Freezing out critical fluctuations

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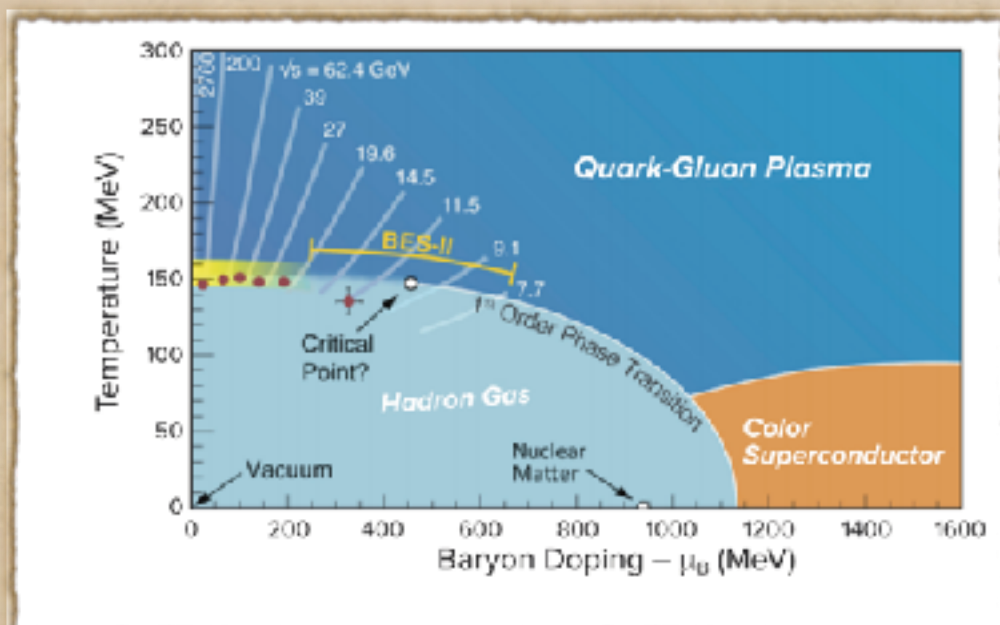
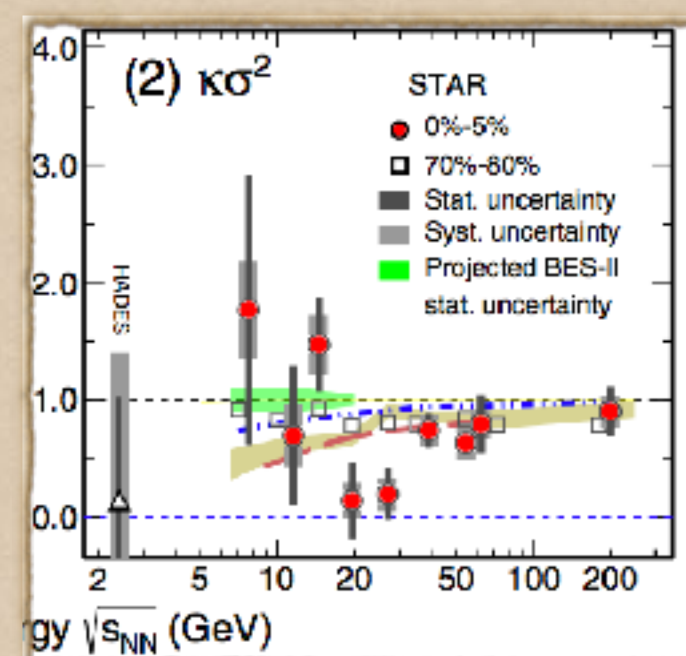
Searching for critical point via cumulants of particle multiplicities





$$\langle \delta N^2 \rangle \sim \xi^2 \, , \, \langle \delta N^3 \rangle \sim \xi^{4.5} \, ,$$

STAR Collaboration, 21

$$\langle \delta N^4 \rangle_c \sim \xi^7$$

Stephanov ,08

There have been estimates made within a grand canonical framework assuming an infinitely large system in equilibrium and using static critical behavior

Athanasiou et al.,10, Mroczek et al., 20

Dynamics of QGP near critical point

- Away from CP : Dynamics of QGP well described by Hydrodynamics
- Near CP, fluctuations of conserved densities fall out of equilibrium

Berdnikov, Rajagopal, 99, Mukherjee, Venugopalan, Yin,15

- Conserved quantities as well as their fluctuations should be treated as dynamical variables

See talks by Nahrgang, Teaney and Yin

- Simulations of these fluctuations in semi-realistic backgrounds also available

Singh et al,18 Rajagopal et al, 19, Du et al., 20

See talks by Nahrgang, Yin and Pihan

Next step : To freeze-out these fluctuations

Overview of the talk

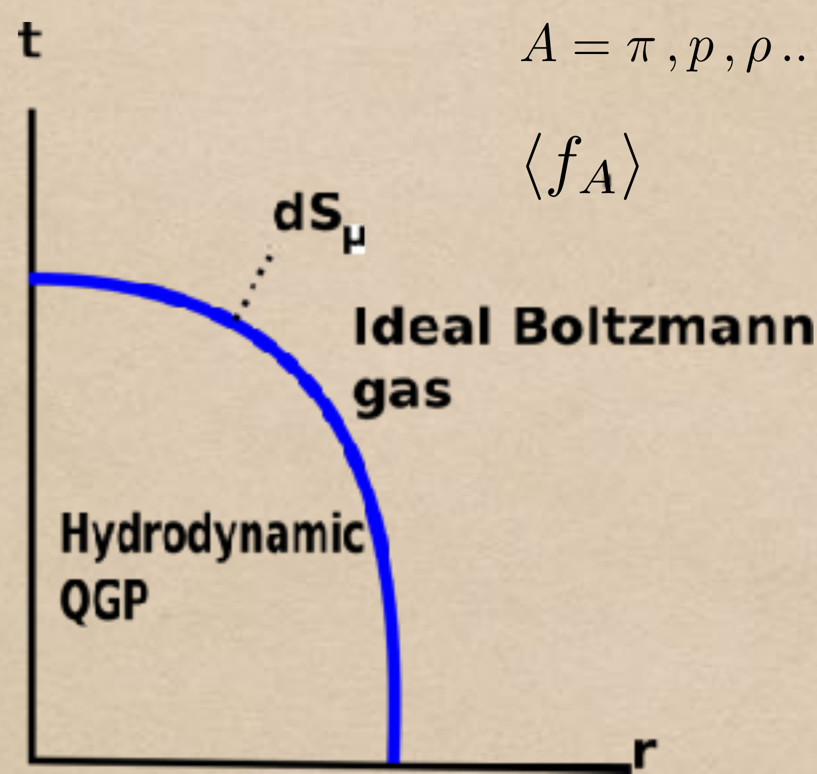
- Introduce an extended Cooper-Frye procedure to freeze-out fluctuations near a critical point
- Demonstrate freeze-out with hydro+ simulation from Rajagopal et al.,19
- Interplay of various effects near the critical point:
  A. Enhancement in fluctuations due to critical point
  B. Suppression due to critical slowing down and charge conservation

In this work, we focus on freezing out two point correlations.

Modified Cooper-Frye freeze-out

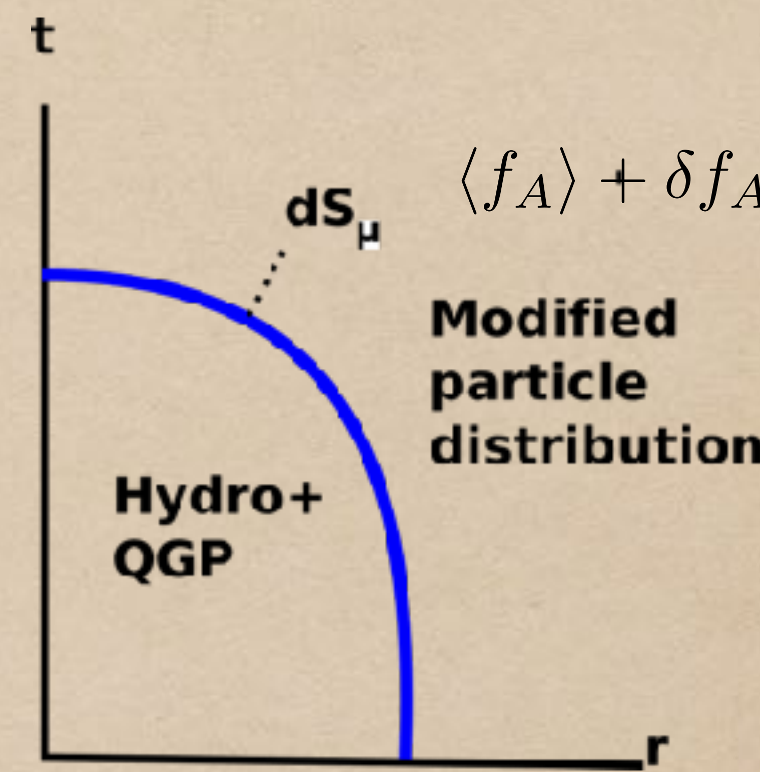
$A = \pi\,, p\,, \rho\,..$

$\langle f_A \rangle$



Cooper,Frye, 74

$\langle f_A \rangle + \delta f_A$



Pradeep et al., 21 (to appear)

$$\langle N_A \rangle = \int dS_\mu \int Dp\, p^\mu \;\langle f_A(x,p) \rangle$$

$$\left\langle \delta N_A^2 \right\rangle = \langle N_A \rangle + \left\langle \delta N_A^2 \right\rangle_\sigma$$

Non-critical - Poisson statistics $\ldots$

Contribution due to critical effects

Critical fluctuations in the particle description

We incorporate the effects of critical fluctuations via the modification of particle masses due to their interaction with the critical sigma field

$$\delta m_A \approx g_A \delta\sigma$$

Modified particle distribution function

$$f_A = \langle f_A \rangle + g_A \frac{\partial \langle f_A \rangle}{\partial m_A} \delta\sigma$$

Sigma field correlations in equilibrium (equal time)

$$\langle \delta\sigma \rangle = 0\,,\ \langle \delta\sigma(x_+)\delta\sigma(x_-) \rangle = \frac{T e^{-\frac{|\Delta x|}{\xi}}}{4\pi |\Delta x|}\,,\ \Delta x = x_+ - x_-\,,\ \Delta x >> \xi_0$$

Stephanov-Rajagopal-Shuryak, *99*

Matching critical fluctuations in the QGP and particle descriptions

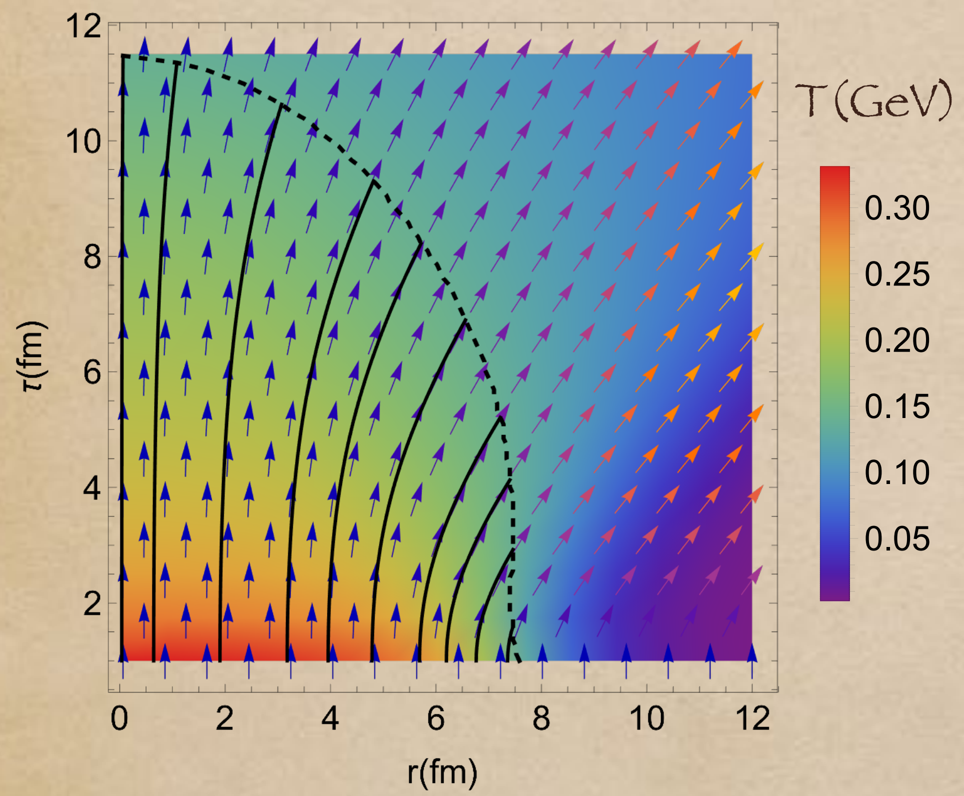
$$\langle \delta\sigma(x_+)\delta\sigma(x_-)\rangle_\sigma \;=\; Z(x)\left\langle \delta\frac{s}{n}(x_+)\delta\frac{s}{n}(x_-)\right\rangle$$

Z is determined by matching to the QCD EoS

$$\left\langle \delta N_A^2\right\rangle_\sigma \;=\; g_A^2 \int dS_\mu J_A^\mu(x_+) \int dS_\nu J_A^\nu(x_-)\, Z(x)\left\langle \delta\frac{s}{n}(x_+)\delta\frac{s}{n}(x_-)\right\rangle$$

$$J_A^\mu = d_A \int Dp\, p^\mu\, \frac{\partial \langle f_A\rangle}{\partial m_A}$$

Freezing out the semi-realistic system discussed in Rajagopal et al.,19



- Flow: Boost invariant azimuthally symmetric
- Evolution equation for 2pt correlation function of s/n - Hydro+
- Effect of back reaction neglected
- Freeze-out condition T(x)=0.14 GeV

Model H relaxation

The fluctuations of the slowest mode

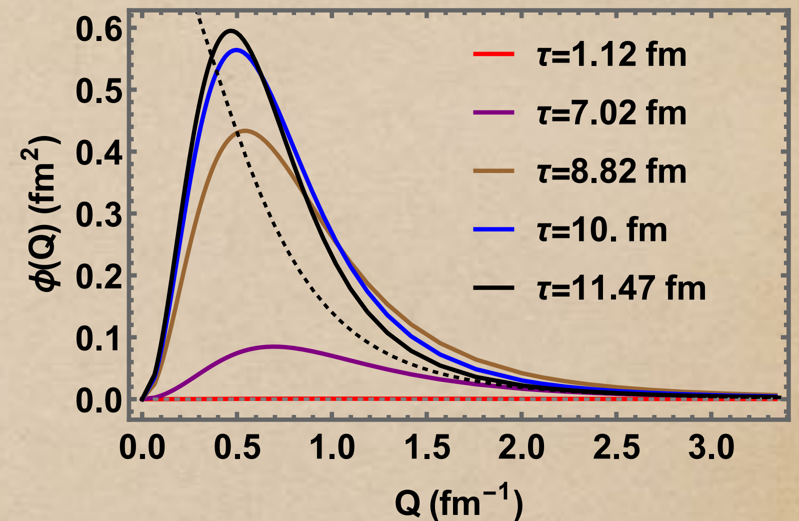
$$\left\langle \delta\frac{s}{n}(x_+)\delta\frac{s}{n}(x_-)\right\rangle = \int_{\mathbf{Q}} e^{i\mathbf{Q}\cdot\Delta x}\phi_{\mathbf{Q}}(x)$$

Evolution equation

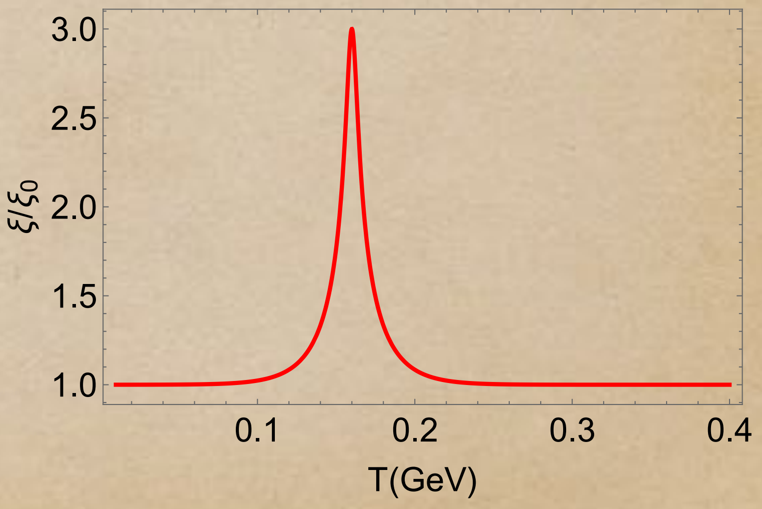
$$u\cdot\partial\phi_{\mathbf{Q}} = -\Gamma(|Q|\xi)\,\left(\phi_{\mathbf{Q}} - \bar{\phi}_{\mathbf{Q}}\right)$$

Relaxation rate

$$\Gamma(x) = \frac{\Gamma_H \xi_0^3}{\xi^3}K(x)\,,K(x)\sim x^2\;\mathrm{for}\;x<<1$$

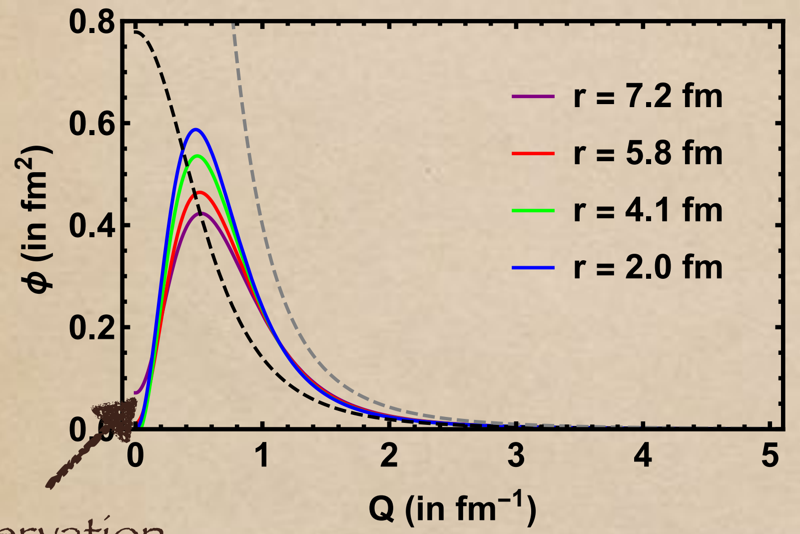


Growth of fluctuations
along a characteristic trajectory

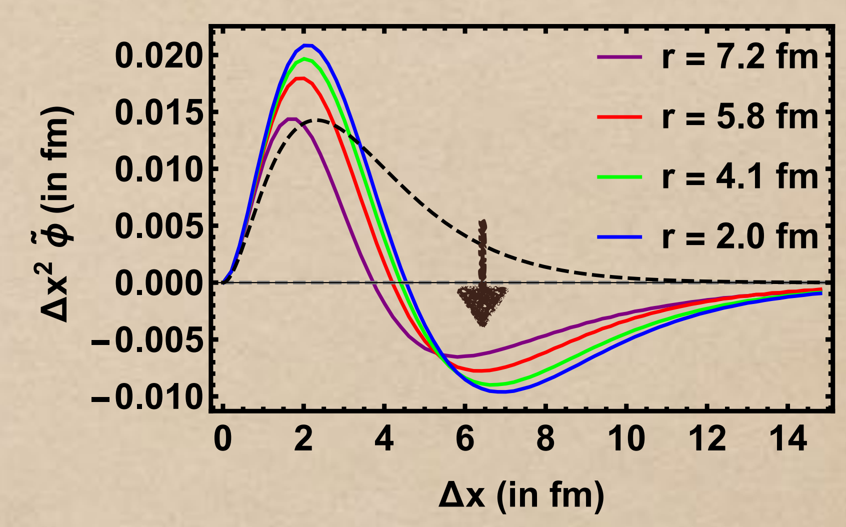


Hydrodynamic fluctuations

Fluctuations on the freeze-out hypersurface

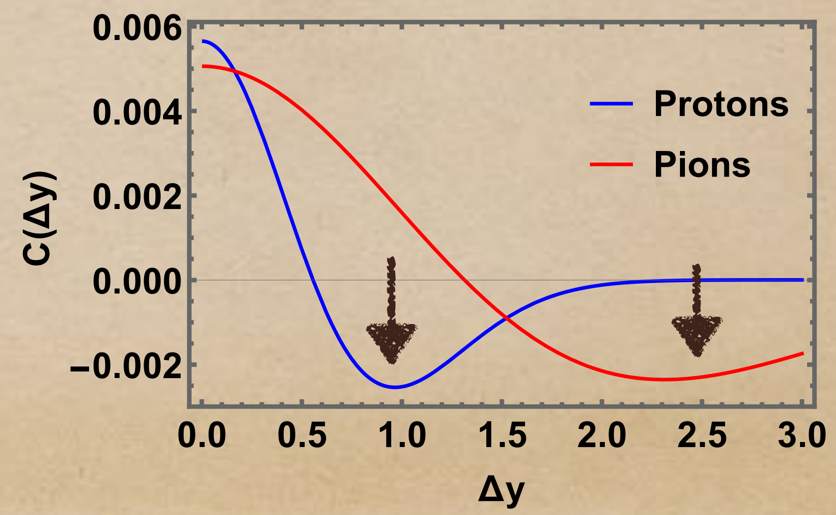


Conservation

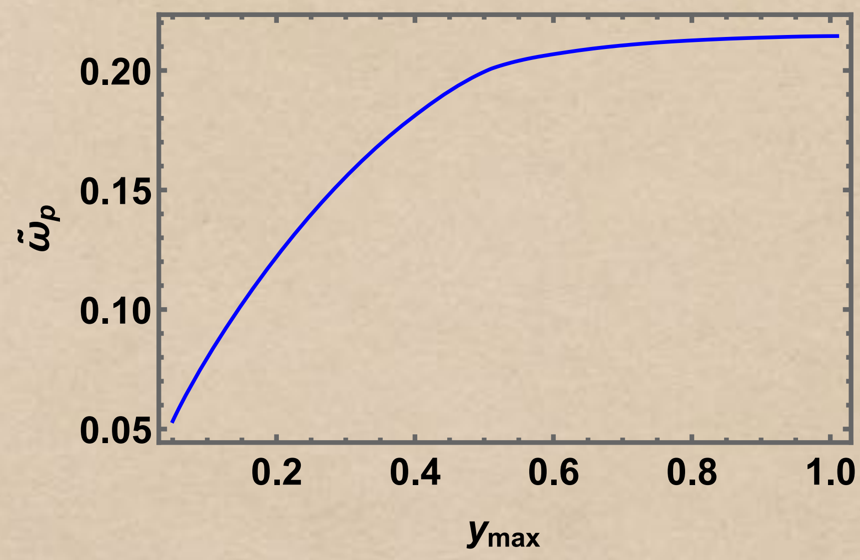


$$\tilde{\phi}(x, \mathbf{\Delta x}) = \left\langle \delta\frac{s}{n}(x_+)\delta\frac{s}{n}(x_-)\right\rangle = \int_{\mathbf{Q}} d^3Q\, e^{i\mathbf{Q}\cdot\mathbf{\Delta x}}\,\phi_{\mathbf{Q}}(x)$$

Fluctuations of particle multiplicity
in rapidity space



Contribution of critical fluctuations to second cumulant of proton multiplicity



$$\tilde{\omega}_A(y_{\max}) = \left(\frac{\langle \delta N_A^2 \rangle_{\sigma,\mathrm{eq}}}{\langle N_A \rangle}\right)^{-1} \frac{\langle \delta N_A^2(y_{\max}) \rangle_\sigma}{\langle N_A(y_{\max}) \rangle}$$

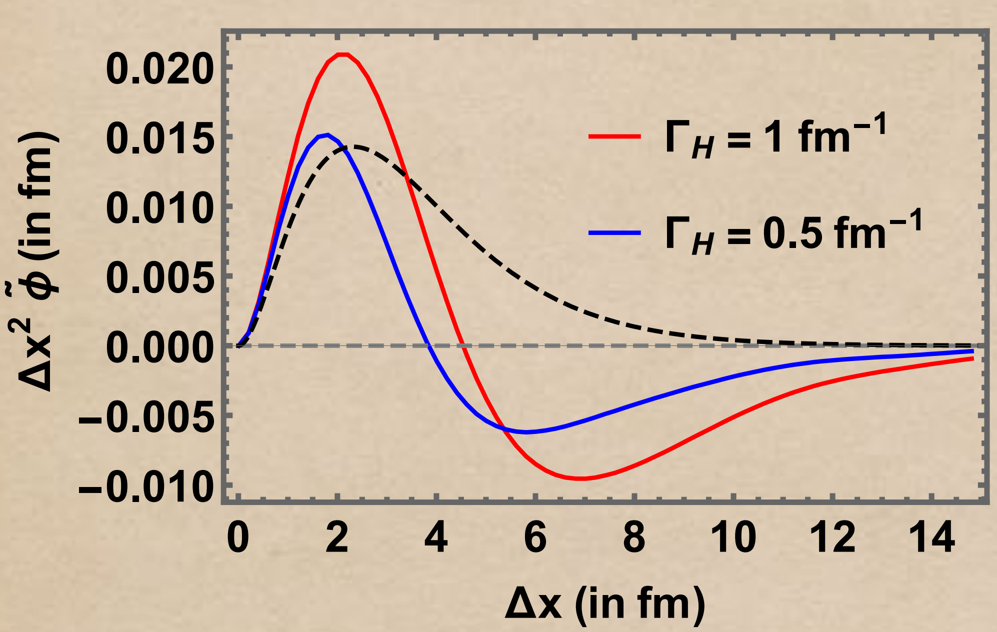
Infinitely large fully equilibrated system
at freeze-out in the GC ensemble

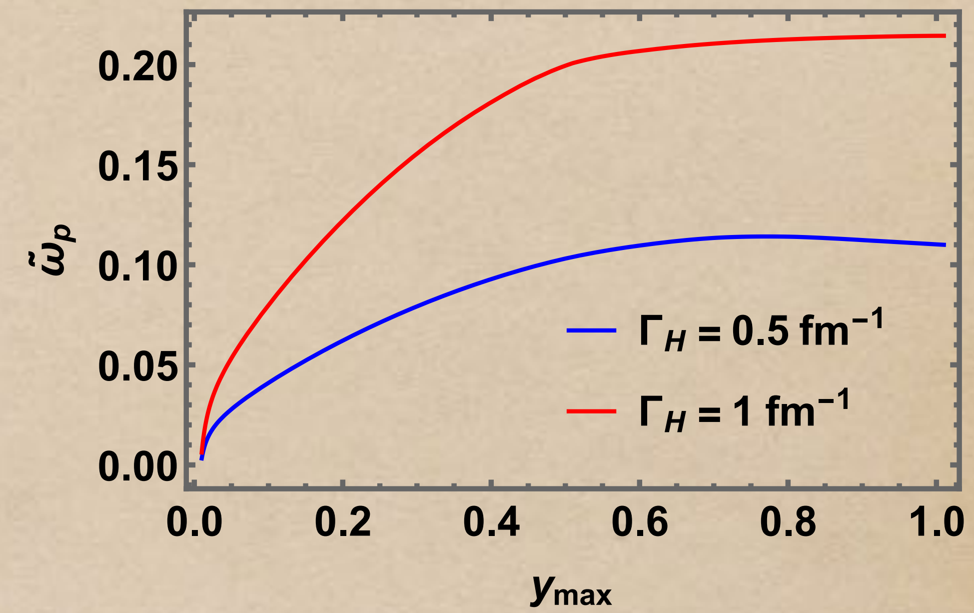
$\omega_A(y_{\max})$

$\Gamma_H = 1\,\mathrm{fm}^{-1}$

Transverse momenta acceptance : 0.4 GeV/c to 2 GeV/c

Interplay of effects in dynamical evolution of QGP





$$\Gamma(x) = \frac{\Gamma_H \xi_0^3}{\xi^3} K(x)\,,\, K(x) \sim x^2 \text{ for } x << 1$$

$$\tilde{\omega}_A(y_{\mathrm{max}}) = \left(\frac{\langle \delta N_A^2 \rangle_{\sigma,\mathrm{eq}}}{\langle N_A \rangle} \right)^{-1} \frac{\langle \delta N_A^2(y_{\mathrm{max}}) \rangle_\sigma}{\langle N_A(y_{\mathrm{max}}) \rangle}$$

Crude estimate for second cumulant of proton multiplicity based on our toy model calculation

$\Gamma_H = 0.5\,\mathrm{fm}^{-1}\,,\ \mu_f = 0.4\,\mathrm{GeV}\,,\ T_f = 0.14\,\mathrm{GeV}$

Transverse momenta acceptance : 0.4 GeV/c to 2 GeV/c

$$\left\langle \delta N_p^2(y_{\max} = 0.9)\right\rangle_\sigma \approx 0.1\left\langle \delta N_p^2\right\rangle_{\sigma,\mathrm{eq}} \approx 0.07\,\langle N_p\rangle \left(\frac{g_p}{7}\right)^2$$

Estimate can be improved by:

- Using more realistic initial conditions and 3D flow profile
- Using more realistic EoS, gA and parametrization for correlation length
- Including sub-leading and non-critical elements to evolution equations
- Including protons from resonance decays
- ....

Summary

- Extended Cooper-Frye procedure to freeze-out critical fluctuations
- Freeze-out of a semi-realistic hydro+ simulation is performed and contribution of critical fluctuations to variance of particle multiplicities calculated.
- Interplay of various effects like critical slowing down and charge conservation leads to a suppression in the cumulants of particle multiplicities relative to equilibrium prediction.

To do

- Extend the analysis to obtain critical contribution to higher order cumulants
- Determine quantitatively the critical contribution to cumulants in equilibrium
- Extend the calculation by including all particles in the hadron resonance gas
- …

Thank you!