Dilepton production rate near the critical temperature of color superconductivity

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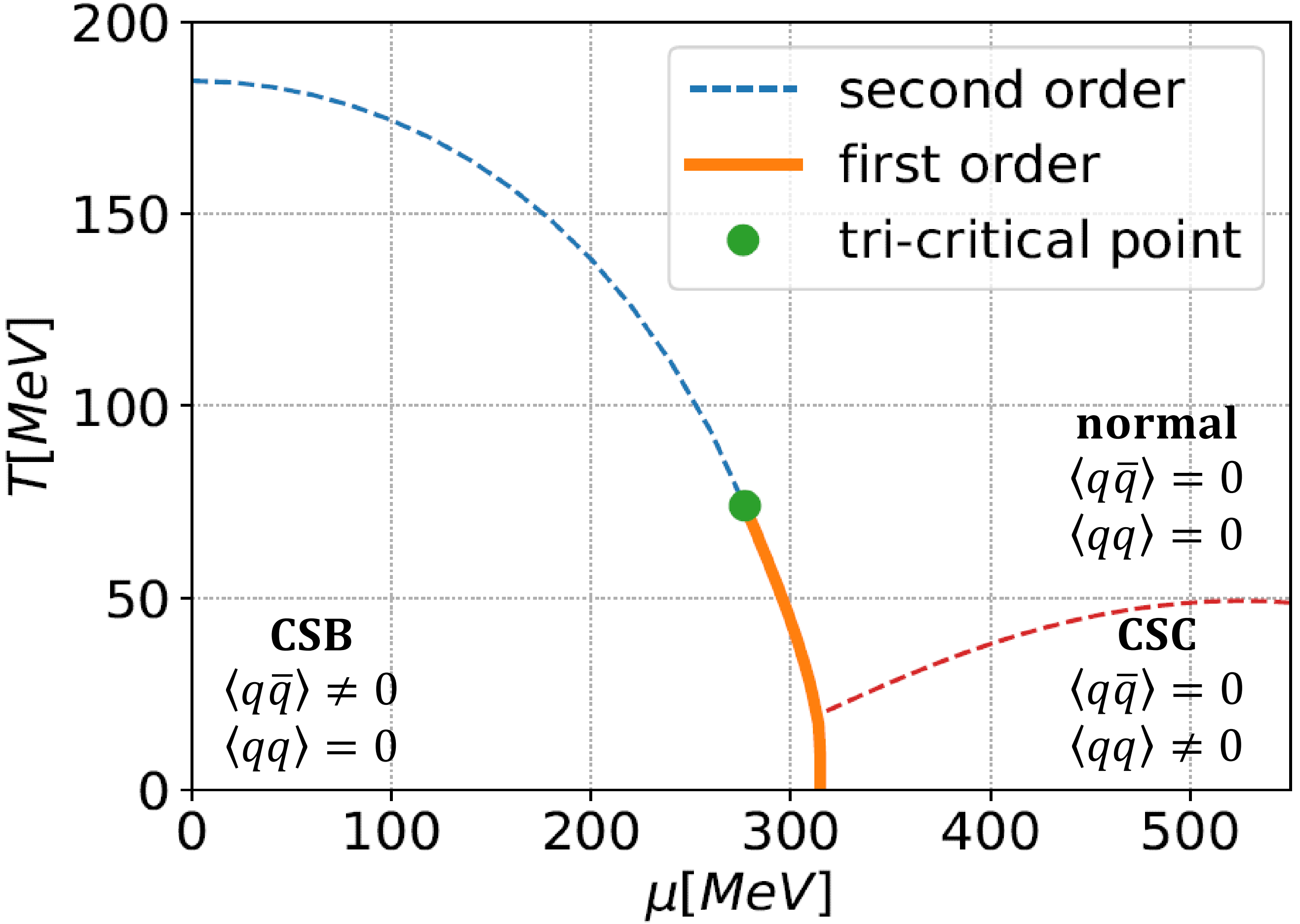
■Color superconductivity (CSC)

