Phenomenological developments for event-by-event fluctuations of conserved charges

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- Correcting lattice QCD susceptibilities for global B,Q,S conservation
- Cooper-Frye particlization for event-by-event fluctuations

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QCD phase diagram

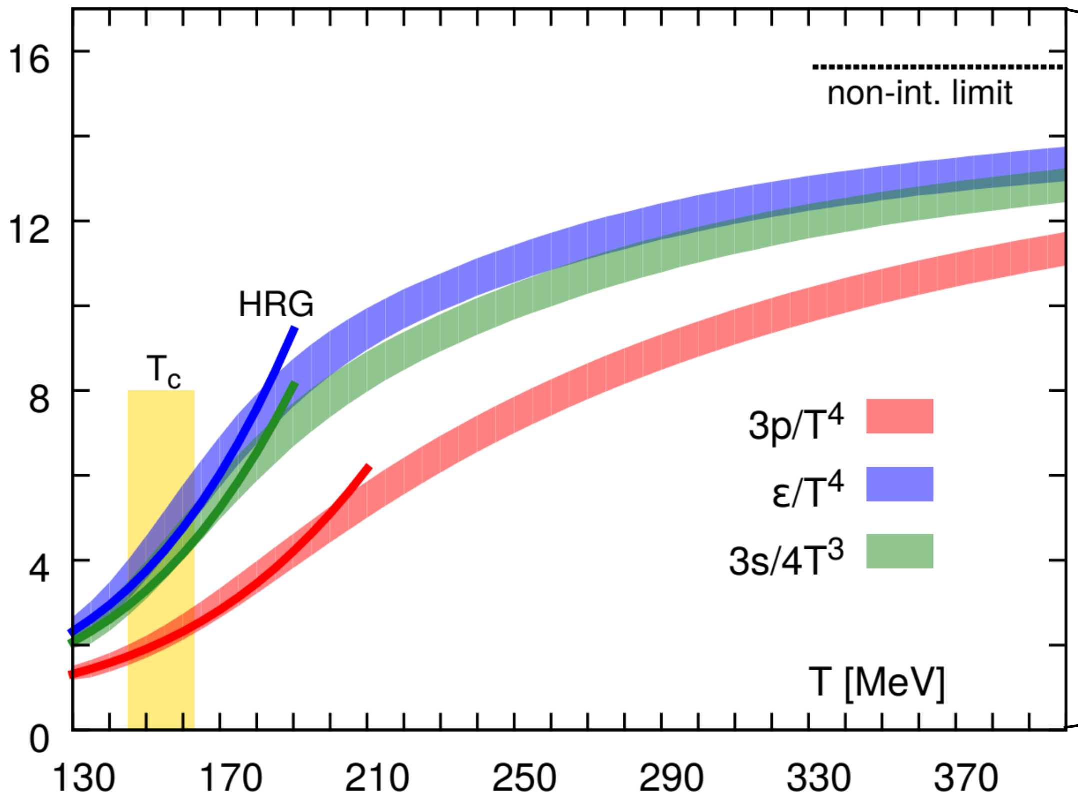


Figure from HotQCD coll., PRD '14

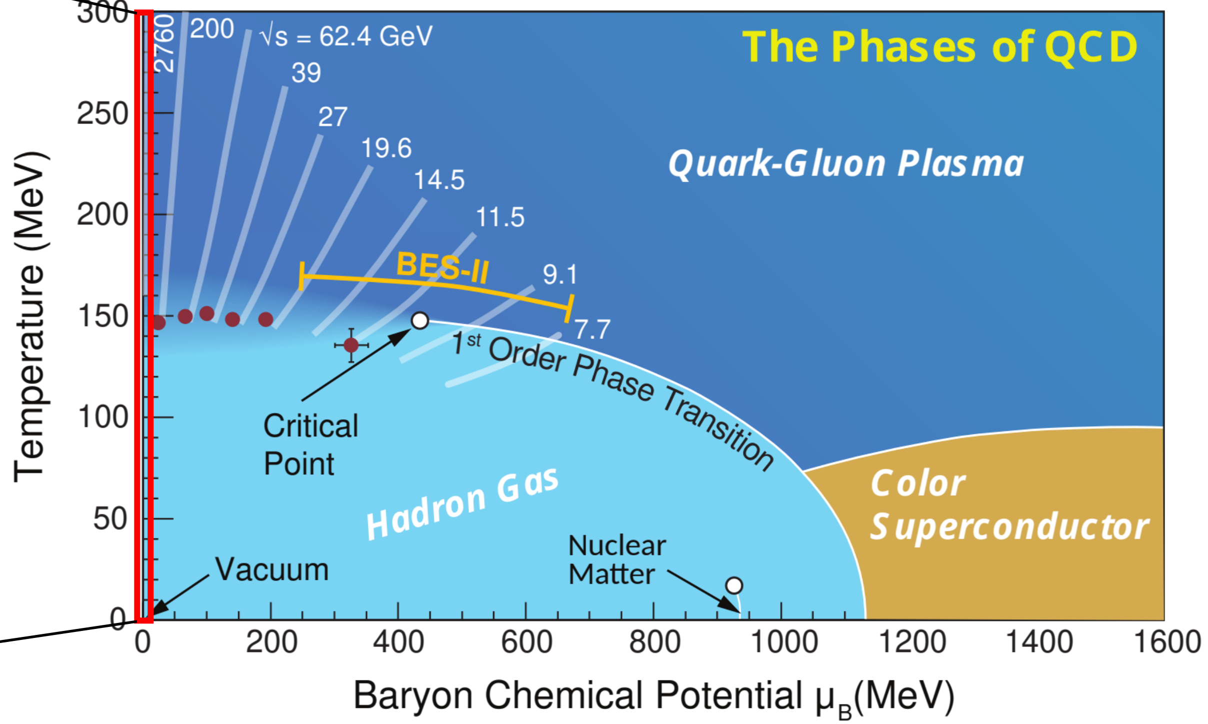


Figure from Bzdak et al., Phys. Rept. '20

- Analytic crossover at vanishing net baryon density – a first-principle result from lattice QCD
- Phase structure at finite density is largely unknown
- Probed by **heavy-ion collisions,** in particular using **event-by-event fluctuations**

Event-by-event fluctuations and statistical mechanics

Cumulant generating function

$$K_N(t) = \ln\langle e^{tN}\rangle = \sum_{n=1}^{\infty} \kappa_n \frac{t^n}{n!}$$

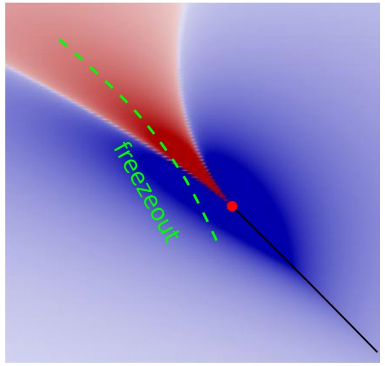
Grand partition function

$$\ln Z^{\text{gce}}(T,V,\mu) = \ln\left[\sum_N e^{\mu N/T} Z^{\text{ce}}(T,V,N)\right]$$

$$\kappa_n \propto \frac{\partial^n(\ln Z^{\text{gce}})}{\partial\mu^n}$$

Cumulants measure chemical potential derivatives of the (QCD) equation of state

- QCD critical point
- Test of (lattice) QCD at $\mu_B \approx 0$
- Freeze-out from fluctuations



M. Stephanov, PRL '09
Energy scans at RHIC (STAR)
and CERN-SPS (NA61/SHINE)

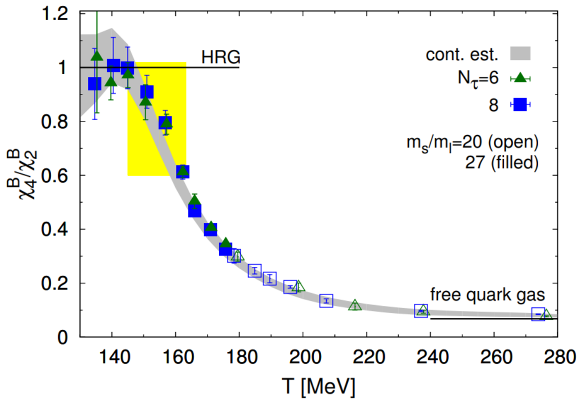
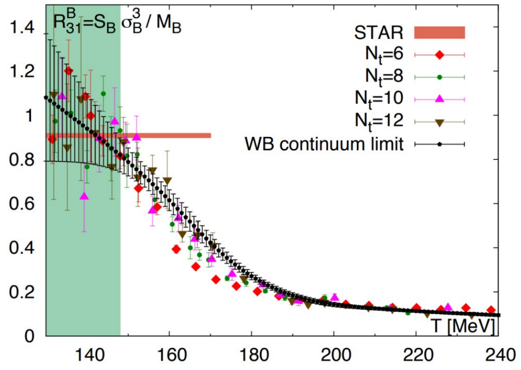


Figure from Bazavov et al. PRD 95, 054504 (2017)
Probed by LHC and top RHIC



Borsanyi et al. PRL 113, 052301 (2014)
Bazavov et al. PRL 109, 192302 (2012)
...

