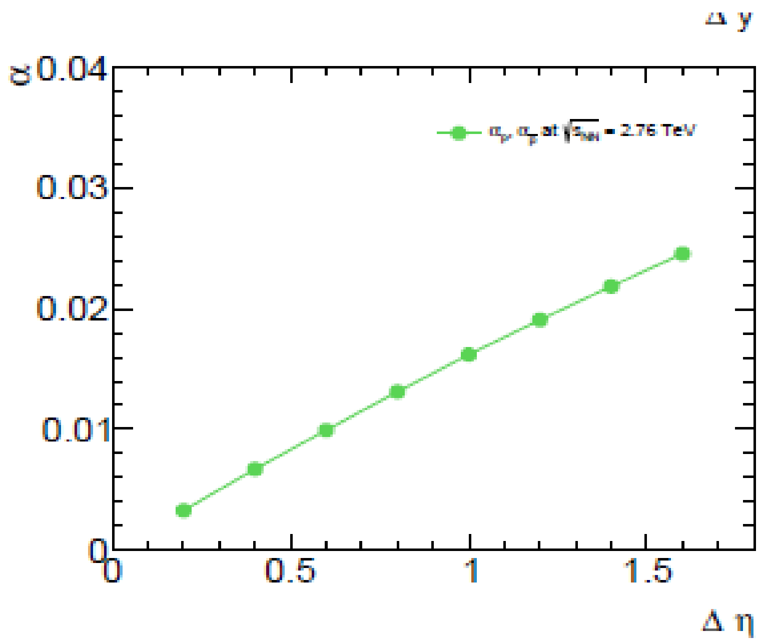
$\Delta y = 1$ centered at y=0

$$0.4 < p_t < 2 \quad GeV$$

Global Baryon number conservation

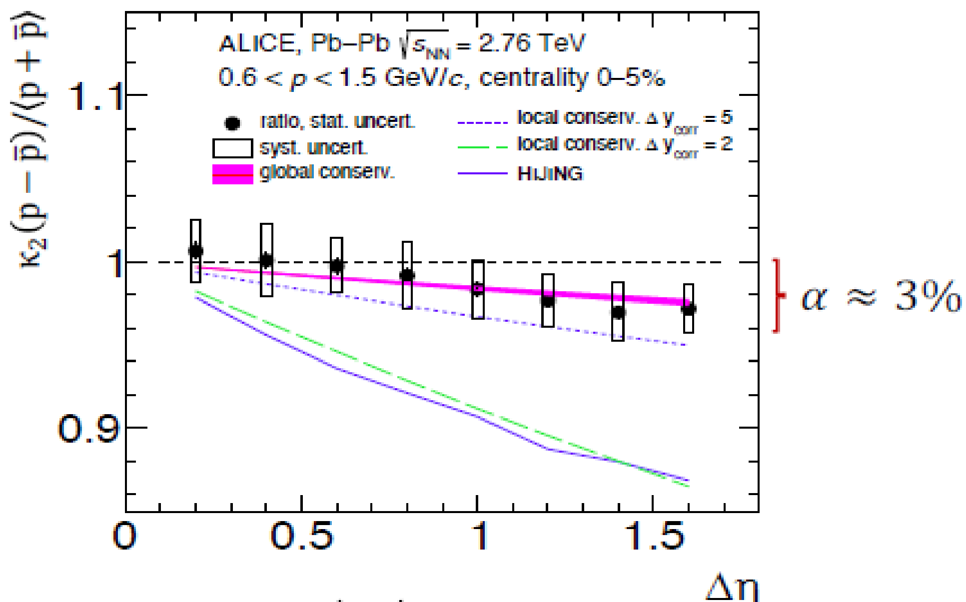
P. Braun-Munzinger, A. Rustamov, J. Stachel
Nucl Phys. A960 (2017) 114

