Fluctuation dynamics near the QCD critical point

1. Evolving critical fluctuations.

2. Freezing out critical fluctuations.

3. Outlook.

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BEST
COLLABORATION

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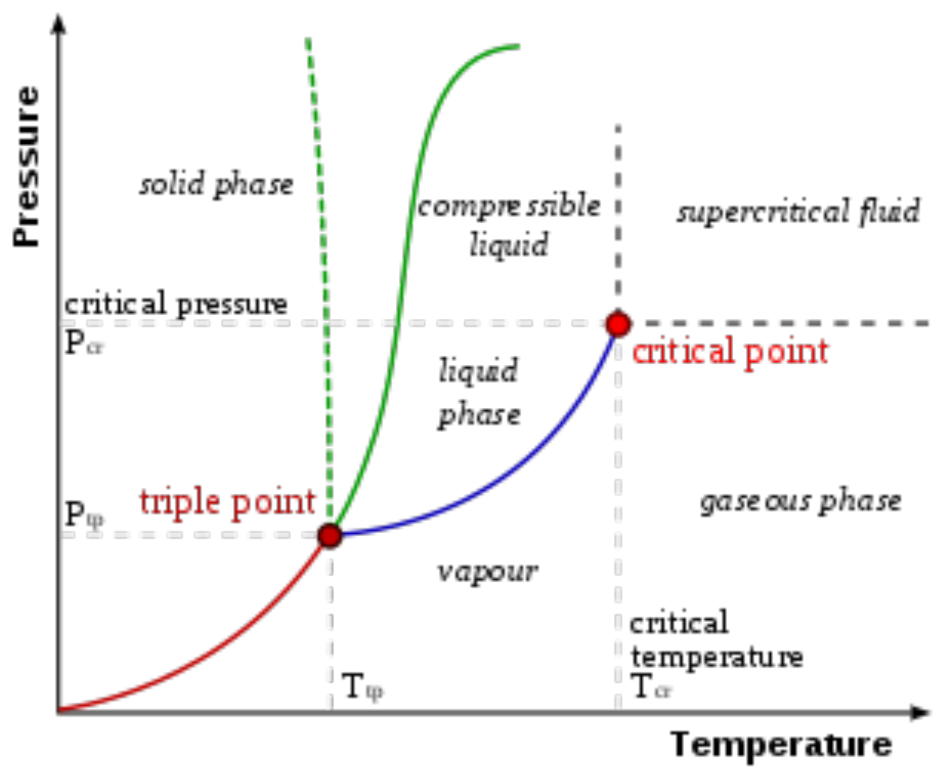
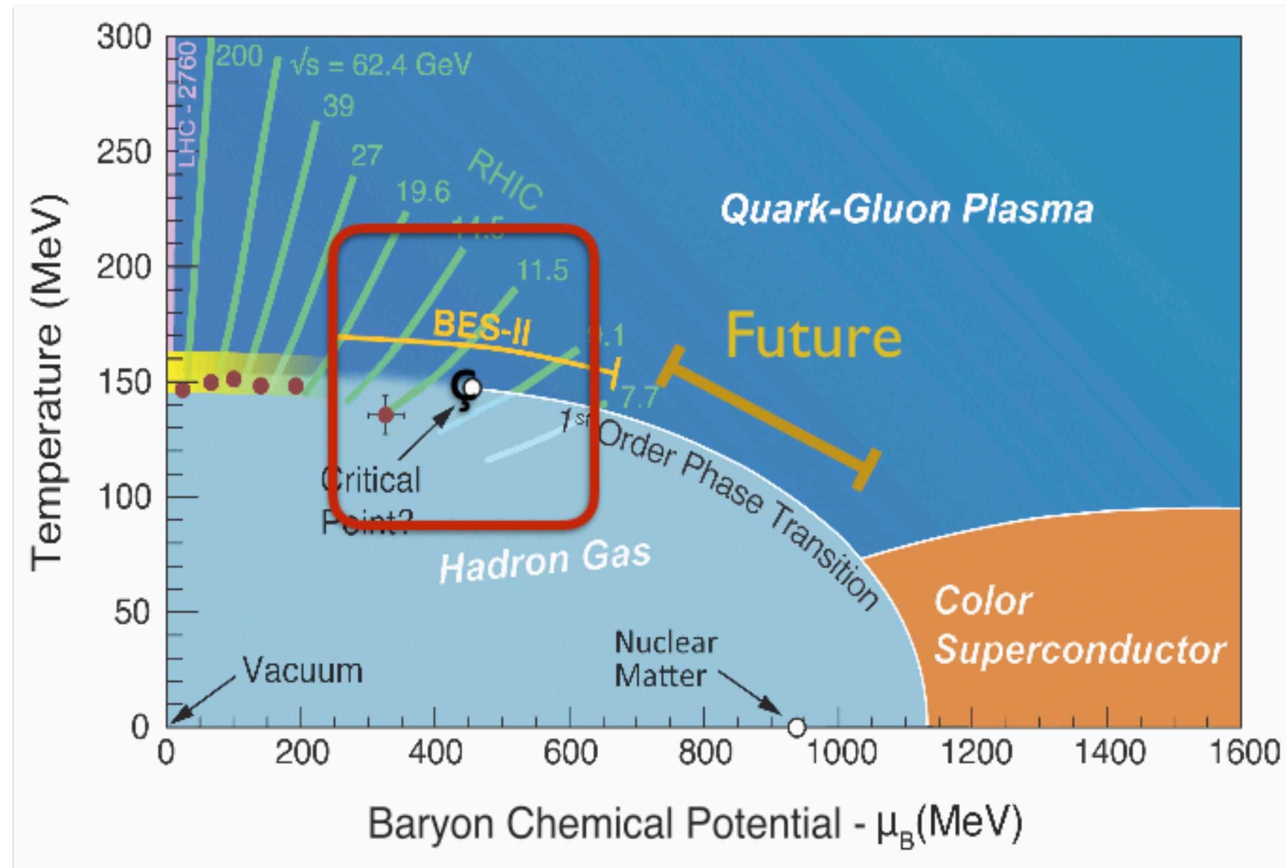
Introduction

Critical point



Phase diagram of water (from wiki)



ubiquitous phenomenon.

the landmark point on the QCD phase diagram (if exists).

The discovery of the QCD critical point and first order transition would break new frontier in the field.

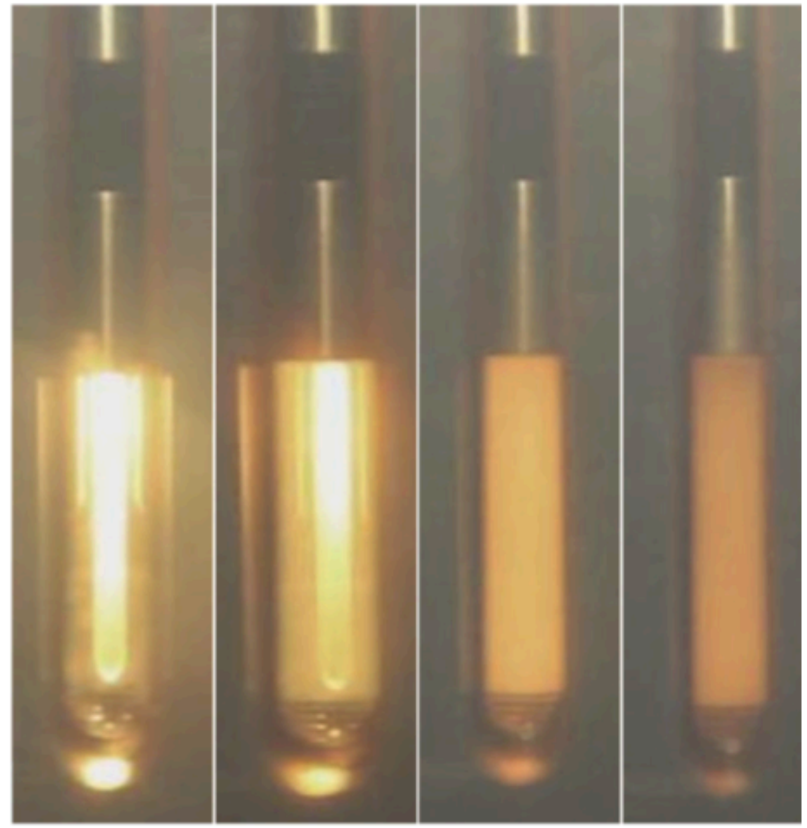
Maximizing the discovery potential of BESII

Theory: building the comprehensive dynamical framework to quantitatively describe the signatures associated with criticality and chirality.

Status: the most important ingredients as well as the connections among them have already been clarified.

e.g., "The BEST framework for the search of the QCD critical point and the chiral magnetic effect", in preparation, BEST collaboration.