Theory vs experiment: Caveats

- **accuracy of the grand-canonical ensemble (global conservation laws)**

  Jeon, Koch, PRL 85, 2076 (2000); Bzdak, Skokov, Koch, PRC 87, 014901 (2013)
  Braun-Munzinger, Rustamov, Stachel, NPA 960, 114 (2017)

- coordinate vs momentum space (thermal smearing)

  Ling, Stephanov, PRC 93, 034915 (2016); Ohnishi, Kitazawa, Asakawa, PRC 94, 044905 (2016)

- proxy observables in experiment (net-proton, net-kaon) vs actual conserved charges in QCD (net-baryon, net-strangeness)

  Kitazawa, Asakawa, PRC 85, 021901 (2012); VV, Jiang, Gorenstein, Stoecker, PRC 98, 024910 (2018)

- volume fluctuations

  Gorenstein, Gazdzicki, PRC 84, 014904 (2011); Skokov, Friman, Redlich, PRC 88, 034911 (2013)
  Braun-Munzinger, Rustamov, Stachel, NPA 960, 114 (2017)

- non-equilibrium (memory) effects

  Mukherjee, Venugopalan, Yin, PRC 92, 034912 (2015)

- hadronic phase

  Steinheimer, VV, Aichelin, Bleicher, Stoecker, PLB 776, 32 (2018)

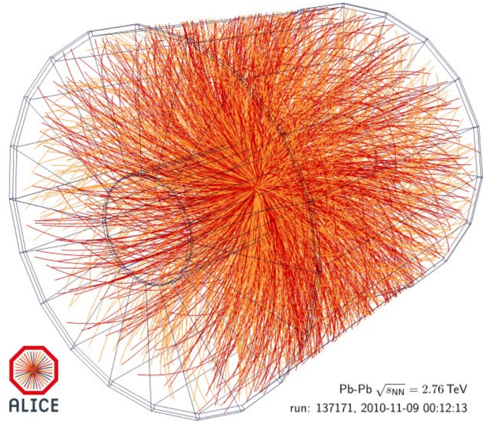
When are the measured fluctuations grand-canonical?

- Consider event-by-event fluctuations of particle number in acceptance ΔY_{accept} around midrapidity

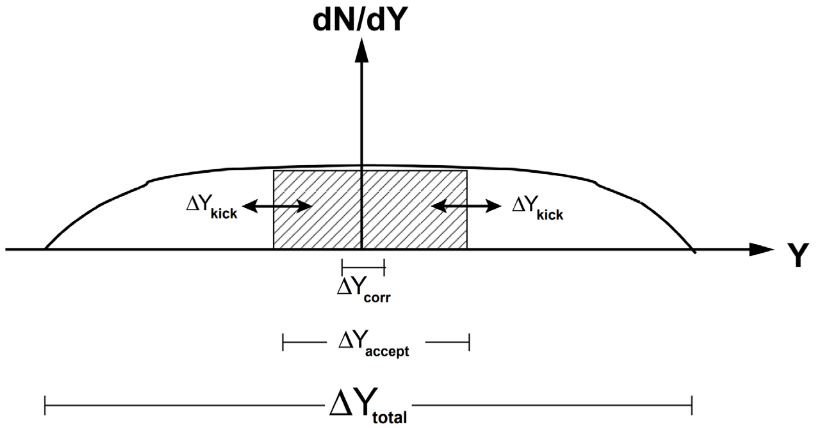
- Scales
  - ΔY_{accept} – acceptance
  - ΔY_{total} – full space
  - ΔY_{corr} – rapidity correlation length (thermal smearing)
  - ΔY_{kick} – diffusion in the hadronic phase

- **GCE applies if** $\Delta Y_{total} \gg \Delta Y_{accept} \gg \Delta Y_{kick}, \Delta Y_{corr}$

- In practice $\Delta Y_{total} \gg \Delta Y_{accept}$ nor $\Delta Y_{accept} \gg \Delta Y_{corr}$ are not simultaneously satisfied
  - Corrections from global conservation are large [Bzdak et al., PRC '13]
  - $\Delta Y_{corr} \sim 1 \sim \Delta Y_{accept}$ [Ling, Stephanov, PRC '16]



ALICE event display



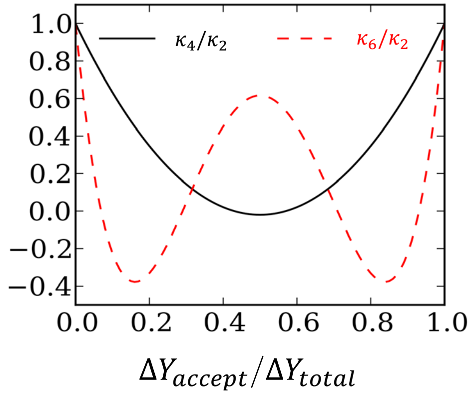
V. Koch, arXiv:0810.2520

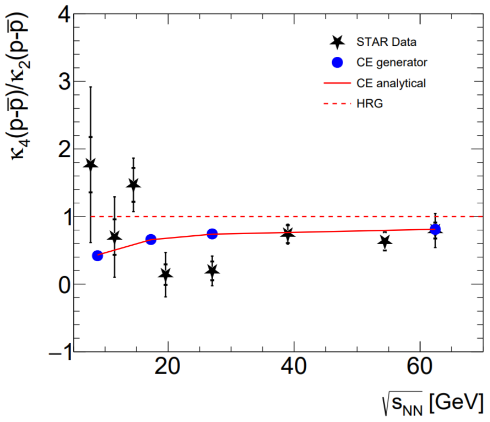
Baryon number conservation and high-order cumulants

- Early studies using UrQMD model suggested strong suppression of fluctuations [M. Nahrgang et al., EPJC 72, 2143 (2012)]

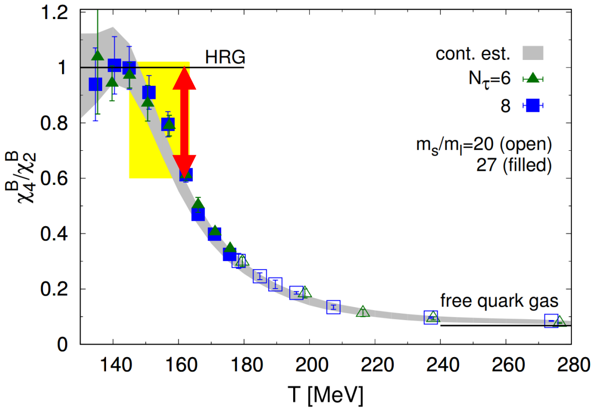
- Analytical framework for the case of free hadron gas developed in [Bzdak, Skokov, Koch, PRC 87, 014901 (2013)]

- Quantitative applications to heavy-ion data by [Braun-Munzinger, Friman, Redlich, Rustamov, Stachel, NPA 1008, 122141 (2021)]
  - Data-driven rapidity distributions and acceptance fractions





