QCD Equation of State and Phase Diagram from Holographic Black Holes

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March, 2021

Outline

1. Introduction
   - The QCD Phase Diagram
2. Holographic Black Hole Model
   - Holography (Gauge/String duality)
   - Fixing the model at $\mu_B = 0$
3. Results
   - Mapping the QCD phase diagram and CEP
   - Equation of State
4. Critical Exponents
5. Summary

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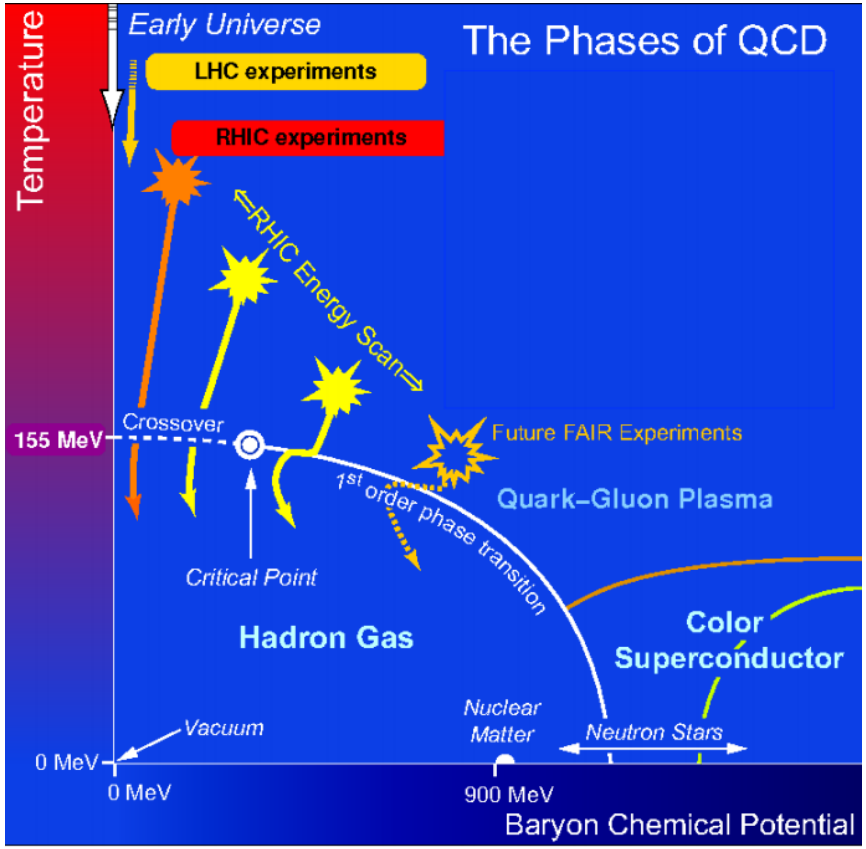
QCD Phase Diagram

We can explore the QCD phase diagram by changing $\sqrt{s}$ in relativistic heavy ion collisions

Many models predict a first order phase transition line with a critical point

Lattice QCD is the most reliable theoretical tool to study the QCD phase diagram.



QCD Phase Diagram

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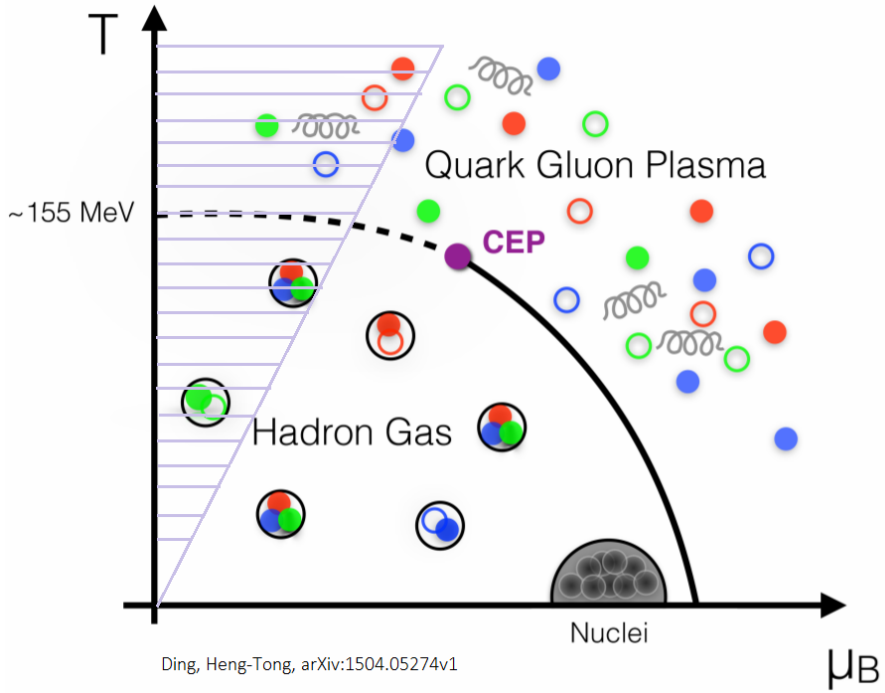
Limitation

Sign problem



Model Requirements

The model should exhibit:

-Deconfinement
-Nearly perfect fluidity
-Agreement with Lattice EoS at $\mu_B = 0$
-Agreement with baryon susceptibilities at $\mu_B = 0$

Taylor Expansion for small μ_B

$$\frac{P(T,\mu_B) - P(T,\mu_B = 0)}{T^4} = \sum_{n=1}^{\infty} \frac{1}{(2n)!} \chi_{2n}(T) \left(\frac{\mu_B}{T}\right)^{2n}$$

where $\chi_n(T,\mu_B) = \frac{\partial^n (P/T^4)}{\partial(\mu_B/T)^n}$

- How can we fulfill these conditions?

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  HOLOGRAPHIC BLACK HOLES!!!

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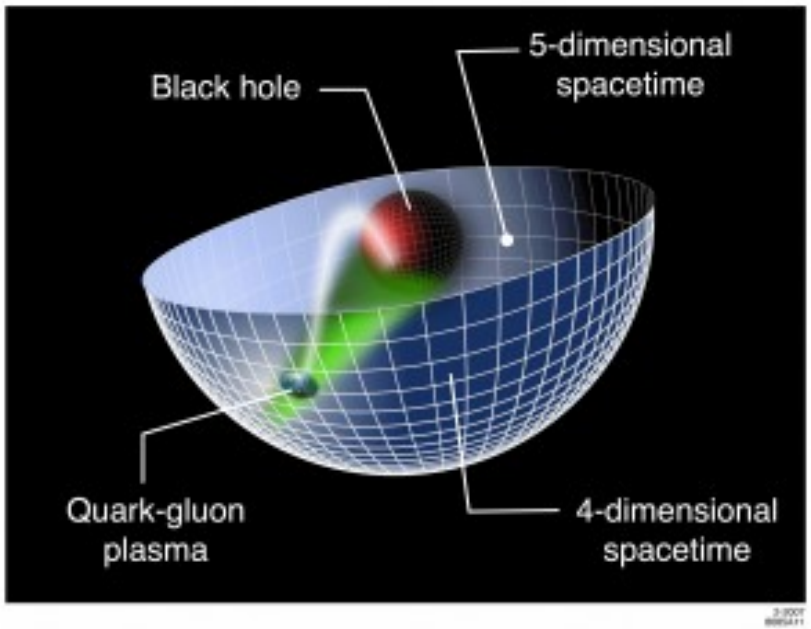
Holography (Gauge/String duality)

Holographic gauge/gravity correspondence

String Theory/Classical Gravity in 5-dimensions $\Longleftrightarrow$ Quantum Field Theory in 4-dimensions

Maldacena 1997; Witten 1998; Gubser, Polyakov, Klebanov 1998



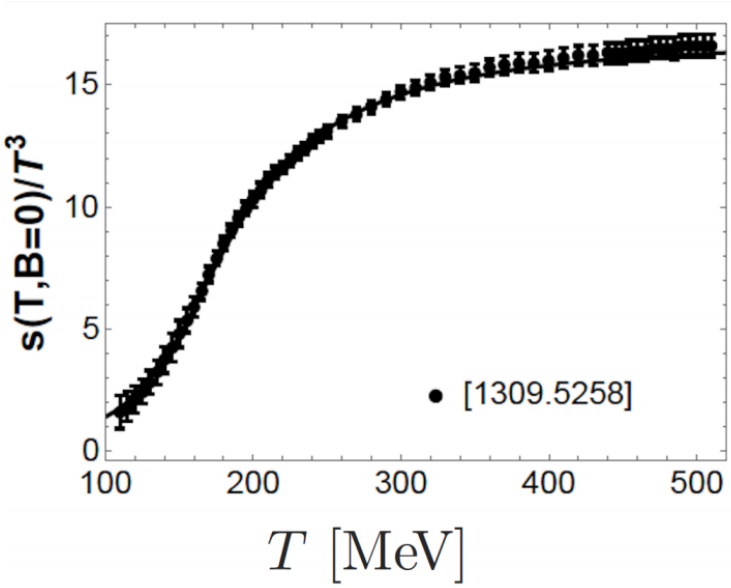
- Near Perfect fluidity
- string theory/classical gravity
  $\Updownarrow$
  strong coupling limit of QFT.
- BH solutions $\rightarrow$ T and μ_B in QFT

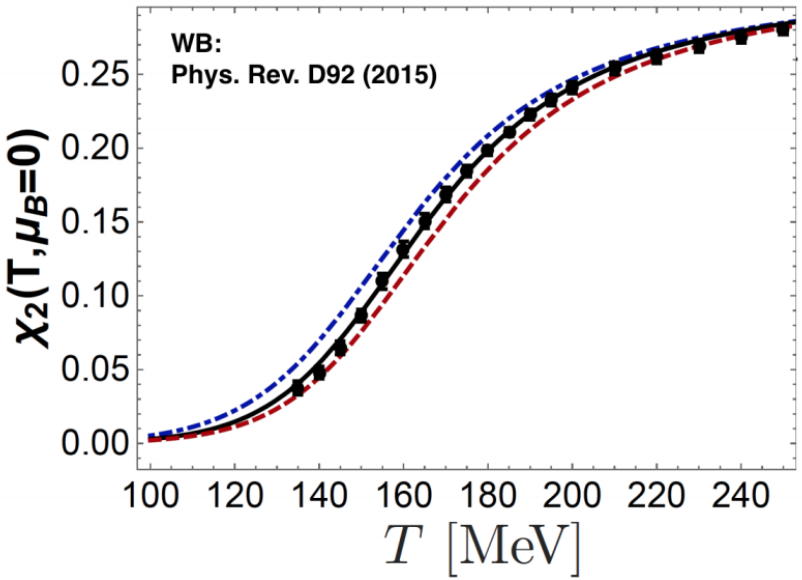
Gravitational Action

O DeWolfe et al. Phys.Rev.D 83, (2011).
R Rougemont et al. JHEP(2016)102.