QCD phase diagram

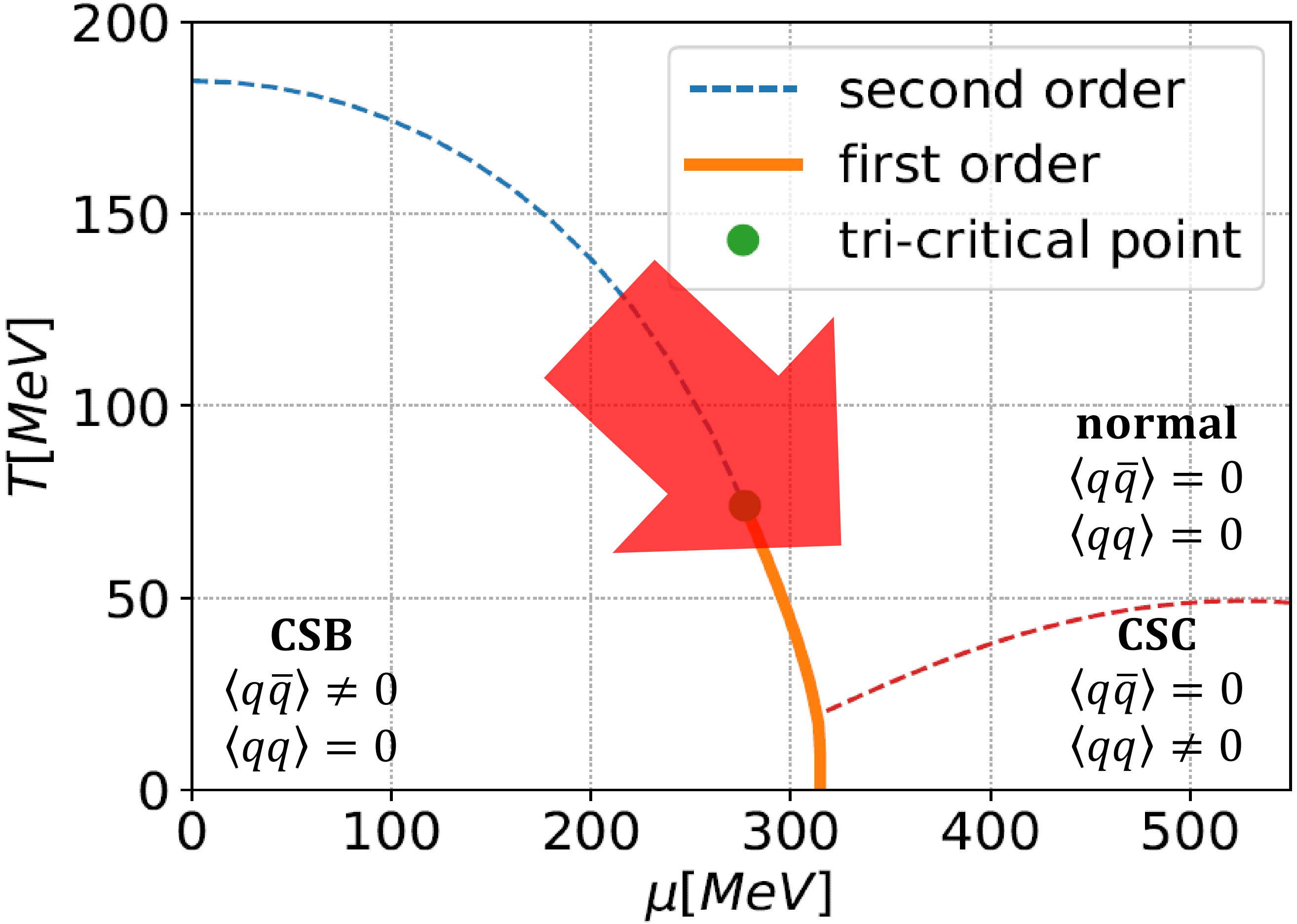


* Induced by **diquark** condensation
* **Low-temperature** and **high-density**
* **It is difficult to observe CSC in experiment.**
* We focus on **2SC**.

Review : Alford, Schmitt, Rajagopal, Schäfer(2008)

■Beam Energy Scan

QCD phase diagram



★**Ongoing**

- **BES II** at RHIC
- **NA61/SHINE** at LHC
- **HADES** at GSI

★**Future**

- **FAIR** at GSI
- **NICA** at JINR
- **J-PARC-HI** (planned)

■ How to observe CSC at HIC?

Problem 1

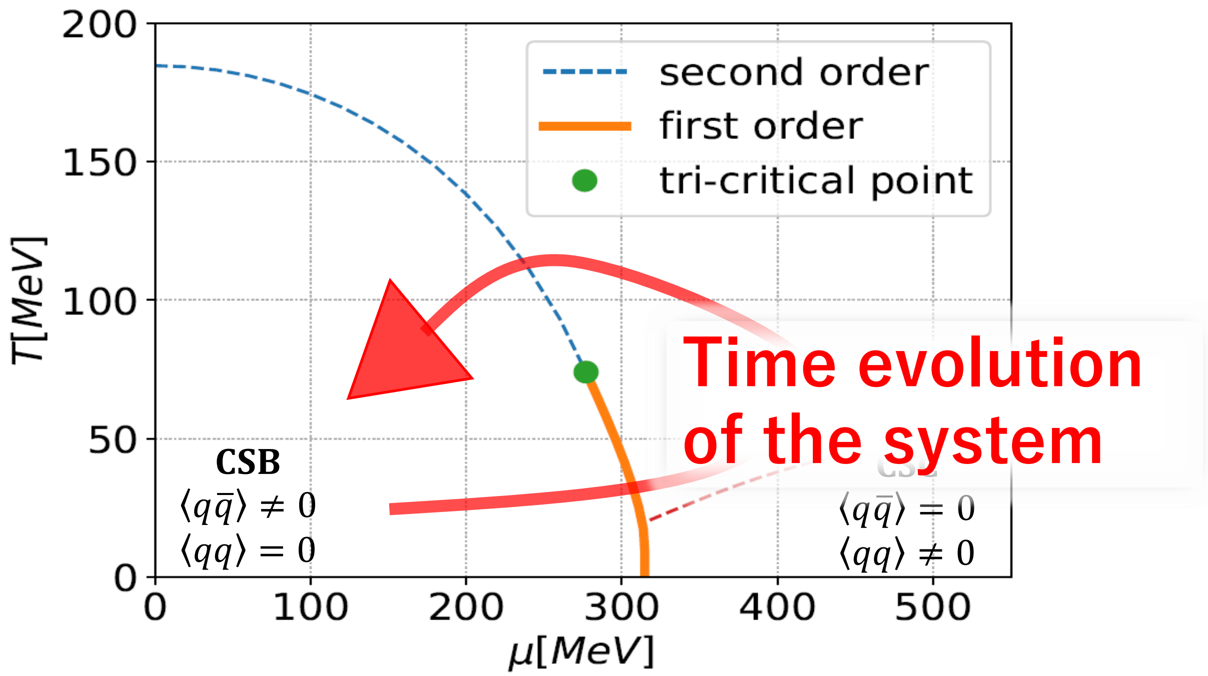
The system produced by HIC is at **high temperature**.

→ Is CSC realised?

Problem 2

CSC is realized only in **the early stage of experiments and for a short time.**

→ It is difficult to employ strongly interacting quantities as probes



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Solution1

Focus on the precursory phenomena due to the fluctuation of the 2nd phase transition of CSC.→**Soft mode**

This modes develop above T_C.

Solution2

Focus on the **dileptons** which don't interact strongly.

→It is possible to detect the initial information directly.

Kitazawa, Koide, Kunihiro, Nemoto, PTP(2005)

Kunihiro, Kitazawa, Nemoto, 0711.4429

■The purpose of this study

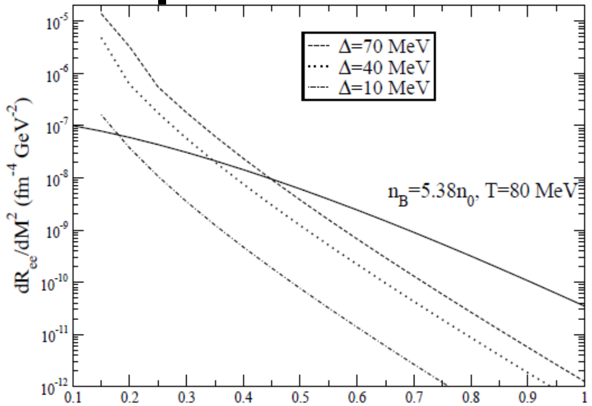
The purpose

Through **"Soft mode" "Dilepton"**,
we research the possibility of observing CSC at HIC.

2SC

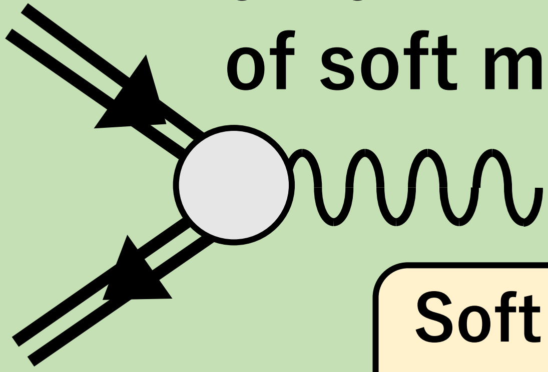
Calculate the effect of this soft mode on **the dilepton production rate.**

Dilepton from CFL

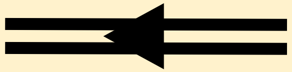


Jaikumar, Rapp, Zahed (2002)

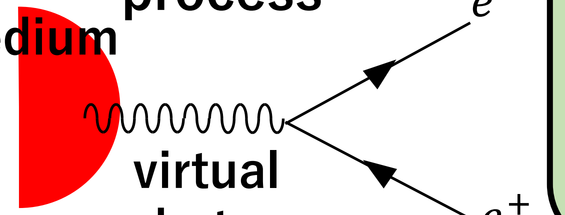
Pair annihilation of soft modes



Soft mode



Dilepton production process



Need "self energy" of photon

$$\frac{d^4\Gamma}{dk^4} = \frac{\alpha}{12\pi^4}\frac{1}{k^2}\frac{1}{e^{\beta\omega}-1}\mathrm{Im}\Pi^{\mu}{}_{\mu}{}^{R}(\boldsymbol{k},\omega)$$

■2-flavor NJL model

$$\mathcal{L} = \bar{\psi} i \partial \psi + \mathcal{L}_S + \mathcal{L}_C$$

$$\mathcal{L}_S = G_S[(\bar{\psi}\psi)^2 + (\bar{\psi} i \gamma_5 \boldsymbol{\tau} \psi)^2]$$

$$\mathcal{L}_C = G_C(\bar{\psi} i \gamma_5 \tau_2 \lambda_A \psi^C)(\bar{\psi}^C i \gamma_5 \tau_2 \lambda_A \psi)$$

quark・antiquark interaction
→$q\bar{q}$ condensation

quark・antiquark interaction
→ qq condensation
(This is related to CSC's fluctuation)

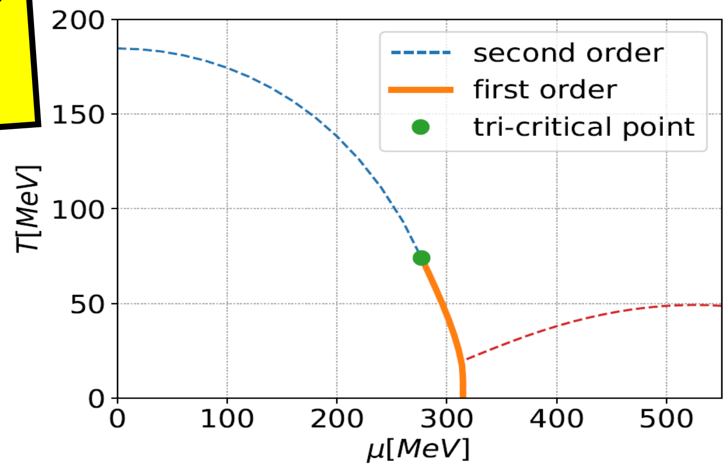
parameter

$\boldsymbol{G_S = 5.01\mathrm{MeV}, G_C = 3.11\mathrm{MeV}, \Lambda = 650\mathrm{MeV}}$

Kitazawa, Koide, Kunihiro, Nemoto (2002)

- **Chiral Symmetry($m_q = 0$)**
- **$SU_c(3)$ Symmetry**
- **Lorentz Symmetry**

MFA

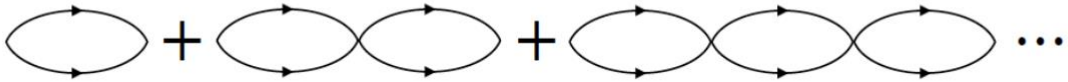


■Soft mode

Diquark fluctuation mode's response function

$D^R(\boldsymbol{k}, \omega)$

random phase approximation

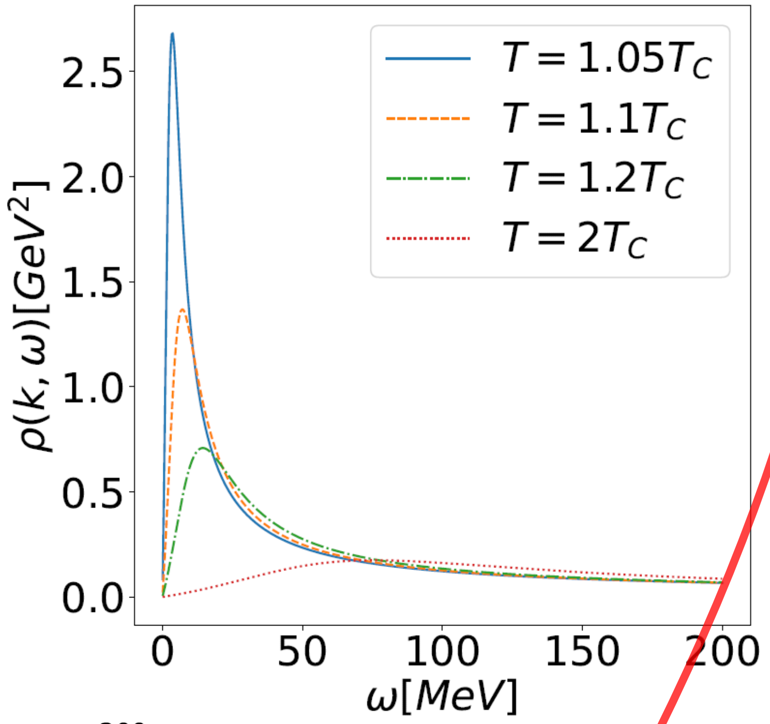
$=$ 

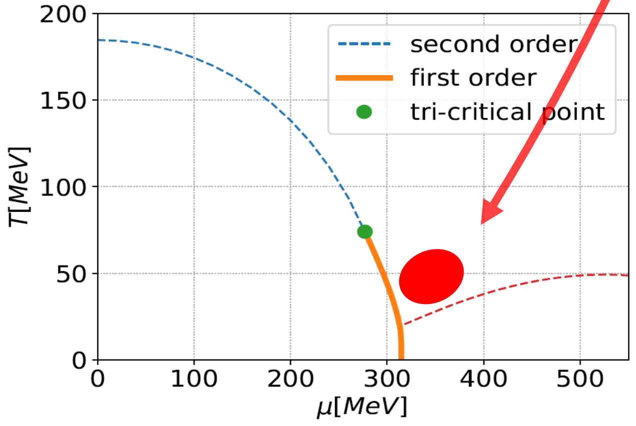
$$= \frac{1}{2} \frac{Q^R(\boldsymbol{k}, \omega)}{1 + G_C Q^R(\boldsymbol{k}, \omega)}$$

Q : diquark correlation

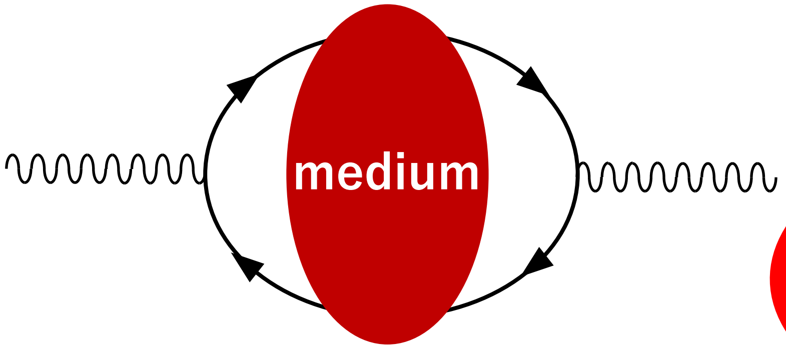
Kitazawa, Koide, Kunihiro, Nemoto (2005)

Spectral function of the soft mode
(momentum $\mathrm{k} = 1\mathrm{MeV}, \mu = 350\mathrm{MeV}$**)**



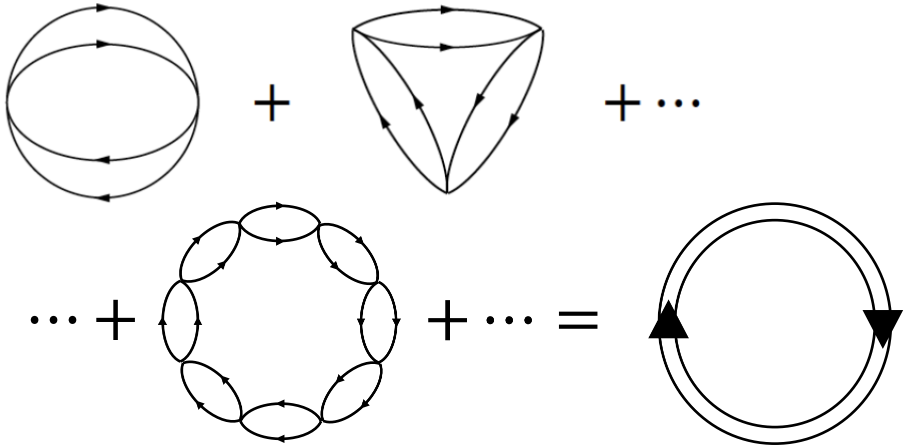


■Photon self energy

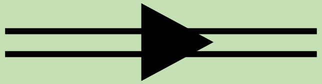


Interacting at two points
→Consider every interaction
→Satisfy Ward Identity

Thermodynamic potential

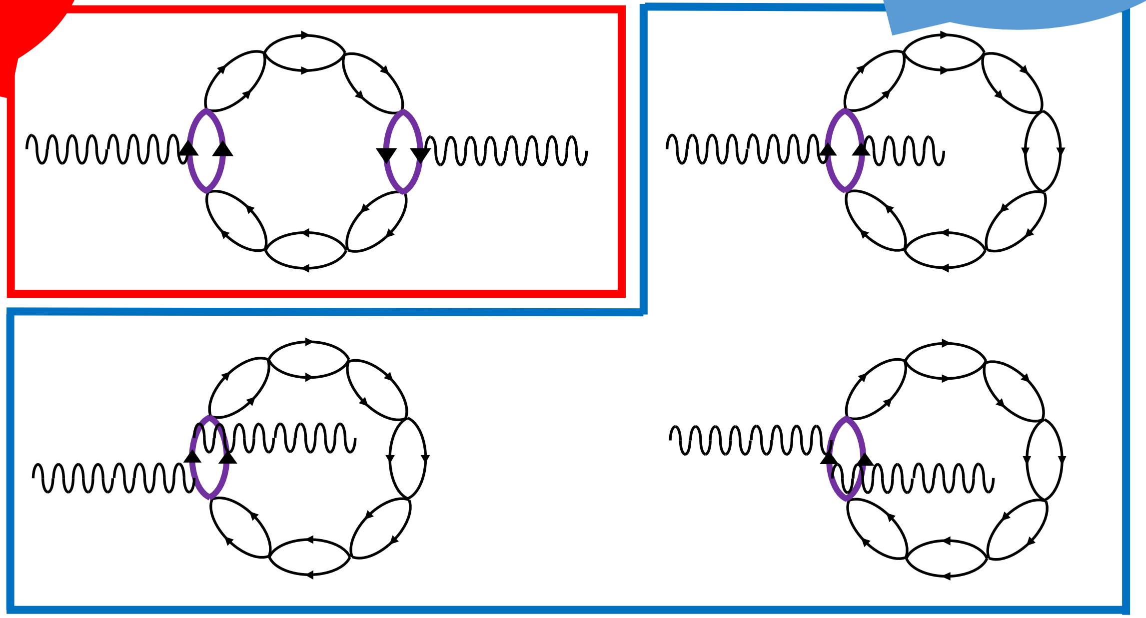


Soft mode



For different two loop

For the same one loop



Four patterns of photon interaction

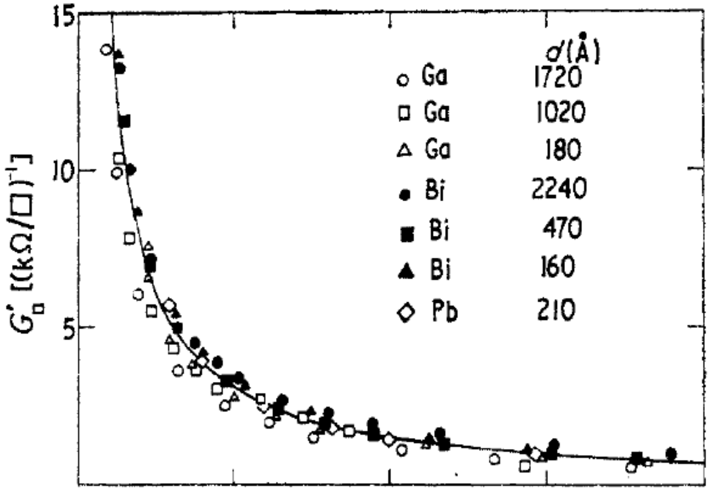
■Aslamasov-Larkin term・Maki-Thompson term

L. G. Aslamasov and A. I. Larkin (1960)　　K. Maki (1968), R. S. Thompson (1970)

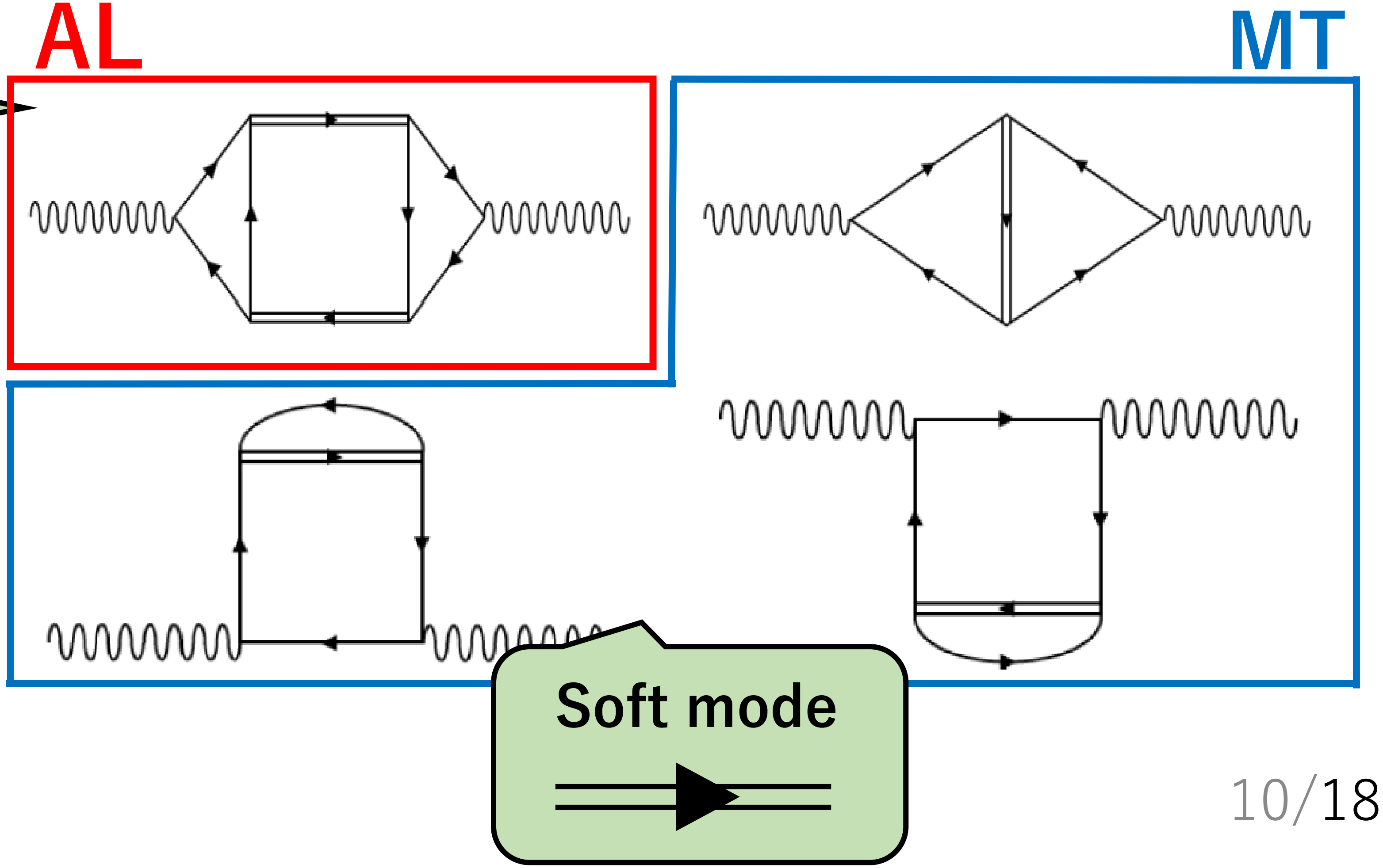
AL term・MT term are well known in the condensed matter theory

Contain the contribution of Soft modes annihilation.

Electric conductivity in metallic SCs



Skopal, Tinkham(1975)



■Time-dependent Ginzburg-Landau(TDGL) approximation

$$\Xi^R(q,\omega) = \frac{G_C}{1+G_C Q^R}$$

$$= \frac{G_C}{c_0\omega + cq^2 + c'q^4 + a\varepsilon}$$

q, ω are momentum and energy of the soft mode

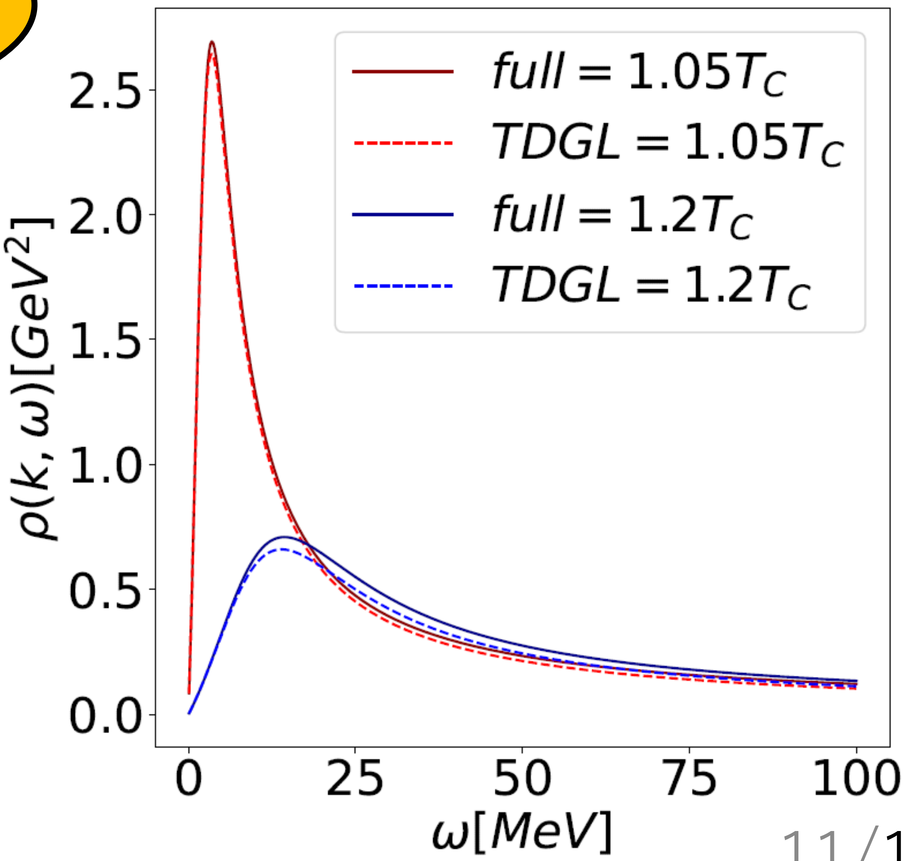
$$\varepsilon = \frac{T - T_C}{T_C}$$

Energy and temperature: first order
Momentum: forth order

⇩

This approximation is valid around T_C in the low energy-momentum region

Spectral function of the oft mode
(momentum $\mathrm{k} = 1\mathrm{MeV}, \mu = 350\mathrm{MeV}$)
before : **full** after : **TDGL**



■The approximation of vertex

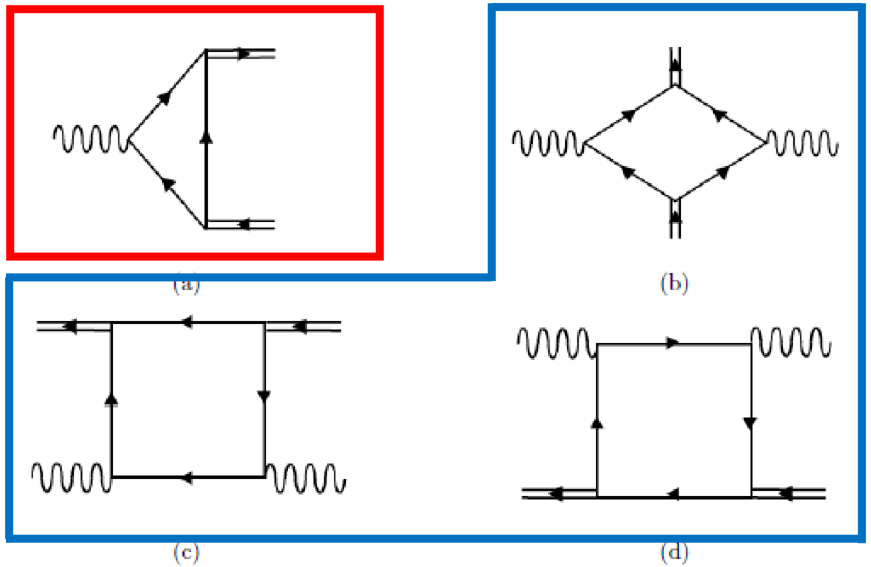
q : soft mode
k : photon

Ward-Takahashi Identity

Strictly valid

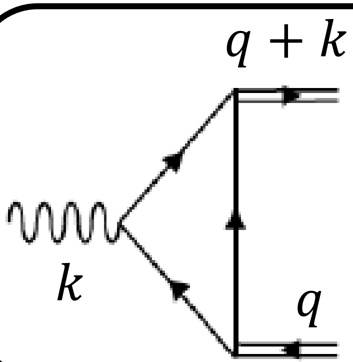
The vertex of AL term
$$k_\mu \Gamma^\mu(q, q+k) = -\frac{1}{3}\left(\Xi^{-1}(q) - \Xi^{-1}(q+k)\right)$$

The vertex of MT term
$$k_\mu k_\nu R^{\mu\nu}(q, k) = \frac{2}{3}\left(2\Xi^{-1}(q) - \Xi^{-1}(q+k) - \Xi^{-1}(q-k)\right)$$



→TDGL approximation for $\boldsymbol{\Xi}$.
→Compare coefficients of $\boldsymbol{k_\mu}$.

For example
$\Gamma^i(q, q+k)$



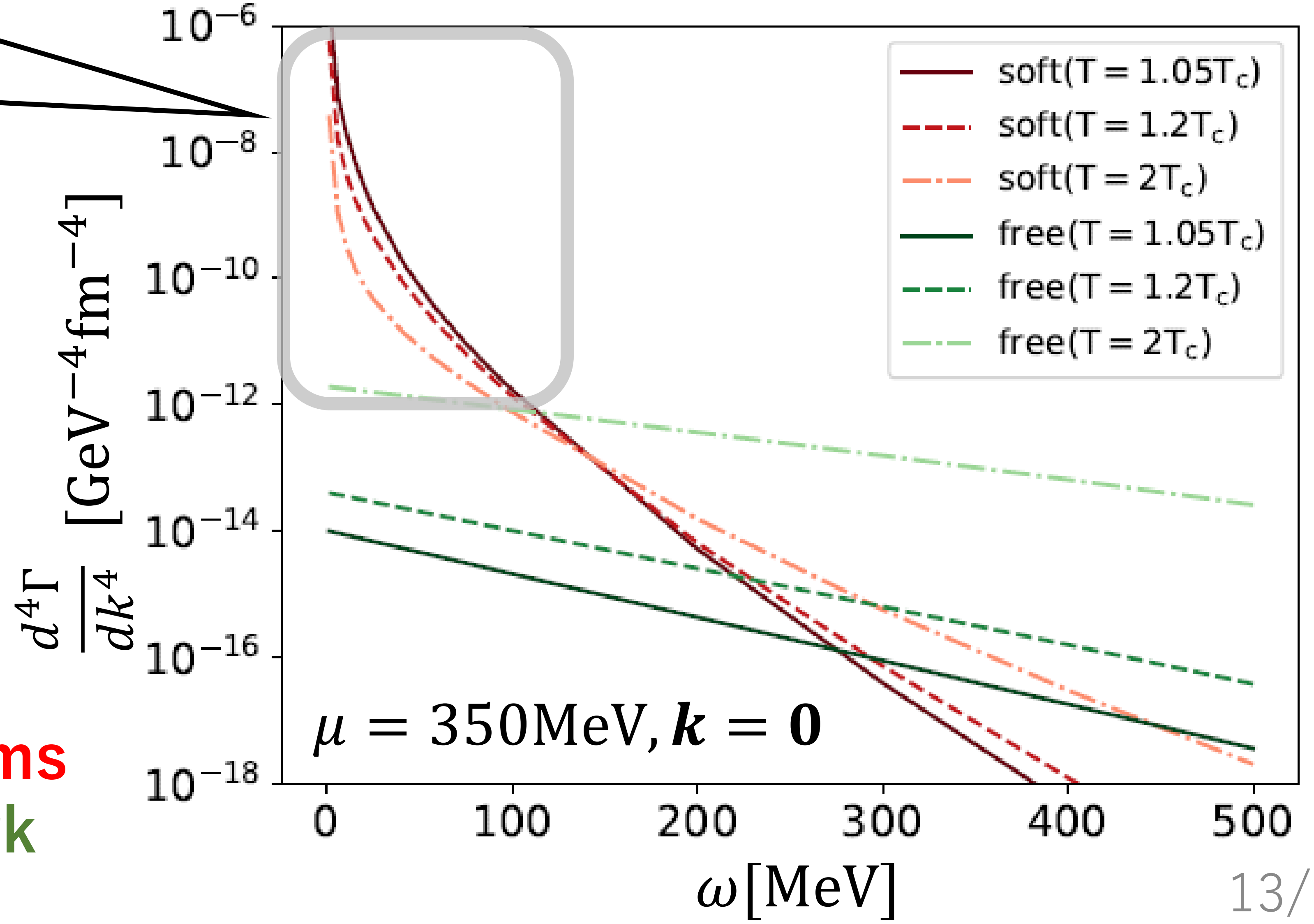
$$= \frac{c + c'(\boldsymbol{q}^2 + (\boldsymbol{q}+\boldsymbol{k})^2)}{3G_C}(2q^i + k^i)$$