Drawback: the formalism is restricted to free gas (HRG), thus not fully compatible with equilibrium QCD at freeze-out



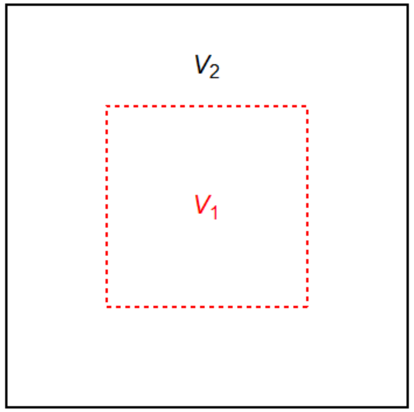
Baryon number conservation 2.0: SAM

VV, Savchuk, Poberezhnyuk, Gorenstein, Koch, PLB 811, 135868 (2020)

Subensemble acceptance method (SAM) – method to correct *any* EoS (e.g. *lattice QCD*) for **charge conservation**

Partition a thermal system with a globally conserved charge B *(canonical ensemble)* into two subsystems which can exchange the charge

$V_1 + V_2 = V$



Assume thermodynamic limit:

$$V, V_1, V_2 \to \infty; \quad \frac{V_1}{V} = \alpha = const; \quad \frac{V_2}{V} = (1-\alpha) = const;$$

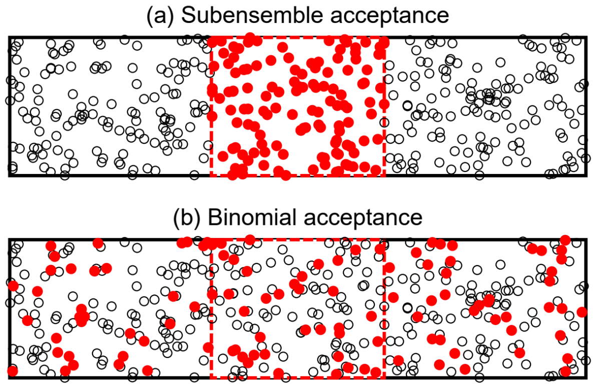
$$V_1, V_2 \gg \xi^3, \qquad \xi = \text{correlation length}$$

The canonical partition function then reads:

$$Z^{ce}(T,V,B) = \mathrm{Tr}\, e^{-\beta \hat{H}} \approx \sum_{B_1} Z^{ce}(T,V_1,B_1)\, Z^{ce}(T,V-V_1,B-B_1)$$

The probability to have charge B_1 is:

$$P(B_1) \propto Z^{ce}(T,\alpha V,B_1)\, Z^{ce}(T,(1-\alpha)V,B-B_1), \qquad \alpha \equiv V_1/V$$



SAM: Computing the cumulants

VV, Savchuk, Poberezhnyuk, Gorenstein, Koch, PLB 811, 135868 (2020)

Cumulant generating function for B_1:

$$G_{B_1}(t) \equiv \ln\langle e^{t\,B_1}\rangle = \ln\left\{\sum_{B_1} e^{t\,B_1}\,\exp\left[-\frac{\alpha V}{T}\,f(T,\rho_{B_1})\right]\exp\left[-\frac{\beta V}{T}\,f(T,\rho_{B_2})\right]\right\} + \tilde{C} \qquad \beta = 1-\alpha$$

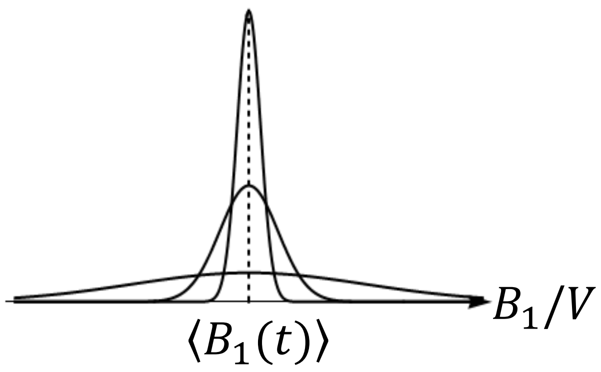
$$\tilde{\kappa}_1[B_1(t)] = \frac{\sum_{B_1} B_1\,\tilde{P}(B_1;t)}{\sum_{B_1}\tilde{P}(B_1;t)} \equiv \langle B_1(t)\rangle \qquad \text{with} \qquad \tilde{P}(B_1;t) = \exp\left\{tB_1 - V\,\frac{\alpha f(T,\rho_{B_1}) + \beta f(T,\rho_{B_2})}{T}\right\}.$$

Thermodynamic limit: $\tilde{P}(B_1;t)$ *highly peaked* at $\langle B_1(t)\rangle$

$\langle B_1(t)\rangle$ is a solution to equation $d\tilde{P}/\mathrm{d}B_1 = 0$:

$$t = \hat{\mu}_B[T,\rho_{B_1}(t)] - \hat{\mu}_B[T,\rho_{B_2}(t)]$$

where $\quad \hat{\mu}_B \equiv \mu_B/T, \qquad \mu_B(T,\rho_B) = \partial f(T,\rho_B)/\partial\rho_B$



$\boldsymbol{t = 0}$**:** $\quad \rho_{B_1} = \rho_{B_2} = B/V$, $B_1 = \alpha B$, i.e. charge uniformly distributed between the subsystems

SAM: Cumulant ratios in terms of GCE susceptibilities

$$\kappa_n[B_1] = \left.\frac{\partial^{n-1}\tilde{\kappa}_1[B_1(t)]}{\partial t^{n-1}}\right|_{t=0} \quad \longleftrightarrow \quad \frac{\partial^n}{\partial t^n}: \; t = \hat{\mu}_B[T, \rho_{B_1}(t)] - \hat{\mu}_B[T, \rho_{B_2}(t)]$$

scaled variance
$$\frac{\kappa_2[B_1]}{\kappa_1[B_1]} = (1-\alpha)\,\frac{\chi_2^B}{\chi_1^B},$$

$$\chi_n^B \equiv \partial^{n-1}(\rho_B/T^3)/\partial(\mu_B/T)^{n-1}$$

skewness
$$\frac{\kappa_3[B_1]}{\kappa_2[B_1]} = (1-2\alpha)\,\frac{\chi_3^B}{\chi_2^B},$$

kurtosis
$$\frac{\kappa_4[B_1]}{\kappa_2[B_1]} = (1-3\alpha\beta)\,\frac{\chi_4^B}{\chi_2^B} - 3\alpha\beta\left(\frac{\chi_3^B}{\chi_2^B}\right)^2.$$

- Global conservation (α) and equation of state (χ_n^B) effects factorize in cumulants up to the 3rd order, starting from κ_4 not anymore
- $\alpha \to 0$ – GCE limit*
- $\alpha \to 1$ – CE limit

