

Search for critical point via intermittency analysis in NA61/SHINE

Pb+Pb at 30A GeV/c ($\sqrt{s_{NN}} \approx 7.5$ GeV)

Ar+Sc at 150A GeV/c ($\sqrt{s_{NN}} \approx 17$ GeV)

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Critical Point and Onset of Deconfinement
March 15-19, 2021

Critical point and intermittency

2nd order phase transition (critical point) \rightarrow scale invariance \rightarrow power-law form of correlation function \rightarrow enhanced multiplicity fluctuations that can be revealed by scaled factorial moments $F_r(\delta)$ of order r :

$$F_r(\delta) = \frac{\left\langle \frac{1}{M} \sum_{i=1}^M n_i(n_i - 1) \dots (n_i - r + 1) \right\rangle}{\left\langle \frac{1}{M} \sum_{i=1}^M n_i \right\rangle^r}$$

δ - size of each of the $M = \frac{\Delta}{\delta}$ subdivision intervals of the momentum region Δ

n_i - number of particles in i -th bin

$\langle \dots \rangle$ - averaging over events

When the system is a simple fractal and $F_r(\delta)$ follows a power-law dependence:

$$F_r(\delta) = F_r(\Delta) \cdot (\Delta/\delta)^{\varphi_r}.$$

Additionally, the exponent (intermittency index) φ_r obeys the relation:

$$\varphi_r = (r - 1) \cdot d_r,$$

where the anomalous fractal dimension d_r is independent of r .

Asakawa, Yazaki NPA 504 (1989) 668

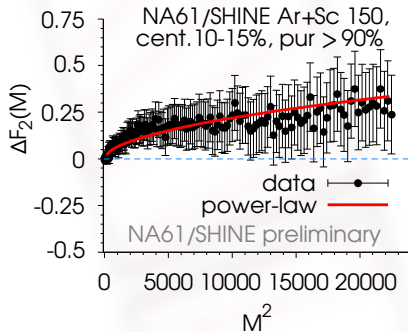
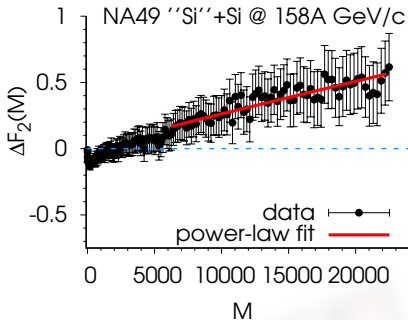
Barducci, Casalbuoni, De Curtis, Gatto, Pettini, PLB 231 (1989) 463

Wosiek, APPB 19 (1988) 863

Satz, NPB 326 (1989) 613

Bialas, Peschanski, NPB 273 (1986) 703

Critical point search with proton intermittency



A deviation of ΔF_2 ($\Delta F_2 = F_2^{\text{data}} - F_2^{\text{mixed}}$) from zero seems to be present in central Si+Si and mid-central Ar+Sc.

However, the data points are correlated which makes the interpretation difficult.

Results presented today were obtained with statistically independent points and cumulative quantities.

Second Scaled Factorial Moment analysis for primary protons (strong and electromagnetic processes) produced in Ar+Sc interactions at 150A GeV/c and Pb+Pb at 30A GeV/c in few centrality windows using cumulative variables and independent points.

$$F_2(M) = \frac{2M^2}{\langle N \rangle^2} \langle N_{pp}(M) \rangle$$

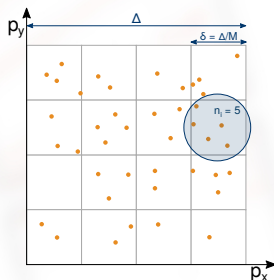
M - number of bins in p_x and p_y

n_i - number of particles in i -th bin

N - event multiplicity

$\langle \dots \rangle$ - averaging over events

$N_{pp}(M)$ - total number of particle pairs in M^2 bins in an event



At the second order phase transition $F_2(M)$ exhibits a power-law dependence on M :

$$F_2(M) \sim (M^2)^{\varphi_2}$$

The intermittency index φ_2 for a system freezing out at the QCD critical endpoint is expected to be $\varphi_2 = 5/6$ assuming that the latter belongs to the 3-D Ising universality class.