P. Braun-Munzinger, A. Rustamov, J. Stachel, arXiv:1907.03032



At the LHC $\alpha_p = \alpha_{\bar{p}}$

$$\kappa_2 = \left(\left\langle N_p \right\rangle_A + \left\langle N_{\bar{p}} \right\rangle_A \right) * (1 - \alpha_B)$$



$$\alpha = \langle n_p \rangle_{\Delta\eta} / \langle n_B \rangle_{4\pi}$$

Data best described by global baryon conservation in full rapidity range

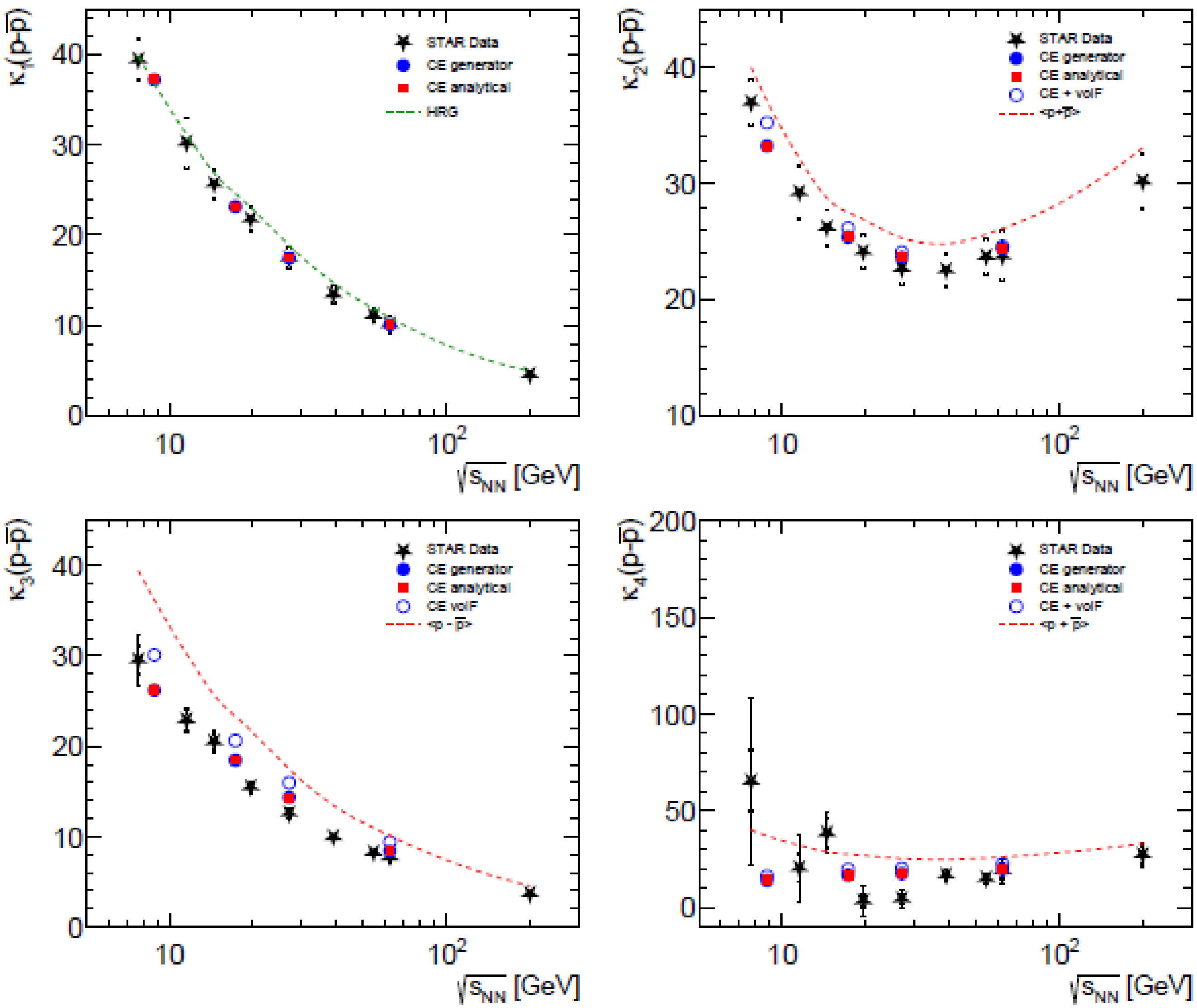
Restricting net-baryon conservation to narrower rapidity window results in strong suppression of κ_2 not consistent with data

ALICE: Phys. Lett. B 807 (2020) 135564

Comparison with STAR data

Stars: STAR data

Adam *et al.* arXiv 2001.02852v2



- Broken lines calculated from Skellam distribution, and are consistent with S-matrix HRG results calculated along the chemical freezeout line extracted from HIC HIC data
- Open circle include volume fluctuations

