Baryon diffusion away from and close to the QCD critical point

Lipei Du

Department of Physics, The Ohio State University, USA

with Xin An (UNC) and Ulrich Heinz (OSU)

International conference on Critical Point and Onset of Deconfinement 2021

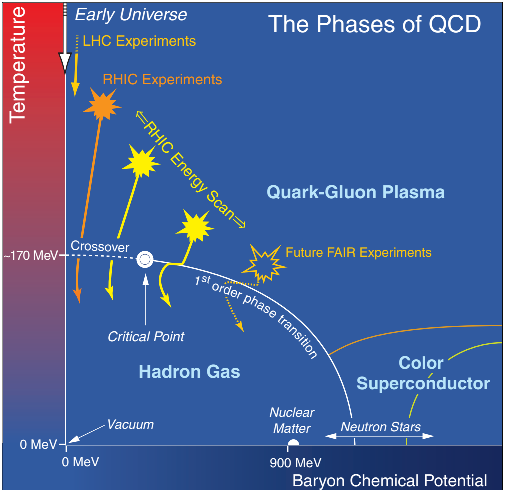
March 17, 2021

Outline

- Motivation
- Hydrodynamics with baryon diffusion current
    - Longitudinal dynamics of baryon density
    - Trajectories in the phase diagram at different space-time rapidities
    - Critical effects on baryon evolution
- Conclusions

Motivation

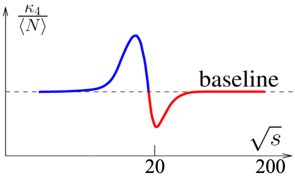
Exploring the QCD phase diagram with heavy-ion collisions

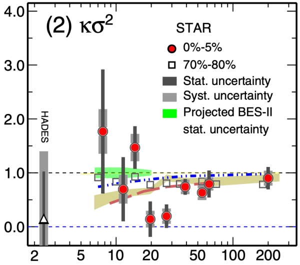


2007 NSAC Long Range Plan

Cumulants of proton multiplicity at various beam energies

- A telltale signature of QCD critical point: non-monotonic dependence of higher-order cumulants on beam energies [M. Stephanov, 0809.3450 and 1104.1627];

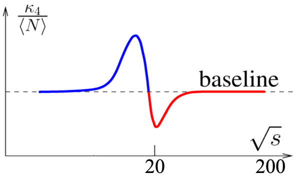


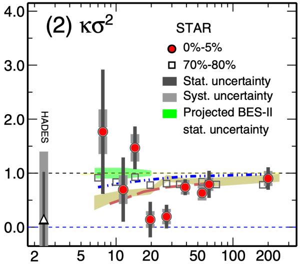


(left) Normalized quartic cumulant of proton multiplicity as a function of μ (equivalently, collision energy $\sqrt{s_{NN}}$) [M. Stephanov, 0809.3450 and 1104.1627; A. Bzdak et al, 1906.00936]; (right) STAR measurement of $\kappa\sigma^2$ at BES energies [PRL 126, 092301 (2021)].

Cumulants of proton multiplicity at various beam energies

- A telltale signature of QCD critical point: non-monotonic dependence of higher-order cumulants on beam energies [M. Stephanov, 0809.3450 and 1104.1627];





(left) Normalized quartic cumulant of proton multiplicity as a function of μ (equivalently, collision energy $\sqrt{s_{NN}}$) [M. Stephanov, 0809.3450 and 1104.1627; A. Bzdak et al, 1906.00936]; (right) STAR measurement of $\kappa\sigma^2$ at BES energies [PRL 126, 092301 (2021)].

- Systematic model-data comparison needs quantitative calculations of off-equilibrium critical fluctuations, which require a calibrated multi-stage theoretical framework.

Constraining a multi-stage theoretical framework

- Question: At low beam energies, does the critical point have non-negligible effects on the bulk dynamics of the fireballs?
  - If yes, calibration of the bulk dynamics needs to consider critical effects (for bulk viscosity, see [A. Monnai, S. Mukherjee and Y. Yin, 1606.00771]);
  - If no, critical effects can be neglected in the calibration.
- This talk focuses on critical effects on baryon diffusion.