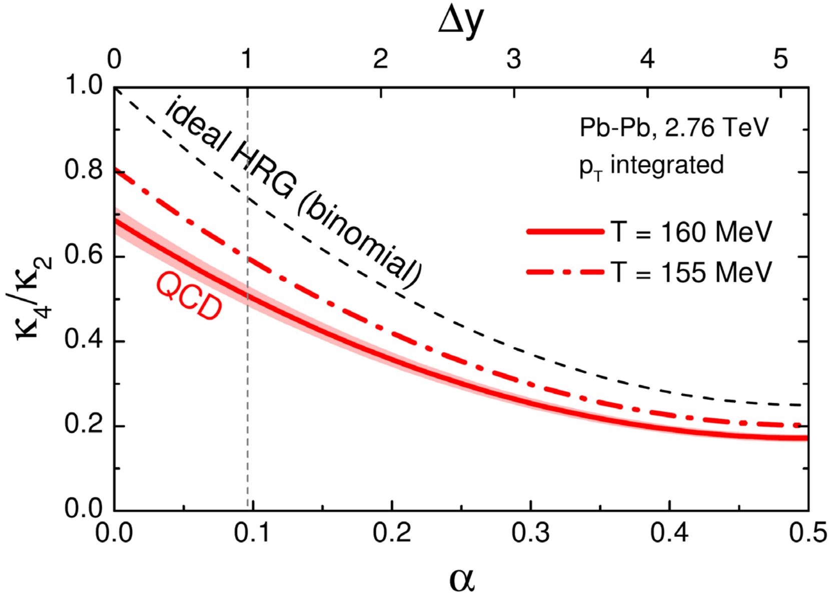
*As long as $V_1 \gg \xi^3$ holds

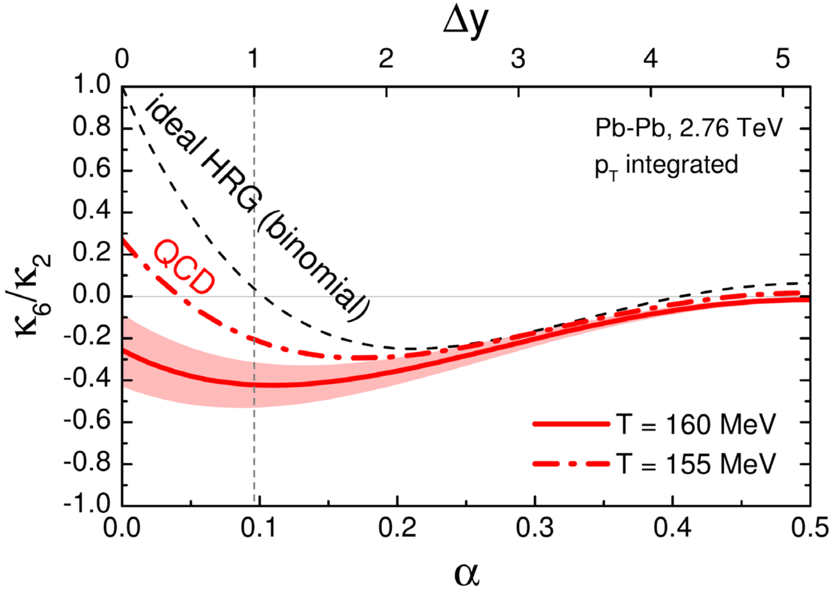
Full result up to κ_6 in VV, Savchuk, Poberezhnyuk, Gorenstein, Koch, PLB 811, 135868 (2020)

Net baryon fluctuations at LHC ($\mu_B = 0$)

VV, Savchuk, Poberezhnyuk, Gorenstein, Koch, PLB 811, 135868 (2020)

$$\left(\frac{\kappa_4}{\kappa_2}\right)_{LHC} = (1-3\alpha\beta)\,\frac{\chi_4^B}{\chi_2^B} \qquad \left(\frac{\kappa_6}{\kappa_2}\right)_{LHC} = [1-5\alpha\beta(1-\alpha\beta)]\,\frac{\chi_6^B}{\chi_2^B} - 10\alpha(1-2\alpha)^2\beta\left(\frac{\chi_4^B}{\chi_2^B}\right)^2$$





Lattice data for χ_4^B/χ_2^B and χ_6^B/χ_2^B from Borsanyi et al., 1805.04445

Theory: negative χ_6^B/χ_2^B is a possible signal of chiral criticality [Friman, Karsch, Redlich, Skokov, EPJC '11]

Experiment: $\alpha \approx \frac{N_{ch}(\Delta y)}{N_{ch}(\infty)} \approx \mathrm{erf}\left(\frac{\Delta y}{2\sqrt{2}\sigma_y}\right)$, for $\Delta y \approx 1$ the κ_6/κ_2 is mainly sensitive to the EoS

Planned measurement in Runs 3 & 4 at the LHC [LHC Yellow Report, 1812.06772]

$N_{ch}(\Delta y)$ measurement: ALICE Collaboration, PLB 726 (2013) 610-622

SAM for multiple conserved charges (B,Q,S)

VV, Poberezhnyuk, Koch, JHEP 10, 089 (2020)

Key findings:

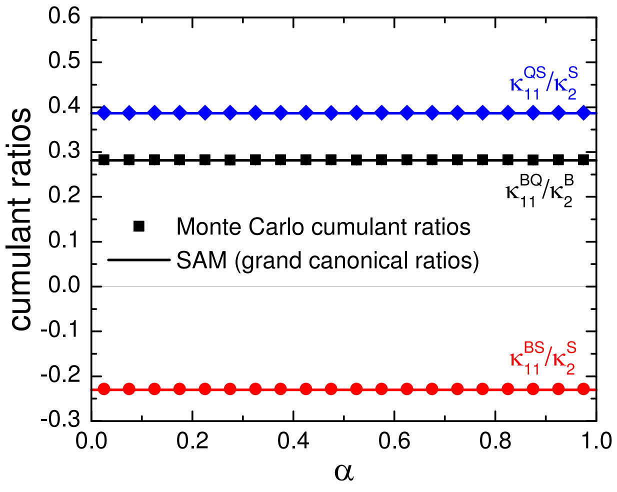
- Cumulants up to 3rd order factorize into product of binomial and grand-canonical cumulants

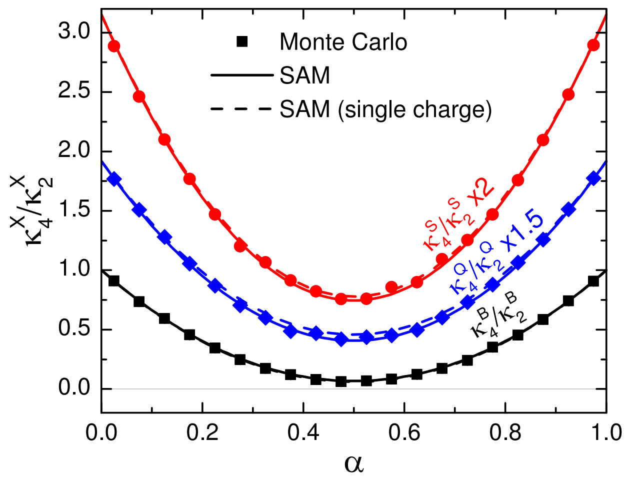
$$\kappa_{l,m,n} = \kappa^{\rm bino}_{l+m+n}(\alpha) \, \times \, \kappa^{\rm gce}_{l,m,n} \, , \qquad l+m+n \leq 3$$

  - Ratios of second and third order cumulants are NOT sensitive to charge conservation
  - Also true for the measurable ratios of covariances involving one non-conserved charge, such as κ_{pQ}/κ_{kQ}

- For order $n > 3$ charge cumulants "mix". Effect in HRG is tiny

$$\kappa_4^B = \kappa_4^{B,\rm gce} \, \beta \left[(1-3\alpha\beta)\, \chi_4^B - 3\,\alpha\,\beta \, \frac{(\chi_3^B)^2 \chi_2^Q - 2\chi_{21}^{BQ}\chi_{11}^{BQ}\chi_3^B + (\chi_{21}^{BQ})^2\chi_2^B}{\chi_2^B\chi_2^Q - (\chi_{11}^{BQ})^2} \right]$$



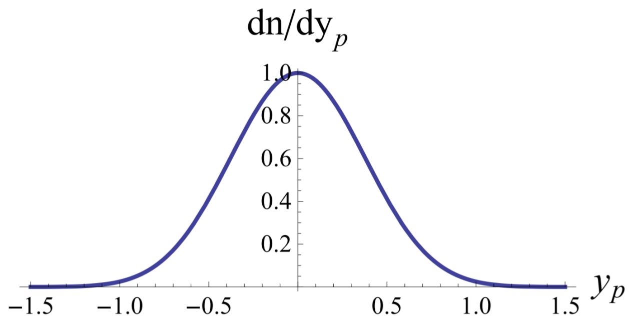


Experiment: Measurements of the off-diagonal cumulants are in progress, e.g. [STAR Collaboration, arXiv:1903.05370]

Mathematica notebook to express any B,Q,S-cumulant of order $n \leq 6$ in terms of grand-canonical susceptibilities available at https://github.com/vlvovch/SAM