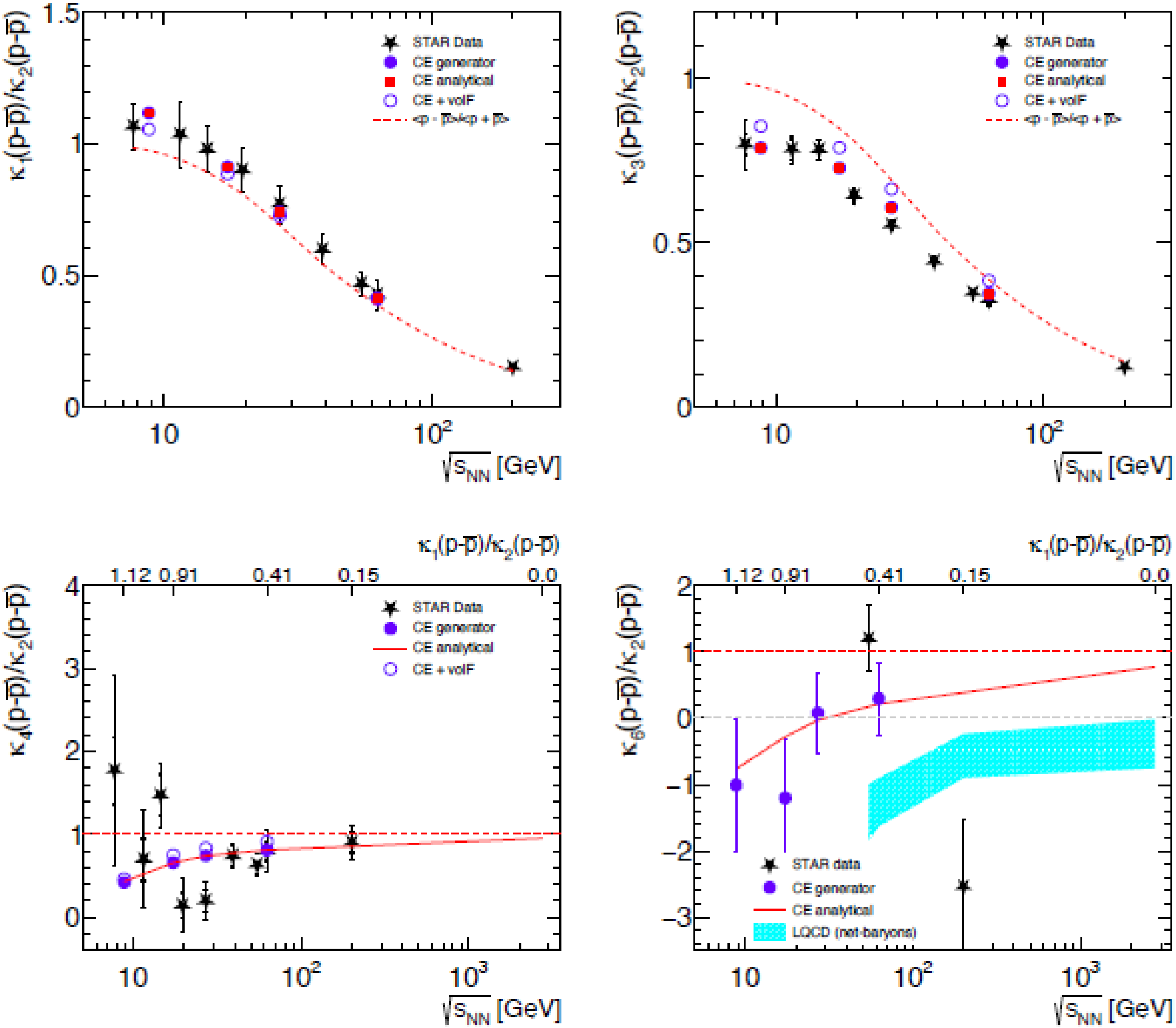
P. Braun-Munzinger, A. Rustamov and J. Stachel, Nucl. Phys. A960, 114 (2017).