■Production rate at k=0

$$\frac{d^4\Gamma}{dk^4} = \frac{\alpha}{12\pi^4}\frac{1}{k^2}\frac{1}{e^{\beta\omega}-1}\mathrm{Im}\Pi^{\mu}{}_{\mu}{}^{R}(\boldsymbol{k},\omega)$$

Low ω : Enhancement as $T \to T_C$

Expected from the property of soft mode



Red：AL, MT terms

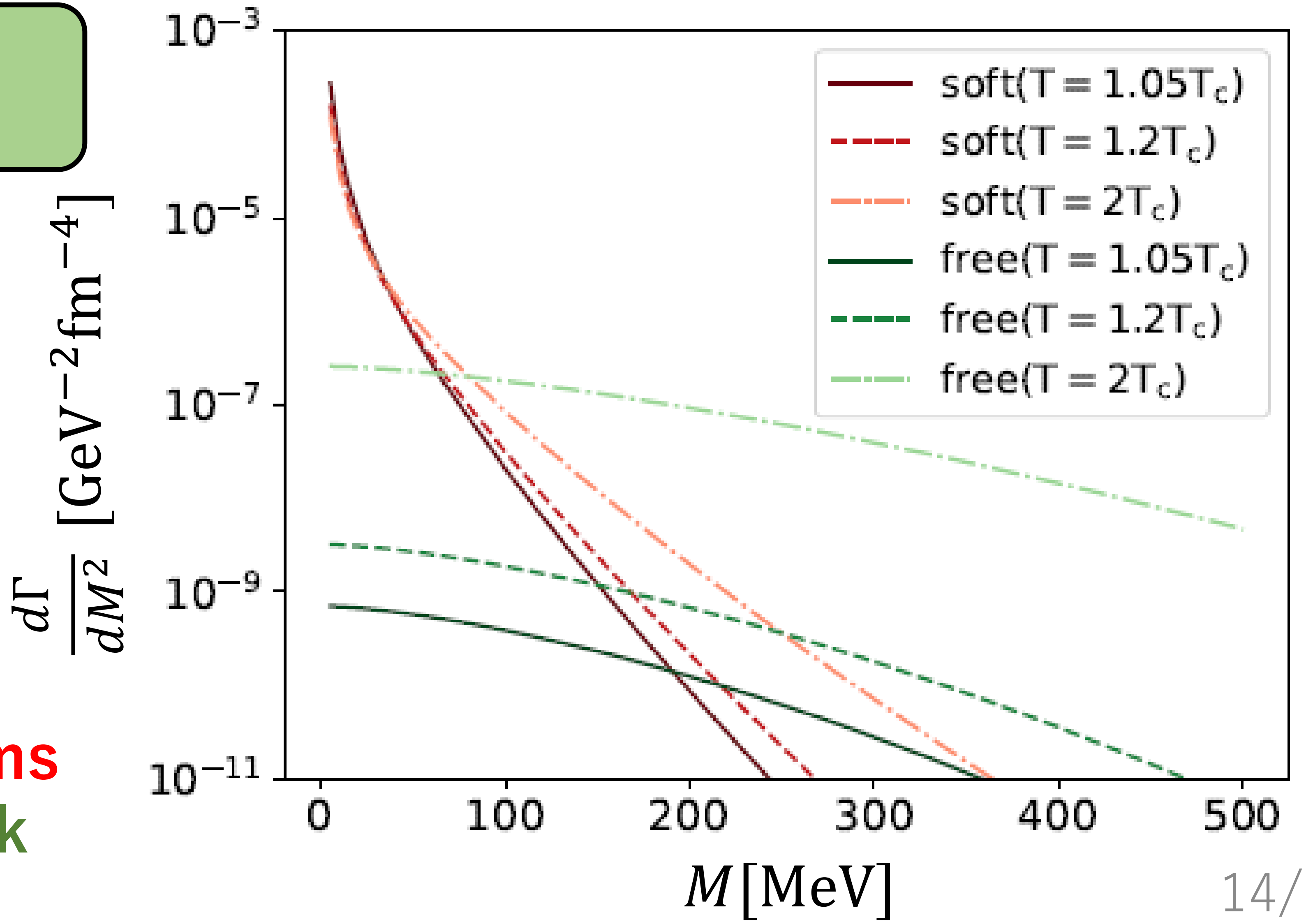
Green：free quark

■Invariant mass spectrum

$$\frac{d\Gamma}{dM^2} = \frac{\alpha}{6\pi^3 M^2}\int dk\,\frac{k^2}{\omega}\frac{\mathrm{Im}\Pi^{\mu}{}_{\mu}{}^{R}(\boldsymbol{k},\omega)}{e^{\beta\omega}-1}$$

