The renormalization of sound and viscosity from non-equilibrium effective field theory

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Many-body system

- Quark Gluon Plasma
  - Hydrodynamics regime

- Dynamics of conserved quantities
  - Basic ingredients
    - Hydro modes – Shear, Sound

  - Fluctuation
    - Long time tail → Thermalization scale
    - Finite size effect

→ Hydro EFT

Hydro EFT

- Non-linear interaction of hydro modes
- Self-energy → Dispersion relation of sound and shear modes
  - c.f. electrons screening in plasma
- Previous studies limited to shear

Idea

- Real time evolution at finite temperature
  - Schwinger-Keldysh formalism (Closed Time Path, CTP)
  - Double degrees of freedom $\phi \to (\phi_r, \phi_a)$

A microscopic action → Integrate out fast modes → EFT

- Impose dynamical Kubo–Martin–Schwinger symmetry
  - Global/Local thermo-equilibrium
  - Fluctuation-dissipation theorem

M. Crossley, P. Glorioso, H. Liu JHEP 1709 (2017) 095
JHEP 1709 (2017) 096

Hydro action

- EFT of stress tensors $(T_r(\lambda, X), T_a(\lambda, X))$
  - Dynamical degrees of freedom $\left(\delta\beta^\mu = \beta_0\lambda^\mu, X_a^\mu\right)$
- Noise X_a^μ and derivative expansion

Viscous (KMS symmetry)

$$S_{eff} = \int d^{d+1}x \sqrt{-g}\left[\left(T^{\mu\nu}_{\text{ideal}} + T^{\mu\nu}_{\text{vis}}\right)\nabla_\mu X^a_\nu + i\Sigma^{\mu\nu\alpha\beta}\nabla_\mu X^a_\nu \nabla_\alpha X^a_\beta\right]$$

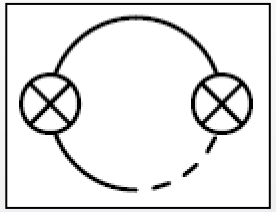
- EOM $\delta X_\nu \to \nabla_\mu T_r^{\mu\nu} = 0$

c.f $S_{MSR} = \int dt\, [-x_a(\partial_t^2 x_r + \gamma\partial_t x_r) + i2\gamma T x_a^2]$

P. C. Martin, E. D. Siggia, H. A. Rose, Phys. Rev. A, 8:423–437, Jul 1973

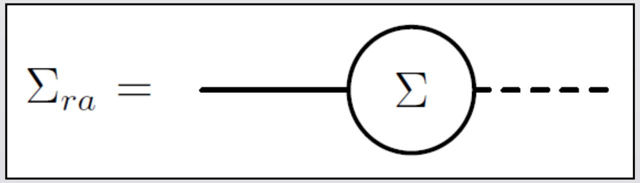
Self-energy – 1-loop

- Transport coefficients
  - $\langle T_r^{xy} T_a^{xy} \rangle$
  - $\langle \Theta_r \Theta_a \rangle$



$\Theta = T_\mu^\mu$

- Dispersion relations
  - $\langle T_r^{00} T_a^{00} \rangle$
  - $\langle T_r^{0x} T_a^{0x} \rangle$



$G_{\mu\nu}^{-1} = G_{0\mu\nu}^{-1} + \Sigma_{\mu\nu}$

Correction to transport coefficients

- Defined by Kubo formula, $k \to 0$
- Shear viscosity
  - $\delta\eta(\omega) = \lim_{k\to 0}\left(\frac{1}{\omega}\mathrm{Im}\langle T_r^{xy} T_a^{xy}\rangle\right)$
- Bulk viscosity (similar story)
- Dispersion relation
  - Need finite k (New)

Y. Akamatsu, A. Mazeliauskas, D. Teaney, *Phys.Rev.C* 95 (2017) 1, 014909
Phys.Rev.C 97 (2018) 2, 024902

Dispersion relation – Shear $\langle T_r^{0x} T_a^{0x} \rangle$

- $\omega = -i\nu_T k^2 - \delta\omega_{\text{sh}}(k)k^2$

Sound | Viscous

$$\delta\omega_{\text{sh}}(k) = \lim_{r\to\infty} \Sigma_T(\omega = -i\nu_T k^2, k) = -i\tilde{g}^2 \sqrt{\frac{c_s k}{\nu_L^3}\frac{4c_s^2}{77}(2-2c_1)}$$

Ideal

- Shear mode only $\rightarrow \delta\omega_{\text{sh}} \propto k$
  - $\delta\omega_{\text{sh}} \sim p_*^{d-2}$
  - $p_*^{shear} = \left(\frac{\omega}{2\nu_T} + i\frac{k^2}{4}\right)^{\frac{1}{2}} \sim k \qquad \leftrightarrow \qquad p_*^{sound} \sim \left(\frac{1}{\nu_L}\max(\omega, c_s k)\right)^{\frac{1}{2}} \sim \sqrt{k}$

$\nu_T = \frac{\eta}{w} \qquad r = \frac{c_s k}{\omega}$

$\nu_L = \frac{\zeta}{w} + \frac{2(d-1)}{d}\nu_T$

X. Chen-Lin, L.V. Delacrétaz, S.A. Hartnoll, *PRL* 122 (2019) 9, 091602
A. Jain, P. Kovtun, A. Ritz, A. Shukla, arXiv: 2011.03691

Dispersion relation – Sound $\langle T_r^{00} T_a^{00} \rangle$

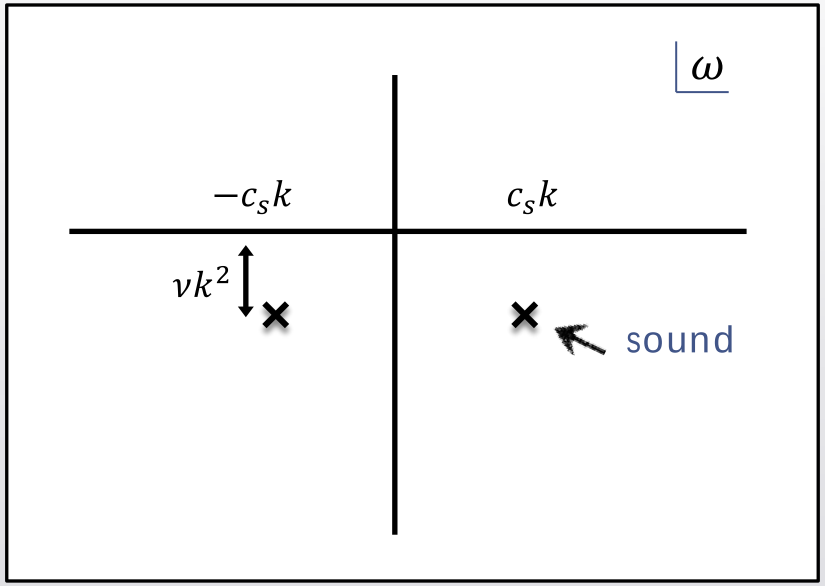
Second order hydro

$$\omega_\pm = \pm c_s k \; - i\frac{\nu_L}{2}k^2 + \delta\omega_1 + i\delta\omega_2 + (\nu k)^2 k$$

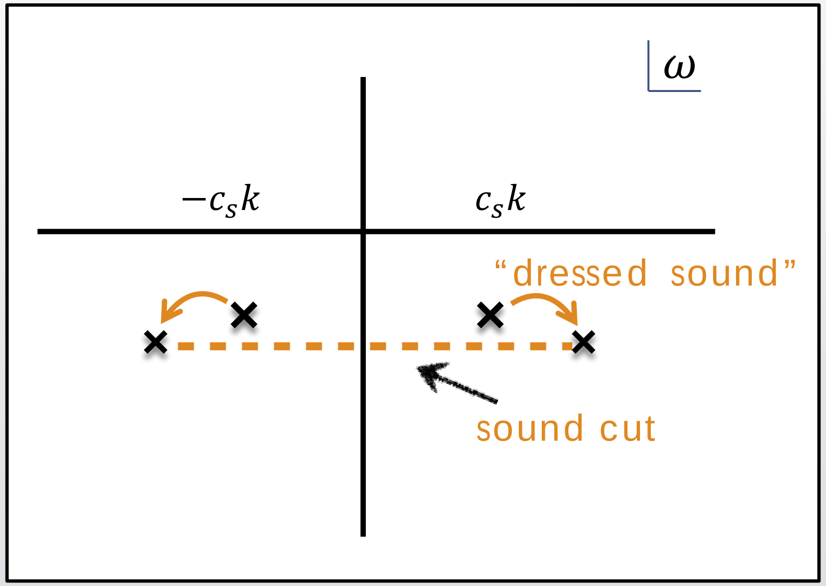
$$\sim \frac{1}{\nu}\left(\sqrt{\frac{c_s k}{\nu}}\right) k^2$$

- Surprising result in $(1+1)$d
  - $\delta\omega_{1,2} \sim \frac{1}{\sqrt{k}} k^2 \sim k^{\frac{3}{2}}$!

Non-analytic structure (preliminary)



w/o fluctuation



with fluctuation

Summary

- Hydro EFT provides a field theory framework for hydrodynamics
    - New results from EFT

- Complete 1-loop study of sound and shear modes
    - Transport coefficients, $k \to 0$
    - Correction to dispersion relations
    - Non-analytic structures

- Experiments
    - Size of QGP → Typical scale ↔ Shift of dispersion