Hydrodynamics with baryon diffusion current

Baryon evolution and diffusion current

- The conservation laws for energy, momentum and the baryon charge are

$$
\begin{aligned}
d_\mu T^{\mu\nu} &= 0\,, \quad \text{with} \quad T^{\mu\nu} = e u^\mu u^\nu - (p+\Pi)\Delta^{\mu\nu} + \pi^{\mu\nu}\,,\\
d_\mu N^{\mu} &= 0\,, \quad \text{with} \quad N^{\mu} = n u^\mu + n^\mu\,.
\end{aligned}
$$

Baryon evolution and diffusion current

- The conservation laws for energy, momentum and the baryon charge are

$$\begin{aligned} d_\mu T^{\mu\nu} &= 0\,, \quad \text{with} \quad T^{\mu\nu} = eu^\mu u^\nu - (p + \cancel{\Pi})\Delta^{\mu\nu} + \cancel{\pi^{\mu\nu}}\,, \\ d_\mu N^\mu &= 0\,, \quad \text{with} \quad N^\mu = nu^\mu + n^\mu\,. \end{aligned}$$

- The relaxation equation for baryon diffusion:

$$u^\nu \partial_\nu n^\mu = -\frac{1}{\tau_n}\left[n^\mu - \kappa_n \nabla^\mu\left(\frac{\mu}{T}\right)\right] + \ldots\,,$$

where the baryon diffusion coefficient, κ_n, controls the response of diffusion current to the driving force, i.e., the Navier-Stokes limit of baryon diffusion

$$n^\mu_{\mathrm{NS}} \equiv \kappa_n \nabla^\mu\left(\frac{\mu}{T}\right)\,,$$

and relaxation time τ_n controls how fast the relaxation happens.

- An active topic: see e.g. [A. Monnai, 1204.4713; C. Shen et al, 1704.04109; G. Moritz et al, 1711.08680; G. Denicol et al, 1804.10557; M. Li and C. Shen, 1809.04034; L. Du and U. Heinz, 1906.11181; J. Fotakis et al, 1912.09103].

Critical behavior: transport coefficients

- Near the QCD critical point [Hohenberg and Halperin, *Rev. Mod. Phys.*, 1977]

$$\kappa_n \sim \xi\,,$$

and we use the following parametrization:

$$\kappa_n = \kappa_{n,0}\left(\frac{\xi}{\xi_0}\right),$$

where ξ_0 is the non-critical correlation length, $\kappa_{n,0}$ is the non-critical value of baryon diffusion coefficient.

Critical behavior: transport coefficients

- Near the QCD critical point [Hohenberg and Halperin, *Rev. Mod. Phys.*, 1977]

$$\kappa_n \sim \xi\,,$$

and we use the following parametrization:

$$\kappa_n = \kappa_{n,0}\left(\frac{\xi}{\xi_0}\right),$$

where ξ_0 is the non-critical correlation length, $\kappa_{n,0}$ is the non-critical value of baryon diffusion coefficient.

- Based on the Hydro++ framework, the slowest mode contributing to n^μ is the diffusive-shear two-point correlator, i.e., $G \sim \langle \delta(s/n)\delta u_\mu \rangle$ [X. An et al, 1912.13456]; thus we expect

$$\tau_n \sim \tau_G \sim \xi^2\,,$$

and use the parametrization

$$\tau_n = \tau_{n,0}\left(\frac{\xi}{\xi_0}\right)^2,$$

where $\tau_{n,0}$ is the non-critical relaxation time.