SAM: Applicability and limitations

- Argument is based on partition in ***coordinate*** space but experiments measure in ***momentum*** space
  - OK at high energies where we have **Bjorken flow** $[Y \sim \eta_s = \mathrm{atanh}(z/t)]$
    - For small $\Delta Y_{acc} < 1$: corrections due to thermal smearing and resonance decays
    - Limited applicability at lower energies
- **Thermodynamic limit** i.e. $V_1, V_2 \gg \xi^3$:
  - OK at LHC where $\frac{dV}{dy} \sim 4000 - 5000$ fm³ vs. $V_{lattice} \sim 125$ fm³
  - Applicability is more limited near the critical point
- Assumes $\boldsymbol{T, \mu_B = const}$ everywhere



[Ling, Stephanov, PRC '16]

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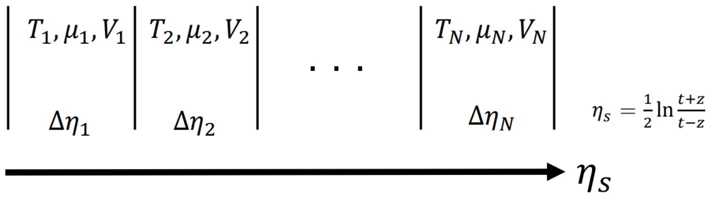
Address these issues with **a novel Cooper-Frye particlization routine**

Subensemble sampler

VV, Koch, arXiv:2012.09954



Momenta: Cooper-Frye formula

$$\omega_p \frac{dN_h}{d^3p} = \int_\sigma d\sigma_\mu \, p^\mu \, f[u^\mu p_\mu; T, \mu_j],$$

Multiplicities: Partition into subensembles

$$Z^{\rm tot} = \prod_{i \in \sigma} \sum_{B_i} e^{\mu_i B_i / T} \, Z^{\rm ce}(T_i, B_i, V_i) \times \delta(B_{\rm tot} - \sum_{i \in \sigma} B_i)$$

1. Partition the Cooper-Frye particlization hypersurface into subvolumes along the space-time rapidity axis
2. Sample each subvolume grand-canonically, using the partition function of an *interacting* HRG
3. Reject the event if global conservation is violated
4. Sample the momenta of particles
5. Do resonance decays or plug into hadronic afterburner

✓ **hydrodynamics**

✓ **locally grand-canonical fluctuations**

✓ **global conservation**

✓ **thermal smearing**

✓ **resonance decays**

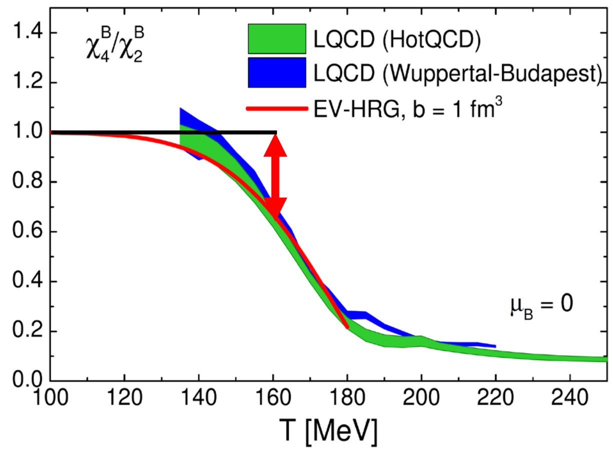
Implementation in the (extended) **Thermal-FIST** package

https://github.com/vlvovch/Thermal-FIST

A case study: net proton/baryon fluctuations at LHC

Central Pb-Pb collisions at $\sqrt{s_{NN}} = 2.76$ TeV

EV-HRG model from [VV, Pasztor, Fodor, Katz, Stoecker, PLB 775, 71 (2017)]



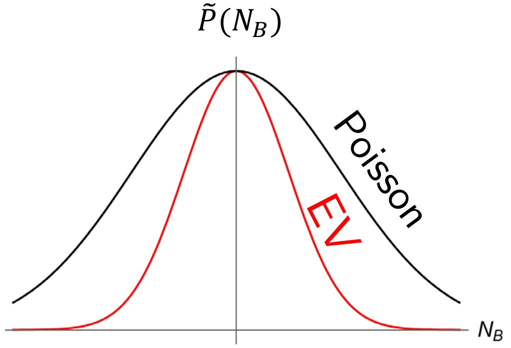
- Particlization at $T = 160$ MeV, $\mu_B = 0$
- Rapidity axis partitioned into 96 slices, $\Delta\eta_s = 0.1$, $|\eta_s| < 4.8$
- Boost-invariant blast-wave hypersurface and flow profile

  BW parameters from [Mazeliauskas, Vislavicius, 1907.11059] to match proton pT spectrum
- Sampling of (anti)baryons from the lattice-based EV-HRG model with global baryon conservation, 10^{10} events

$$P(N) \sim \frac{(V-bN)^N}{N!}\,\theta(V-bN)$$

Poisson + rejection sampling

details in VV, Gorenstein, Stoecker, 1805.01402



GCE baseline: $\frac{\kappa_2^B}{\langle B+\bar{B}\rangle} = 0.94$, $\frac{\chi_4^B}{\chi_2^B} = 0.69$, $\frac{\chi_6^B}{\chi_2^B} = -0.18$ $\leftarrow$ compatible with lattice

VV, Koch, arXiv:2012.09954

Net baryon fluctuations at LHC

- Global baryon conservation distorts the cumulant ratios already for one unit of rapidity acceptance

e.g. $\left.\frac{\chi_4^B}{\chi_2^B}\right|_{T=160MeV}^{GCE} \simeq 0.67$ ("lattice QCD") $\neq$ $\left.\frac{\chi_4^B}{\chi_2^B}\right|_{\Delta Y_{acc}=1}^{HIC} \simeq 0.56$ (experiment)

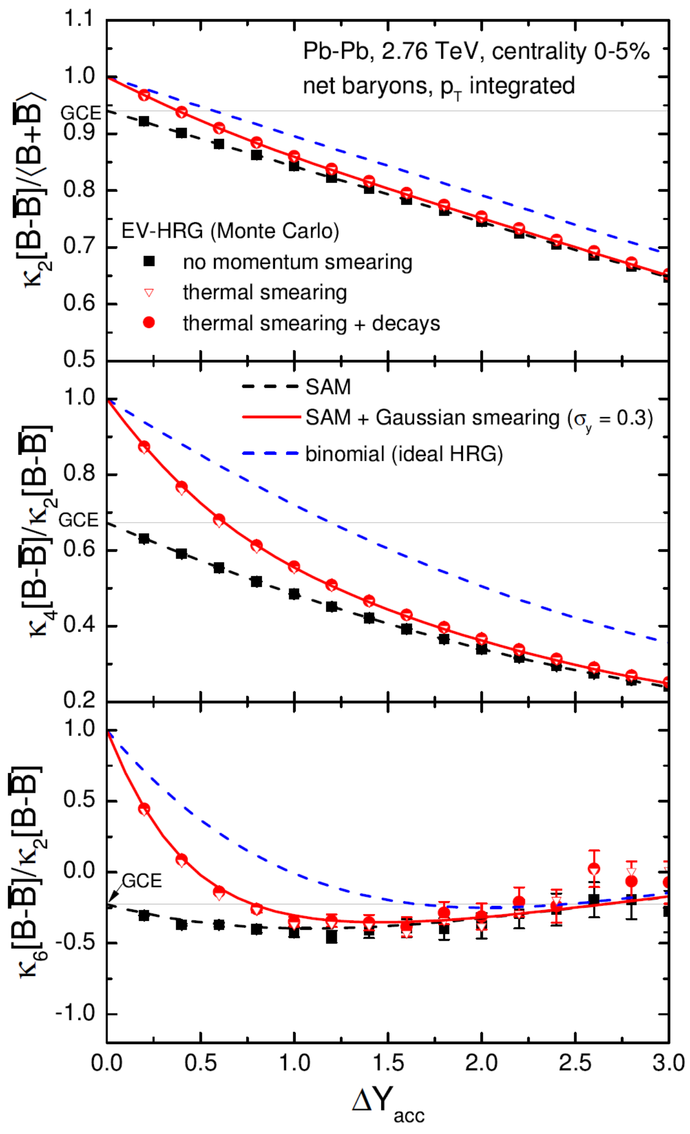
- Neglecting thermal smearing, effects of global conservation can be described analytically via SAM

$$\frac{\kappa_2}{\langle B+\bar{B}\rangle} = (1-\alpha)\frac{\kappa_2^{gce}}{\langle B+\bar{B}\rangle}, \qquad \alpha = \frac{\Delta Y_{acc}}{9.6}, \quad \beta \equiv 1-\alpha$$

$$\frac{\kappa_4}{\kappa_2} = (1-3\alpha\beta)\,\frac{\chi_4^B}{\chi_2^B},$$

$$\frac{\kappa_6}{\kappa_2} = [1-5\alpha\beta(1-\alpha\beta)]\,\frac{\chi_6^B}{\chi_2^B} - 10\alpha(1-2\alpha)^2\beta\left(\frac{\chi_4^B}{\chi_2^B}\right)^2$$