Wosiek, APPB 19 (1988) 863

Bialas, Hwa, PLB 253 (1991) 436

Bialas, Peschanski, NPB 273 (1986) 703

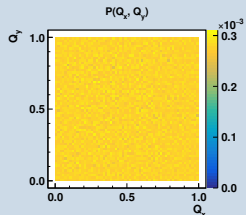
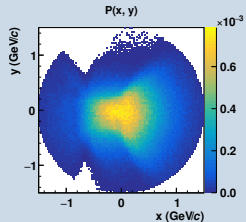
Antoniu, Diakonou, Kapoyannis, Kousouris, PRL 97 (2006) 032002

Cumulative variables

Instead of using p_x and p_y , one can use cumulative quantities:

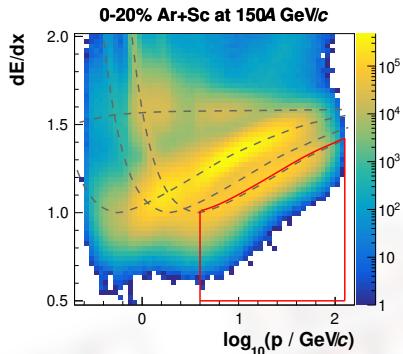
$$Q_x = \int_{\min}^x \rho(x) dx / \int_{\min}^{\max} \rho(x) dx \quad Q_y(x) = \int_{y_{\min}}^y P(x, y) dy / P(x)$$

- transform any distribution into uniform one (0,1)
- remove the dependence of F_2 on the shape of the single-particle distribution
- intermittency index of an ideal power-law correlation function system described in two dimensions in momentum space was proven to remain approximately invariant after the transformation



(example for 0-5% Ar+Sc at 150A GeV/c)

Proton selection



Selection of protons is based on dE/dx measurements in TPCs:

- $0.60 < \log_{10}(p / \text{GeV}/c) < 2.10$
- $0.5 < dE/dx < BB_p + 0.15(BB_K - BB_p)$

Within the selected range, the cut selects more than 50% of protons and a few percent kaons.

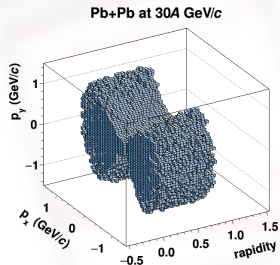
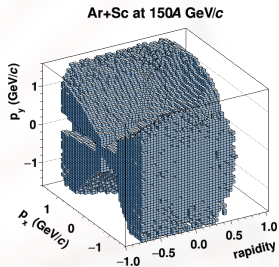
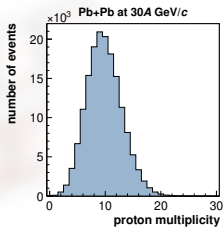
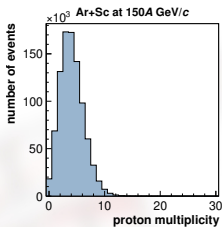
Analyses acceptance

$$|p_x| < 1.5 \text{ GeV/c}$$

$$|p_y| < 1.5 \text{ GeV/c}$$

$$\text{Ar+Sc: } -0.75 < \text{rapidity} < 0.75$$

$$\text{Pb+Pb: } 0.00 < \text{rapidity} < 0.75$$



Two-Track Distance cut

Time Projection Chambers do not allow to reconstruct tracks too close to each other.

A momentum-based cut was introduced.

Parameters adjusted for Ar+Sc and Pb+Pb separately.

$$s_x = p_x / p_{xz} = \cos(\psi)$$

$$s_y = p_y / p_{xz} = \tan(\lambda)$$

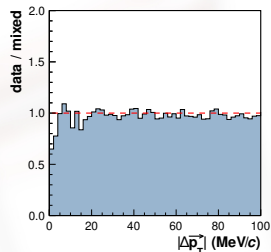
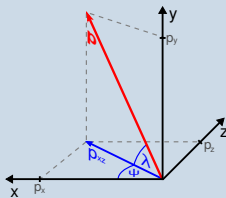
$$\rho = 1 / p_{xz}$$

For pairs:

$$\Delta s_x = s_{x,2} - s_{x,1}$$

$$\Delta s_y = s_{y,2} - s_{y,1}$$

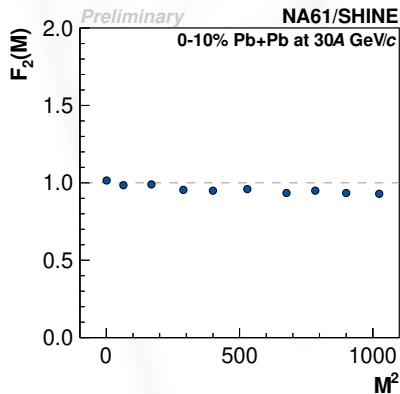
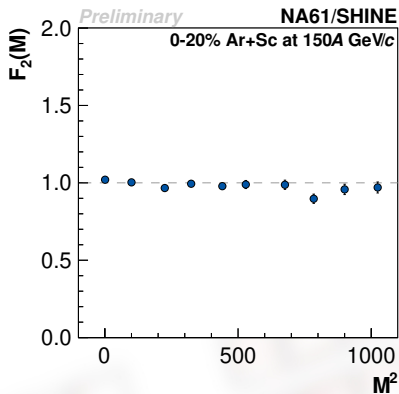
$$\Delta \rho = \rho_2 - \rho_1$$



Effect seen for $|\Delta \vec{p}_T| < 10$ MeV/c

$F_2(M)$ results

for $M = 1 \dots 32$

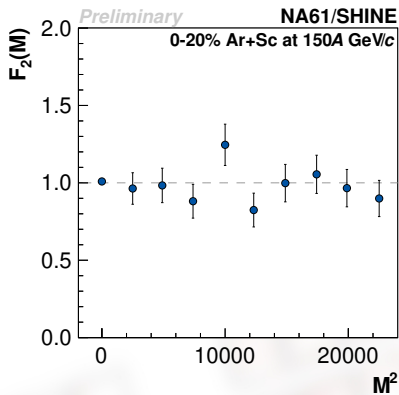


statistical uncertainties only

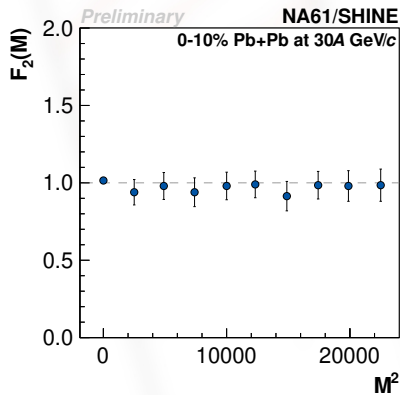
No indication for power-law increase with bin size

$F_2(M)$ results

for $M = 1 \dots 150$



statistical uncertainties only



No indication for power-law increase with bin size

Simple power-law model

Comparison with simple power-law model

A simple model that generates momentum of particles for a given number of events with a given multiplicity distribution.

It has two main parameters:

- ratio of correlated to uncorrelated particles,
- power-law exponent.

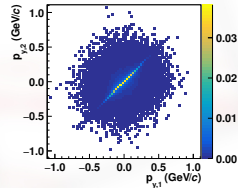
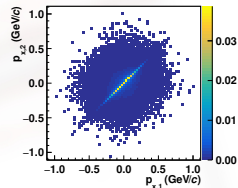
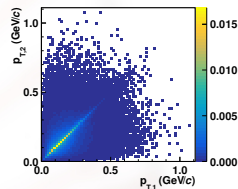
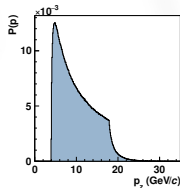
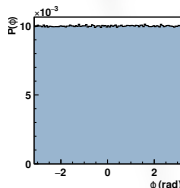
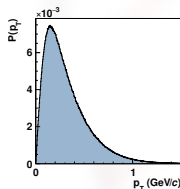
Uncorrelated particles (background)

$$\rho_B(p_T) = p_T \cdot e^{-6p_T}$$

Correlated pairs (signal)

$$\rho_S(p_{T,1}, p_{T,2}) = \rho_B(p_{T,1}) \cdot \rho_B(p_{T,2}) \cdot \left[|\Delta p_x|^\phi + \epsilon \right]^{-1} \cdot \left[|\Delta p_y|^\phi + \epsilon \right]^{-1}$$