Critical behavior: Equation of State

- We rewrite the Navier-Stokes limit

$$n^\mu_{\rm NS} \equiv \kappa_n \nabla^\mu(\mu/T) = D_B \nabla^\mu n + D_T \nabla^\mu T\,,$$

where

$$D_B = \frac{\kappa_n}{T\chi}, \quad D_T = \frac{\kappa_n}{Tn}\left[\left(\frac{\partial p}{\partial T}\right)_n - \frac{e+p}{T}\right].$$

Here χ is the isothermal susceptibility

$$\chi \equiv \left(\frac{\partial n}{\partial \mu}\right)_T.$$

Critical behavior: Equation of State

- We rewrite the Navier-Stokes limit

$$n^\mu_{\rm NS} \equiv \kappa_n \nabla^\mu(\mu/T) = D_B \nabla^\mu n + D_T \nabla^\mu T\,,$$

where

$$D_B = \frac{\kappa_n}{T\chi}, \quad D_T = \frac{\kappa_n}{Tn}\left[\left(\frac{\partial p}{\partial T}\right)_n - \frac{e+p}{T}\right].$$

Here χ is the isothermal susceptibility

$$\chi \equiv \left(\frac{\partial n}{\partial \mu}\right)_T.$$

- Since $\chi \sim \xi^2$ [Hohenberg and Halperin, *Rev. Mod. Phys.*, 1977], we apply the following parametrization for χ:

$$\chi = \chi_0 \left(\frac{\xi}{\xi_0}\right)^2,$$

where χ_0 is the isothermal susceptibility evaluated in the non-critical region.

- We use the NEOS [A. Monnai et al, 1902.05095]. One can also use an EoS exhibiting singularities to incorporate contribution from a critical point, see e.g. [P. Parotto et al, 1805.05249].

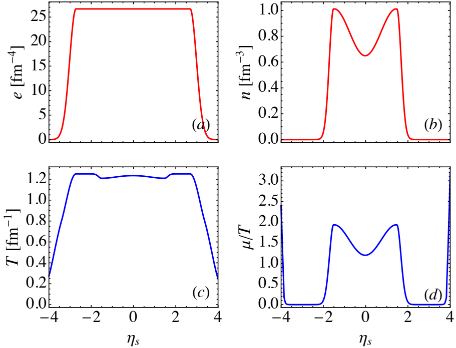
Setup of the framework

- We use transport coefficients from a kinetic approach [G. Denicol et al, 1804.10557] for non-critical values:

$$\kappa_{n,0} = C_n \frac{n}{T} \left(\frac{1}{3} \coth\left(\frac{\mu}{T}\right) - \frac{nT}{e+p} \right), \quad \tau_{n,0} = \frac{C_n}{T},$$

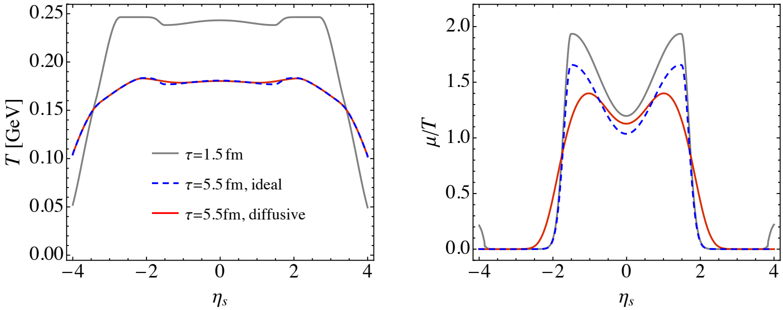
where C_n is a free parameter, and $C_n \sim \mathcal{O}(1)$.

- For the longitudinal initial profile at $\sqrt{s_{NN}} = 19.6$ GeV, we use [G. Denicol et al, 1804.10557]



Results and discussion

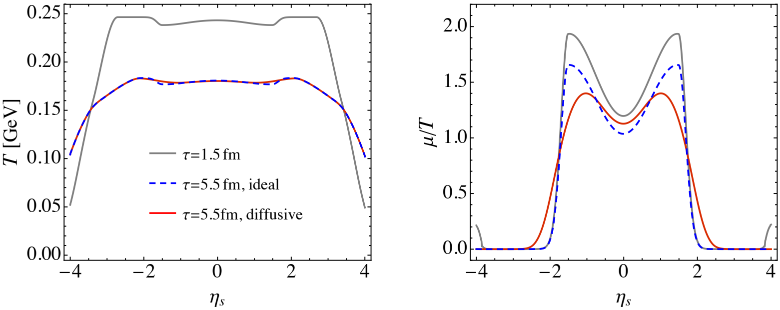
- Longitudinal dynamics with and without baryon diffusion current (no critical effects, (1+1)-dimensional system, using BEShydro [L. Du and U. Heinz, 1906.11181])