Comparable with experiments

Enhancement in low-invariant mass region



Red：AL, MT terms

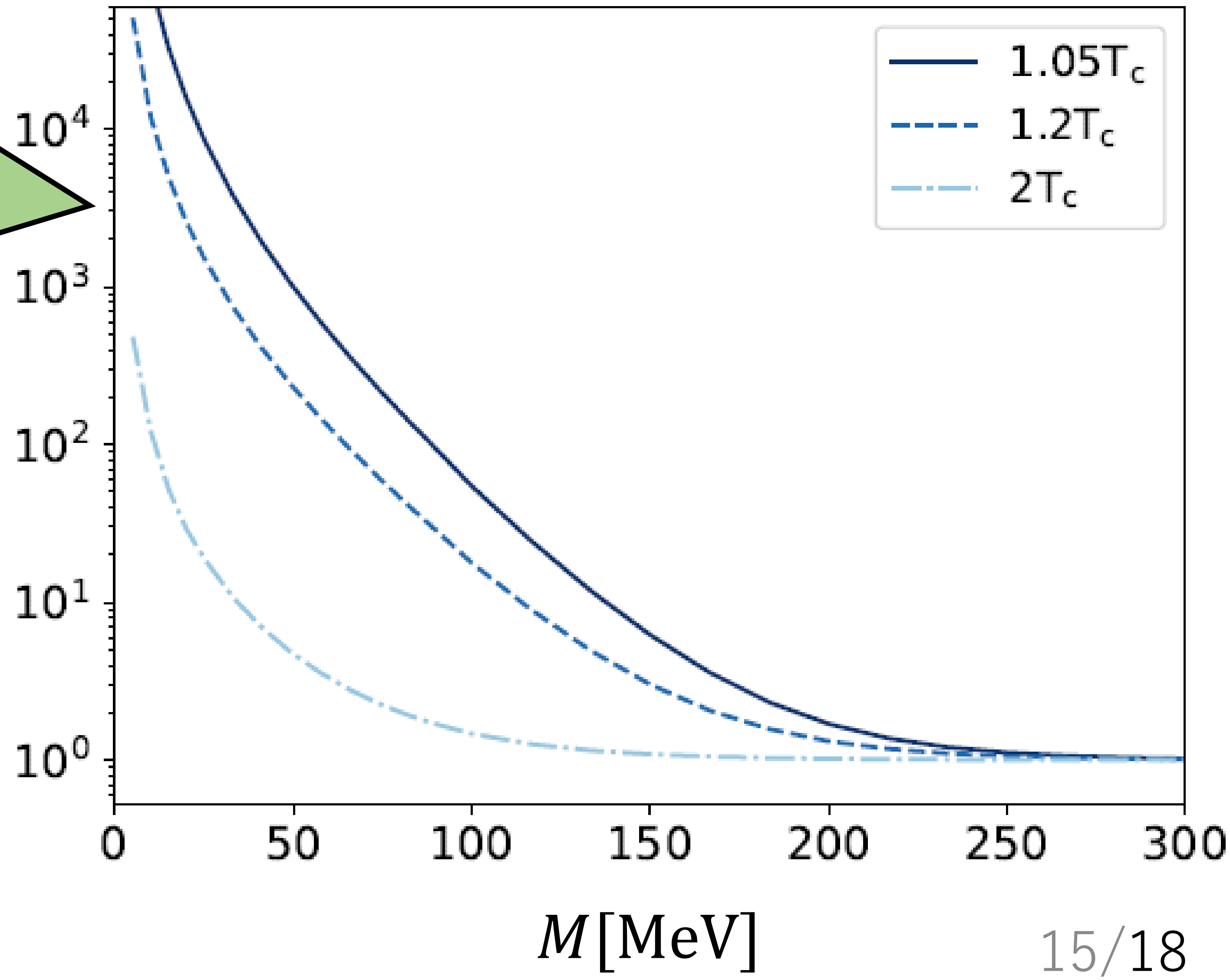
Green：free quark

■ Ratio with free quark contribution

$$\frac{\text{Total rate}}{\text{rate from free quark}}$$

(Total = AL + MT + Free quark)

As T is bigger, the contribution from soft mode decreases.



■Summary

★Calculate the contoribution of **“soft mode”** to the **“dilepton production rate”** in order to observe CSC at HIC.

＊ Consider AL term and MT term.

＊ Satisfy Ward Identity.

＊ Simplify momentum integration by TDGL approximation

＊ The production rate per invariant mass

=>**Enhancement in low-invariant mass!!**

■Outlook

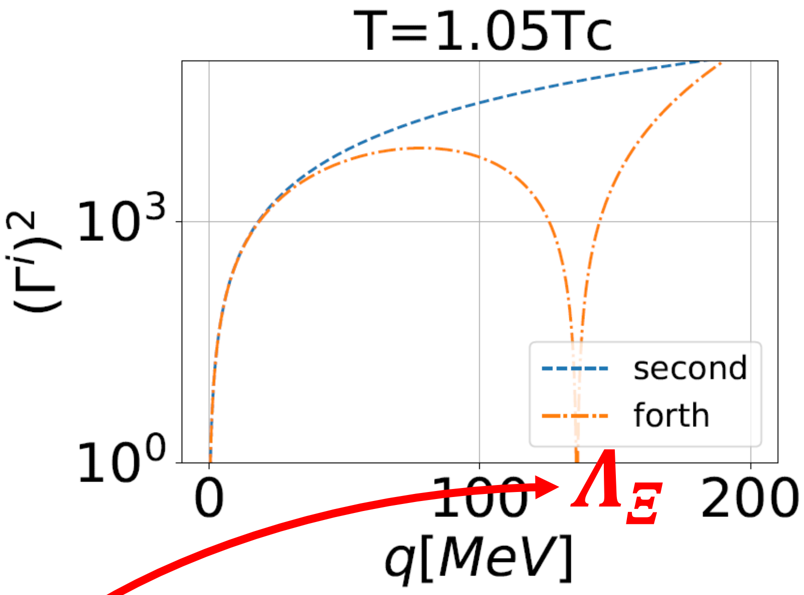
★**The possibility of observing CSC at HIC.**

＊Apply our result to dynamical model

＊Consider the competiton with other dilepton production process (Dalitz decay etc..)

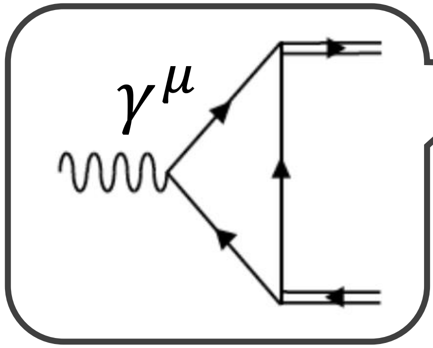
■Vertex and the momentum cutoff

The previous approximations are valid in the low energy-momentum region



momentum of soft mode

AL vertex



Focus on the space composition of AL term' vertex $(\boldsymbol{k} = 0)$

$$(\Gamma^i)^2 \sim (c + 2c'\boldsymbol{q}^2)^2\boldsymbol{q}^2, c > 0, c' < 0$$

Introduce the cutoff for q integration in order to take the contribution of soft mode.

cutoff : $\Lambda_\Xi = \sqrt{-\frac{c}{2c'}}$

TDGL approximation

$$\Xi^R(q, \omega') = \frac{G_C}{c_0\omega + cq^2 + c'q^4 + a\varepsilon}$$

q : momentum of soft mode
ω : energy of soft mode