This talk: evolving and freezing out critical fluctuations.



Hou et al, Journal of Chemistry 16'.

"As the density fluctuations become of a size comparable to the wavelength of light, the light is scattered and causes the normally transparent liquid to appear cloudy." -- wiki

Fluctuations, correlations and criticality

The correlation function of the order parameter field δM (Fourier momentum Q ~ inverse of the size of the fluctuation)

$$\phi_{\mathrm{eq}}(Q) \sim \langle \delta M \delta M \rangle \sim \frac{1}{\xi^{-2} + Q^2}$$

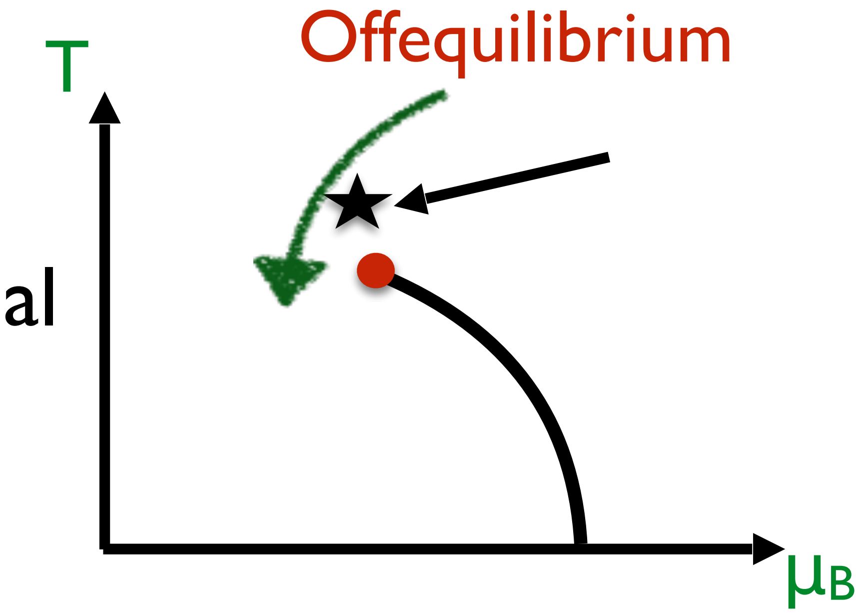
Enhanced $\phi_{eq}(Q)$ near the critical point for Q~1/ ξ $\Rightarrow$ *phenomenon of the critical opalescence*.

QCD critical point: $\phi_{eq}(Q)$ $\Rightarrow$ i) the critical scaling of E.o.S. ii) the growth of non-Gaussian fluctuations of proton numbers

Two keys to make connections to observables: non-equilibrium evolution and freezeout prescription.

Evolving critical fluctuations

Real-time critical fluctuations



Inescapably fall out of equilibrium near the critical point. ("Critical slowing down")

Could be different from the equilibrium expectation both quantitatively and qualitatively !

e.g. S. Mukherjee, R. Venugopalan and YY, PRC15' ; see YY, 1811.06519 for a concise review on related development.

Back-reacts on bulk evolution.

gradient of p (?) ≃ acceleration of flow

How to describe the interplay among fluctuations and bulk evolutions?

Approaches to fluctuating hydro.

Stochastic approach: adding noise to hydro. eqns

$$\partial_t \vec{u} = -(\vec{u} \cdot \vec{\partial})\vec{u} - \nu \nabla^2 \vec{u} + \vec{F} \qquad \langle F(t, \vec{x})F(t', \vec{x}') \rangle \sim 2T\eta\delta(t-t')\delta(\vec{x} - \vec{x}')$$

Landau-Lifshitz; Kapusta-Mueller-Stephanov; ..

Deterministic approach: formulating and solving a set of deterministic equations, which couple fluctuations with hydro modes.

$$\partial_\mu \left[T^{\mu\nu}_{\rm ave}(\epsilon, n, u^\mu) + \Delta T^{\mu\nu}(\text{2pt,3pt}, \ldots) \right] = 0$$

E.o.Ms for 2pt, 3pt,…

Kawasaki, Ann. Phys. '70; Andreev, JTEP, '1971; …
Akamatsu-Mazeliauskas-Teaney, PRC 16' & 18'; Stephanov-YY PRD 18'; Mauricio-Schaefer PRC 19; Xin An-Basar-Stephanov -H.-U. Yee, PRC19'&20'& 2009.10742;

EFT approaches: based on action principle.

See Teaney's talk

$$Z = \int D\psi_{\rm hydro}\, e^{iI_{hydro}[\psi_{\rm hydro}]}$$

Kovtun-Moore-Romatschke, JHEP 14'; Glorioso-Crossley-Liu, JHEP 17'; Haehl-Loganayagam-Rangamani, 1803.11155, …

Hydro+

Key idea: couples hydro. modes with critical fluctuations using deterministic equations. *Stephanov-YY, PRD '18;*

Further development: e..g. additional slow modes, non-Gaussian fluctuations. Xin An-Basar-Stephanov -H.-U. Yee, PRC19'&20'& 2009.10742.

Broad application: systems with slow modes in additional to conserved densities, e.g. hydro with chiral anomaly.

A generalized hydro+ model to describing jet-medium interaction in non-hydro. non-perturbative regime of QGP. *Weiyao- Ke YY, in preparation*

(Stochastic hydro: i. Implementation for expanding system are challenging.
ii Results depend on cut-off (lattice space). Significant progress recently.)

see Nahrgang's talk

The construction of "hydro+"

"+": (Wigner transform of) the two point function of δM (For QCD critical point, M ~ s/n):

$$\phi(t,x;Q) = \int d\Delta x e^{-i\Delta x Q} \left\langle \delta M(t, x+\Delta x/2)\delta M(t, x-\Delta x/2) \right\rangle$$

The evolution of "+" is modelled by relaxation rate equation.

$$u^\mu \partial_\mu \phi = \Gamma_\phi(Q)\left(\phi(Q) - \phi_{\rm eq}(e,n;Q)\right)$$

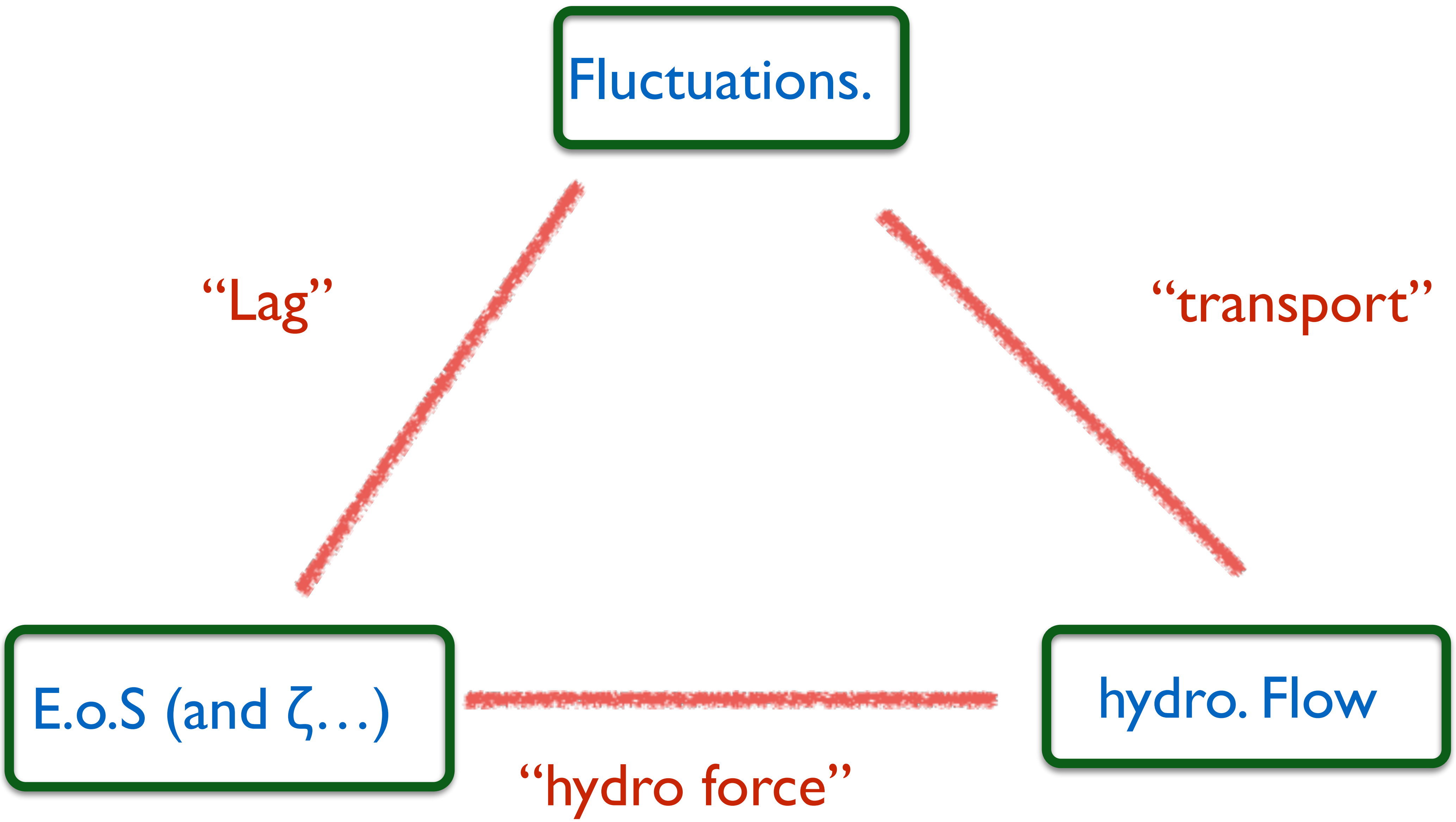
Feedback to hydro. $\partial_\mu T^{\mu\nu} = 0; \partial_\mu J^\mu = 0$:

$$T^{\mu\nu} = e\, u^\mu u^\nu + p_{(+)}\left(g^{\mu\nu} + u^\mu u^\nu\right) + \mathcal{O}(\partial) \qquad p(e,n) \to p_{(+)}(e,n,\phi)$$

Similar for transport coefficients.

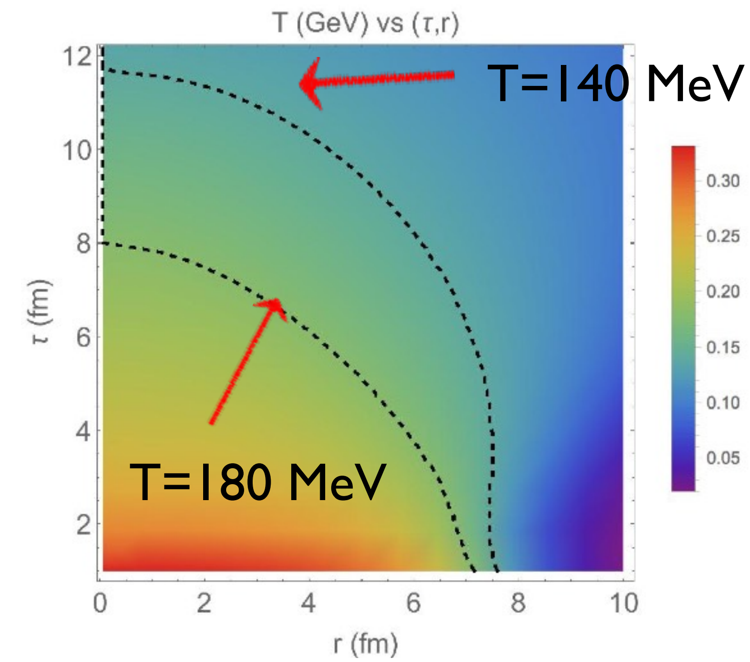
Generalized entropy $s_{(+)}$ and generalized pressure $p_{(+)}$ can be derived, e.g.,

$$\Delta s[\phi] = \frac{1}{2}\int_Q \left[\log(\frac{\phi}{\phi_{eq}}) - \frac{\phi}{\phi_{eq}} + 1\right] + \ldots$$


Fluctuations.
"Lag"
"transport"
E.o.S (and ζ…)
hydro. Flow
"hydro force"

The simulation of Hydro+:

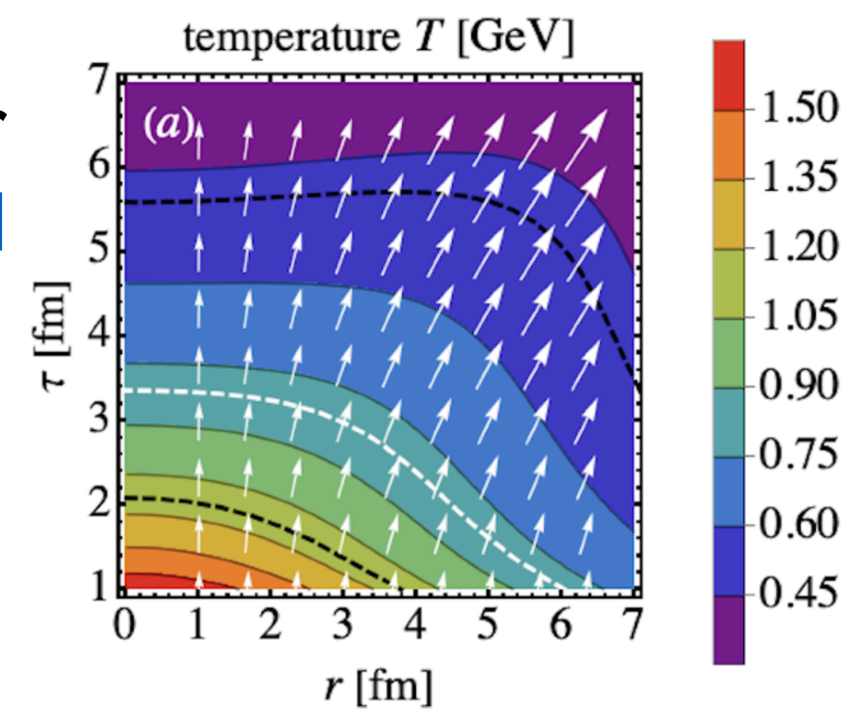
Simulation A: placing a hypothetical C.P. near μ=0 (no eqn for baryon density): showcase the intertwined dynamics.



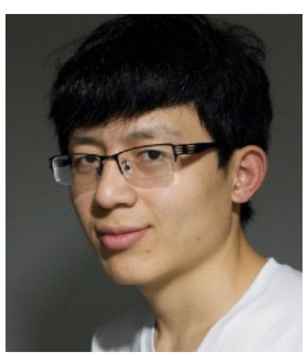
Rajagopal-Ridgway-Weller-YY, 1908.08539

Ridgway, graduate@MIT

Simulation B: Solving eqn. for ϕ on top of Gubser flow at finite μ; "anatomy" of the intertwined dynamics by analytic manipulations.



Lipei Du-Heinz-Rajagopal-YY, 2004.02719

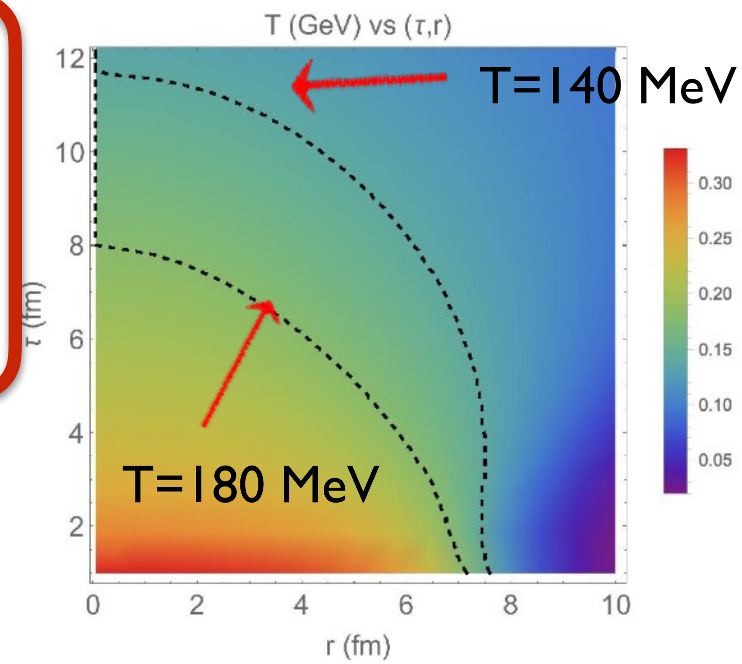


Lipei Du, graduate@OSU

NB: simulations are done in boost-invariant and transverse symmetric flow at relative low baryon density. In future, full 3d simulation at BES energy will be pursued.

(In boost-invariant and transverse symmetric flow.)

Simulation A: placing a hypothetical C.P. near μ=0 (no eqn for baryon density): showcase the intertwined dynamics.



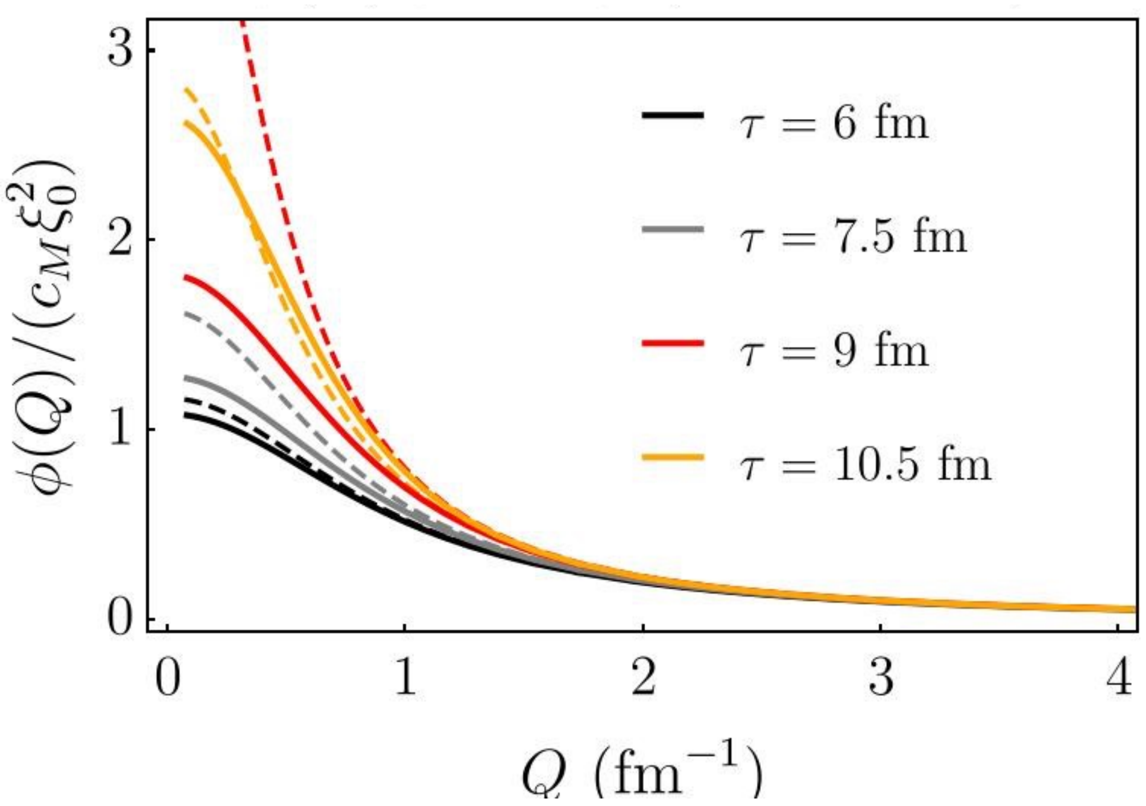
Rajagopal-Ridgway-Weller-YY, 1908.08539

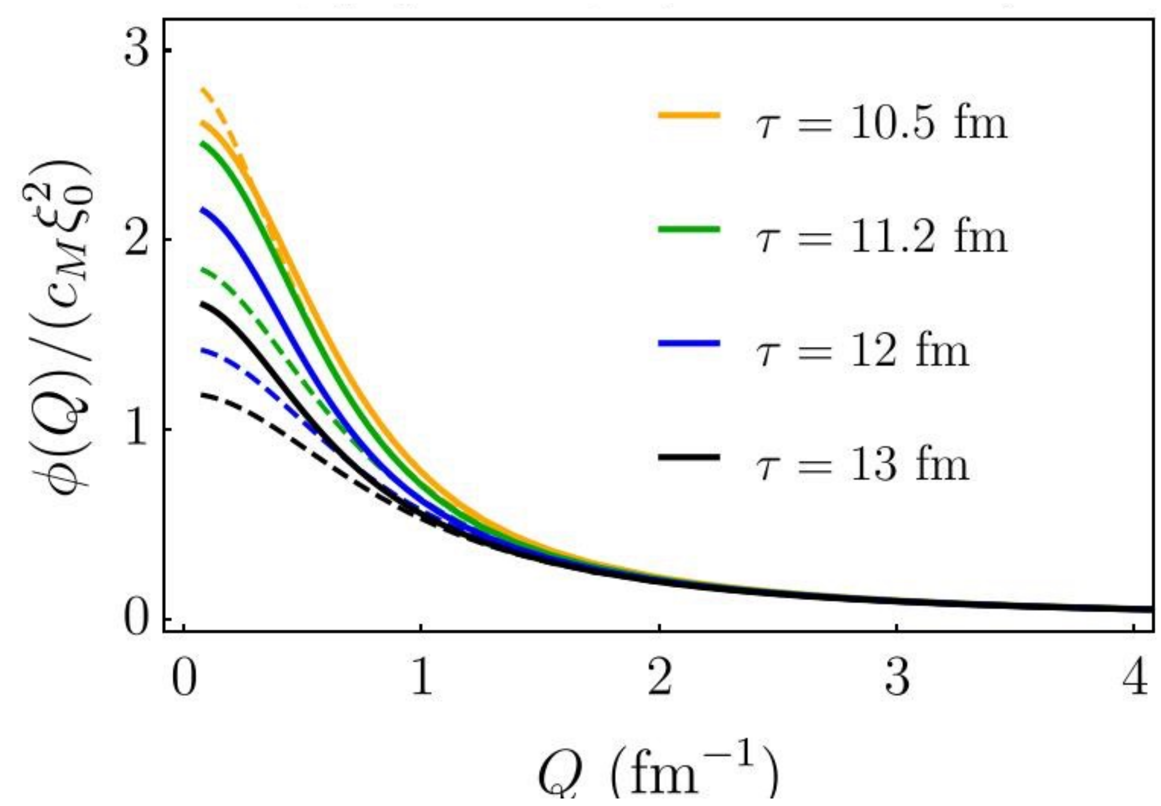
Ridgway, graduate@MIT

Q-dependent off-equilibrium fluctuation at r=1fm.

Off-equilibrium : solid ; Equilibrium: dashed.

See also: Berdnikov-Rajagopal.



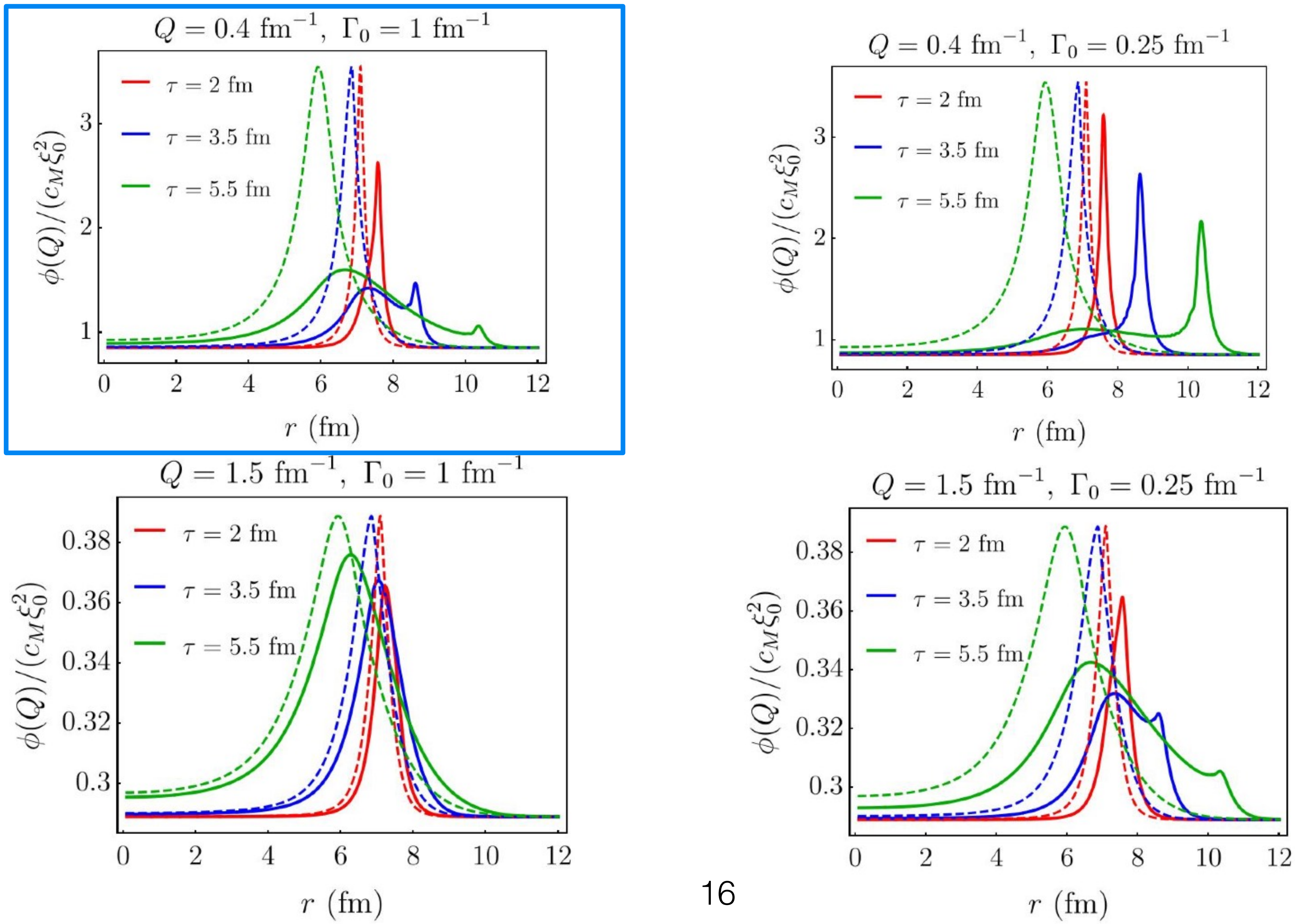


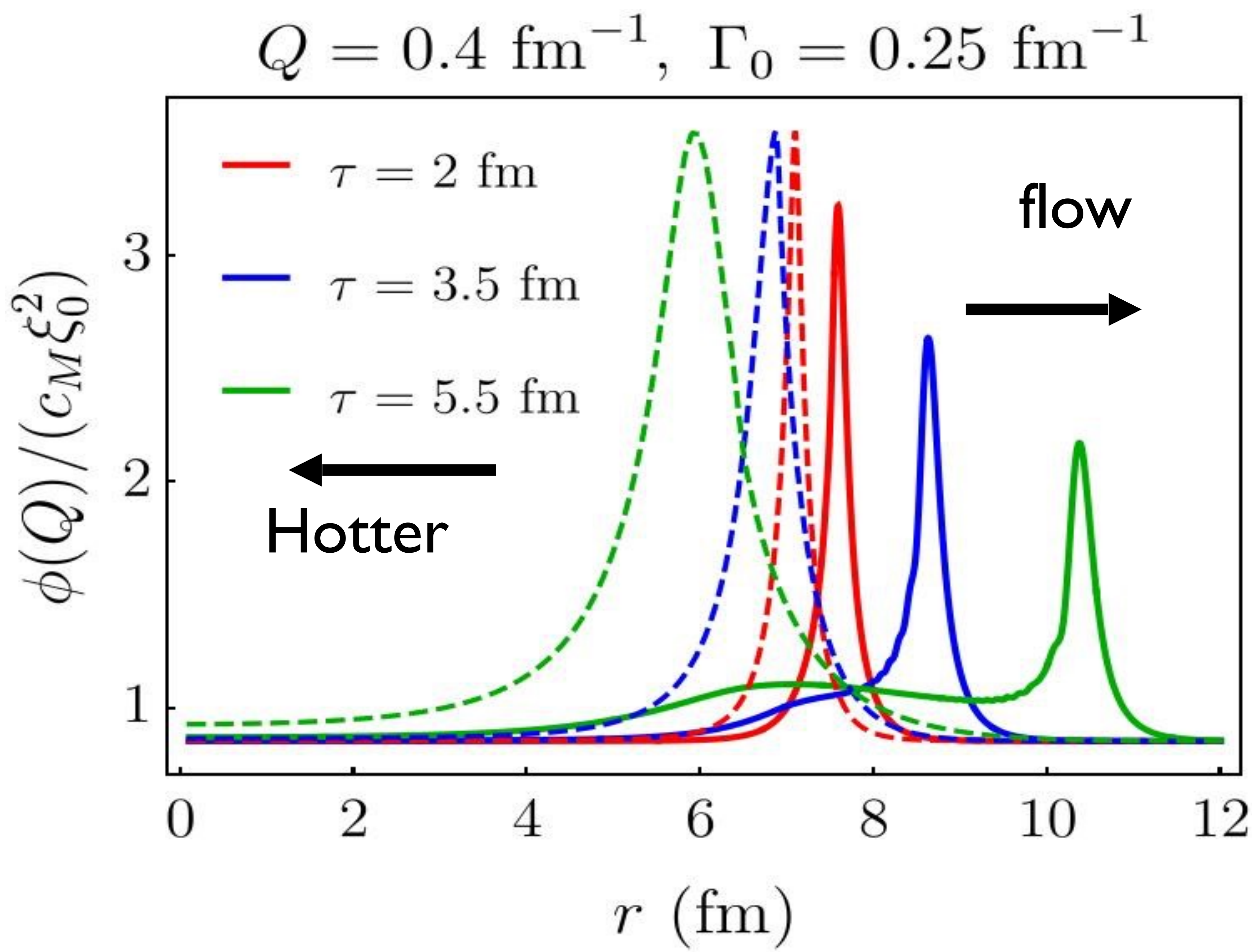
Large Q (shortwave length) modes are in equilibrium while small Q (long wavelength) modes are not.

Critical slowing down leads to the jet-lag of critical fluctuations: the information of the criticality is encoded in offequilibrium effect!

Previously, the "jet-lag" effect has been studied by many others, but mostly for homogeneous expanding fireball.

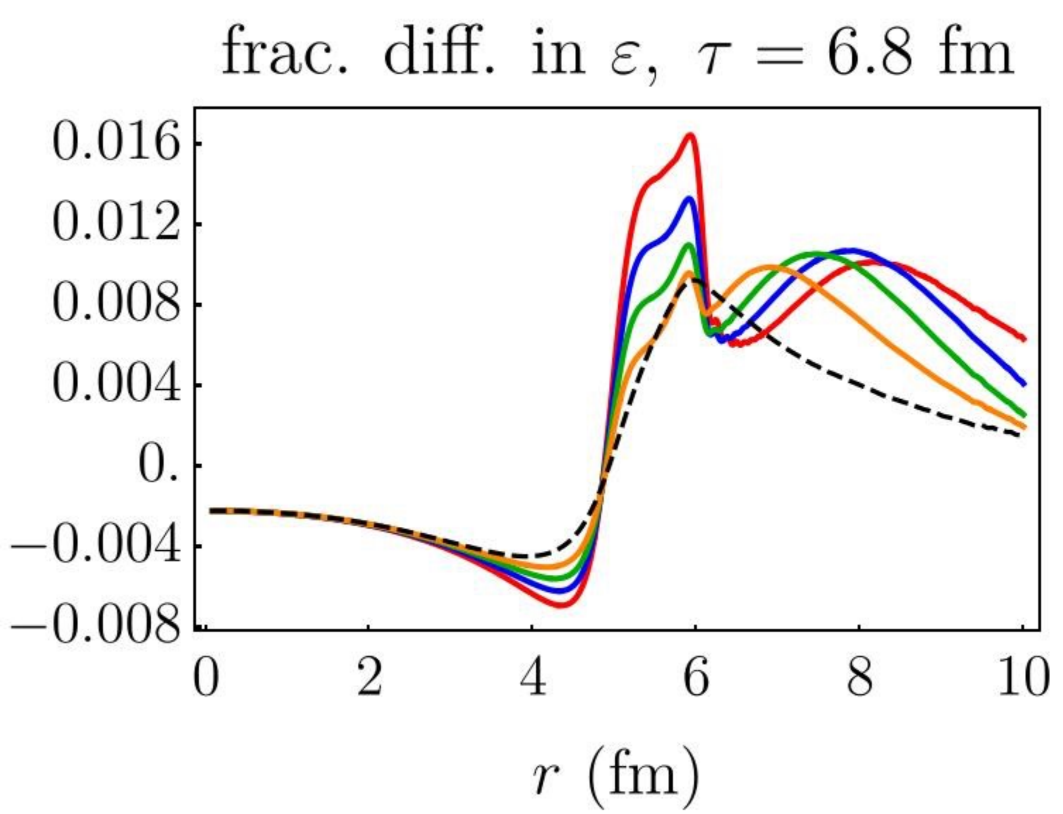
We further watch the nontrivial *spatial distribution* of long wavelength fluctuations.

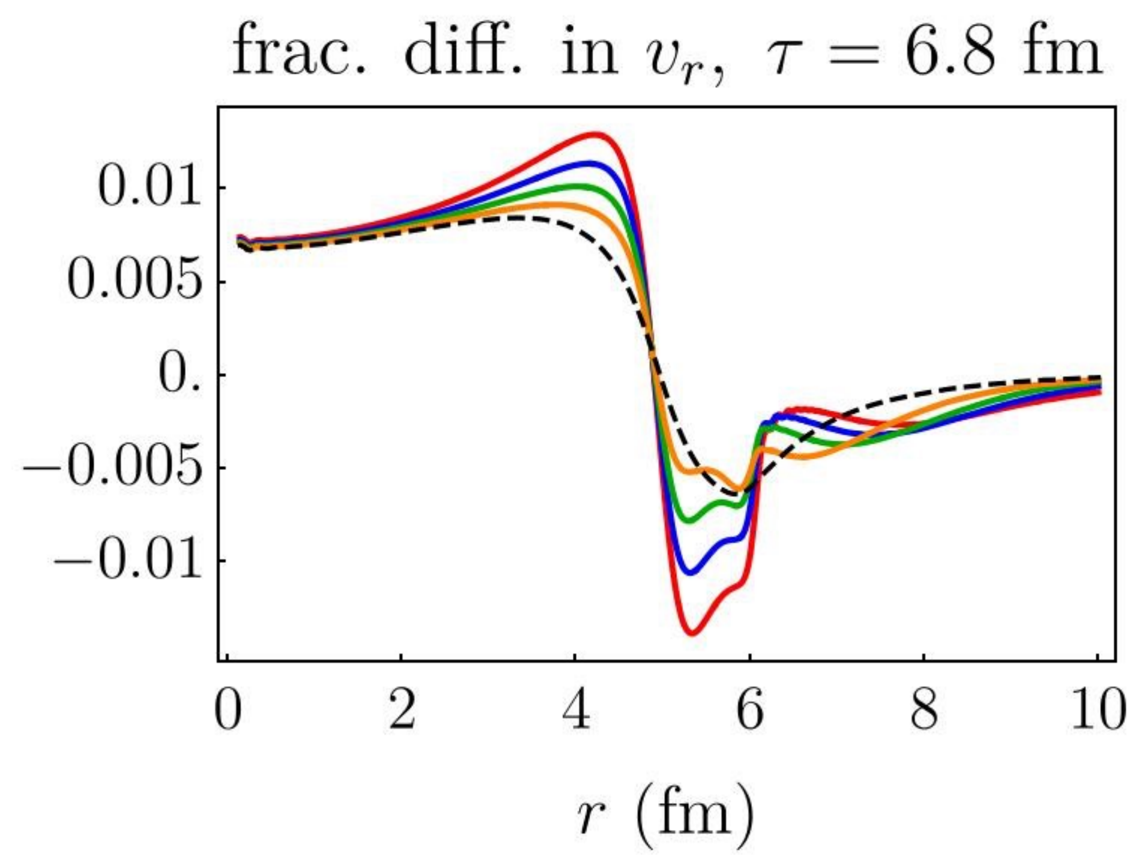




Driven by the critical slowing down and by the advection of the flow.

The snapshot of energy density and radial flow vs r





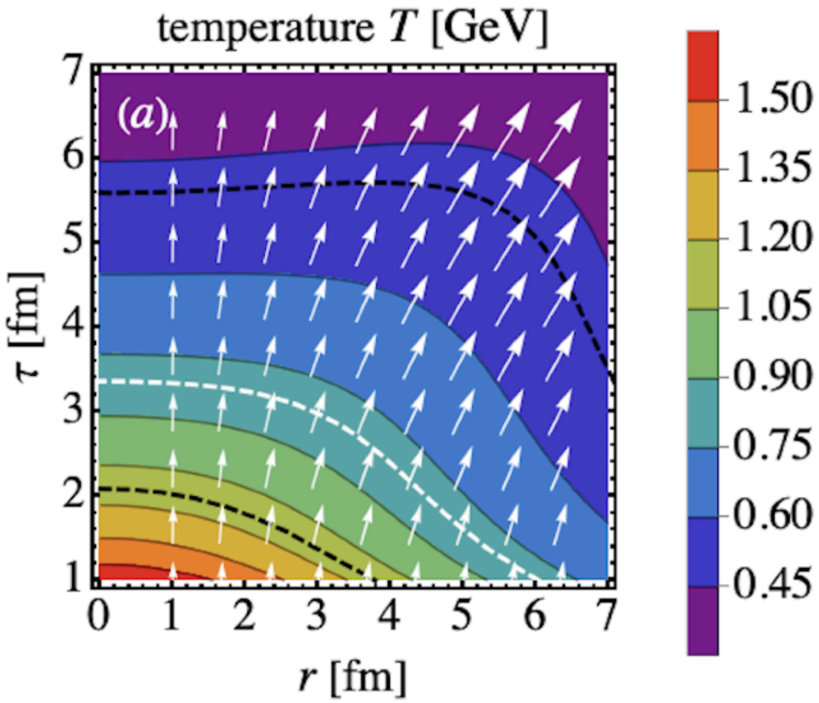
Dashed, no back-reaction.

From: red, blue, green and orange, results including back-reaction with decreasing relaxation rate.

The slower relaxation, the stronger back-reaction.

Within our model, the back-reaction effects are small (within 1 percent).

Simulation B: Solving eqn. for ϕ on top of Gubser flow at finite μ; "anatomy" of the intertwined dynamics by analytic manipulations.



Lipei Du-Heinz-Rajagopal-YY, 2004.02719

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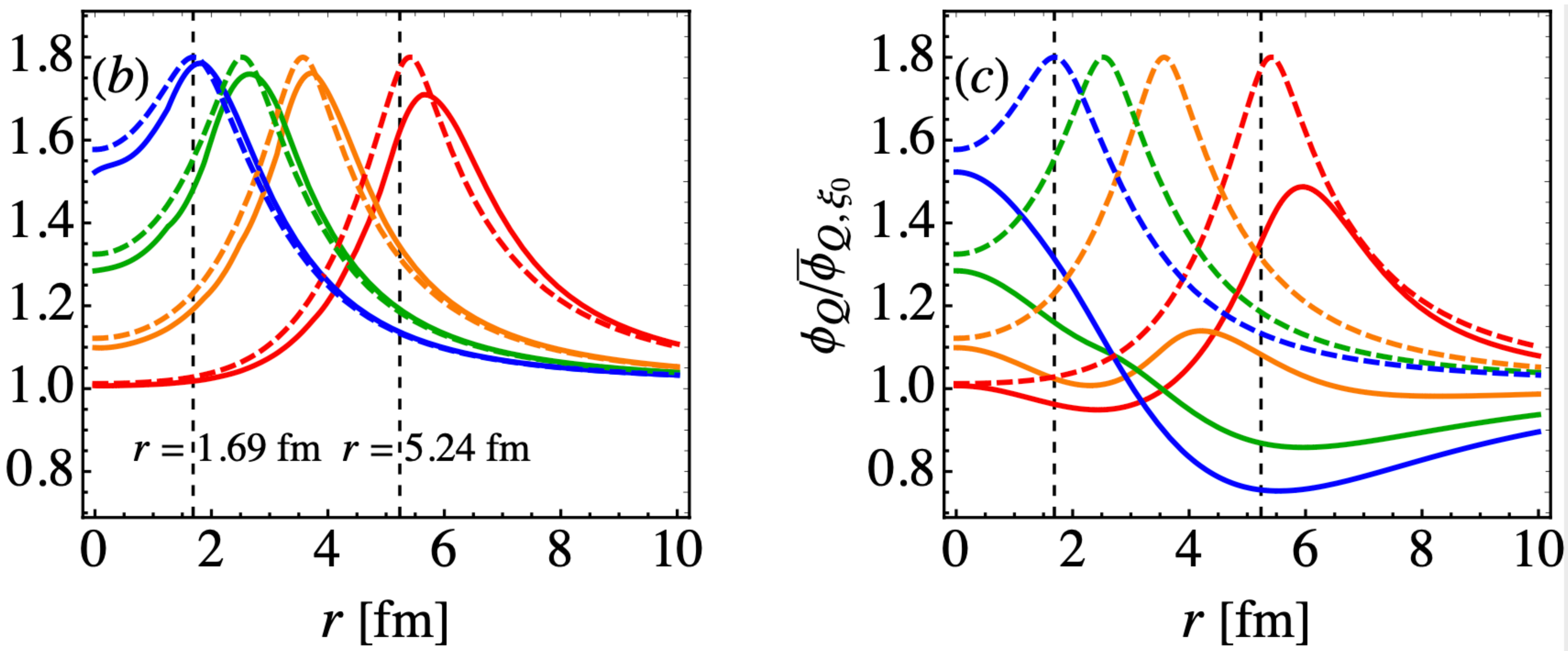
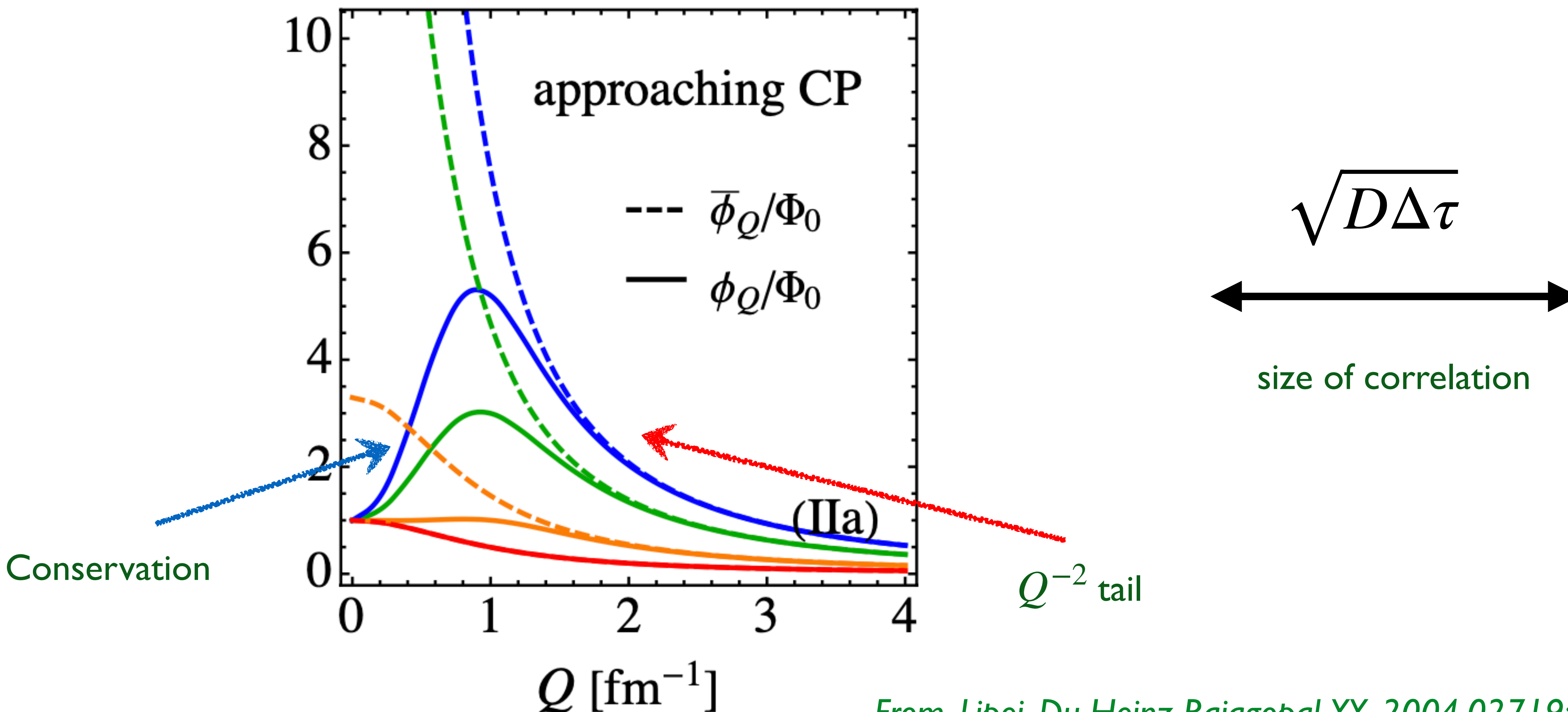
Critical slowing down and advection