Dynamics in longitudinal direction with and without baryon diffusion current

Baryon evolution: longitudinal dynamics

- Longitudinal dynamics with and without baryon diffusion current (no critical effects, (1+1)-dimensional system, using BEShydro [L. Du and U. Heinz, 1906.11181])

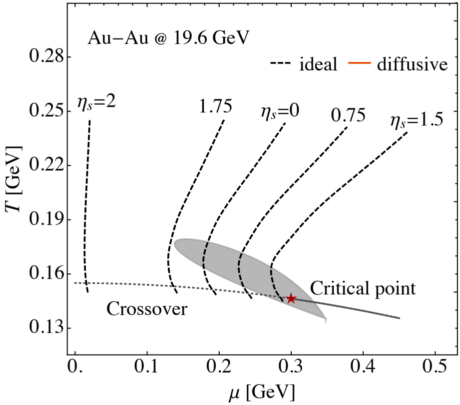


Dynamics in longitudinal direction with and without baryon diffusion current

- Baryon diffusion leaves no pronounced signatures in the evolution of the temperature (energy density) but smoothes out gradients in baryon chemical potential (baryon density).
- See also, e.g. [C. Shen et al, 1704.04109; G. Denicol et al, 1804.10557; M. Li and C. Shen, 1809.04034; L. Du and U. Heinz, 1906.11181; J. Fotakis et al, 1912.09103].

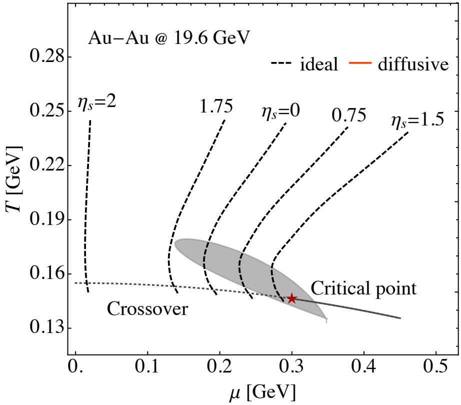
Phase diagram trajectories at different rapidities: without critical effects

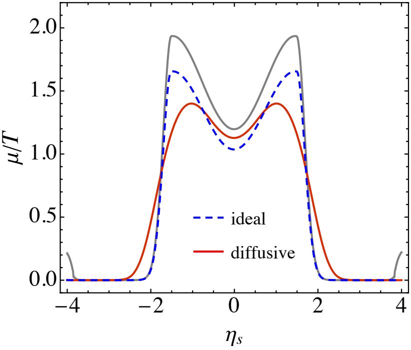
- At different η_s, the fireball locates at different (μ, T) and follows different trajectories:



Phase diagram trajectories at different rapidities: without critical effects

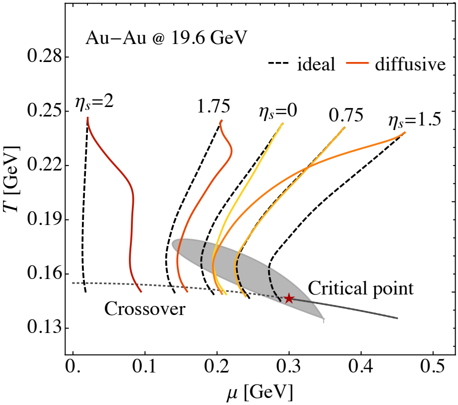
- At different η_s, the fireball locates at different (μ, T) and follows different trajectories:

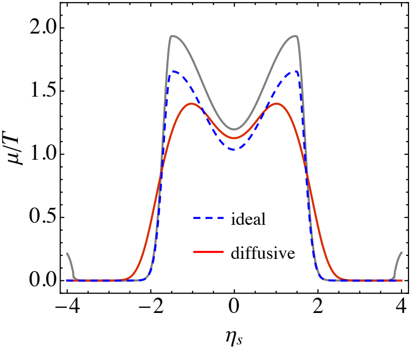




Phase diagram trajectories at different rapidities: without critical effects

- At different η_s, the fireball locates at different (μ, T) and follows different trajectories:

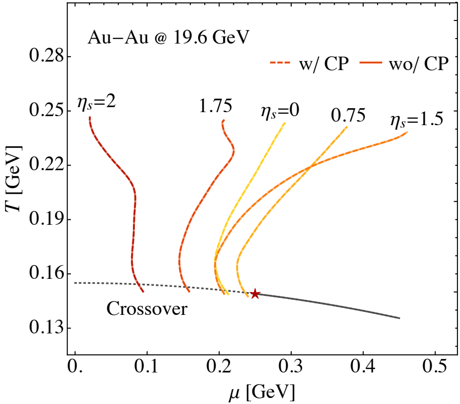




- A fireball corresponds to a set of phase diagram trajectories, scanning different regions of the phase diagram, and baryon diffusion introduces interactions among them.
- See hydrodynamic studies on trajectories in (μ, T): e.g. [C. Nonaka and M. Asakawa, PRC71, 044904 (2005); C. Shen et al, 1704.04109; A. Monnai et al, 1606.00771; T. Dore et al, 2007.15083; A. Monnai et al, 2101.11591;].

Phase diagram trajectories at different rapidities: with critical effects

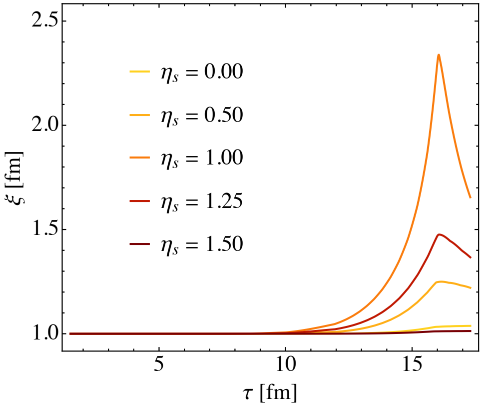
- Put a critical point at $\mu_c = 250$ MeV and $T_c = 149$ MeV ($\xi_m/\xi_0 = 10$ ▶▶ parametrization $\xi(\mu, T)$):



- No effect is seen from the critical point. Why?

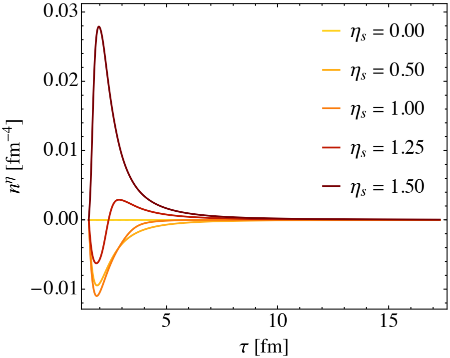
Phase diagram trajectories at different rapidities: with critical effects

- Put a critical point at $\mu_c = 250$ MeV and $T_c = 149$ MeV ($\xi_{\rm m}/\xi_0 = 10$ ▶▶ parametrization $\xi(\mu, T)$):