Examples for: $\phi = 0.80$
 $\epsilon = 1e^{-5}$
 $N_B = \text{Poisson}(30)$
 $N_S = 2$



Example of $F_2(M)$ results

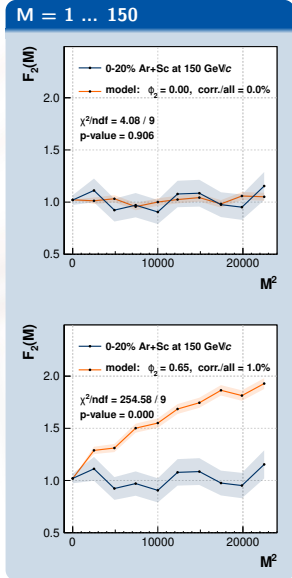
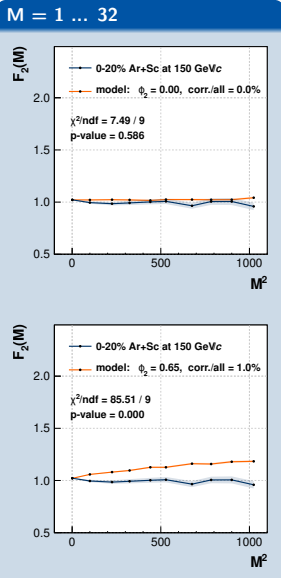
Comparison with simple power-law model

Lots of model data sets generated:

- correlated-to-all ratio: vary from 0.0 to 4.0% (with 0.2 step)
- power-law exponent: vary from 0.00 to 1.00 (with 0.05 step)

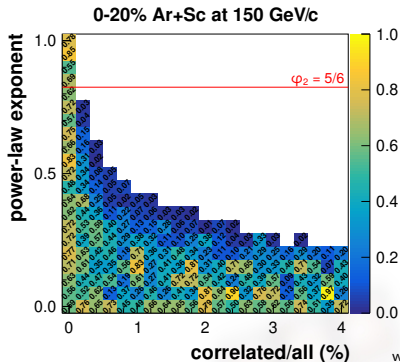
and compared with the experimental data

For the construction of exclusion plots, statistical uncertainties were calculated using model with statistics corresponding to the data.

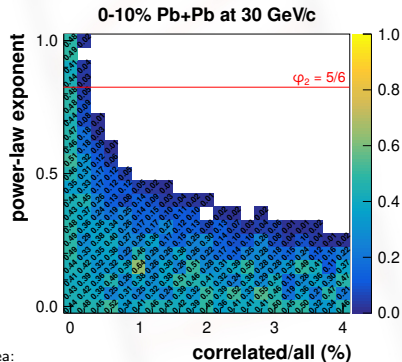


Exclusion plot

Comparison with simple power-law model



white area:
p-value < 0.01



exclusion plots for parameters of simple power-law model

The intermittency index φ_2 for a system freezing out at the QCD critical endpoint is expected to be $\varphi_2 = 5/6$ assuming that the latter belongs to the 3-D Ising universality class.

- results on the dependence of the second scaled factorial moments of proton multiplicity on cumulative momentum bin size ($F_2(M)$) for
 - Ar+Sc at 150A GeV/c
 - Pb+Pb at 30A GeV/care presented
- no indication for a power-law increase is observed
- exclusion plots for parameters of a simple model (ratio of correlated to background particles and power-law exponent) are shown

Thank You!

Additional slides

Cumulative transformation

1) normalization:

$$P(x, y) = \rho(x, y) / \int_{x,y} \rho(x, y) dx dy$$

2) projection on x:

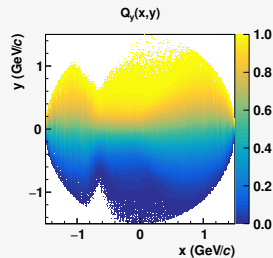
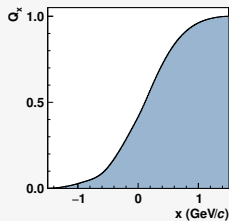
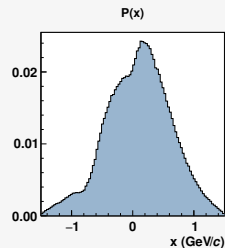
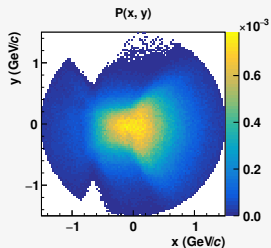
$$P(x) = \int_y P(x, y) dy$$

3) cumulative x:

$$Q_x = \int_{x_{\min}}^x P(x) dx$$

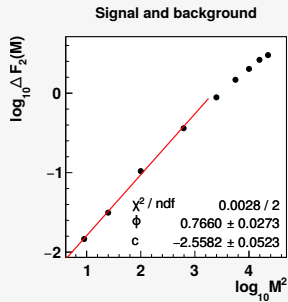
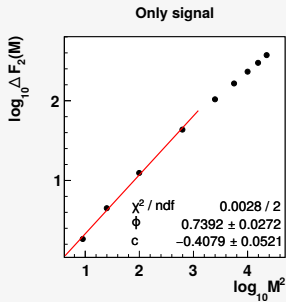
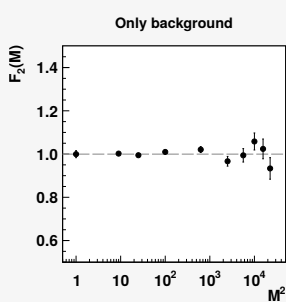
4) cumulative y:

$$Q_y(x) = \int_{y_{\min}}^y P(x, y) dy / P(x)$$



(examples for 0-5% Ar+Sc at 150A GeV/c, $x=p_x$, $y=p_y$)

Simple power-law model results



Event and single-track selection

Ar+Sc at 150A GeV/c

- full 'good' target-inserted data set (029_17c_v1r8p1_pA_slc6_phys_PP)
- in total, 1.10M events with $\langle N_{\text{proton}} \rangle \approx 3.8$ left for analysis

Event selection

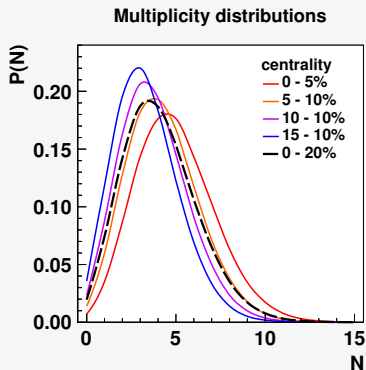
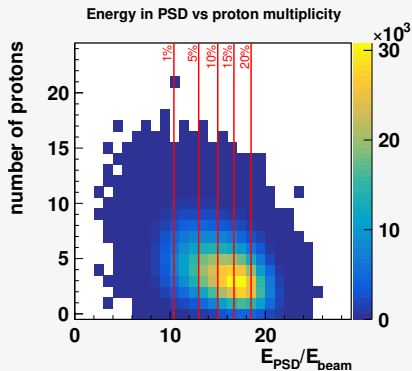
- WFA beam ($4 \mu\text{s}$)
- WFA interaction ($25 \mu\text{s}$)
- standard BPD cut
- trigger T2
- main vertex fit quality
- main vertex z-position (± 10 cm around target)
- energy in small PSD modules < 2800
- $800 < \text{energy in big PSD modules} < 5000$
- if number of tracks in fit < 50 , tracks in fit / all tracks > 0.25
- 'cloud no. 5'

Track selection

- good vertex track fit
- total number of measured clusters > 30
- $0.5 < \text{measured clusters} / \text{potential clusters} < 1.1$
- clusters in VTPCs > 15
- dE/dx clusters > 30
- $|b_x| < 2$ cm, $|b_y| < 4$ cm
- charge > 0
- $0.60 < \log_{10}(p / \text{GeV}/c) < 2.10$
- $0.5 < dE/dx < BB_p + 0.15(BB_K - BB_p)$
- $p_x < 1.5$ GeV/c, $p_y < 1.5$ GeV/c
- $|y_{\text{proton}}^{\text{CMS}}| < 0.75$
- acceptance map

Centrality selection

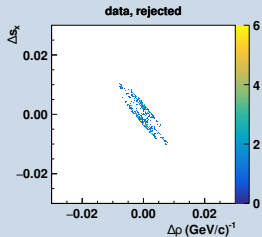
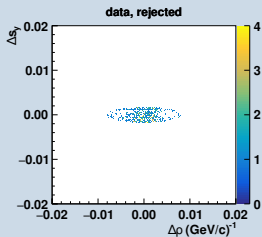
Ar+Sc at 150A GeV/c



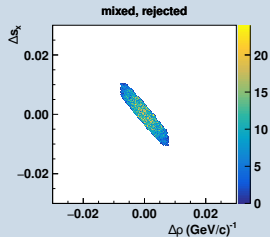
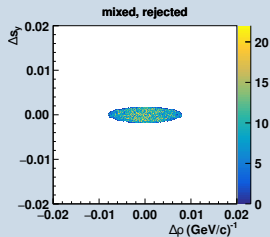
Two-Track Distance cut – rejected pairs

Ar+Sc at 150A GeV/c

Momentum-based TTD cut

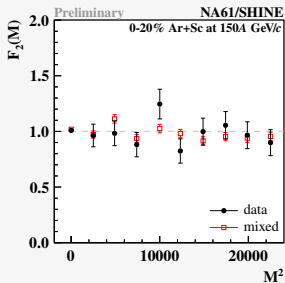
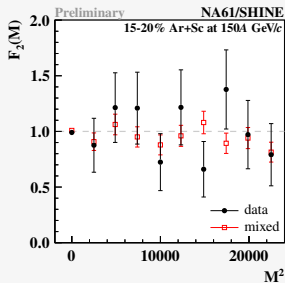
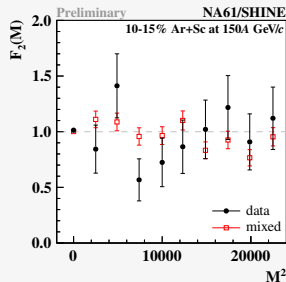
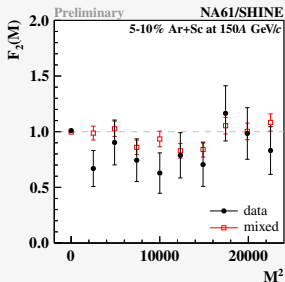
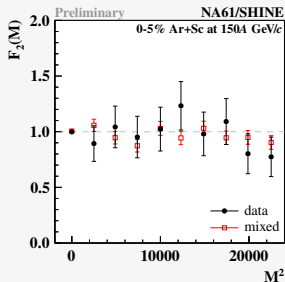


Momentum-based TTD cut



Results - $F_2(M)$

for $M = 1 \dots 150$

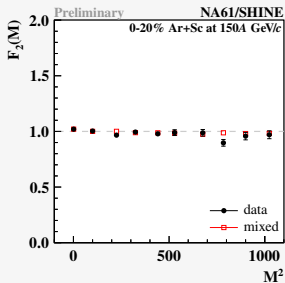
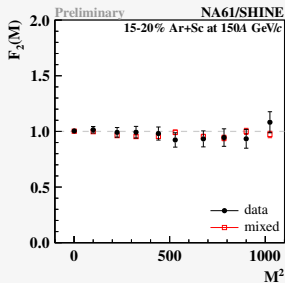
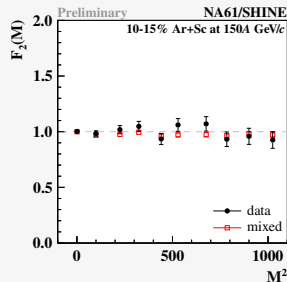
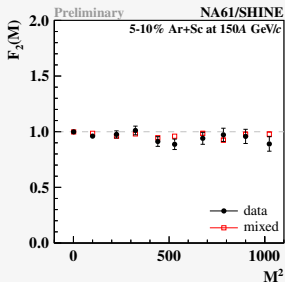
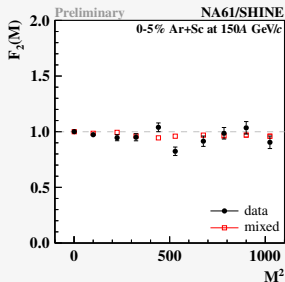


cumulative variables
uncorrelated points

No signal observed in
Ar+Sc at 150A GeV/c

Results - $F_2(M)$

for $M = 1 \dots 32$



cumulative variables
uncorrelated points

No signal observed in
Ar+Sc at 150A GeV/c