V. Skokov, B. Friman and K. Redlich, Phys. Rev. C88, 034911 (2013)

Ratios of cumulants: STAR data versus model

Stars: STAR data

Adam *et al.* arXiv 2001.02852v2



- Broken lines calculated from

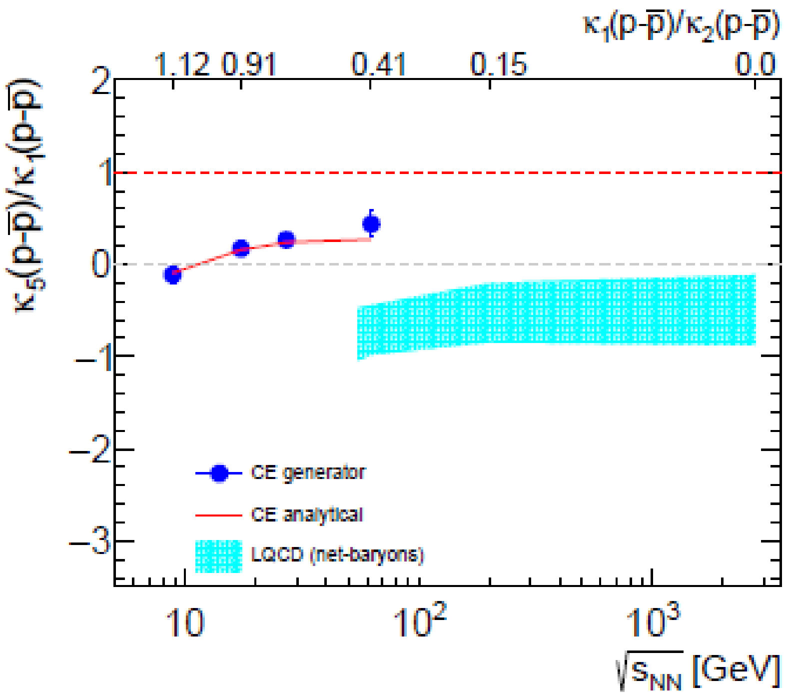
  Skellam distribution

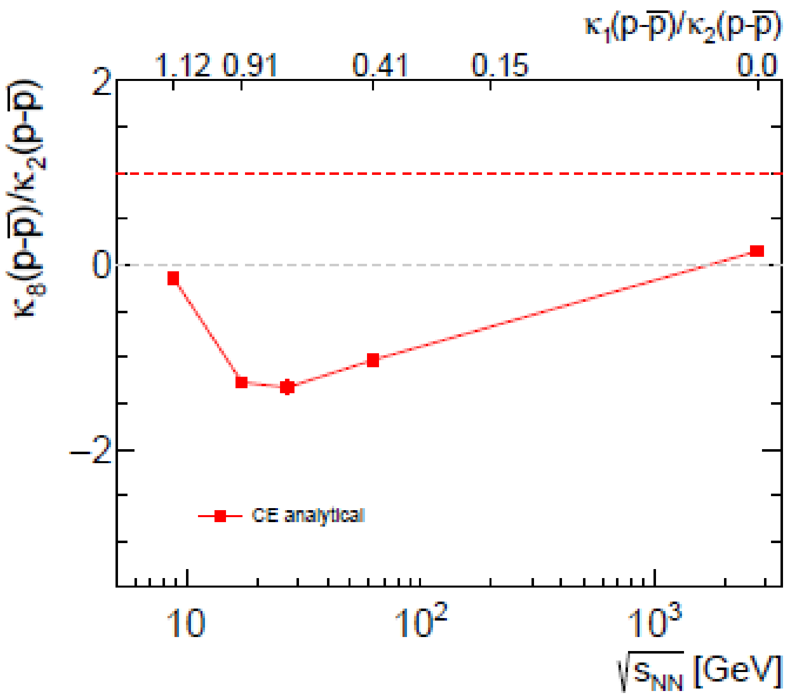
$$\frac{\kappa_1}{\kappa_2} = \frac{\kappa_3}{\kappa_2} = \frac{< p - \bar{p} >}{< p + \bar{p} >}$$

- Open circle include MC volume fluctuations

  P. Braun-Munzinger, A. Rustamov and J. Stachel, Nucl. Phys. A960, 114 (2017).
  V. Skokov, B. Friman and K. Redlich, Phys. Rev. C88, 034911 (2013)

- Cumulants up to n<4 order follow the SATR data
- Kurtosis data exhibit interesting deviations, however *not necessarily* of statistical significant

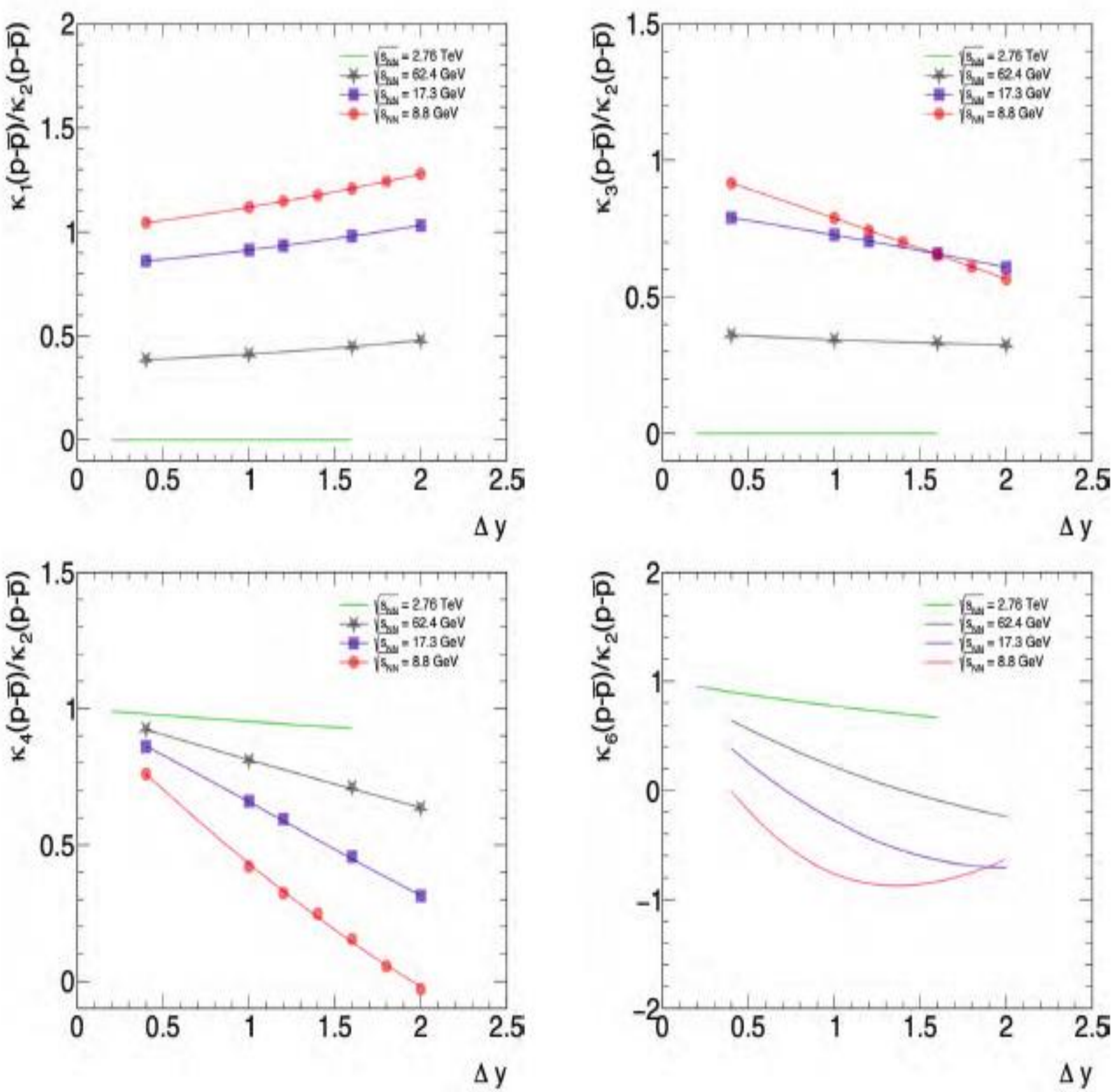




Higher order cumulants

- The effects of baryon number conservation for higher order cumulants are sizeable, however κ_5 / κ_1 stays positive whereas LQCD shows negative ratio
- Expected negative values for the 8th order cumulant at RHIC up to LHC already due to exact baryon number conservation

Dependence on rapidity gap:



- At lower energies, strong dependence on Δy and essential differences from Skellam distribution where $\kappa_1 / \kappa_2 = \kappa_3 / \kappa_2$ and $\kappa_4 / \kappa_2 = \kappa_6 / \kappa_2$
- At higher energies for $\sqrt{s} > 60\ GeV$ these differences are smaller for n<4 as well as for n> 4 at LHC

CONCLUSIONS:

- QCD thermodynamic potential is encoded in nuclear collision data

- S-matrix (Hadron Resonance Gas) thermodynamic potential provides good approximation of the QCD equation of states in confined phase and also a noncritical baseline for fluctuation observables

- The exact conservation of net strange number is essential to quantify the particle yields and their observed scaling with charged particle multiplicities

- To establish a noncritical baseline for net proton (baryon) number fluctuations in the acceptance, the canonical formulation of the conservation law must be accounted for in a full phase space