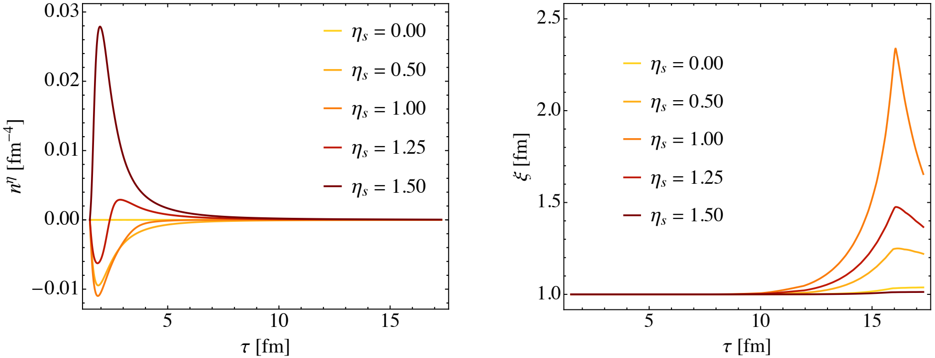
- No effect is seen from the critical point. Why? Note: Maximum $\xi \simeq 2.5$ fm, $\tau_n \simeq 6\,\tau_{n,0}$, $\kappa_n \simeq 2.5\,\kappa_{n,0}$.

Time evolution of baryon diffusion current



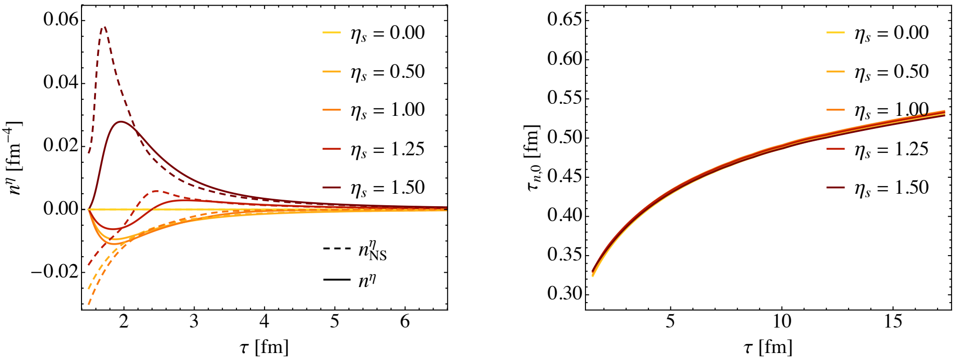
- Longitudinal component of baryon diffusion current grows and decays quickly;

Time evolution of baryon diffusion current

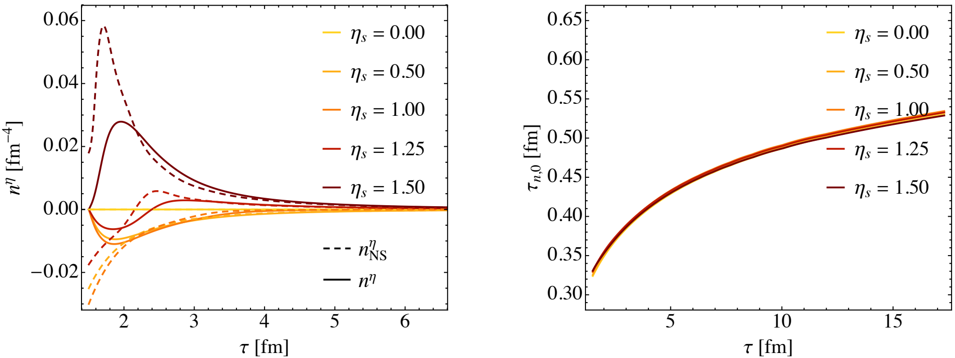


- Longitudinal component of baryon diffusion current grows and decays quickly;
- Correlation length grows at the late stage of the evolution, when the system enters the critical regime;
- Critical effect couldn't manifest itself through baryon diffusion which already approaches zero.