Turning off radial flow

Turning on radial flow



From Lipei Du-Heinz-Rajagopal-YY, 2004.02719;see also Akamatsu-Teaney-Yan-Yin, PRC 19'

Hydro+ has the flexibility of incorporating different equilibration dynamics : simulation A assumes an non-conserved order parameter ($\Gamma(Q=0)\neq 0$) while simulation B assumes the conserved one ($\Gamma(Q\to 0)\sim Q^2$)

Diffusive equilibration leads to emergent length scale (off-equilibrium correlation length).

Discussion

Lesson learned: the non-trivial spatial distribution of fluctuation induced by advection.

Small back-reaction => hydro modes decouples from fluctuation modes (i.e."+") => significant simplifications for numerical simulations.

$$\text{backreaction} \propto \frac{Q^3_{\text{non-eq}}}{s} \to 10^{-3} \sim 10^{-4}$$

Ratio between phase space volume of critical modes and micro. d.o.f.

How about off-equilibrium effects on baryon diffusion?

See Lipei Du's talk today

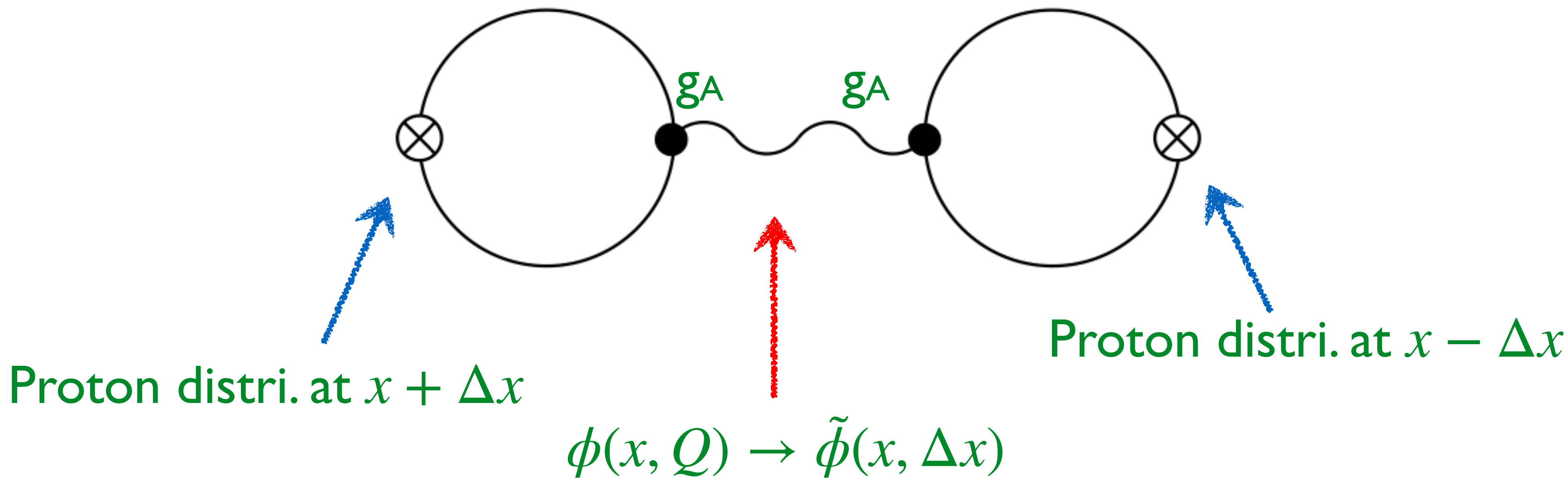
Crucial next step: converting density fluctuations into observable correlations/fluctuations.

Freeze out critical fluctuations

Freezeout of Gaussian fluct.

Pradeep-Rajagopal-Stephanov-Weller-YY, to appear

see Pradeep's talk this Thurs.



$$\langle \delta N_A^2 \rangle_\sigma = g_A^2 Z \int dS_\mu J_A^\mu(x_+) \int dS'_\nu J_A^\nu(x_-) \tilde{\phi}(x, \Delta\tilde{x})$$

(Expression for non-Gaussian cumulants is similar)

Key idea: the fluctuation of the order parameters induce the fluctuation of hadron mass, including proton mass.

Stephanov-Rajagopal-Shuryak 1999'

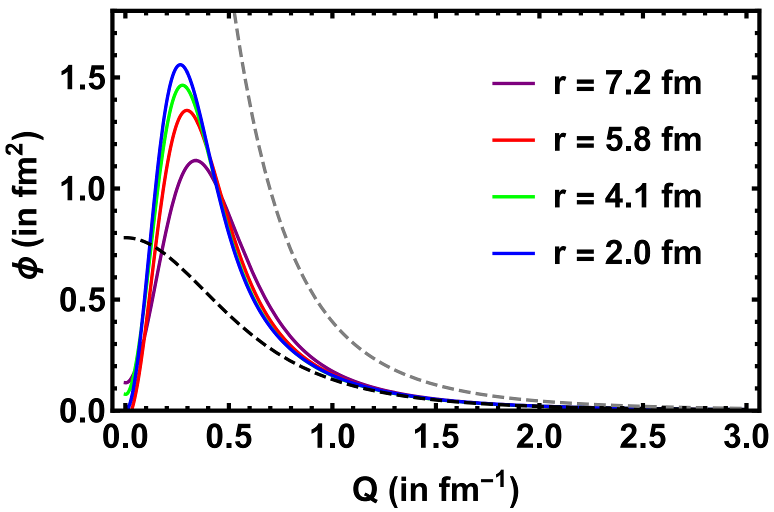
Freezing out non-critical fluctuations is also important.

see Vovchenko' s talk this Tues.

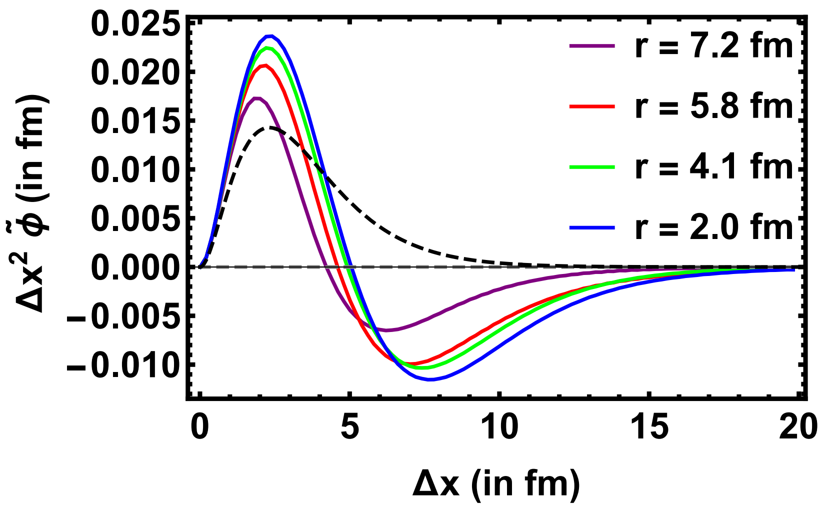
Oliinychenko-Koch PRL 19'; Oliinychenko-Shuzhe Shi-Koch PRC 20'

From hydro+ simulation to freezout

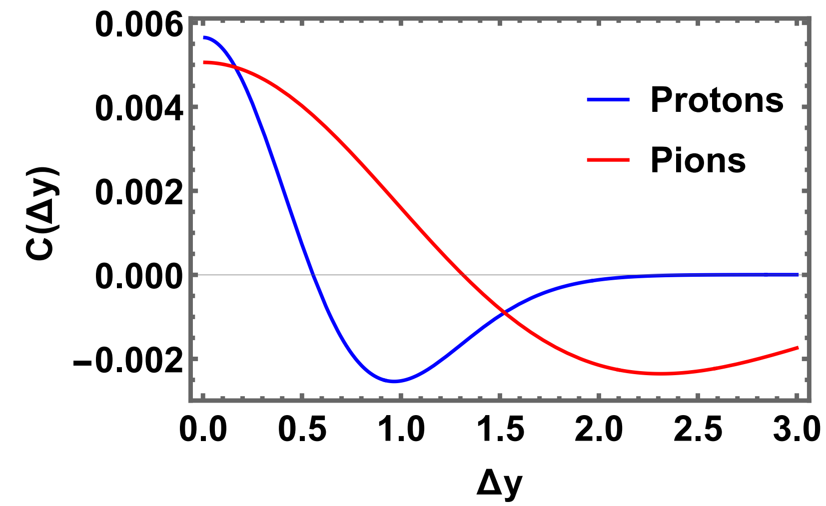
see Pradeep's talk this Thurs.



$\phi(Q)$: from Hydro+ calculation, before freezeout



$\tilde{\phi}(\Delta x)$: correlation in position space, before freezeout

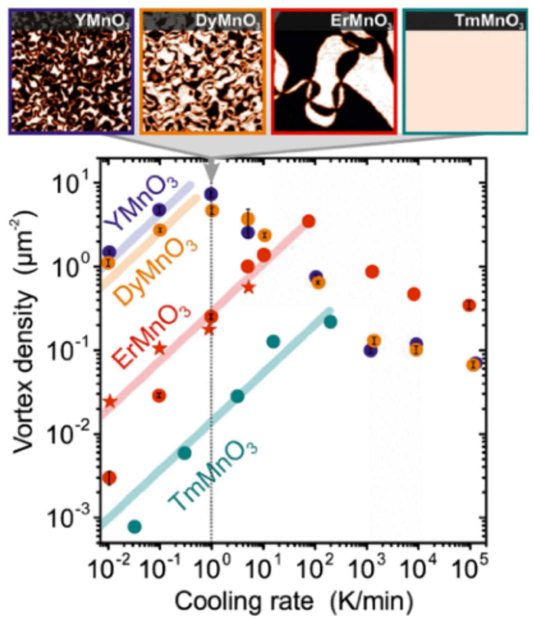


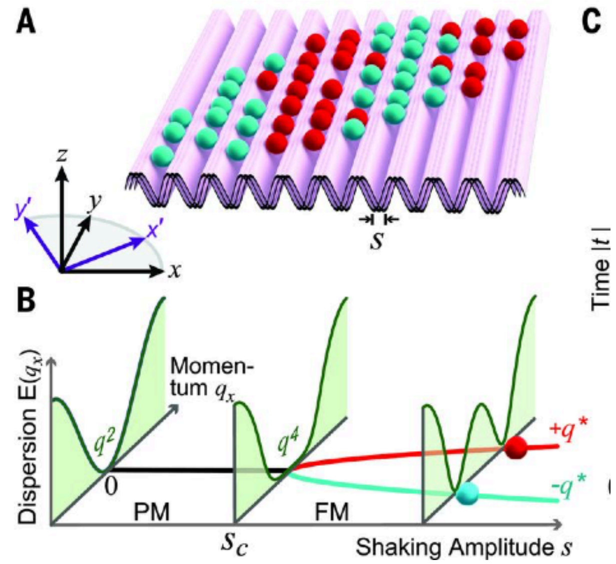
Observable correlation in rapidity, after freezeout.

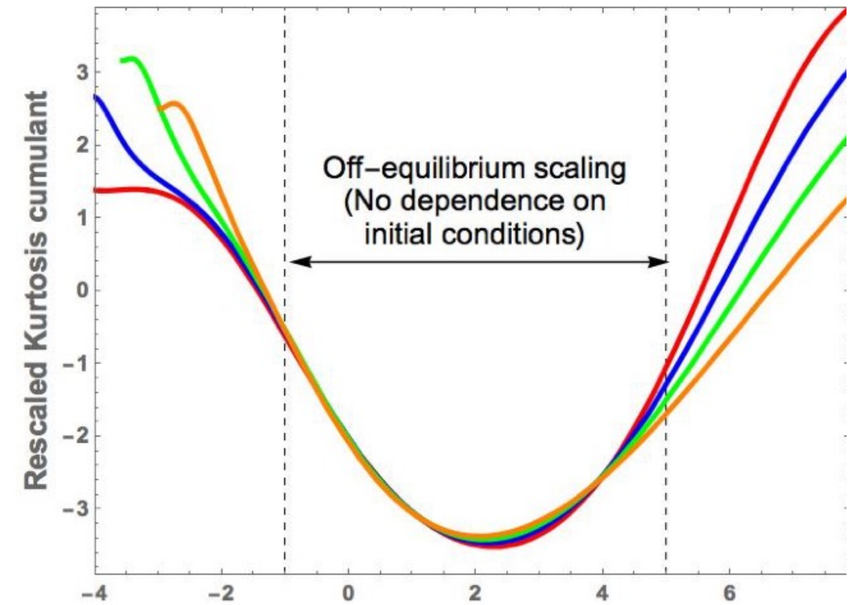
Effects of conservation included in the dynamics, immediately visible in position and rapidity space.

This is an alternative to putting in sub-volume scale by hand; here the charge conservation scale (analogue of sub-volumes) determined directly by the dynamics.

Kibble-Zurek scaling revisited







(Bosons in a shaken optical lattice, W. Clark et al, Science' 16)

Hexagonal Manganites, M. Griffin et al, Phys.Rev.X ' 12

(Conjecture for the QCD point, Mukherjee, Venugopalan and YY, PRL, 16.)

Kibble-Zurek scaling: critical fluctuations and correlations scale with off-equilibrium correlation length.

Because of the advances in dynamical modeling and upcoming high statistic BESII data, we have good chance to test the scaling hypothesis for QCD critical point explicitly.

Once observed, a beautiful demonstration of the unity of physics.

Conclusion

Summary and outlook

The evolution and freeze out of critical fluctuations are ready for implementation in realistic simulation.

Future challenges: non-Gaussian fluctuations? first-order transitions?

Broad view: we are embracing the quantitative era of BES dynamics.

Other applications of fluctuation dynamics: scale-dependent transport properties; hydrodynamization/thermalization.

see Chris Lau's talk this Tues

Not covered in this talk: the potential of spin polarization in probing the phase boundary where the change of "spin carriers" happens.

Shuai Liu-YY, 2103.09200; Baochi Fu-Shuai Liu-Huichao Song-Longgang Pang-YY, to appear.

Many exciting physics ahead!

see Huichao Song's talk this Friday

Back-up