R. Critelli et al., Phys.Rev.D96(2017).

$$S = \frac{1}{2\kappa_5^2}\int_{M_5} d^5x\sqrt{-g}\left[R - \frac{(\partial_\mu\phi)^2}{2} - \underbrace{V(\phi)}_{\text{nonconformal}} - \underbrace{\frac{f(\phi)F_{\mu\nu}^2}{4}}_{\mu_B\neq 0}\right]$$





$$\chi_2(T,\mu_B) = \frac{\partial^2(P/T^4)}{\partial(\mu_B/T)^2} = \frac{\partial(\rho/T^3)}{\partial(\mu_B/T)}$$

$V(\phi)$ and $f(\phi)$

R. Critelli et al., Phys.Rev.D96(2017).

J. G et al. arXiv:2102.12042 [nucl-th].

Free Parameters for the Holographic Model

$$\kappa_5^2 = 8\pi G_5 = 8\pi(0.46), \qquad \Lambda = 1053.83 MeV,$$

$$V(\phi) = -12\cosh(0.63\phi) + 0.65\phi^2 - 0.05\phi^4 + 0.003\phi^6,$$

$$f(\phi) = \frac{\mathrm{sech}(c_1\phi + c_2\phi^2)}{1+c_3} + \frac{c_3}{1+c_3}\mathrm{sech}(c_4\phi),$$

where

$$c_1 = -0.27, \quad c_2 = 0.4, \quad c_3 = 1.7, \quad c_4 = 100$$

Equations of Motion

$$S = \frac{1}{2\kappa_5^2}\int_{M_5} d^5x\sqrt{-g}\left[R - \frac{(\partial_\mu\phi)^2}{2} - \underbrace{V(\phi)}_{\text{nonconformal}} - \underbrace{\frac{f(\phi)F_{\mu\nu}^2}{4}}_{\mu_B\neq 0}\right]$$

$$\begin{aligned} ds^2 &= e^{2A(r)}[-h(r)dt^2 + d\vec{x}^2] + \frac{e^{2B(r)}dr^2}{h(r)} \\ \phi &= \phi(r) \\ A_\mu dx^\mu &= \Phi(r)dt \end{aligned}$$

Equations of Motion

$$\phi''(r) + \left[\frac{h'(r)}{h(r)} + 4A'(r)\right]\phi'(r) - \frac{1}{h(r)}\left[\frac{\partial V(\phi)}{\partial\phi} - \frac{e^{-2A(r)}\Phi'(r)^2}{2}\frac{\partial f(\phi)}{\partial\phi}\right] = 0$$

$$\Phi''(r) + \left[2A'(r) + \frac{d[\ln f(\phi)]}{d\phi}\phi'(r)\right]\Phi'(r) = 0$$

$$A''(r) + \frac{\phi'(r)^2}{6} = 0$$

$$h''(r) + 4A'(r)h'(r) - e^{-2A(r)}f(\phi)\Phi'(r)^2 = 0$$

$$h(r)[24A'(r)^2 - \phi'(r)^2] + 6A'(r)h'(r) + 2V(\phi) + e^{-2A(r)}f(\phi)\Phi'(r)^2 = 0$$

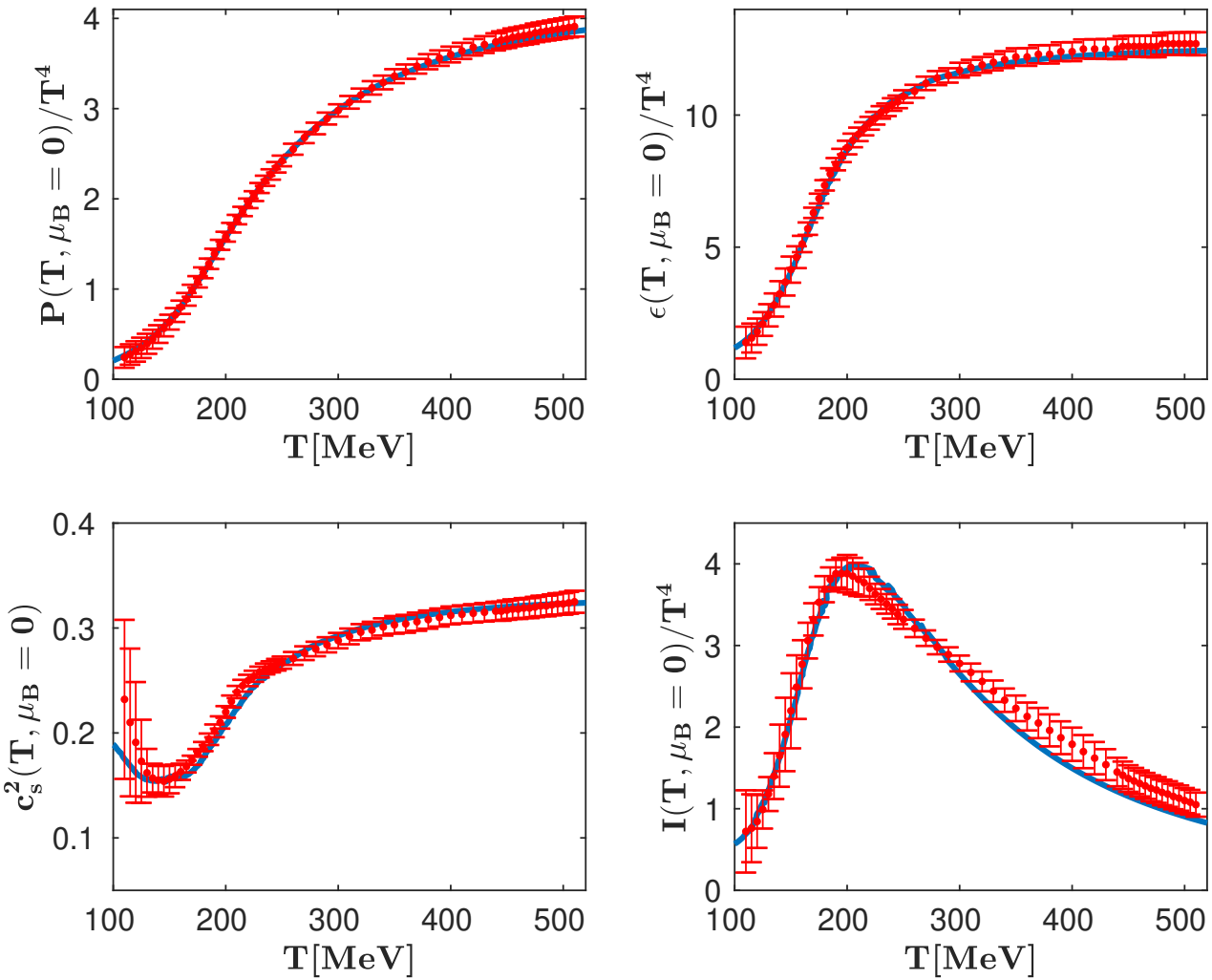
Far-Region asymptotic:

$$A(r) = \alpha(r) + \mathcal{O}\left(e^{-2\nu\alpha(r)}\right), \quad \text{where:} \quad \alpha(r) = A_{-1}^{\text{far}} r + A_0^{\text{far}}$$
$$h(r) = h_0^{\text{far}} + \mathcal{O}\left(e^{-4\alpha(r)}\right),$$
$$\phi(r) = \phi_A e^{-\nu\alpha(r)} + \mathcal{O}\left(e^{-2\nu\alpha(r)}\right),$$
$$\Phi(r) = \Phi_0^{\text{far}} + \Phi_2^{\text{far}} e^{-2\alpha(r)} + \mathcal{O}\left(e^{-(2+\nu)\alpha(r)}\right),$$

Thermodynamics:

$$T = \frac{1}{4\pi\phi_A^{1/\nu}\sqrt{h_0^{\text{far}}}}\Lambda \qquad s = \frac{2\pi}{\kappa_5^2\,\phi_A^{3/\nu}}\Lambda^3$$
$$\mu_B = \frac{\Phi_0^{\text{far}}}{\phi_A^{1/\nu}\sqrt{h_0^{\text{far}}}}\Lambda \qquad \rho_B = -\frac{\Phi_2^{\text{far}}}{\kappa_5^2\,\phi_A^{3/\nu}\sqrt{h_0^{\text{far}}}}\Lambda^3$$

Thermodynamics at $\mu_B = 0$



Lattice Results: [WB] S Borsanyi et al.Phys. Lett. B730.99.

BH curves: J. G et al. arXiv:2102.12042 [nucl-th].

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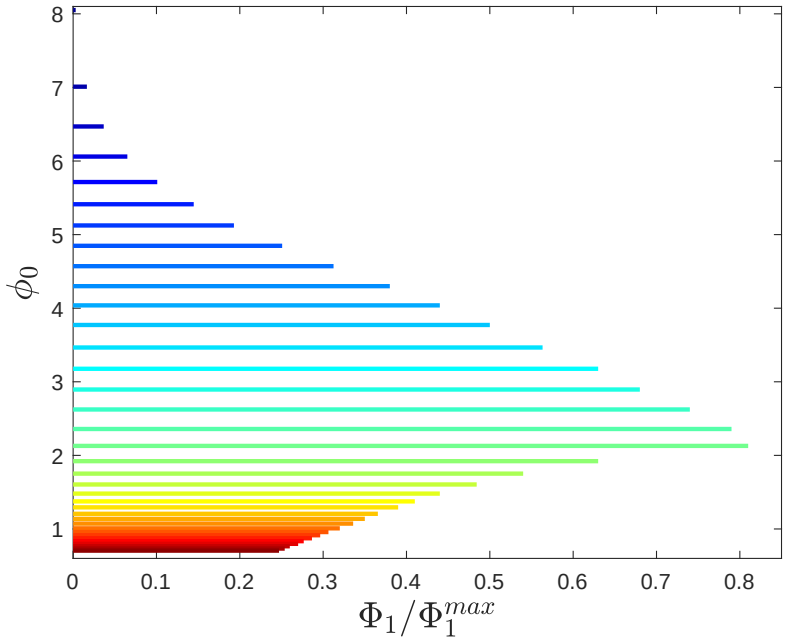
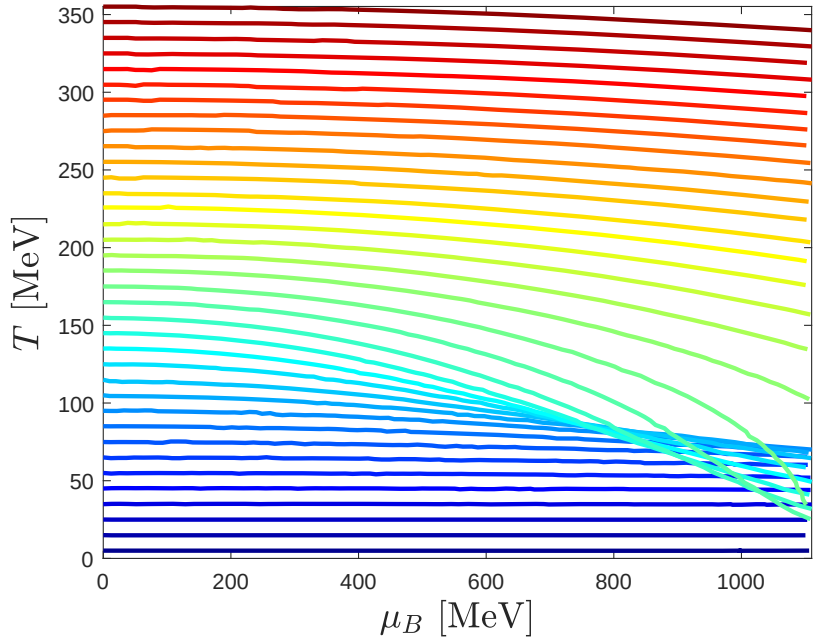
Mapping the QCD phase diagram from Black Hole solutions

The BH solutions are parametrized by (ϕ_0, Φ_1), where

$\phi_0 \rightarrow$ value of the scalar field at the horizon, and

$\Phi_1 \rightarrow$ electric field in the radial direction at the horizon



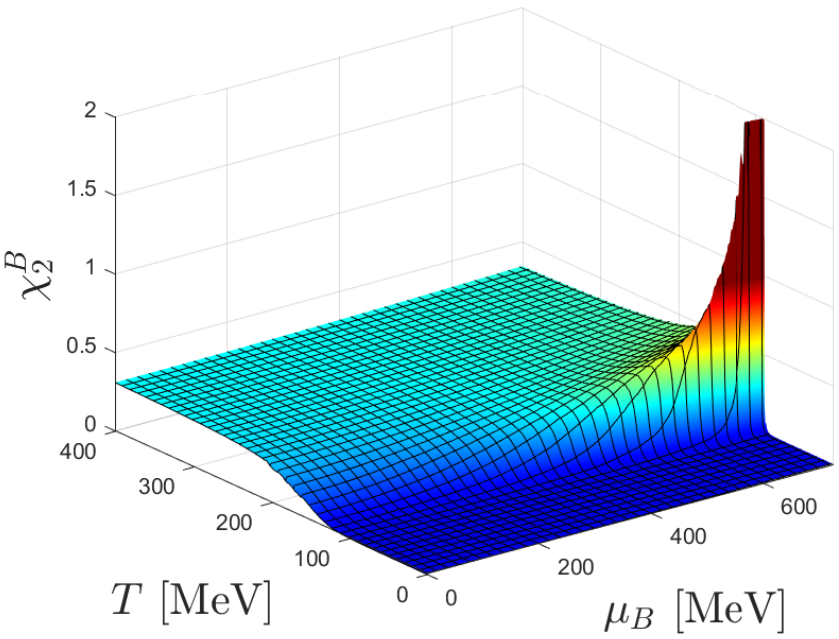


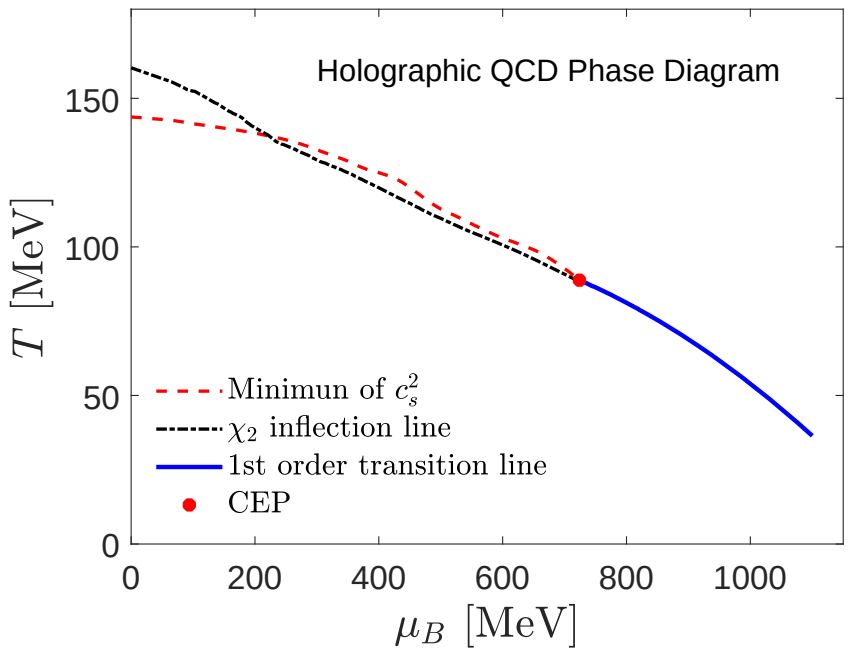
J. G et al. arXiv:2102.12042 [nucl-th].

Locating the Critical End Point (CEP)

$T_{CEP} = 89$ MeV

$\mu_B^{CEP} = 724$ MeV





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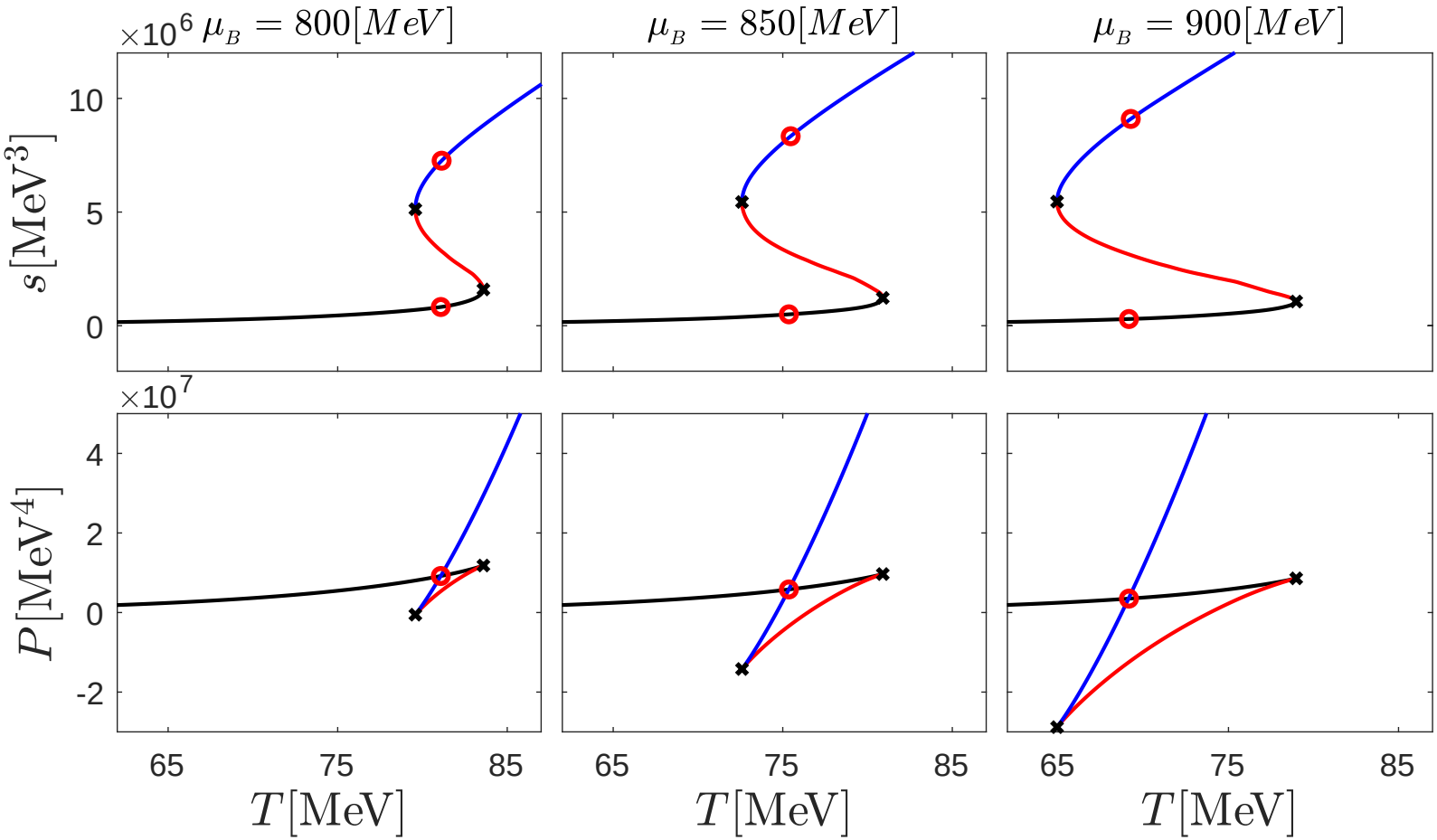
Locating the first order phase transition line



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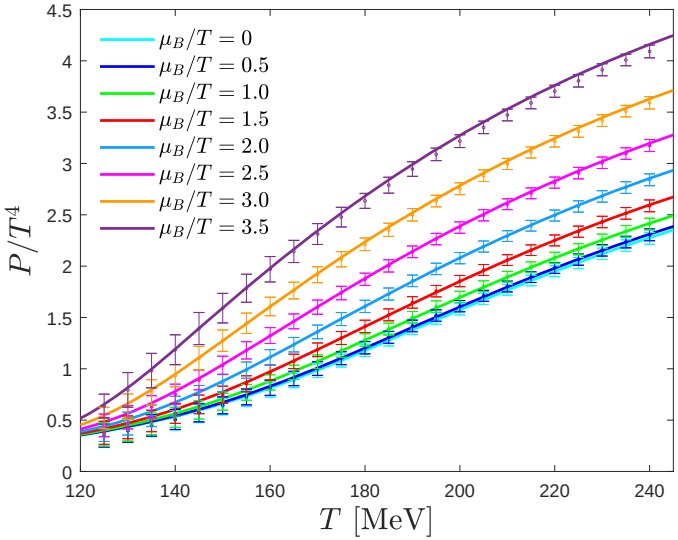
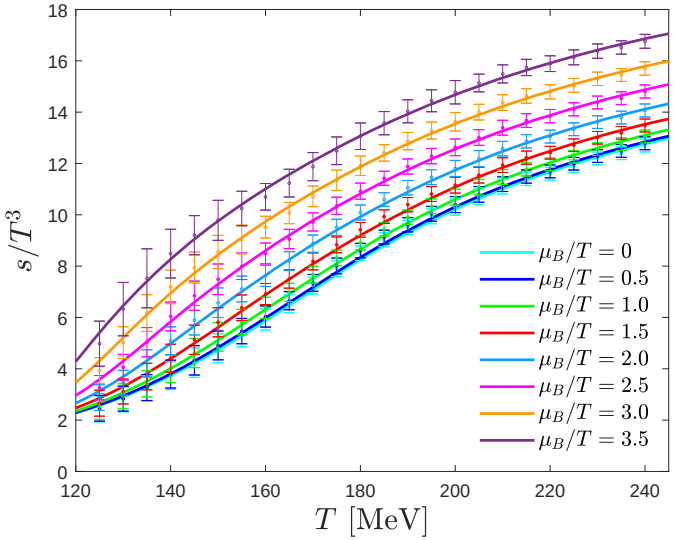
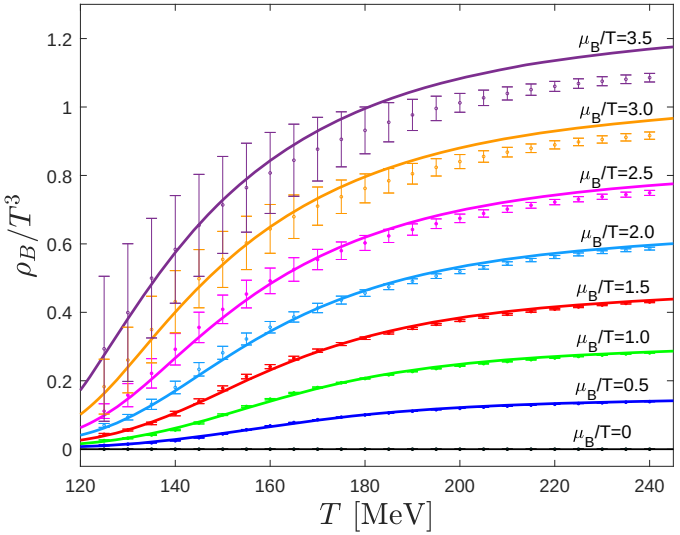
Comparison with the state-of-the-art lattice QCD thermodynamics



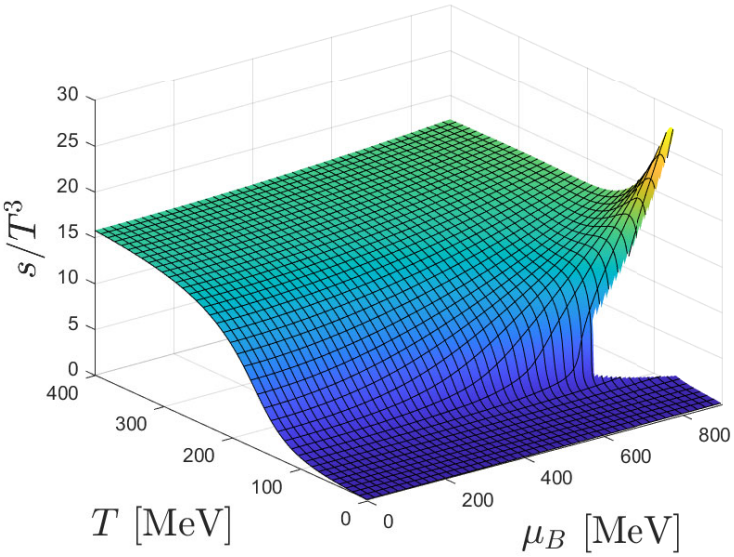


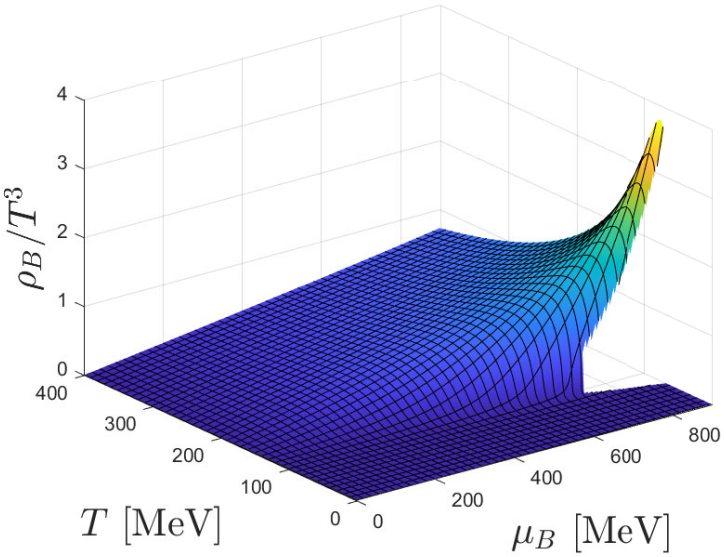


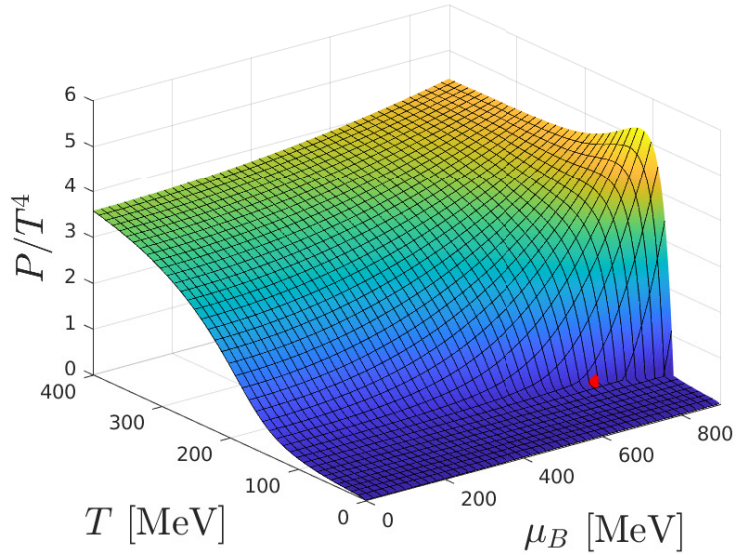
Lattice results: S. Borsanyi et al. arXiv:2102.06660 [hep-lat].

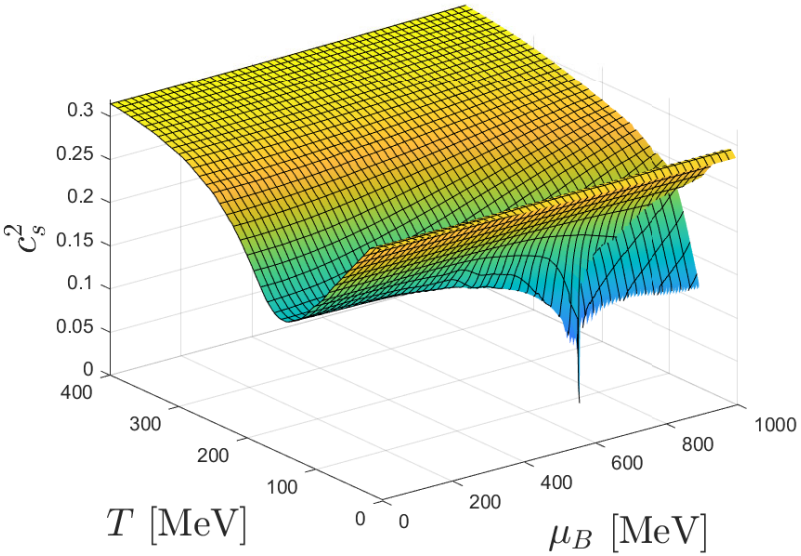
BH curves: J. G. et al. arXiv:2102.12042 [nucl-th].

Equation of State



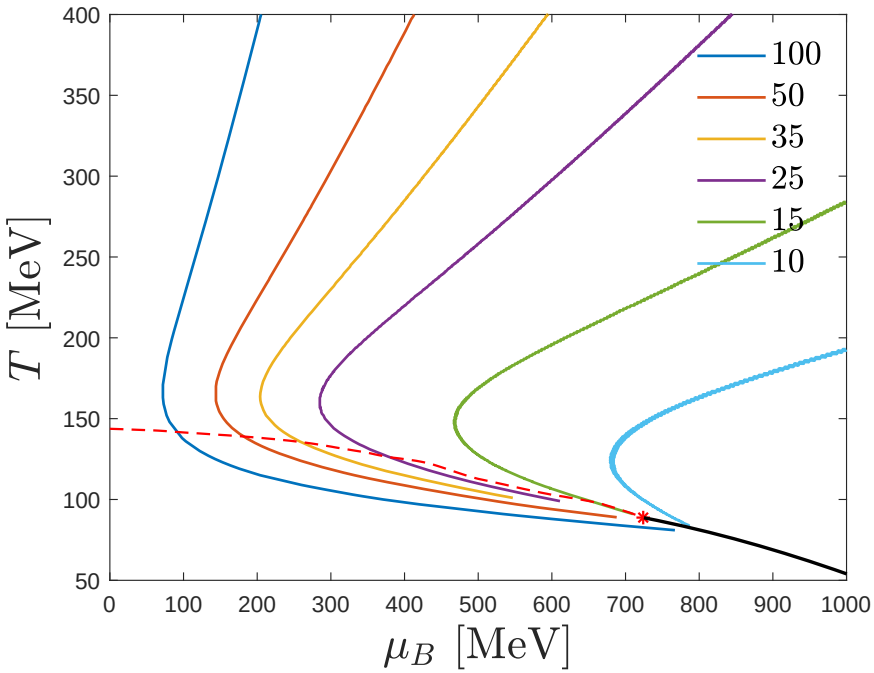


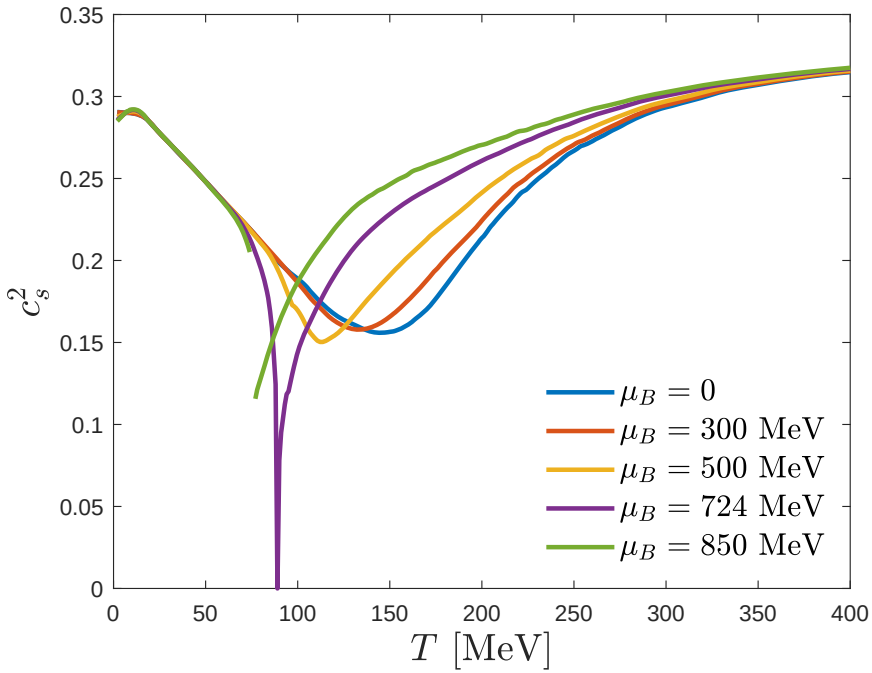




J. G et al. arXiv:2102.12042 [nucl-th].

Speed of sound and Isentropes





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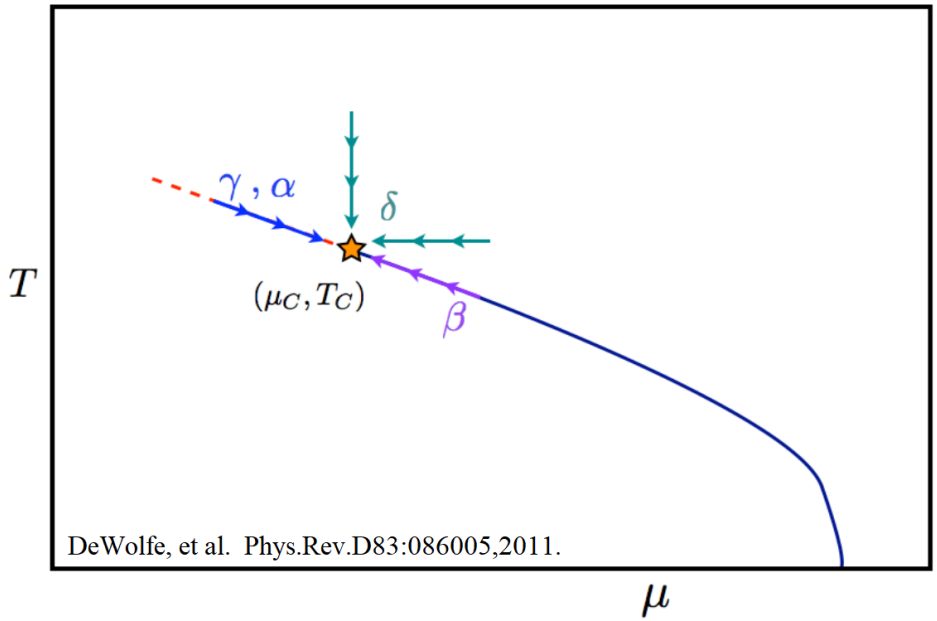
Critical Exponents



The critical point of QCD is expected to lie in the universality class of the 3D Ising model (see e.g., Rajagopal, Wilczek, NPB 399 (1993) 395).

α along the first order axis

$$C_\rho \sim |T - T_c|^{-\alpha}$$

β along first order line

$$\Delta\rho \sim (T_c - T)^{\beta}$$

γ along first order axis

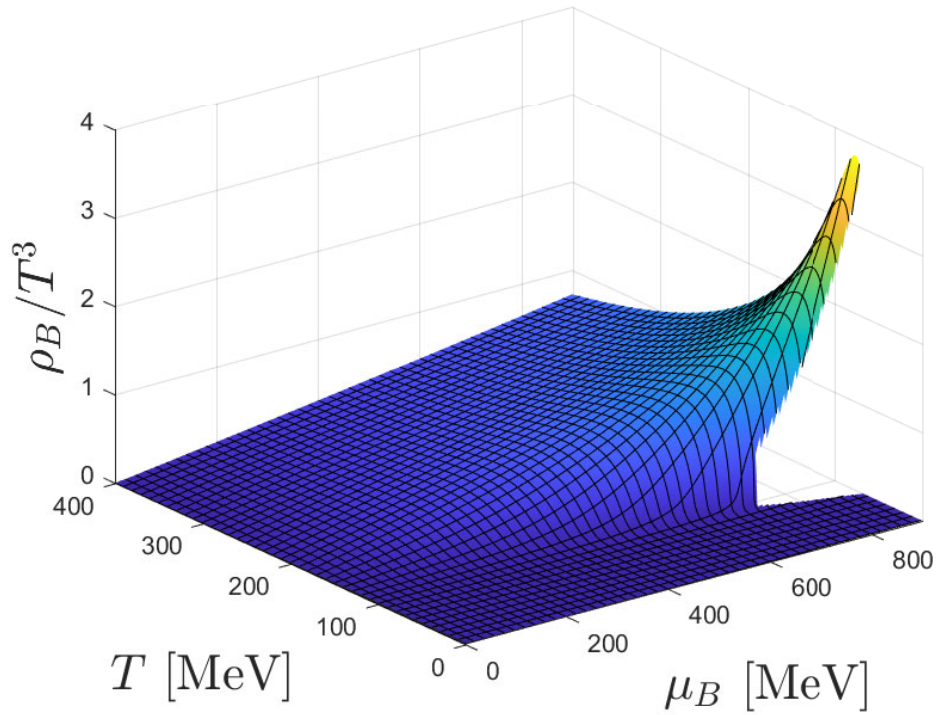
$$\chi_2 \sim |T - T_c|^{-\gamma}$$

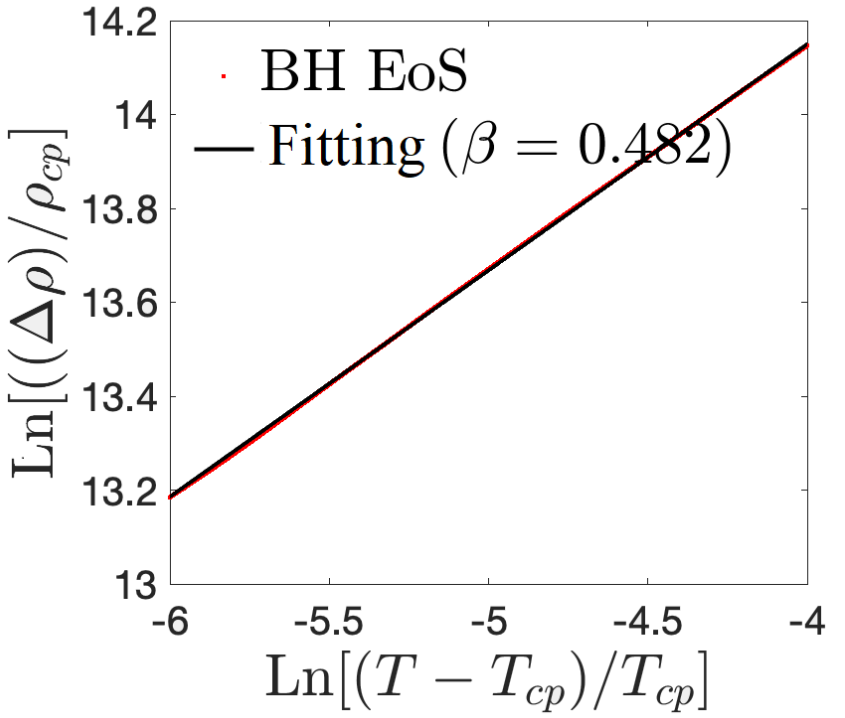
δ for $T = T_c$

$$\rho - \rho_c \sim |\mu - \mu_c|^{1/\delta}$$

$\Delta\rho$ and β

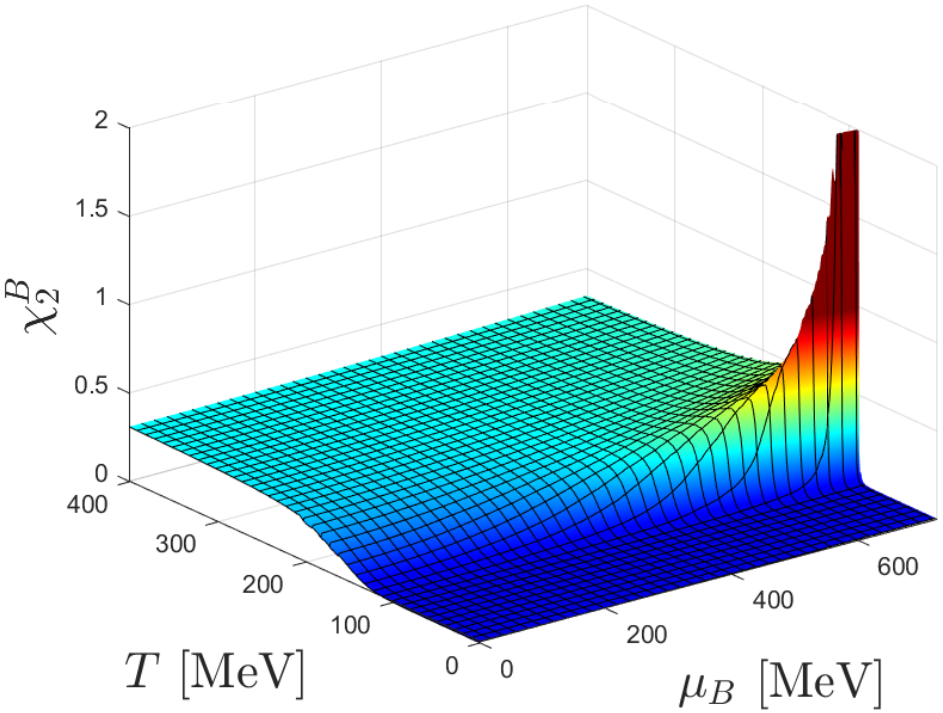
$$\Delta\rho \sim (T_c - T)^{\beta}, \qquad \text{along the first order line}$$

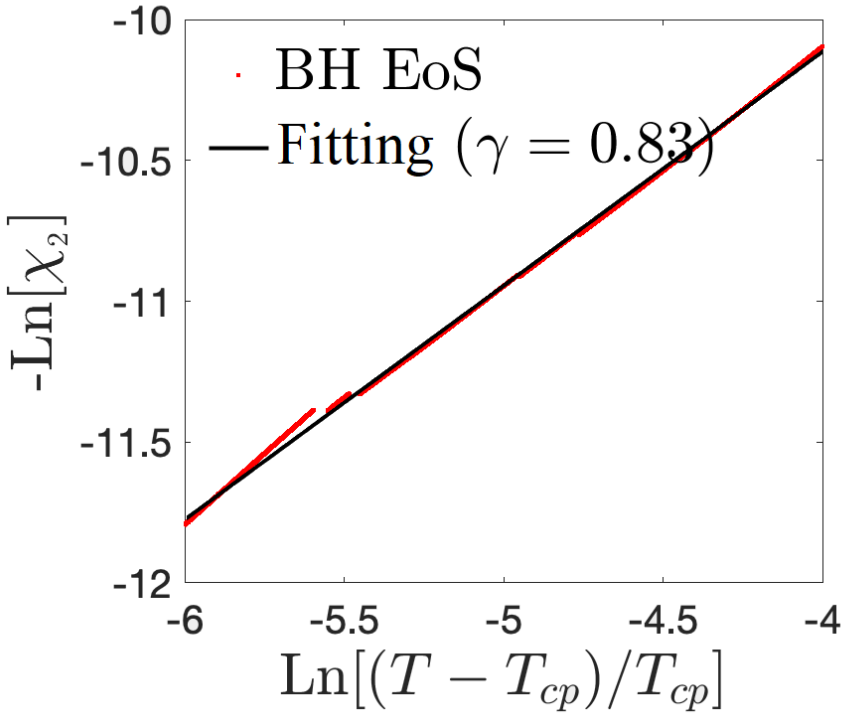




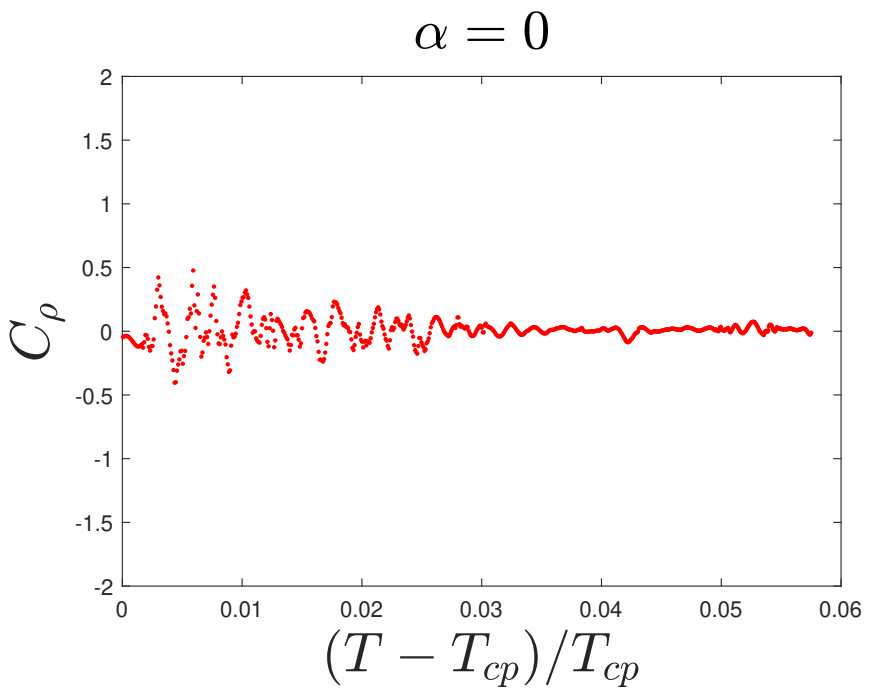
χ_2 and γ

$$\chi_2 \sim |T - T_c|^{-\gamma}, \qquad \text{along first order axis}$$





C_ρ, α and δ



$$C_\rho \equiv T \left(\frac{\partial s}{\partial T} \right)_\rho$$

Scaling relations

$$\alpha + 2\beta + \gamma = 2$$

$$\alpha + \beta(1 + \delta) = 2$$

$$\delta = 3.149$$

	3D Ising	Mean Field	Our Model*
α	0.110(5)	0	0
β	0.325±0.0015	0.5	0.482
γ	1.2405±0.0015	1	0.83
δ	4.82(4)	3	3.149

*Consistent with mean-field scaling

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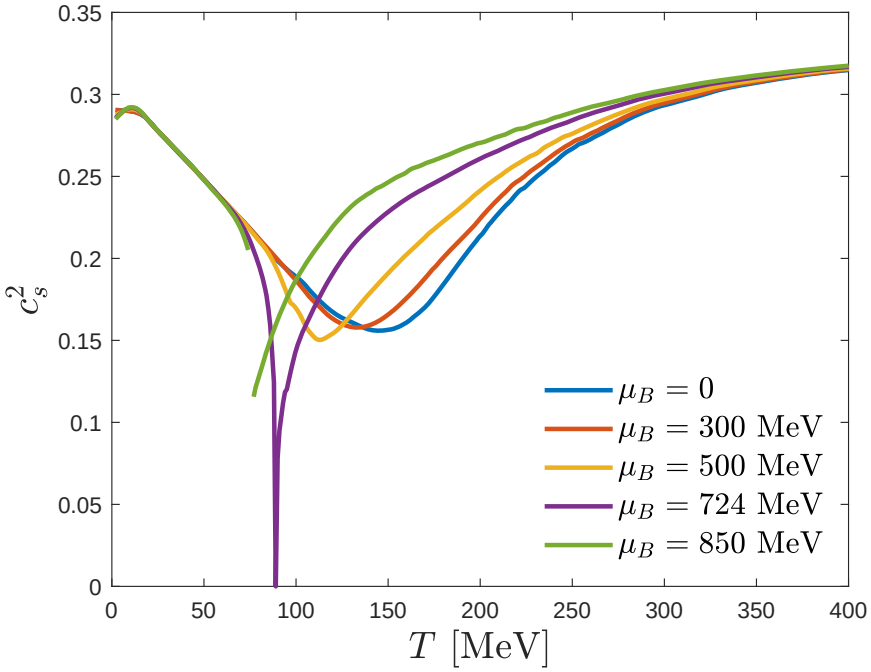
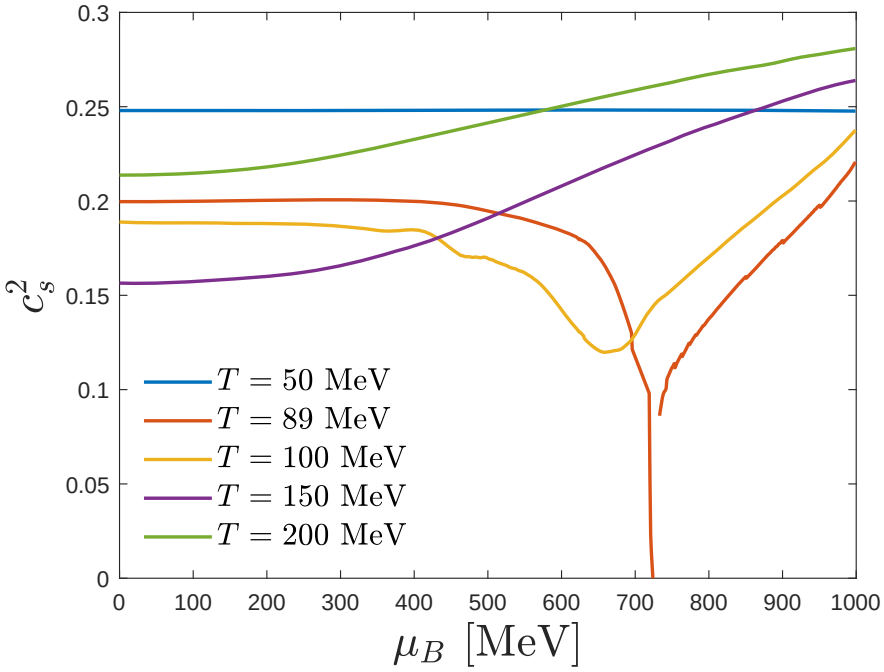
Summary

- The holographic model, which is fixed to mimic the Lattice EoS for $\mu_B = 0$, predicts a CEP:
$$T^{CEP} = 89 \text{ MeV}, \ \mu_B^{CEP} = 724 \text{ MeV}$$
- With the first order phase transition line located in the QCD phase diagram, we considerably extended the baryon chemical potential coverage of the EoS in the QCD phase diagram.
- The holographic critical exponents are consistent with mean field theory scaling,
$$\alpha = 0, \quad \beta \approx 0.482, \quad \gamma \approx 0.83, \quad \delta \approx 3.149$$
- Future work:
-From the present model, we expect to compute the QCD transport coefficients across the entire phase diagram covered in this work.

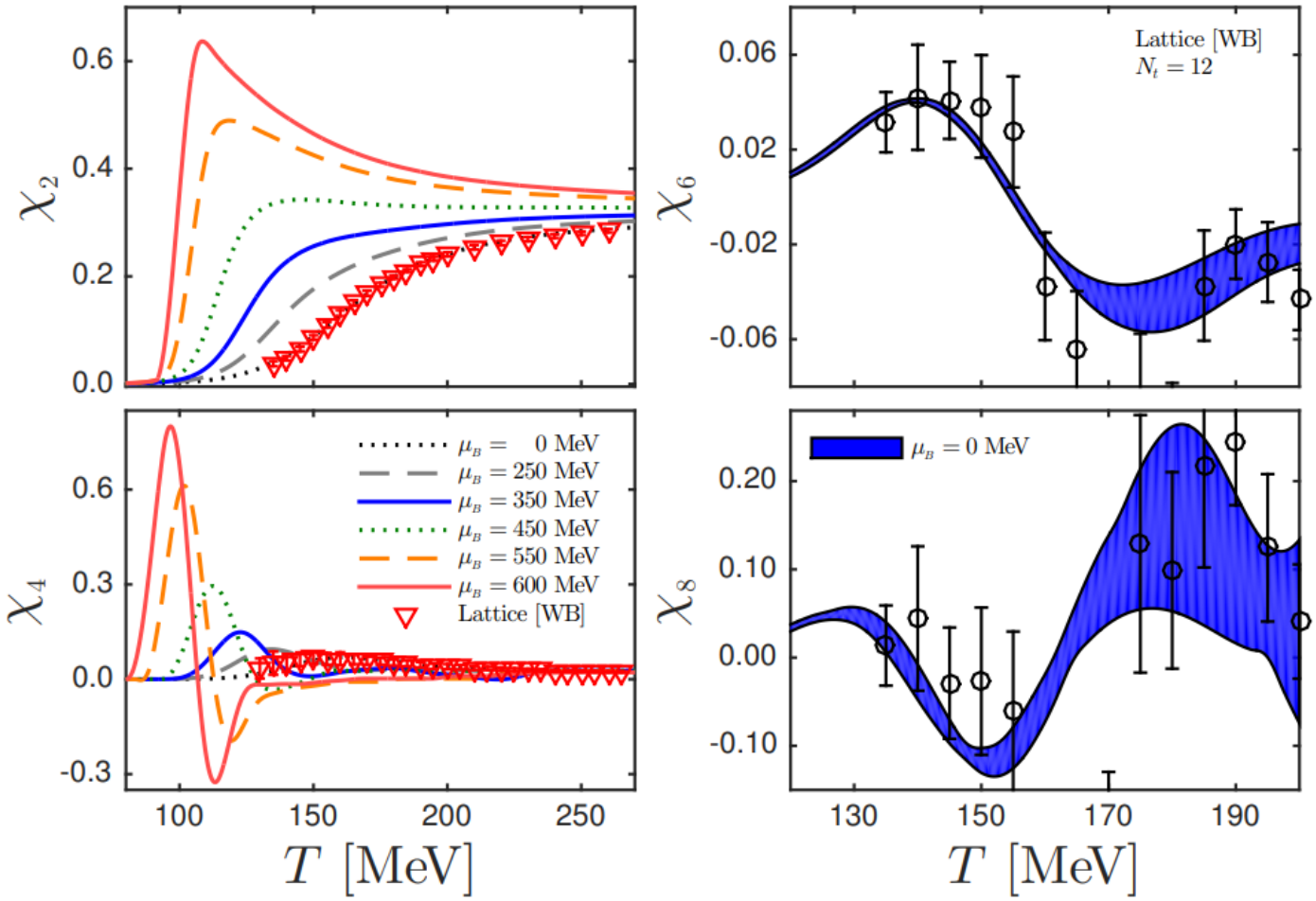
THANKS!

Speed of sound





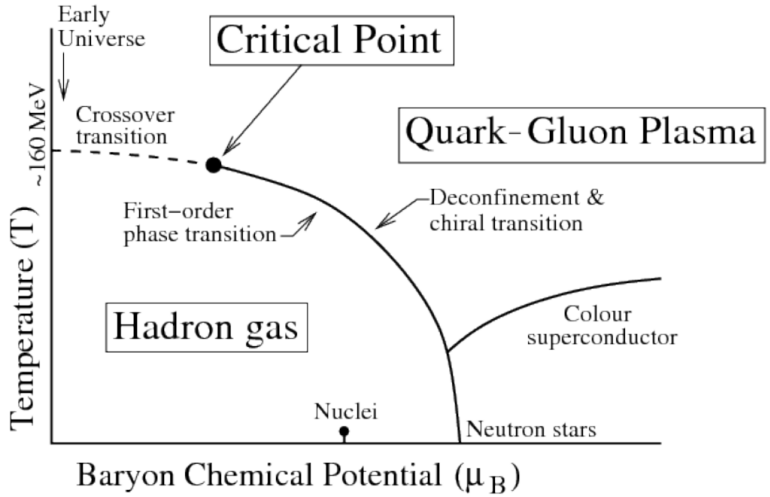
Susceptibilities at finite μ_B

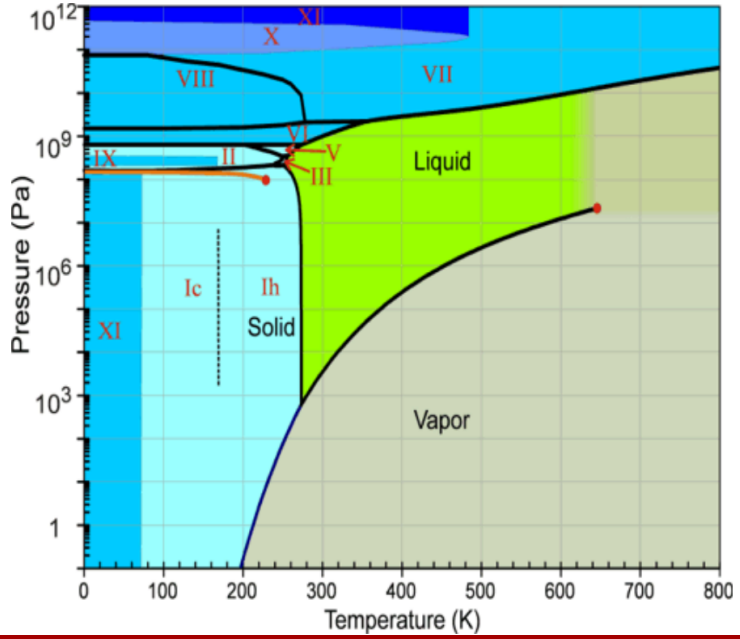


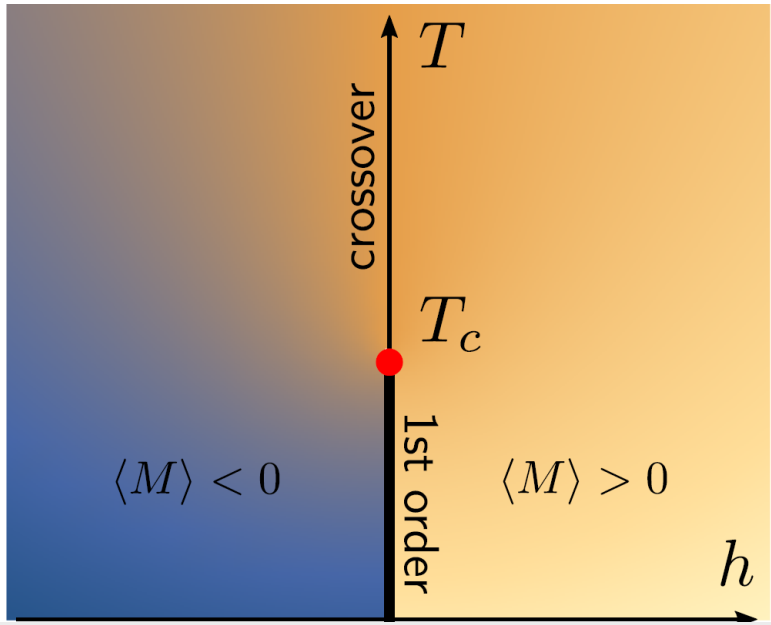
Lattice Results: [WB] S Borsanyi et al. arXiv:1805.04445v1.

BH curves: R. Critelli et al., Phys.Rev.D96(2017).

The QCD Critical Point



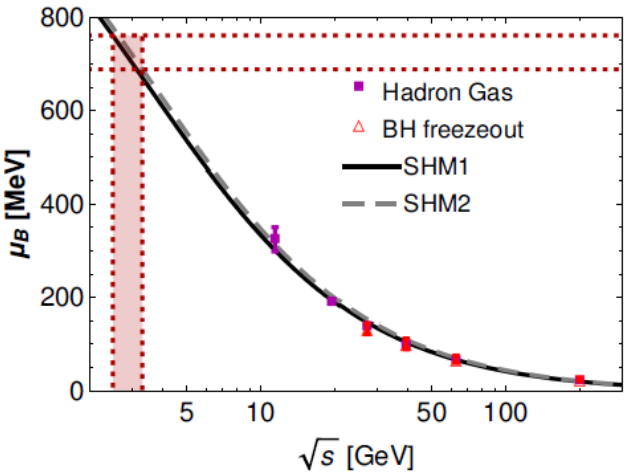


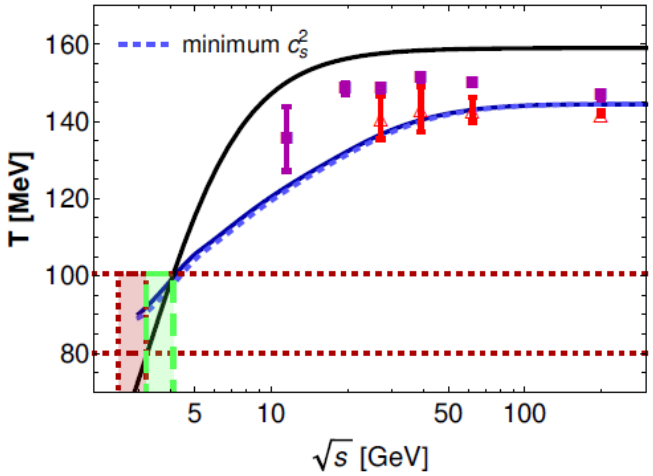


Collision Energy Estimates

We estimate a collision energy needed to hit the CEP

- $\sqrt{s} = 2.5 - 4.1$ GeV





- The collision energy is reachable by the next generation of experiments.

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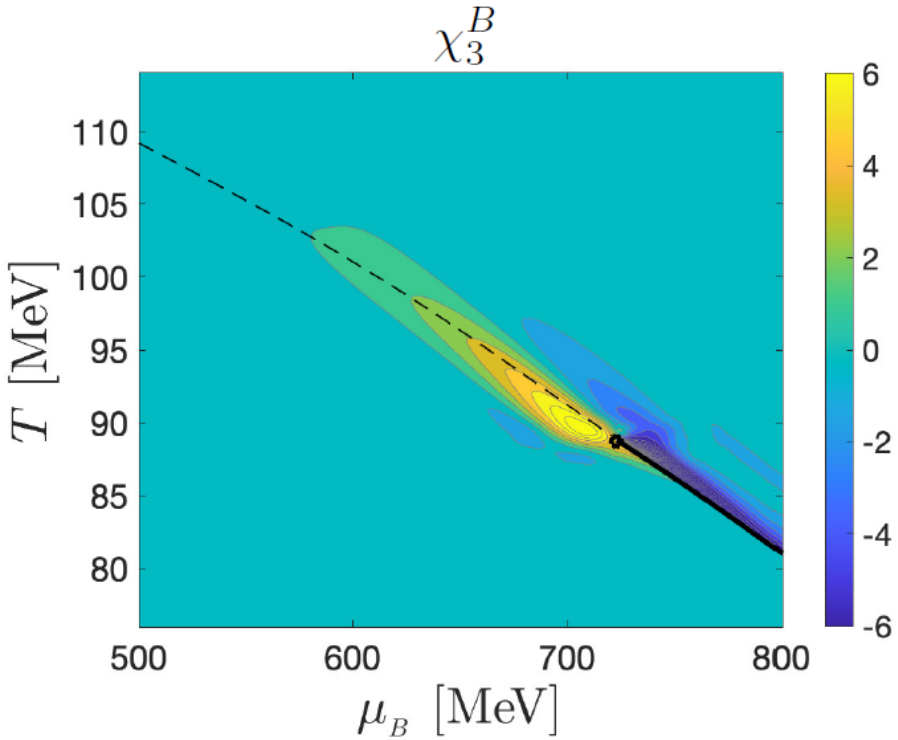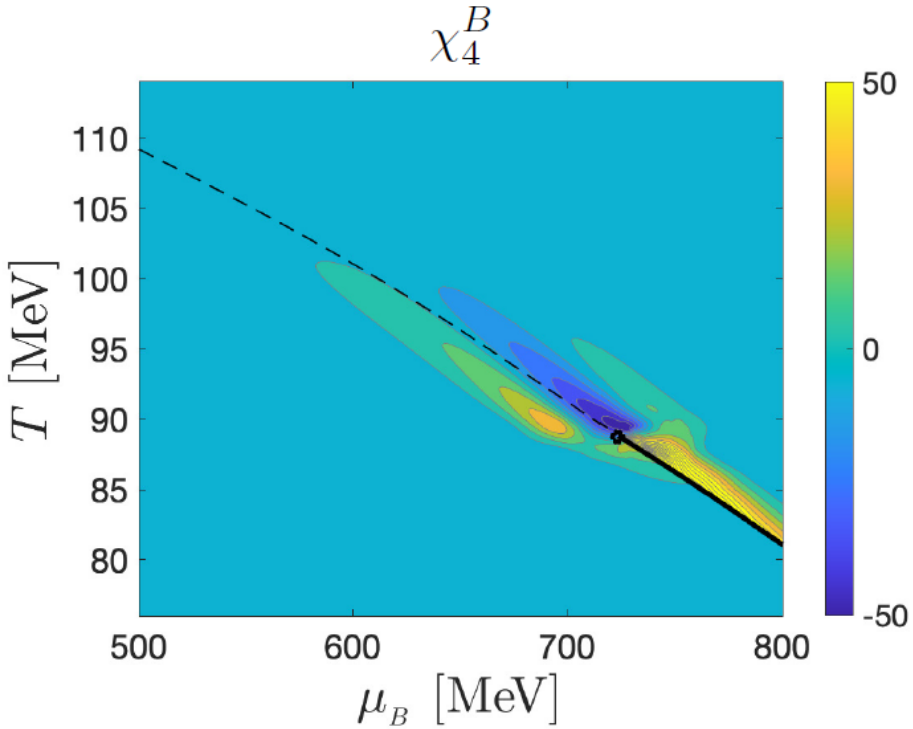
Higher Order Baryon Susceptibilities