Time evolution of baryon diffusion current

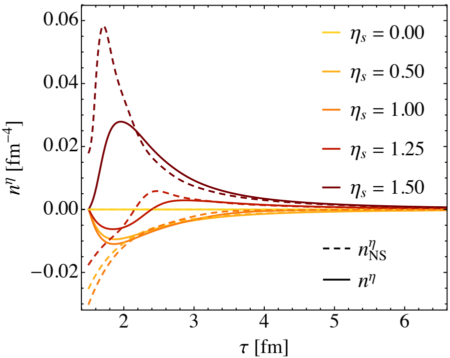


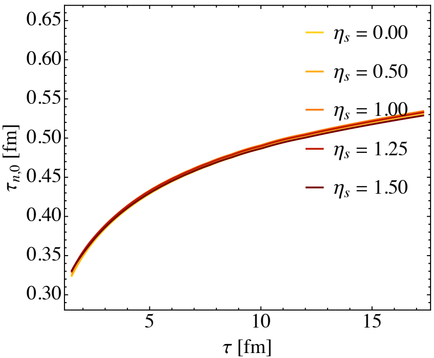
- With the fireball cooling down, the driving force of diffusion current ($\kappa_n \nabla(\mu/T)$) decreases:
  - Two reasons: (a) gradient $\nabla(\mu/T)$ gets smoothened; (b) κ_n decreases.



- With the fireball cooling down, the driving force of diffusion current ($\kappa_n \nabla(\mu/T)$) decreases:
  - Two reasons: (a) gradient $\nabla(\mu/T)$ gets smoothened; (b) κ_n decreases.
- Response to the driving force also gets slower, because of the growing relaxation time;

Time evolution of baryon diffusion current





- With the fireball cooling down, the driving force of diffusion current ($\kappa_n \nabla(\mu/T)$) decreases:
  - Two reasons: (a) gradient $\nabla(\mu/T)$ gets smoothened; (b) κ_n decreases.
- Response to the driving force also gets slower, because of the growing relaxation time;
- Critical slowing-down ($\tau_n \simeq 6\,\tau_{n,0}$) would help n^η to stay at (almost) zero, even if $\kappa_n \nabla(\mu/T)$ got affected by the critical point.

Critical effect on the net proton distribution

- Final particle distribution has off-equilibrium correction from baryon diffusion current [G. Denicol et al, 1804.10557; M. McNelis et al, 1912.08271; M. McNelis, U. Heinz, 2103.03401]

$$\delta f^i_{\text{diffusion}} = f^i_{\text{eq}}(1 \pm f^i_{\text{eq}}) \left(\frac{n}{e+p} - \frac{b_i}{u \cdot p_i} \right) \frac{p^\nu_i \, n_\nu}{\kappa_n/\tau_n} \, ;$$

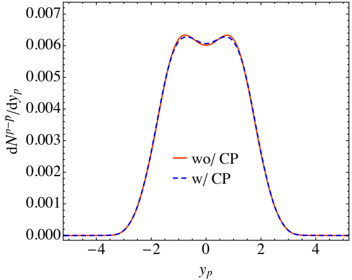
- Critical effects through critical scaling $\kappa_n/\tau_n \sim \xi^{-1}$;

Critical effect on the net proton distribution

- Final particle distribution has off-equilibrium correction from baryon diffusion current [G. Denicol et al, 1804.10557; M. McNelis et al, 1912.08271; M. McNelis, U. Heinz, 2103.03401]

$$\delta f^i_{\text{diffusion}} = f^i_{\text{eq}}(1 \pm f^i_{\text{eq}}) \left(\frac{n}{e+p} - \frac{b_i}{u \cdot p_i} \right) \frac{p_i^\nu n_\nu}{\kappa_n / \tau_n} ;$$

- Critical effects through critical scaling $\kappa_n/\tau_n \sim \xi^{-1}$;



Net proton distribution in rapidity with and without critical effects (central cell only, using iS3D [M. McNelis et al, 1912.08271])

- Final particle distribution is not strongly affected, because of the small diffusion current on the freeze-out surface.

Conclusions

Conclusions

- A fireball corresponds to a set of trajectories, scanning different regions of the phase diagram, and baryon diffusion introduces interactions among them;
- Baryon diffusion grows and decays at the early stage of the evolution, before the system enters the critical regime, at the late stage of the evolution;
- Critical effects on baryon diffusion during the hydrodynamic stage, and on particle distribution at freeze-out, via diffusion's viscous correction, are found to be small.

Thank you very much!

Parametrization of correlation length

- Critical point at ($\mu_c = 250\text{MeV}$, $T_c = 149\text{MeV}$) and $\xi_\text{m}/\xi_0 = 10$: