

Baryon diffusion away from and close to the QCD critical point

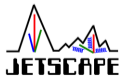
Lipei Du

Department of Physics, The Ohio State University, USA

with Xin An (UNC) and Ulrich Heinz (OSU)

International conference on Critical Point and Onset of Deconfinement 2021

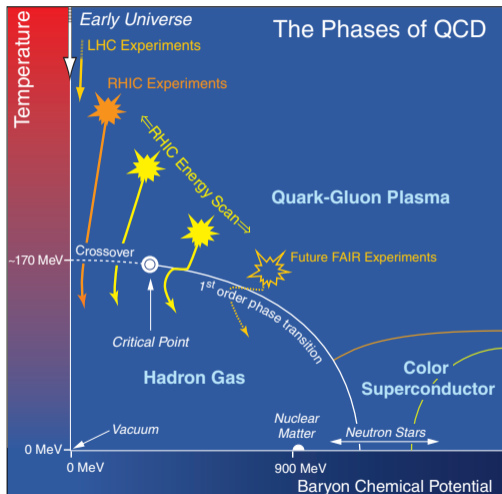
March 17, 2021



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- Hydrodynamics with baryon diffusion current
 - Longitudinal dynamics of baryon density
 - Trajectories in the phase diagram at different space-time rapidities
 - Critical effects on baryon evolution
- Conclusions

Motivation

Exploring the QCD phase diagram with heavy-ion collisions



2007 NSAC Long Range Plan

Cumulants of proton multiplicity at various beam energies

- A telltale **signature of QCD critical point**: non-monotonic dependence of higher-order cumulants on beam energies [M. Stephanov, 0809.3450 and 1104.1627];

(left) Normalized quartic cumulant of proton multiplicity as a function of $\sqrt{s_{NN}}$ (equivalently, collision energy $\sqrt{s_{NN}}$) [M. Stephanov, 0809.3450 and 1104.1627; A. Bzdak et al, 1906.00936]; (right) STAR measurement of κ_4 at BES energies [PRL 126, 092301 (2021)].

Cumulants of proton multiplicity at various beam energies

A telltale **signature of QCD critical point**: non-monotonic dependence of higher-order cumulants on beam energies. [M. Stephanov, 0809.3450 and 1104.1627]

(left) Normalized quartic cumulant of proton multiplicity as a function of (equivalently, collision energy $\sqrt{s_{NN}}$) [M. Stephanov, 0809.3450 and 1104.1627; A. Bzdak et al, 1906.00936]; (right) STAR measurement at BES energies [PRL 126, 092301 (2021)].

Systematic model-data comparison needs **quantitative calculations of μ -equilibrium critical fluctuations**, which require a **calibrated multi-stage theoretical framework**.

Question: At low beam energies, does the critical point have non-negligible effects on the bulk dynamics of the reballs?

If yes, calibration of the bulk dynamics needs to consider critical effects (for bulk viscosity, see [A. Monnai, S. Mukherjee and Y. Yin, 1606.00711](#));

If no, critical effects can be neglected in the calibration.

This talk focuses on [critical effects on baryon di usion](#).

Hydrodynamics with baryon di usion current

The conservation laws for energy, momentum and the baryon charge are

$$d T = 0; \text{ with } T = e u + (p +) + ;$$

$$d N = 0; \text{ with } N = n u + n :$$

The conservation laws for energy, momentum and the baryon charge are

$$\begin{aligned} d T &= 0; & \text{with } T &= \epsilon u + (p - \mathcal{Z}) \quad \mathcal{H} \quad \mathcal{H}; \\ d N &= 0; & \text{with } N &= n u + \mathbf{n} : \end{aligned}$$

The relaxation equation for baryon diffusion:

$$u \cdot \nabla n = \frac{1}{n} \nabla n \cdot \mathbf{r} \quad \mathbf{T} \quad \mathbf{i} + \dots;$$

where the **baryon diffusion coefficient**, n , controls the response of diffusion current to **the driving force**, i.e., the Navier-Stokes limit of baryon diffusion

$$n_{NS} \quad \mathbf{n} \quad \mathbf{T} ;$$

and **relaxation time** τ_n controls how fast the relaxation happens.

An active topic: see e.g. [A. Monnai, 1204.4713](#); [C. Shen et al, 1704.04109](#); [G. Moritz et al, 1711.08680](#); [G. Denicol et al, 1804.1055](#) and [C. Shen, 1809.04034](#); [L. Du and U. Heinz, 1906.11181](#); [J. Fotakis et al, 1912.09103](#)

Near the QCD critical point [Hohenberg and Halperin, Rev. Mod. Phys. 1977]

$$\chi \sim \xi^{-n};$$

and we use the following parametrization:

$$\chi = \chi_{n;0} \xi^{-n};$$

where ξ_0 is the non-critical correlation length, $\chi_{n;0}$ is the non-critical value of baryon diffusion coefficient.

Critical behavior: transport coefficients

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Based on the **Hydro++ framework**, the slowest mode contributing to the diffusive-shear two-point correlator, i.e. $G_{hh}(s, n) \sim \chi_{n;0} \xi^{-n}$ [X. An et al, 1912.13456], thus we expect

$$\chi \sim G \xi^{-2};$$

and use the parametrization

$$\chi = \chi_{n;0} \xi^{-2};$$

where $\chi_{n;0}$ is the non-critical relaxation time.