- Effect of resonance decays is negligible



VV, Koch, arXiv:2012.09954

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"lattice QCD" *experiment*

- Neglecting thermal smearing, effects of global conservation can be described analytically via SAM

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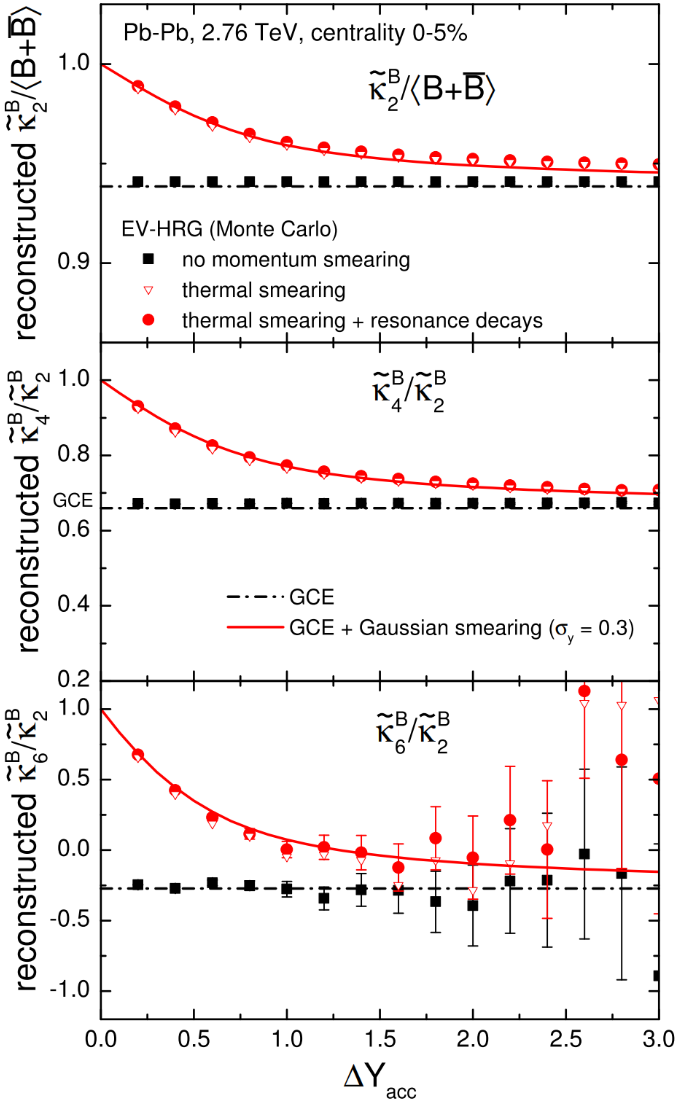
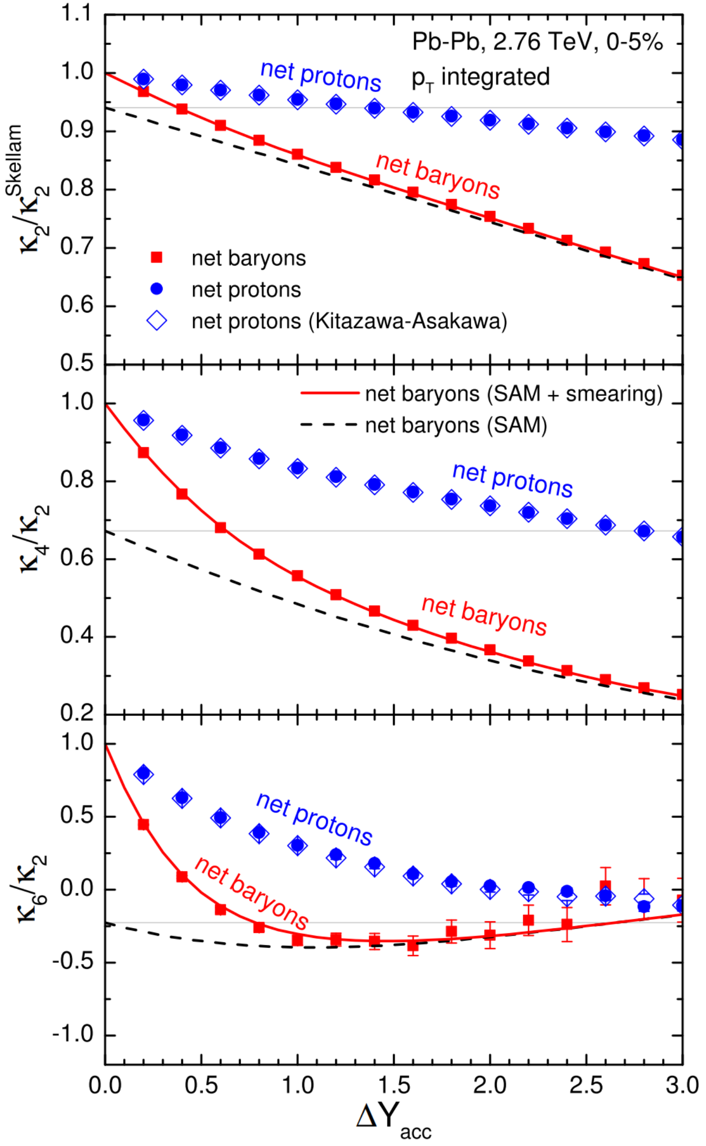
- Effect of resonance decays is negligible

Cumulants corrected for baryon conservation



VV, Koch, arXiv:2012.09954

Net baryon vs net proton 



- Thermal smearing distorts the signal at $\Delta Y_{accept} \leq 1$. Net baryons converge to model-independent SAM result at larger ΔY_{accept}

- net baryon ≠ net proton, e.g.

$$\left.\frac{\chi_4^B}{\chi_2^B}\right|_{\Delta Y_{acc}=1}^{HIC} \simeq 0.56 \neq \left.\frac{\chi_4^p}{\chi_2^p}\right|_{\Delta Y_{acc}=1}^{HIC} \simeq 0.83$$

- Baryon cumulants can be reconstructed from proton cumulants via binomial (un)folding based on isospin randomization [Kitazawa, Asakawa, Phys. Rev. C 85 (2012) 021901]
  - Requires the use of joint factorial moments, only experiment can do it model-independently

unfolding

VV, Koch, arXiv:2012.09954

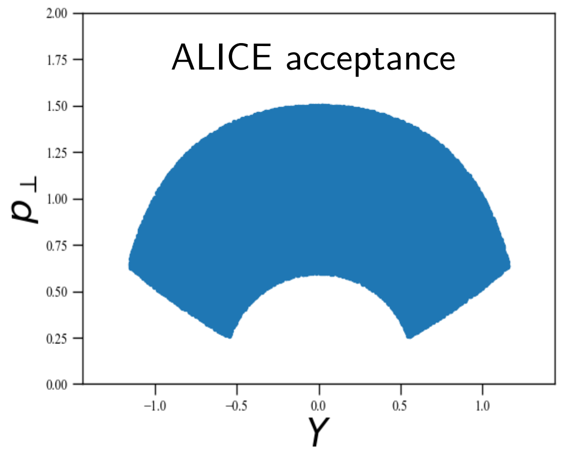
Comparison to ALICE data

VV, Koch, arXiv:2012.09954

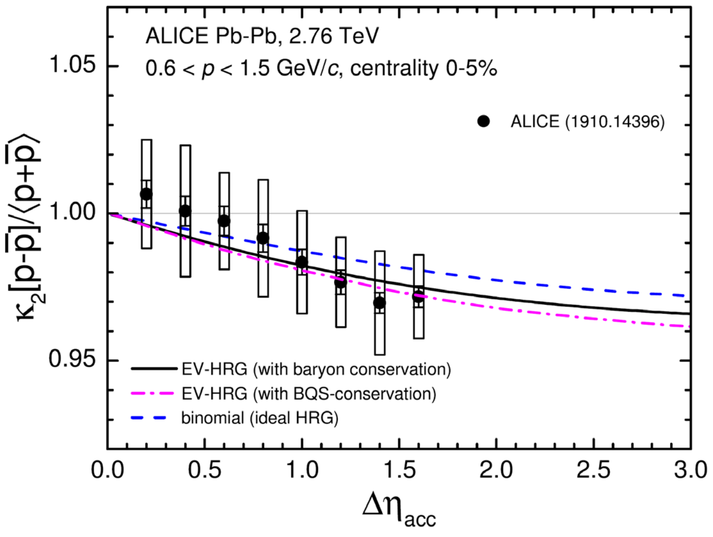
- Net protons described within errors but not sensitive to the equation of state for the present experimental acceptance

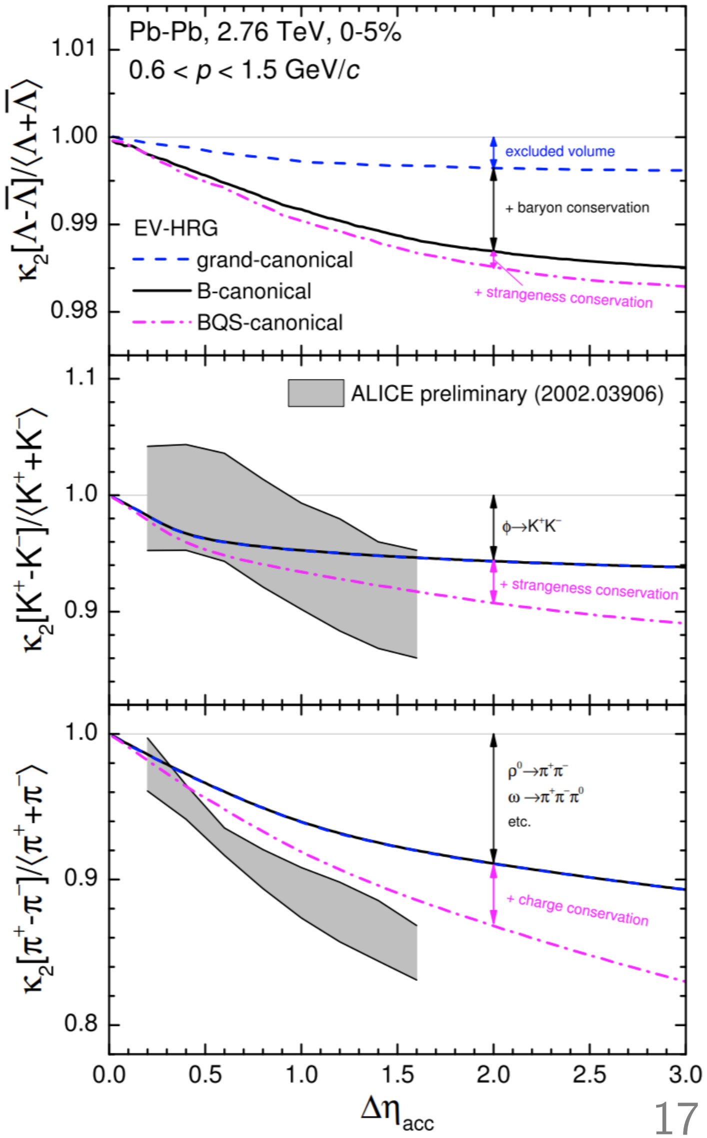
- Large effect from resonance decays for lighter particles

- Future measurements will require larger acceptance



$0.6 < p < 1.5$ GeV$/c$, $\Delta\eta_{acc} = 1.6$

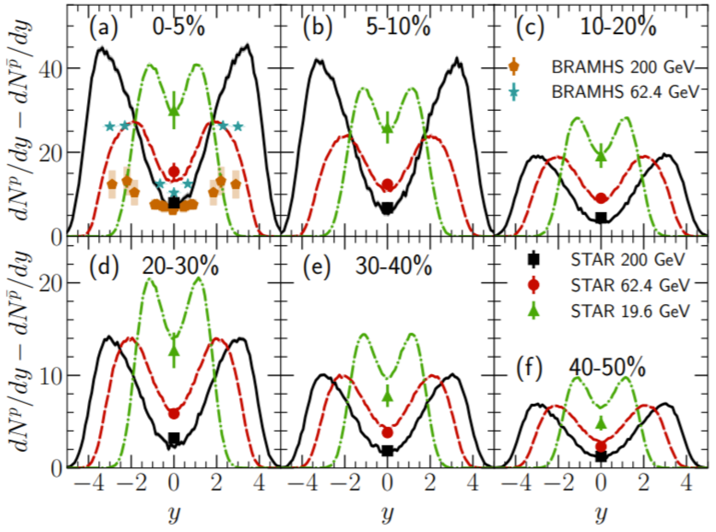




Net proton fluctuations in beam energy scan

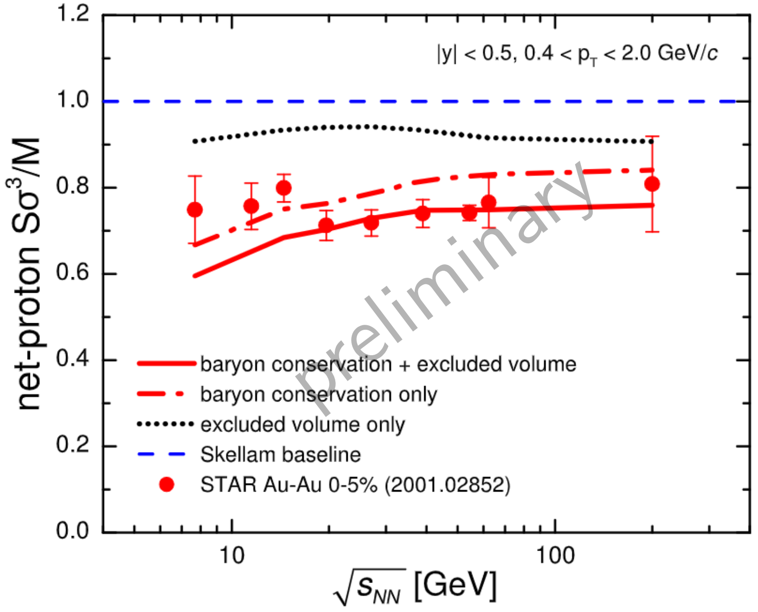
VV, C. Shen, V. Koch, in preparation

- Use Cooper-Frye hypersurface from full (3+1)D numerical hydro solution (MUSIC) at $\sqrt{s_{NN}} = 7.7 - 200$ GeV constrained to measured dN/dY
- Calculation of net-proton fluctuations in the "standard model" for dynamical description of heavy-ion collisions

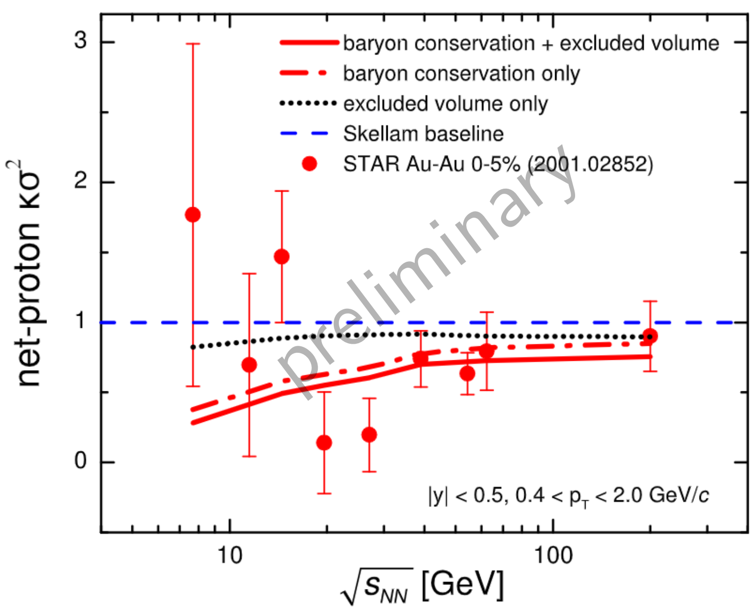


Shen, Alzhrani, 2003.05852

net-proton κ_3/κ_1



net-proton κ_4/κ_2



STAR data at $\sqrt{s_{NN}} > 20$ GeV consistent with simultaneous effects of global baryon conservation and excluded-volume interactions

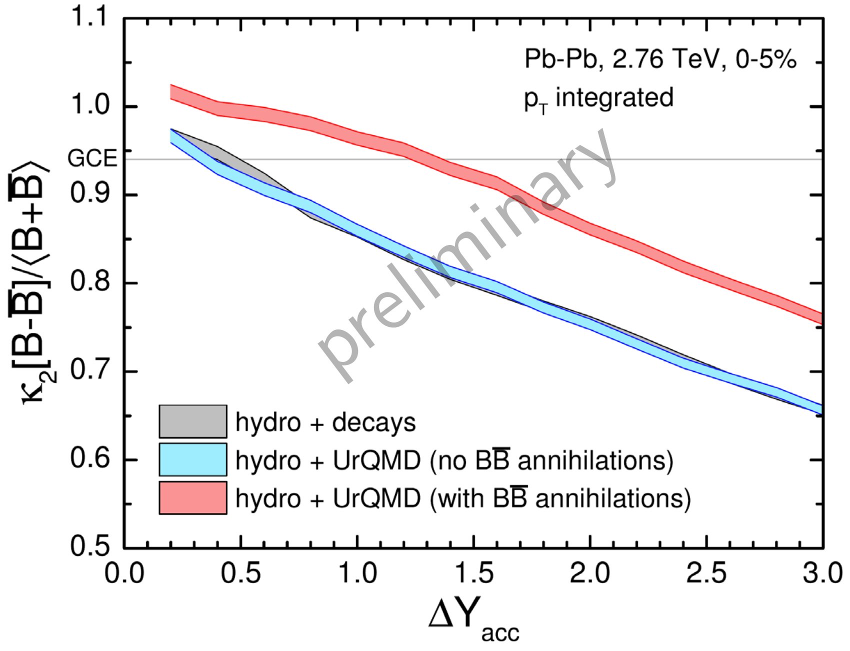
More interesting data recently became available [STAR Collaboration, arXiv:2101.12413]

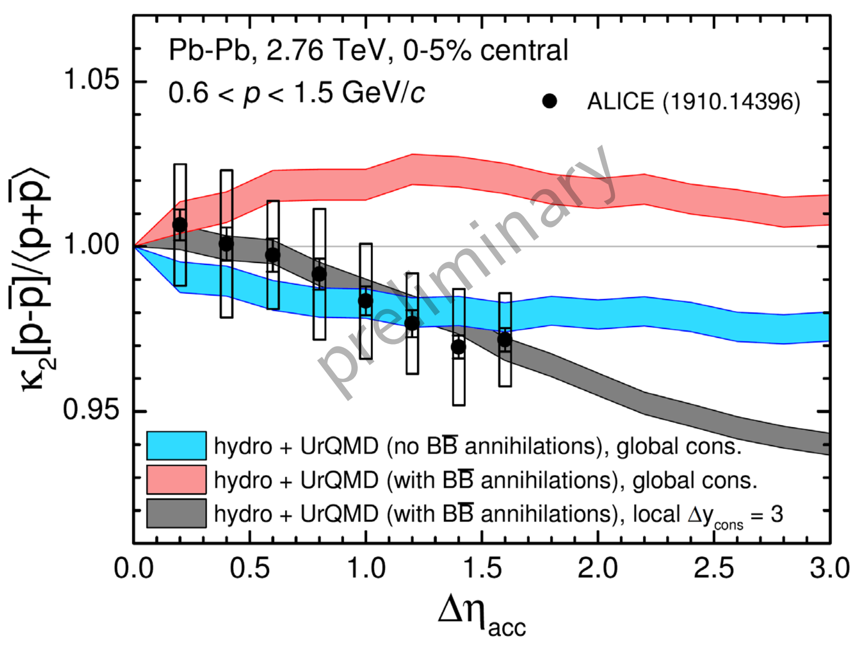
Constraining baryon annihilation with fluctuations

VV, V. Koch, **O. Savchuk**, J. Steinheimer, H. Stoecker, in preparation

Baryon annihilation $B\bar{B} \to n\pi$ in afterburners (UrQMD, SMASH) suppresses baryon yields

$$\langle B + \bar{B} \rangle \searrow \quad \Longrightarrow \quad \frac{\kappa_2[B - \bar{B}]}{\langle B + \bar{B} \rangle} \nearrow$$

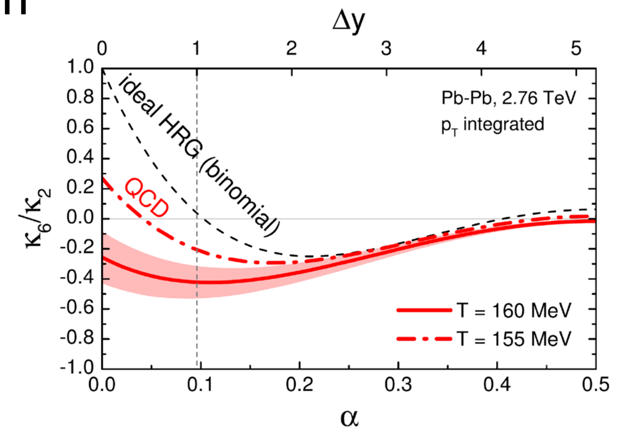


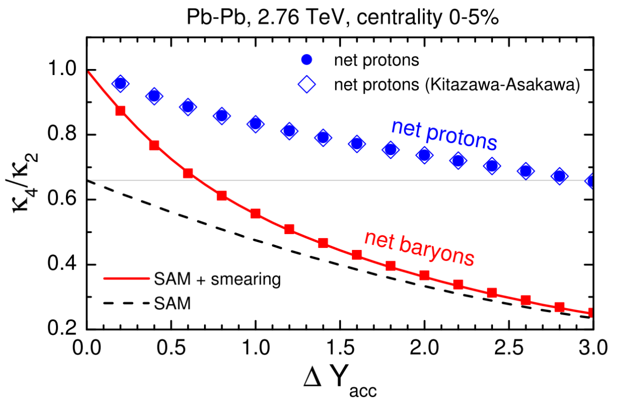


Data on net-proton fluctuations can constrain the effect of annihilations in the hadronic phase

Summary

- Fluctuations are a powerful tool to explore the QCD phase diagram
  - test of lattice QCD and equilibration
  - probe of chiral criticality and QCD critical point
- SAM corrects QCD cumulants in heavy-ion collisions for global conservation of (multiple) charges
  - important link between theory and experiment
- Quantitative analysis of fluctuations at LHC, RHIC, SPS via new particlization routine
  - Need to unfold baryon cumulants from measured protons
  - Data at $\sqrt{s_{NN}} > 20$ GeV consistent with baryon conserv. + excluded volume
  - New way to constrain baryon annihilation





Thanks for your attention!