Critical behavior: Equation of State

We rewrite the Navier-Stokes limit

$$n_{NS} \quad n^r \quad (= T) = D_B r \quad n + D_T r \quad T;$$

where

$$D_B = \frac{n}{T}; \quad D_T = \frac{n}{Tn} \quad \frac{\partial n}{\partial T} \quad \frac{e+p}{T} :$$

Here $\frac{\partial n}{\partial T}$ is the isothermal susceptibility

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Here χ is the isothermal susceptibility

$$\chi = \left(\frac{\partial n}{\partial \mu} \right)_T ;$$

Since ² [Hohenberg and Halperin *Rev. Mod. Phys.* 1977], we apply the following parametrization for :

$$\chi = \chi_0 \frac{1}{t} ;$$

where χ_0 is the isothermal susceptibility evaluated in the non-critical region.

We use the NEOS [A. Monnai et al, 1902.05095](#). One can also use an EoS exhibiting singularities to incorporate contribution from a critical point, see e.g. [G. Parotto et al, 1805.05249](#)

Setup of the framework

We use transport coefficients from a kinetic approach [S. Denicol et al, 1804.10557] for non-critical values:

$$n_{i,0} = C_n \frac{n}{T} \left[\frac{1}{3} \coth \frac{p}{T} - \frac{nT}{e+p} \right] ; \quad n_{i,0} = \frac{C_n}{T} ;$$

where C_n is a free parameter, and $C_n \ll 1$.

For the longitudinal initial profile at $\sqrt{s_{NN}} = 196 \text{ GeV}$, we use [S. Denicol et al, 1804.10557]

Results and discussion

Baryon evolution: longitudinal dynamics

Longitudinal dynamics with and without baryon diffusion current (no critical effects, (1+1)-dimensional system, using BEShydro [L. Du and U. Heinz, 1906.11181])

Dynamics in longitudinal direction with and without baryon diffusion current

Longitudinal dynamics with and without baryon diffusion current (no critical effects, (1+1)-dimensional system, using BEShydro [L. Du and U. Heinz, 1906.11181])

Dynamics in longitudinal direction with and without baryon diffusion current

Baryon diffusion leaves no pronounced signatures in the evolution of the temperature (energy density) but **smoothes out gradients in baryon chemical potential (baryon density)**.

See also, e.g. [C. Shen et al, 1704.04109; G. Denicol et al, 1804.10557; M. Li and C. Shen, 1809.04034; L. Du and U. Heinz, 1906.11181 et al, 1912.09103]

Phase diagram trajectories at different rapidities: without critical effects

At different η_s , the reball locates at different $(\mu; T)$ and follows different trajectories:

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At different \sqrt{s} , the reball locates at different $(\mu; T)$ and follows different trajectories:

A reball corresponds to [a set of phase diagram trajectories](#), scanning different regions of the phase diagram and baryon di usion introduces interactions among them.

See hydrodynamic studies on trajectories $(\mu; T)$: e.g. [C. Nonaka and M. Asakawa, PRC71, 044904 (2005); C. Shen et al, 1704.04109; A. Monnai et al, 1606.00771; T. Dore et al, 2007.15083; A. Monnai et al, 2101.11591;]

Phase diagram trajectories at different rapidities: with critical effects

Put a critical point at $\mu_c = 250$ MeV and $T_c = 149$ MeV ($m = 0 = 10^3$ parametrization (μ ; T)):

No effect is seen from the critical point. Why?

Phase diagram trajectories at different rapidities: with critical effects

Put a critical point at $\mu_c = 250$ MeV and $T_c = 149$ MeV ($m = 0 = 10$ parametrization ($\mu; T$)):

No effect is seen from the critical point. Why? Note: Maximum $\mu = 2.5$ fm, $n = 6$ $n; 0$; $n = 2.5$ $n; 0$.

Longitudinal component of **baryon di usion current grows and decays quickly**;

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Correlation length grows at the late stage of the evolution, when the system enters the critical regime;

Critical e ect couldn't manifest itself through baryon di usion which already approaches zero.

With the reball cooling down, the driving force of di usion current ($n_r (= T)$) decreases:
Two reasons: (a) gradient ($= T$) gets smoothened; (b) n decreases.

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Response to the driving force also gets slower, because of the growing relaxation time;

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Two reasons: (a) gradient ($= T$) gets smoothened; (b) n decreases.

Response to the driving force also gets slower, because of the growing relaxation time;

Critical slowing-down ($n \sim 6_{n;0}$) would help n to stay at (almost) zero, even if $n_r (= T)$ got affected by the critical point.

Critical effect on the net proton distribution

Final particle distribution has **0-equilibrium correction** from baryon diffusion current. [G. Denicol et al, 1804.10557; M. McNelis et al, 1912.08271; M. McNelis, U. Heinz, 2103.03401]

$$f_{\text{diffusion}}^i = f_{\text{eq}}^i \left(1 - f_{\text{eq}}^i \frac{n}{e+p} - \frac{b_i}{u-p_i} \frac{p_i n}{n} \right);$$

Critical effects through critical scaling $n = n^*$;

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Critical effects through critical scaling $n = n^*$;

Net proton distribution in rapidity with and without critical effects (central cell only, using iS3D [M. McNelis et al, 1912.08271])

Final particle distribution is not strongly affected, because of the small diffusion current on the freeze-out surface.

Conclusions

A reball corresponds to a set of trajectories, scanning different regions of the phase diagram, and baryon diusion introduces interactions among them;

Baryon diusion grows and decays at the early stage of the evolution, before the system enters the critical regime, at the late stage of the evolution;

Critical effects on baryon diusion during the hydrodynamic stage, and on particle distribution at freeze-out, via diusion's viscous correction, are found to be small.

Thank you very much!

Parametrization of correlation length

- Critical point at ($\mu_c = 250\text{MeV}; T_c = 149\text{MeV}$) and $m = 0 = 10$:

