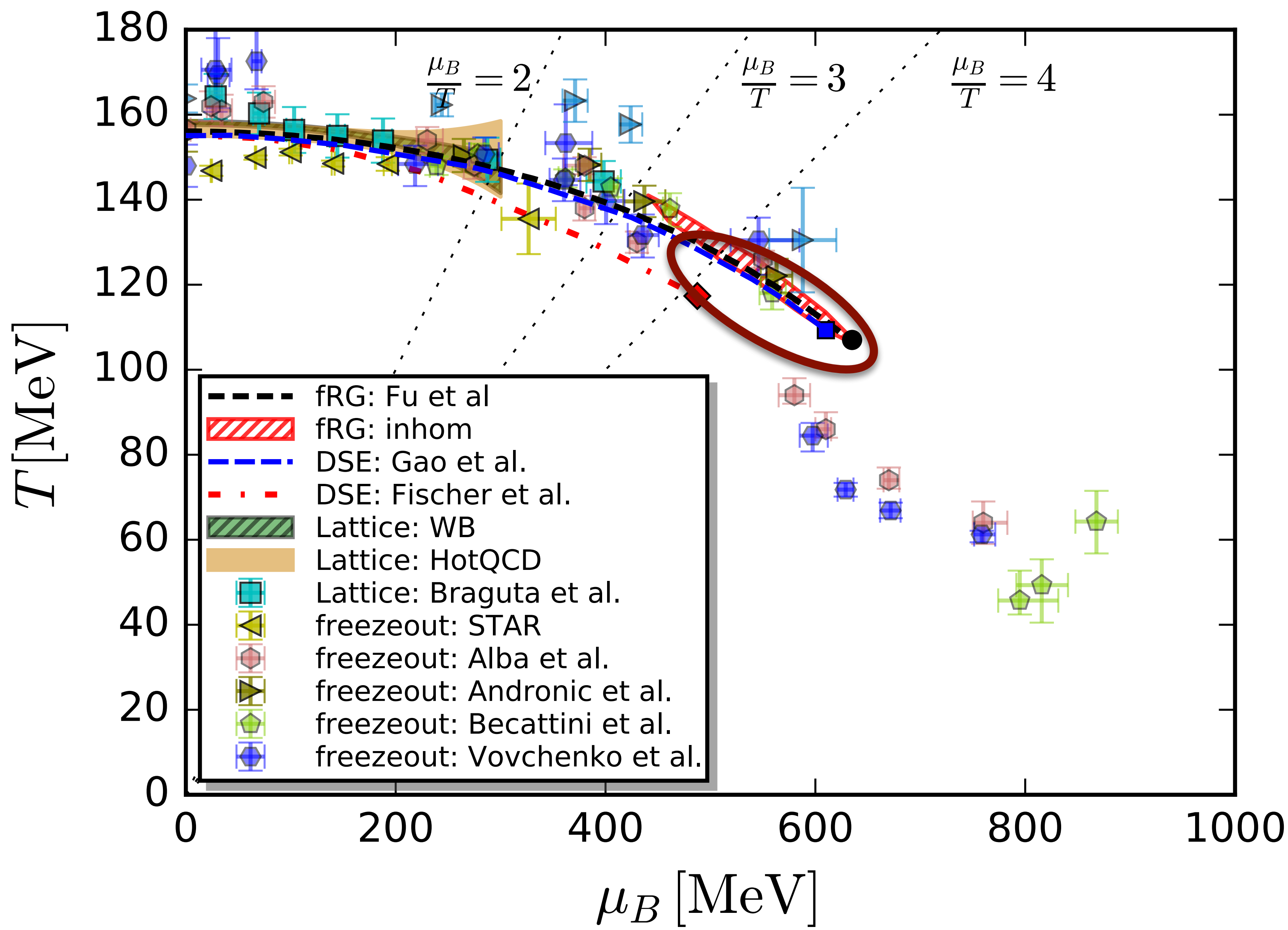
QCD phase structure



$$(135\,,\,450\,)\,\mathrm{MeV} \lesssim (T_{\mathrm{CEP}}\,,\,\mu_{B_{\mathrm{CEP}}}) \lesssim (100\,,\,650)\,\mathrm{MeV}$$

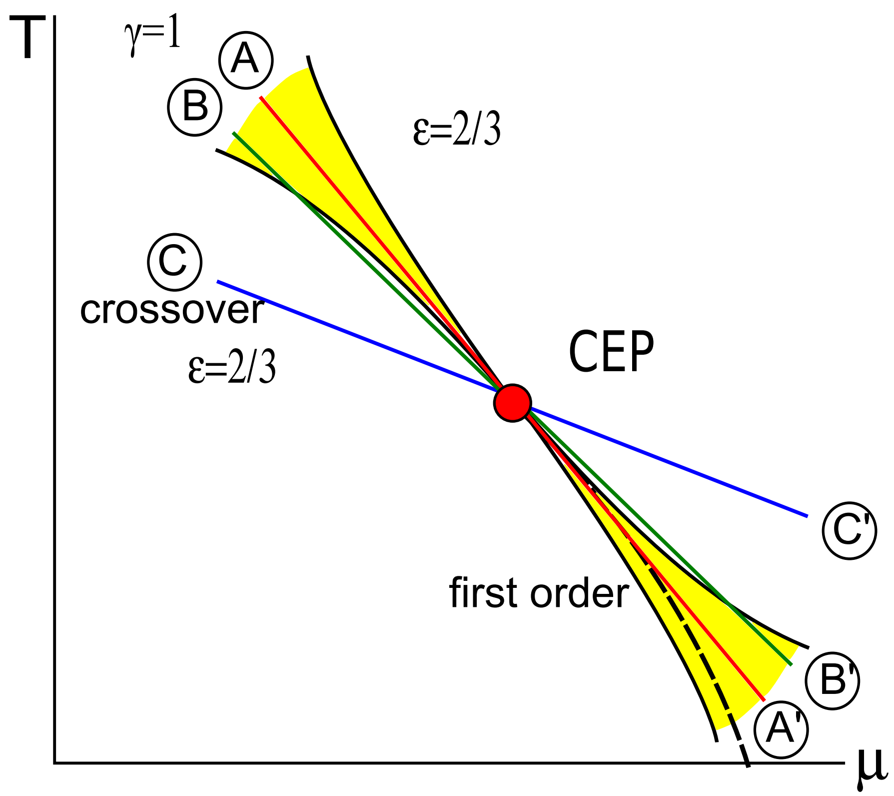
Suggests: no criticality for $\frac{\mu_B}{T_c} \lesssim 4$

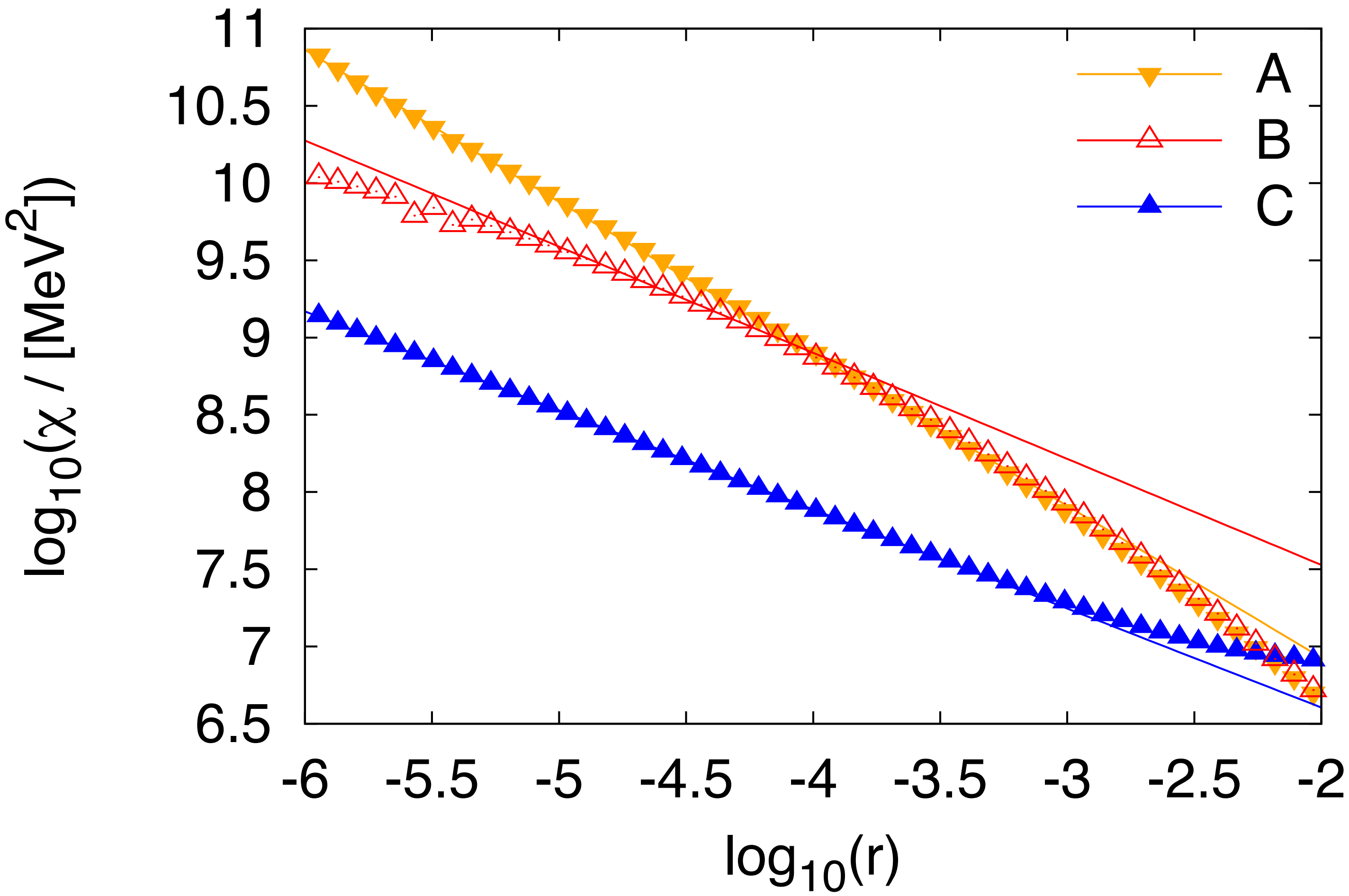
fRG: Fu, JMP, Rennecke, PRD 101, (2020) 054032

DSE: Fischer, PPNP 105 (2019) 1, Gao, JMP, arXiv:2010.137005 by courtesy of Fabian Rennecke

SIZE OF THE CRITICAL REGION

$r = \sqrt{\hat{t}^2 + \hat{\mu}^2}$



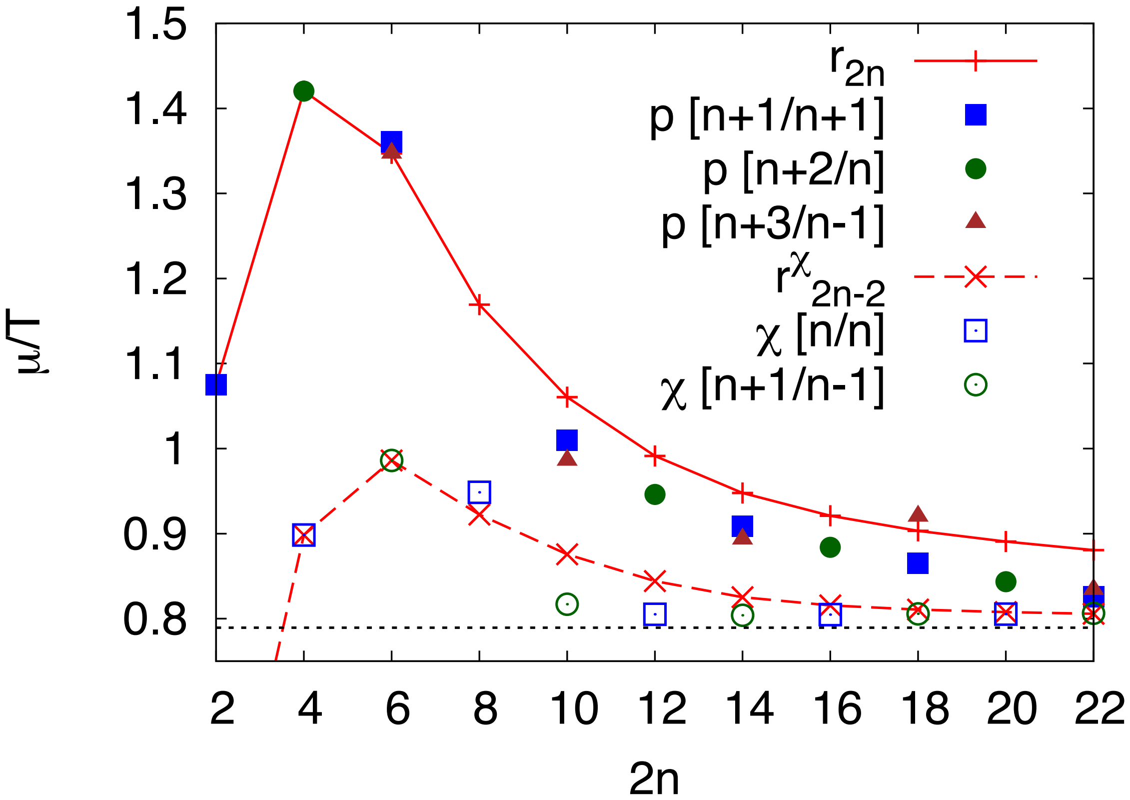


- mean-field PQM: critical region $\lesssim 10\,\mathrm{MeV}$ (tangential to transition line (A)) [Schaefer, Wagner (2012)]
- including quantum fluctuations: critical region shrinks to $< 1\,\mathrm{MeV}$ in T and μ_B direction

[Schaefer, Wambach (2006)]
[Chen, Wen, Fu (2021)]

by courtesy of Fabian Rennecke

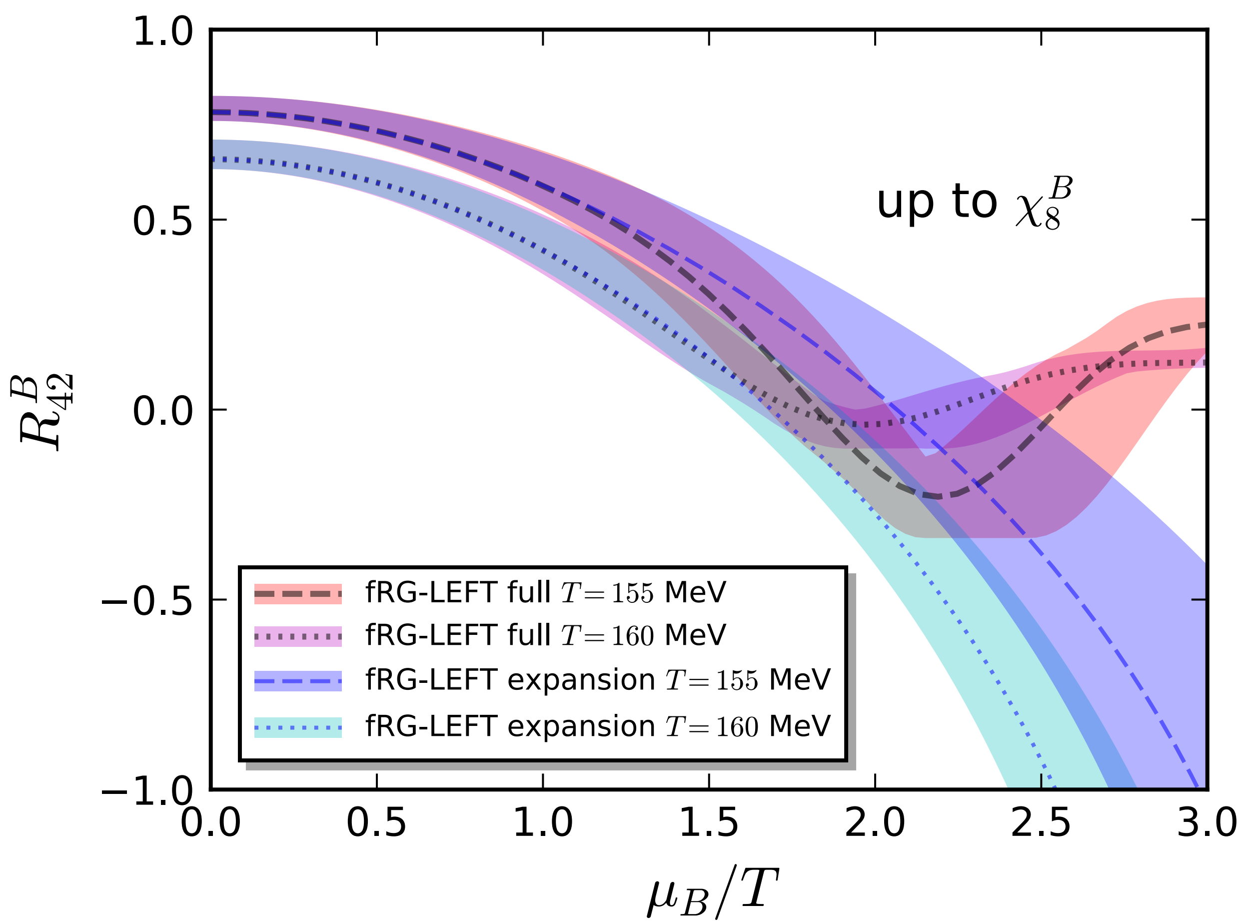
VALIDITY OF THE TAYLOR EXPANSION



- estimate of location of 2nd order phase transition based on Taylor expansion about $\mu = 0$ for different expansion orders n

very high orders necessary for good estimates

[Karsch, Schaefer, Wagner, Wambach (2011)]



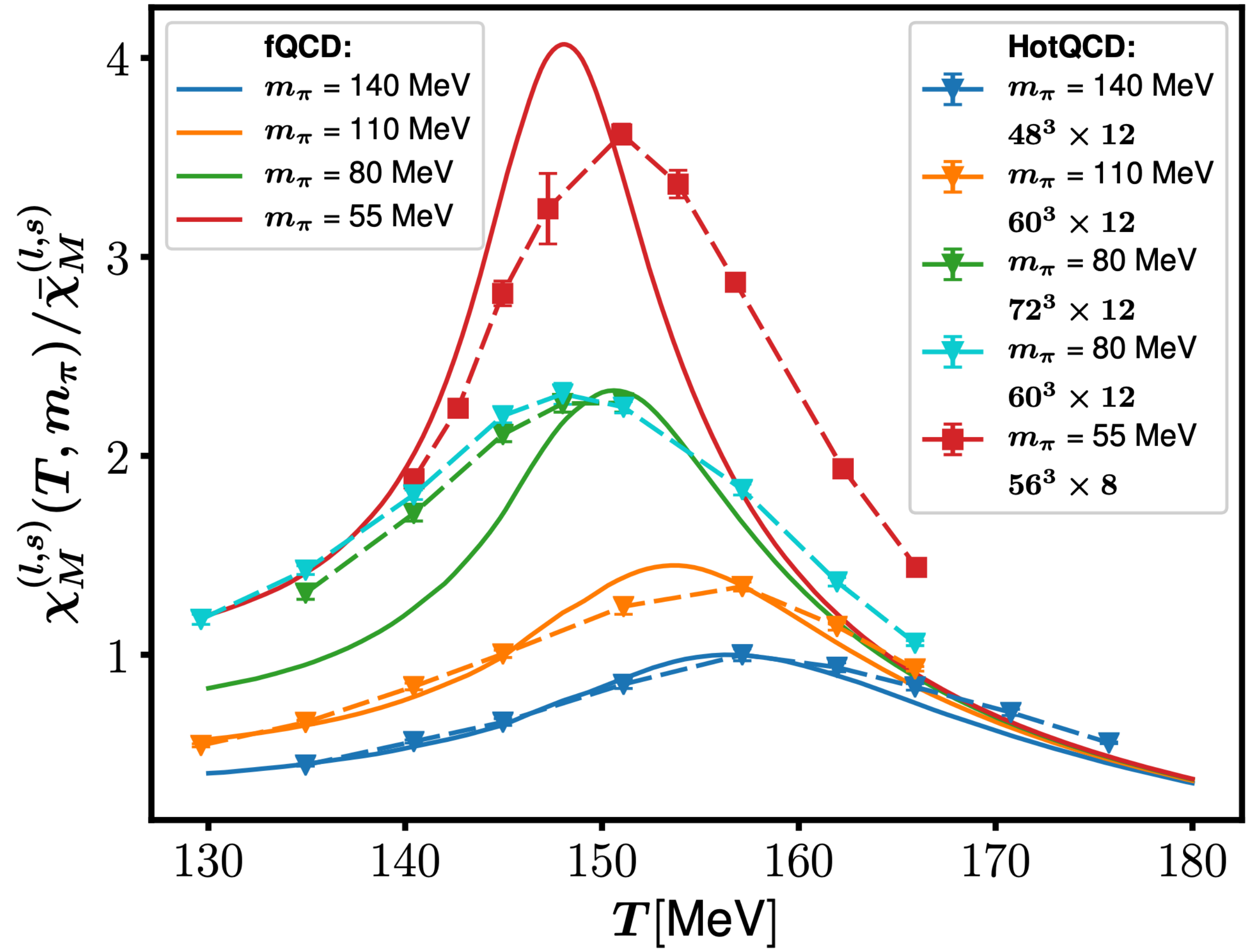
- Taylor expansion about $\mu_B = 0$ vs **direct computation**

Taylor expansion fails to capture qualitative features at finite μ_B

[Fu, Luo, Pawlowski, Rennecke, Wen (2021)] by courtesy of Jens Braun

Scaling analysis from functional QCD

Braun, Fu, Pawlowski, Rennecke, Rosenblüh, Yin, PRD 102 (2020) 056010

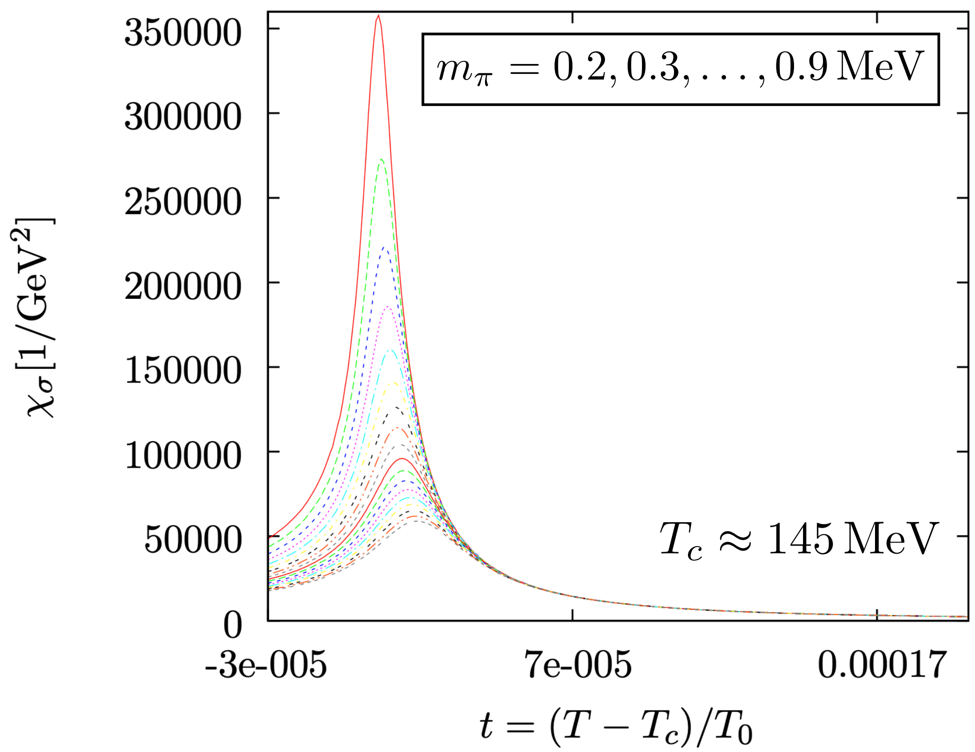


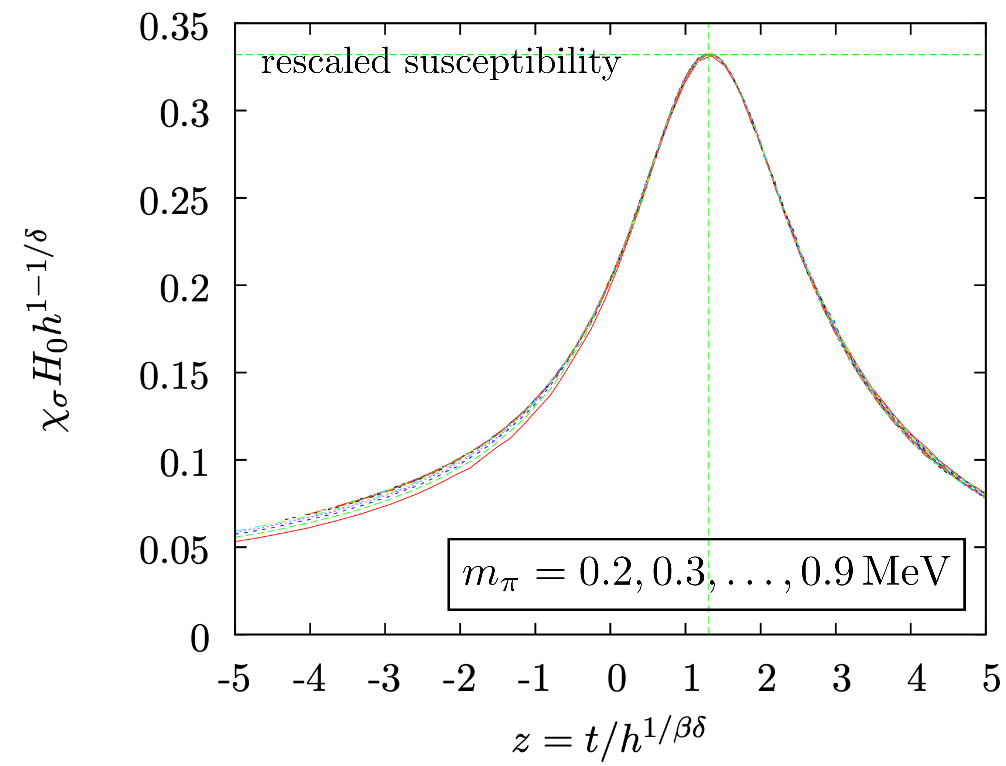
- Very good agreement with lattice QCD results from the hotQCD collaboration around physical pion masses and above
- Susceptibilities do not show indications for scaling for $m_\pi \gtrsim 30\,\mathrm{MeV}$

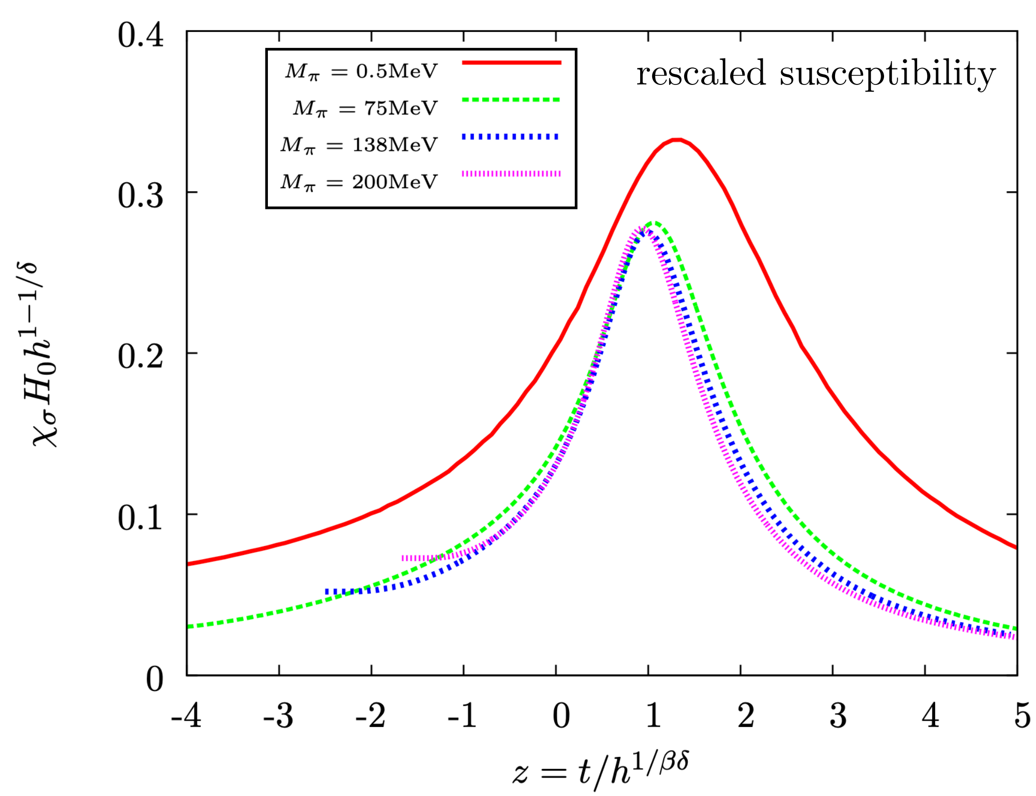
by courtesy of Jens Braun

Size of the scaling region: quark-meson model

Braun, Klein, Piasecki, EPJ C 71 (2011) 1576







- Size of the true scaling region:

$$m_\pi \lesssim 1\,\mathrm{MeV}$$

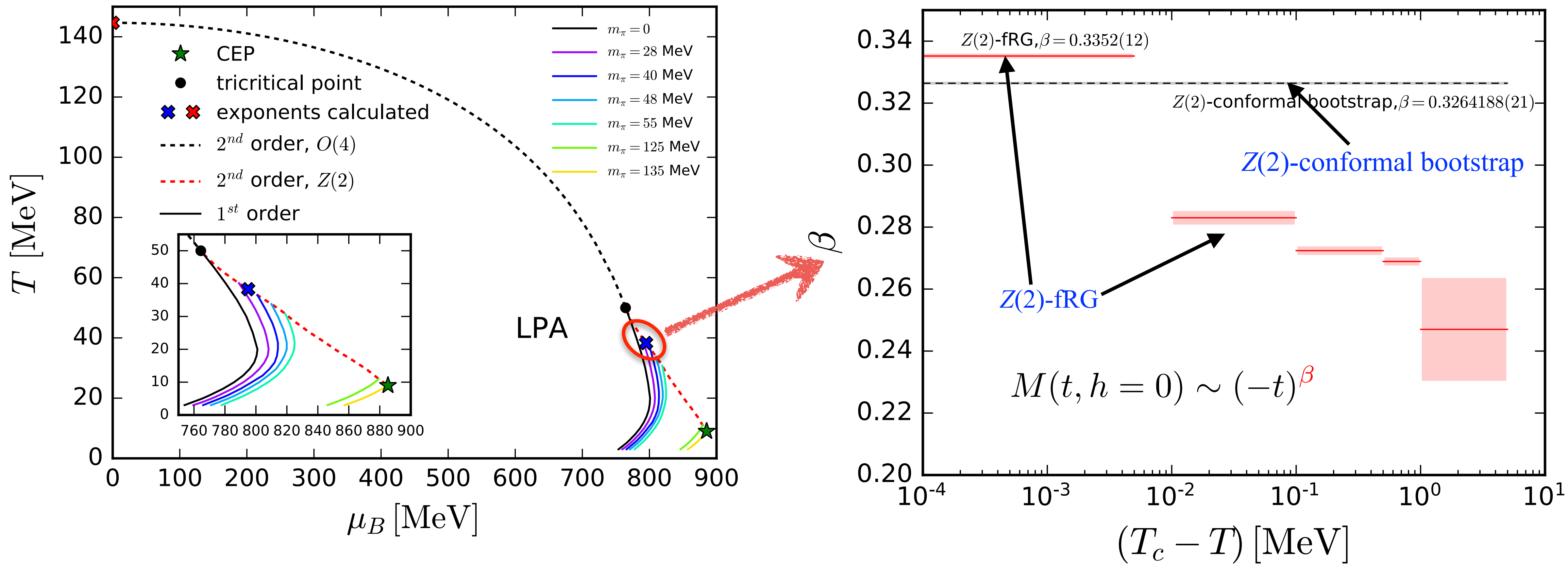
- "**Seeming**" scaling region:

$$75\,\mathrm{MeV} \lesssim m_\pi \lesssim 200\,\mathrm{MeV}$$

by courtesy of Wei-die Fu

Critical region near the CEP with Z(2) symmetry

Chen, Wen, Fu, arXiv: 2101.08484



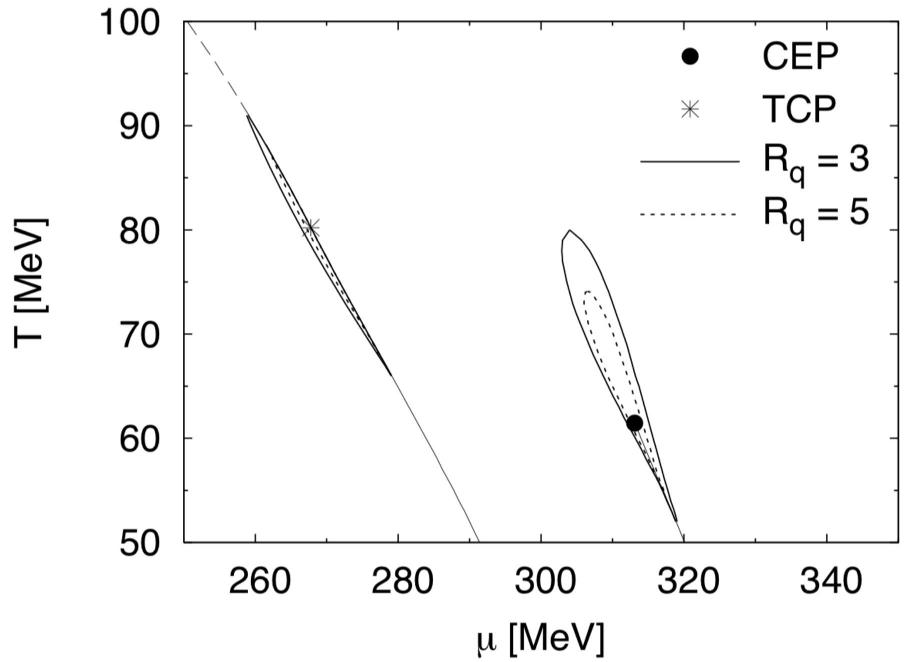
- Critical exponent β is close to the CEP
- Critical region (in temperature direction) smaller than **1 MeV**!

See also: Schaefer, Wambach, PRD 75 (2007) 085015
Schaefer, Wagner, PRD 85 (2012) 034027

by courtesy of Bernd-Jochen Schaefer

Critical region from RG flows

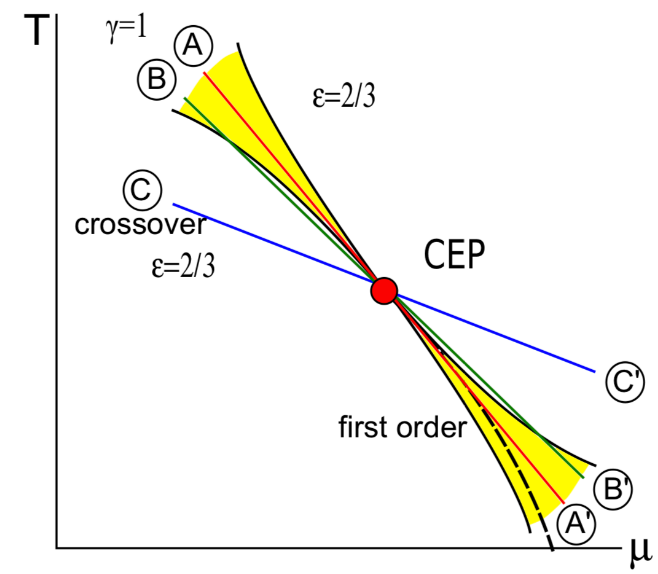
[B.-J. Schaefer, J. Wambach, Phys. Rev. D 75 (2007)]



- Contour regions for quark number susceptibility

(chiral limit: TCP physical pion mass: CEP)

[B.-J. Schaefer, M. Wagner, Phys. Rev. D (2012)]



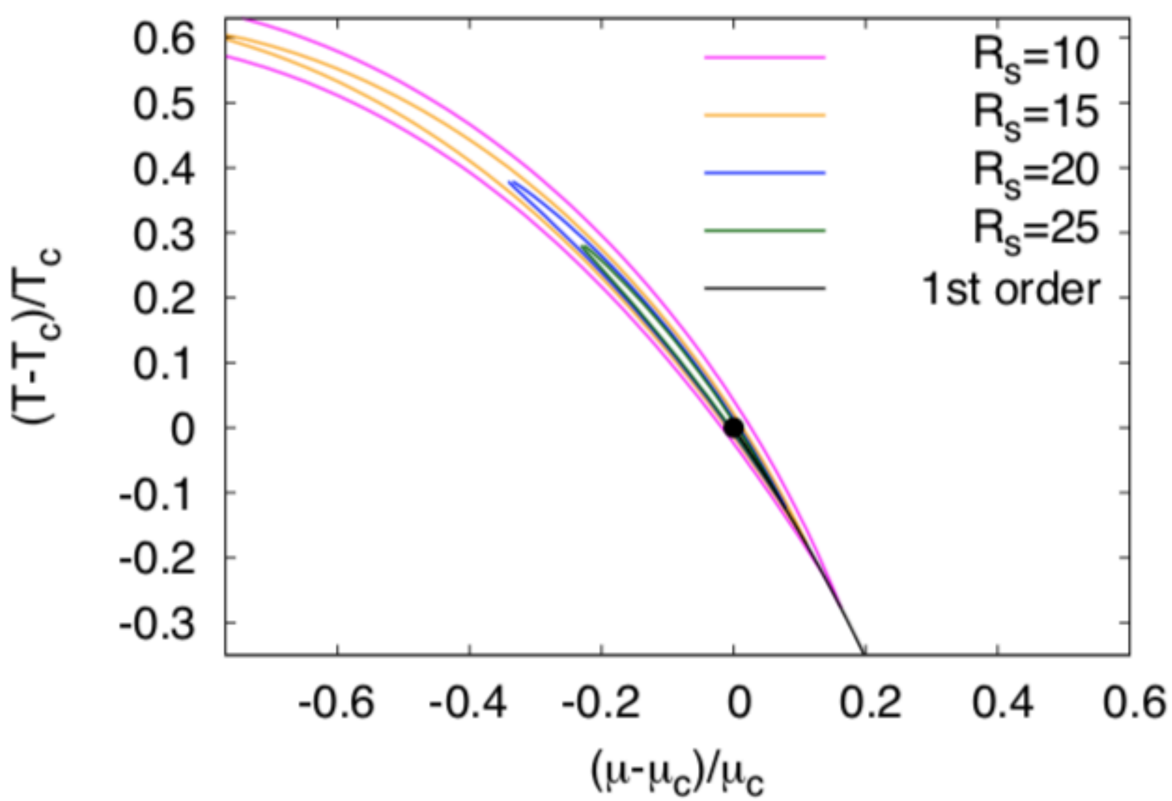
- Critical region around CEP
  ➜ criticality depends on the path towards CEP

  ➜ crossover between different universality classes by courtesy of Bernd-Jochen Schaefer

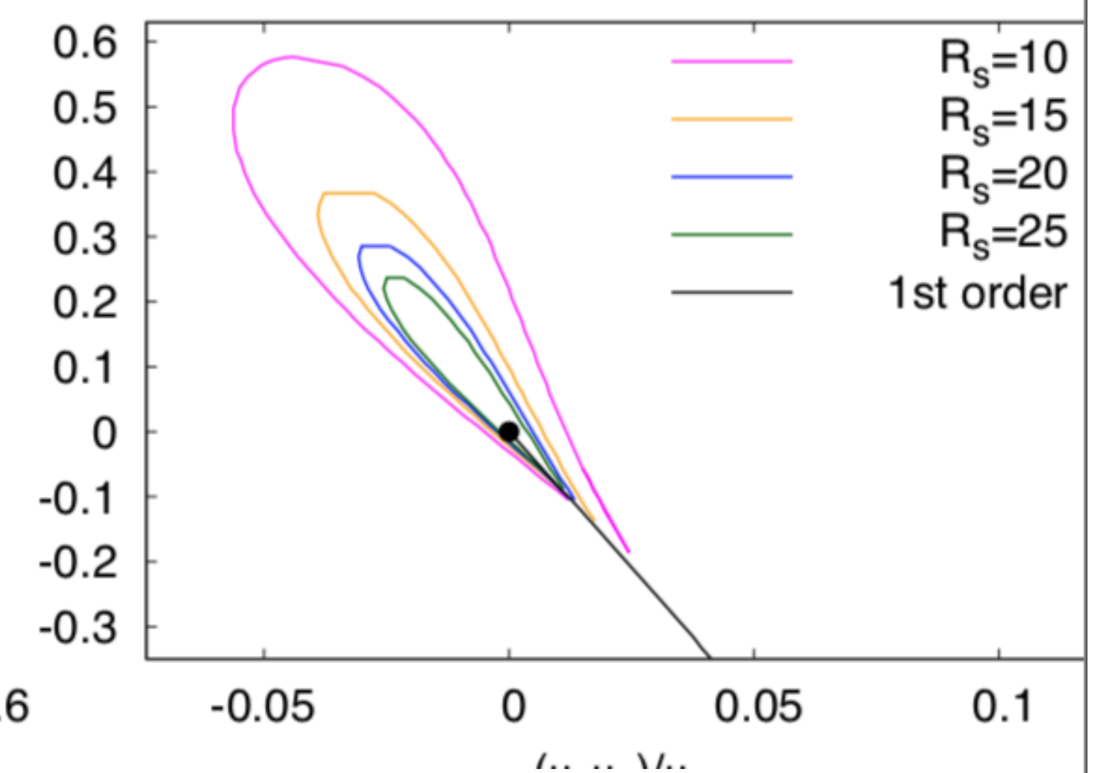
Size of the critical region: quark-meson model

[B.-J. Schaefer, J. Wambach, Phys. Rev. D 75 (2007)]

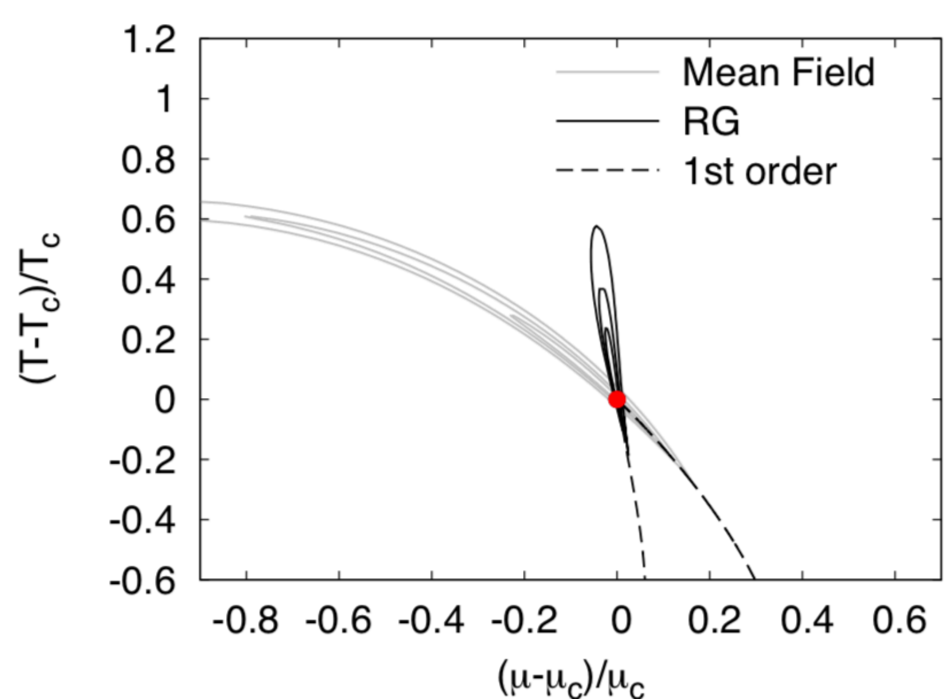
Mean-field



FRG

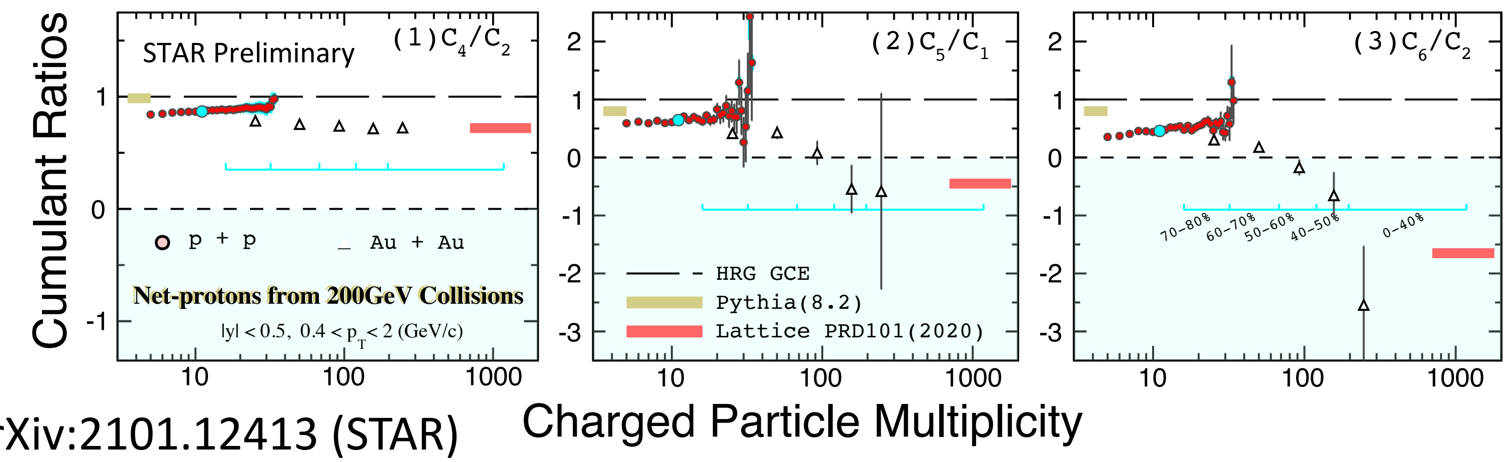


- Size much more compressed with FRG



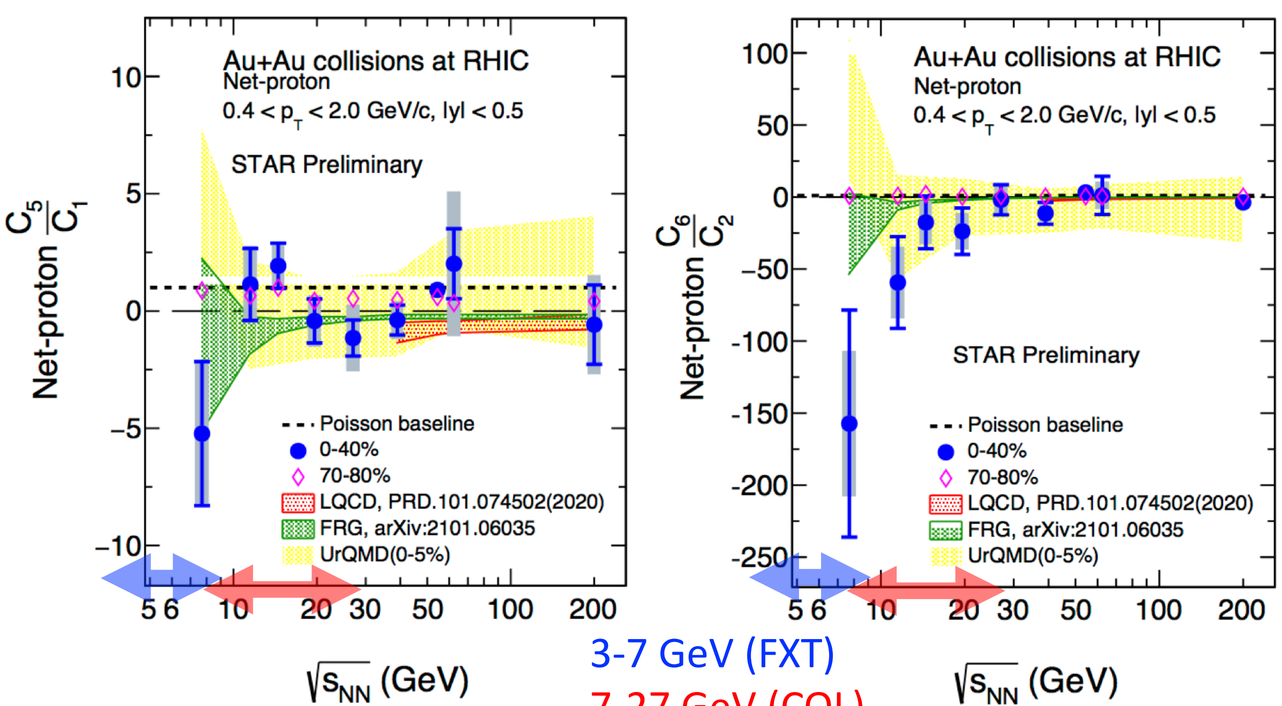
The 5th, 6th order fluctuation of net-proton including pp collisions

New data of net-p cumulants at 200 GeV pp collisions will be shown by Risa N.



arXiv:2101.12413 (STAR)

New data of net-proton C_5, C_6 at BES-I will be shown by Ashish P.

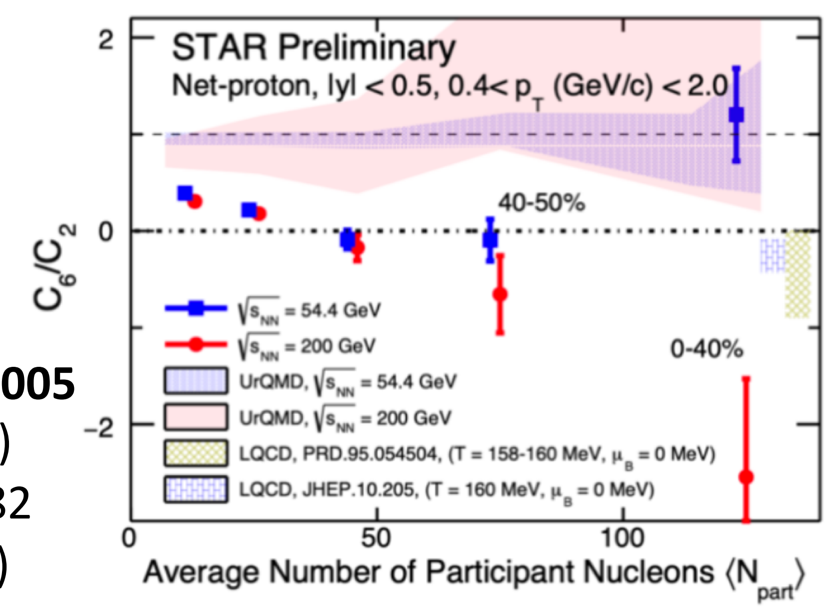


3-7 GeV (FXT)

7-27 GeV (COL)

negative c6 could be taken as an indication of cross-over transition at small μ_B

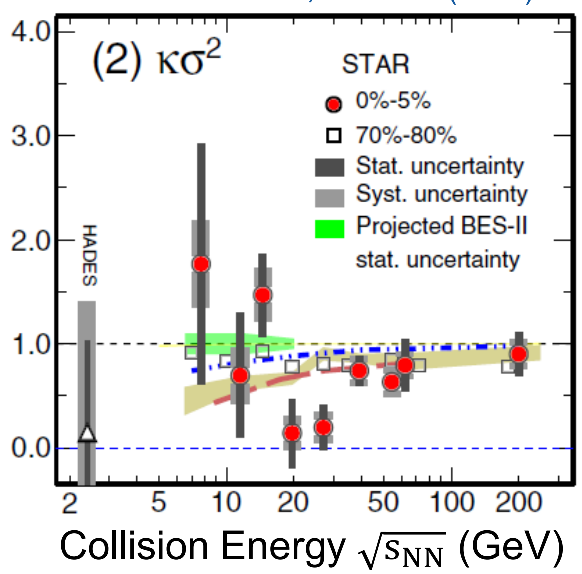
NPA **1005** (2021) 121882 (STAR)



Critiacl Fluctuations

TECHNISCHE UNIVERSITÄT DARMSTADT

STAR Collaboration, PRL 126 (2021) 092301



Direct link to EoS $$\frac{1}{VT^3}k_n = \frac{\partial^n \hat{p}}{\partial \hat{\mu}^n}$$

$\hat{p} = \frac{p}{T^4}$ reduced pressure

$\hat{\mu} = \frac{\mu}{T}$ reduced chemical potential

cf. B. Friman *et al.*, EPJC 71 (2011) 1694
M. Stephanov, PRL107 (2011) 052301

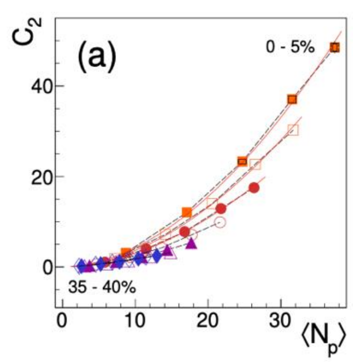
Ling, Stephanov, PRC 93, 034915 (2016)

Cumulants k_n hold information on multi-particle correlators C_n

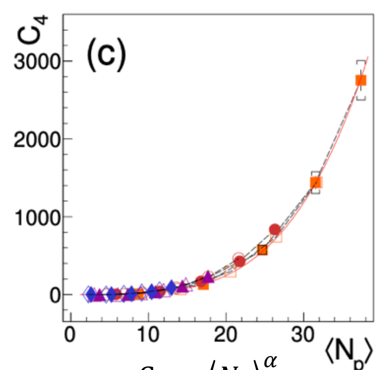
Bzdak, Koch, Strodthoff, PRC 95, 054906 (2017)

Investigate C_n vs. $\langle N_p \rangle$ to isolate relevant physics, $C_n \propto \langle N_p \rangle^\alpha$

HADES Collaboration, PRC 102 (2020) 2, 024914
https://www.hepdata.net/record/ins1781493



$C_2 \propto \langle N_p \rangle^\alpha$
$\alpha = 1.86 \pm 0.04$



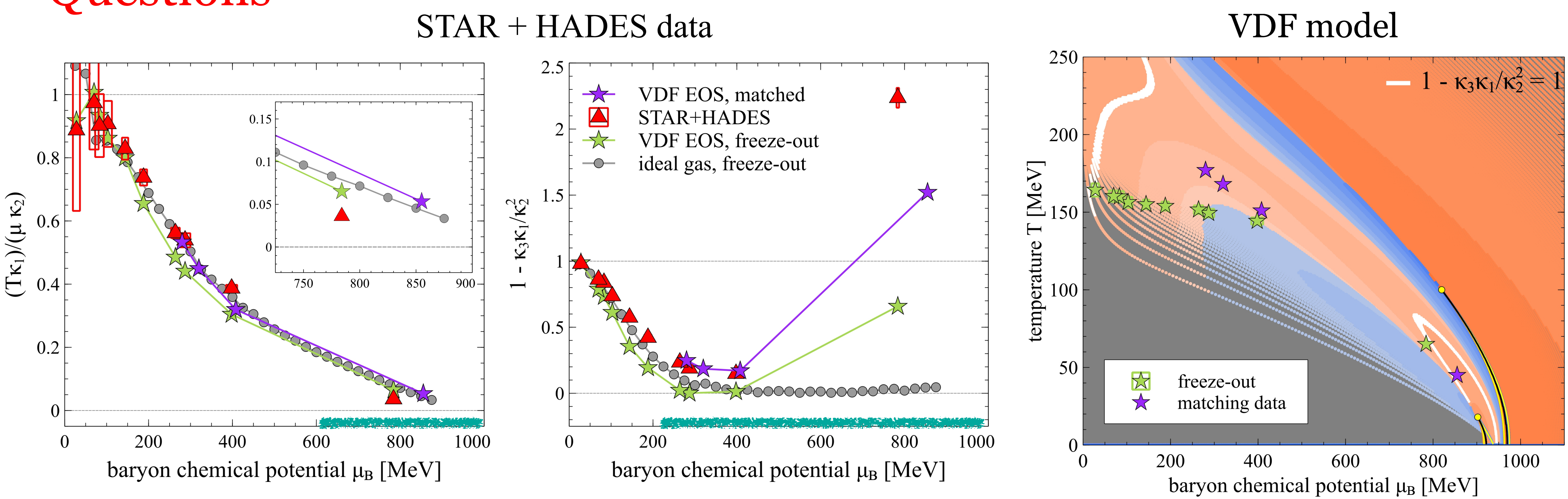
$C_4 \propto \langle N_p \rangle^\alpha$
$\alpha = 3.89 \pm 0.14$

$\alpha \simeq n \rightarrow$ **signature of multi-particle correlations** $(\Delta y_{corr} > 1)$

Further progress with data:

- Applying identity method A. Rustamov, M.I. Gorenstein, PRC 86 (2012) 044906
- Analyzing new Ag+Ag data

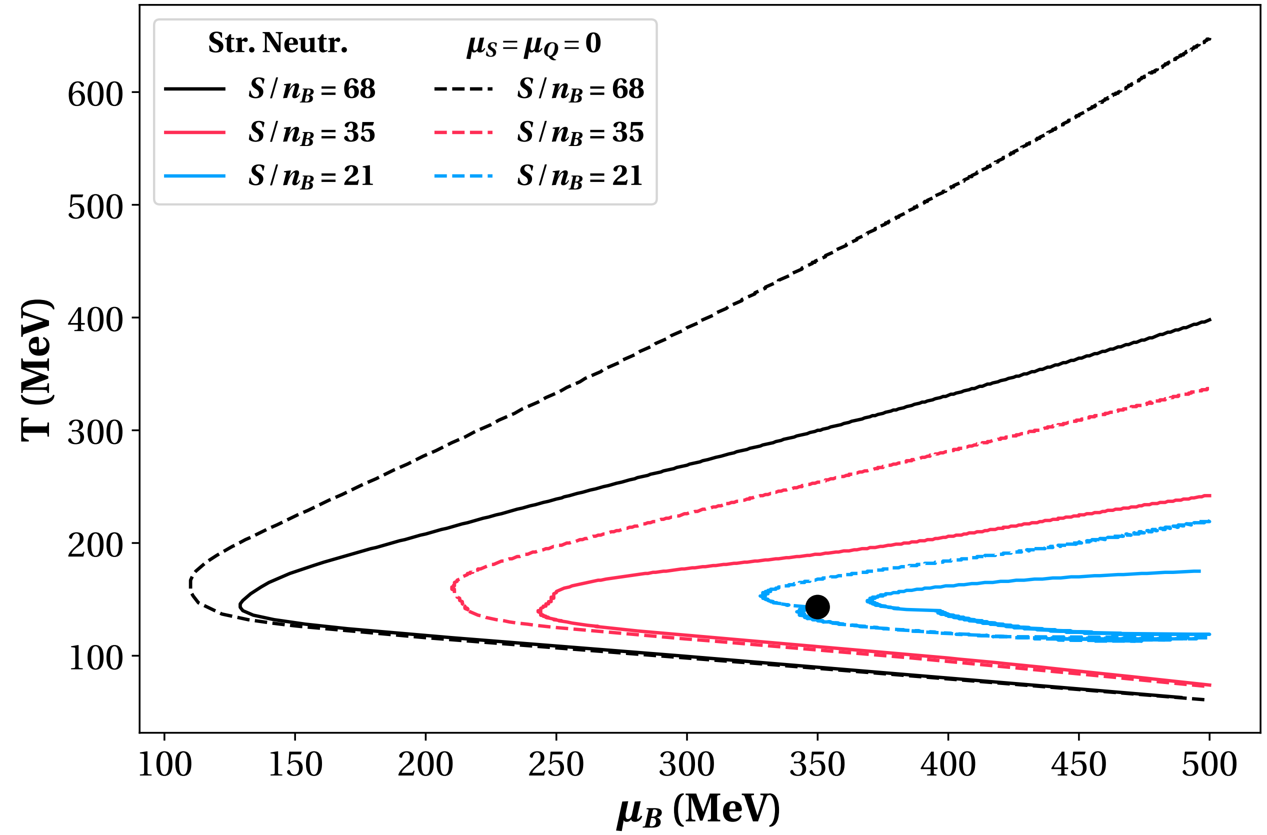
Questions



- what happens in the STAR BES fixed target region?
- is behavior of the cumulants at low energies dominated by hadronic effects and the nuclear liquid-gas phase transition?
- can we study c_T^2 below $\sqrt{s} = 2.4$ GeV?

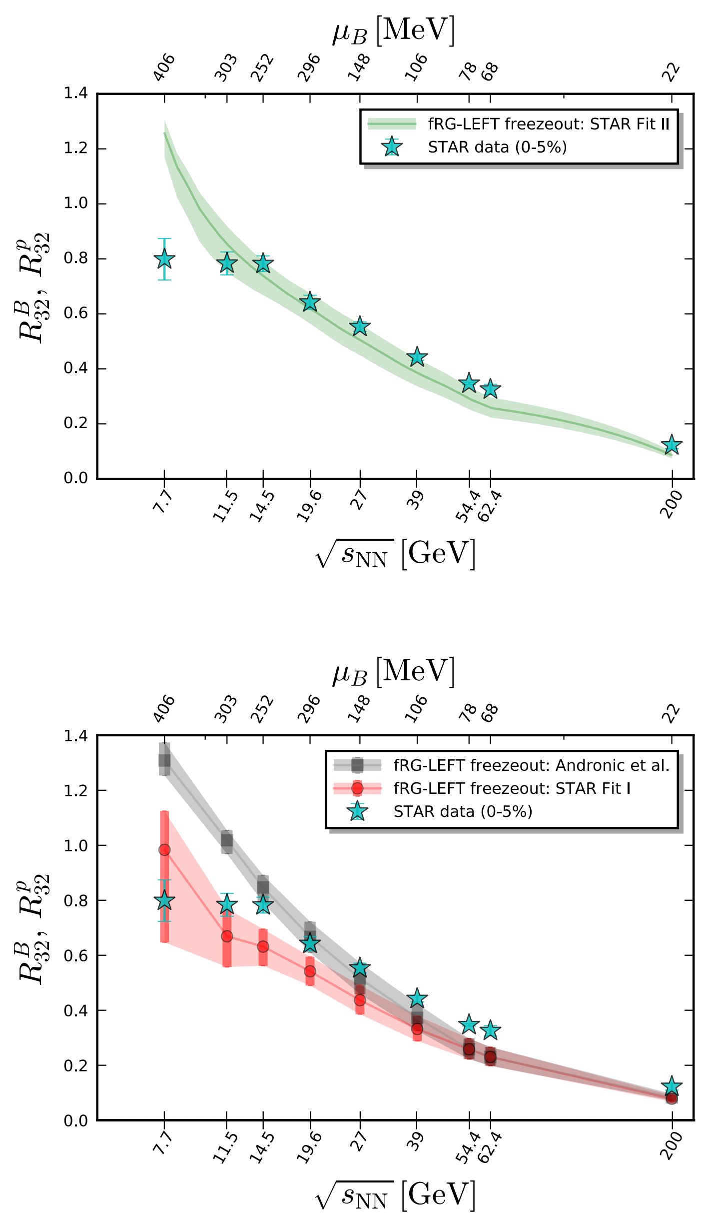
Isentropic Trajectories

- Isentropes show the path of the HIC system through the phase diagram in the absence of dissipation
  - Different path when conserved charge conditions applied



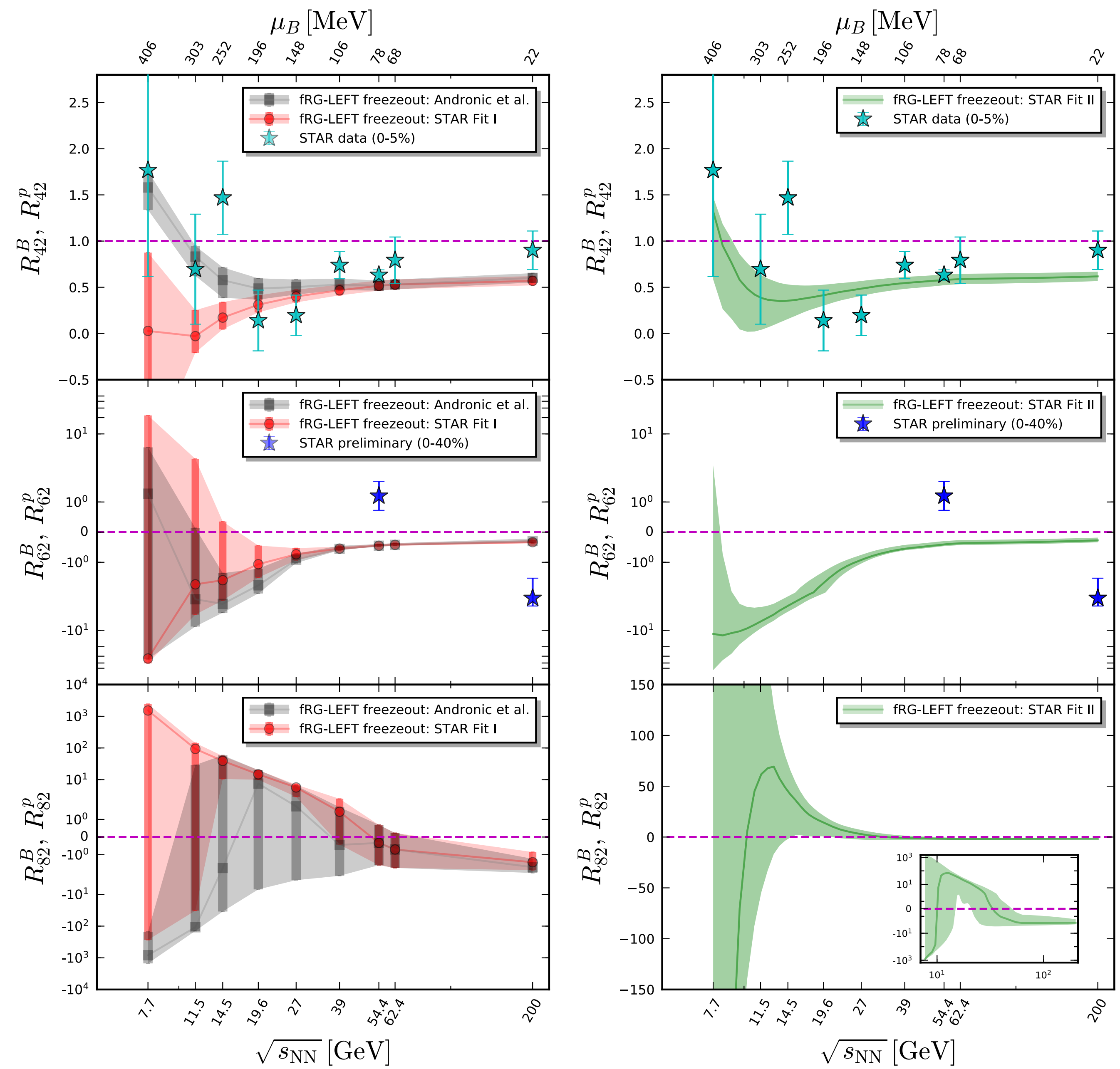
JS et al, arXiv:2103.08146

Fluctuations on the freeze-out curve



WF, Luo, Pawlowski, Rennecke, Wen, Yin, arXiv:2101.06035

J. Adam *et al.* (STAR), *PRL* 126 (2021), 092301
M. Abdallah *et al.* (STAR), arXiv:2101.12413

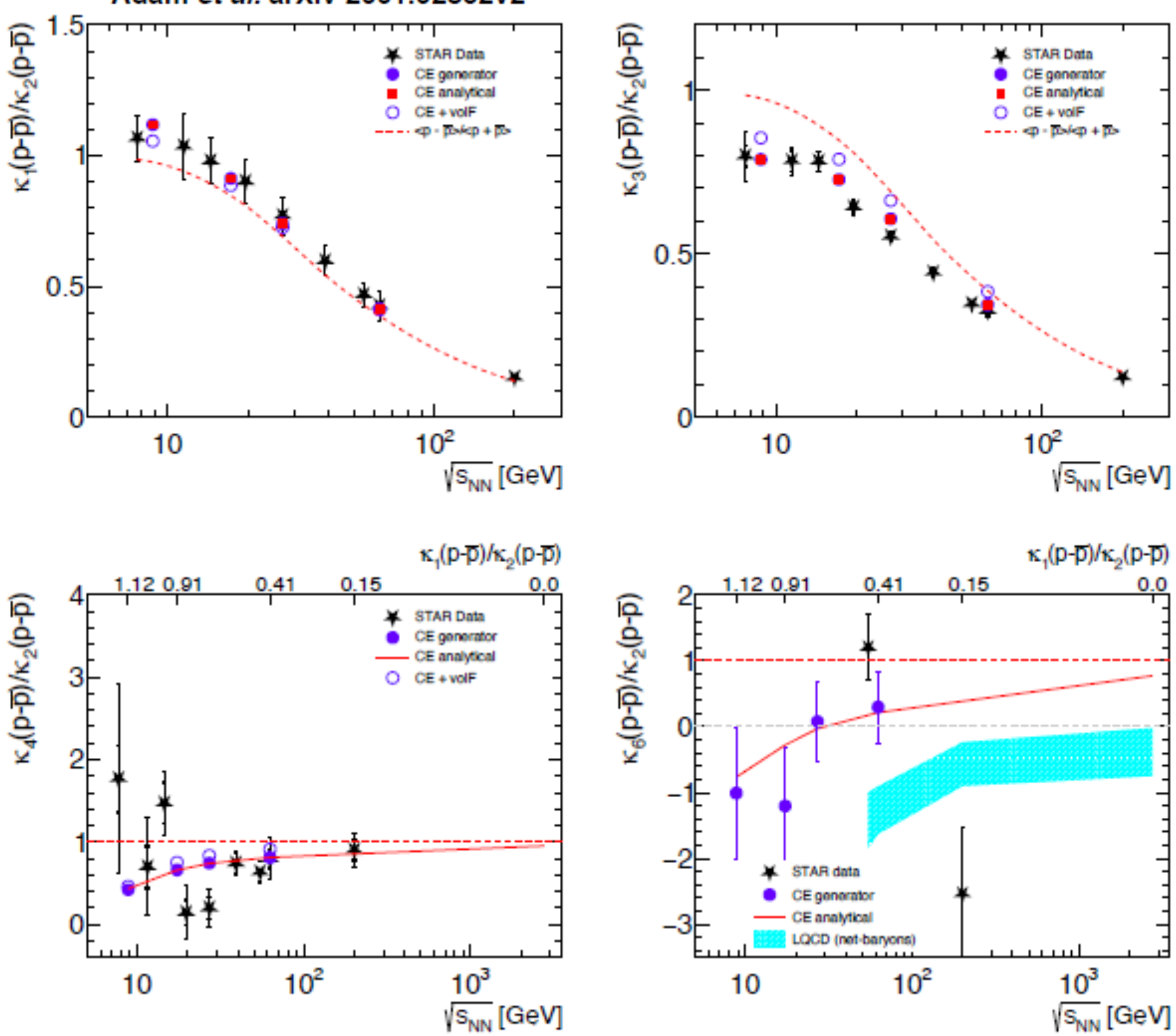


T. Nonaka (STAR), *NPA* 1005 (2021) 121882; A. Pandav (STAR), arXiv: *NPA* 1005 (2021) 121936

Ratios of cumulants: STAR data versus model

Stars: STAR data

Adam *et al.* arXiv 2001.02852v2



- Broken lines calculated from

  Skellam distribution

$$\frac{\kappa_1}{\kappa_2} = \frac{\kappa_3}{\kappa_2} = \frac{< p - \bar{p} >}{< p + \bar{p} >}$$

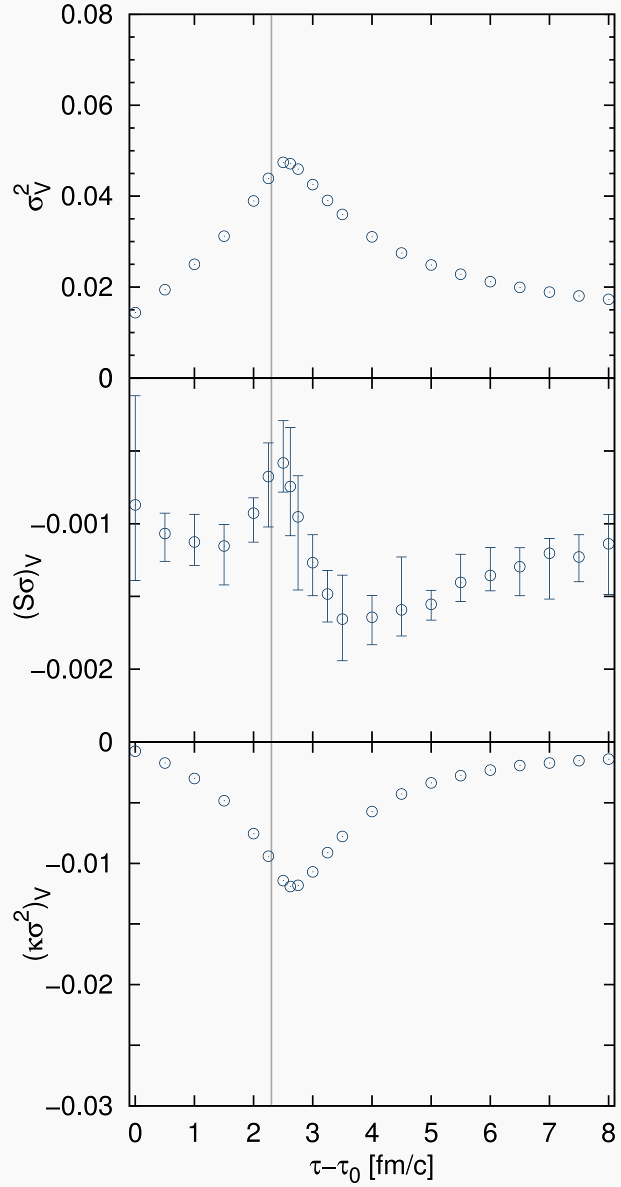
- Open circle include MC volume fluctuations

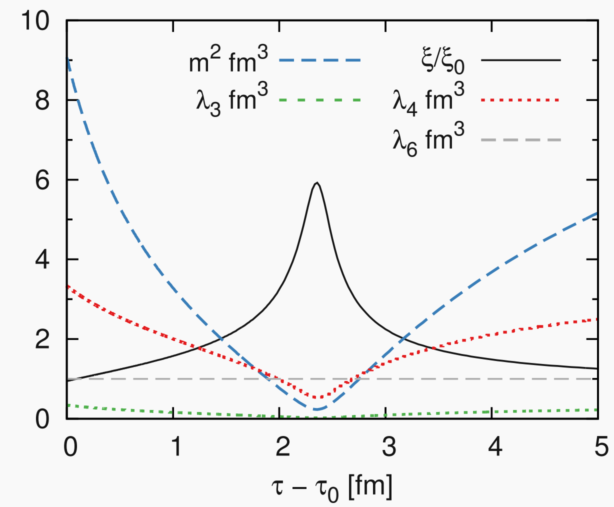
P. Braun-Munzinger, A. Rustamov and J. Stachel, Nucl. Phys. A960, 114 (2017).
V. Skokov, B. Friman and K. Redlich, Phys. Rev. C88, 034911 (2013)

- Cumulants up to n<4 order follow the SATR data
- Kurtosis data exhibit interesting deviations, however *not necessarily* of statistical significant

Dynamics: time evolution of critical fluctuations

For a Bjorken-like temperature evolution:





- shift of extrema for variance/kurtosis (retardation effects) to later times corresponding to $T(\tau) < T_c$
- |extremal values| in dyn simulations < equilibrium values (nonequilibrium effects):

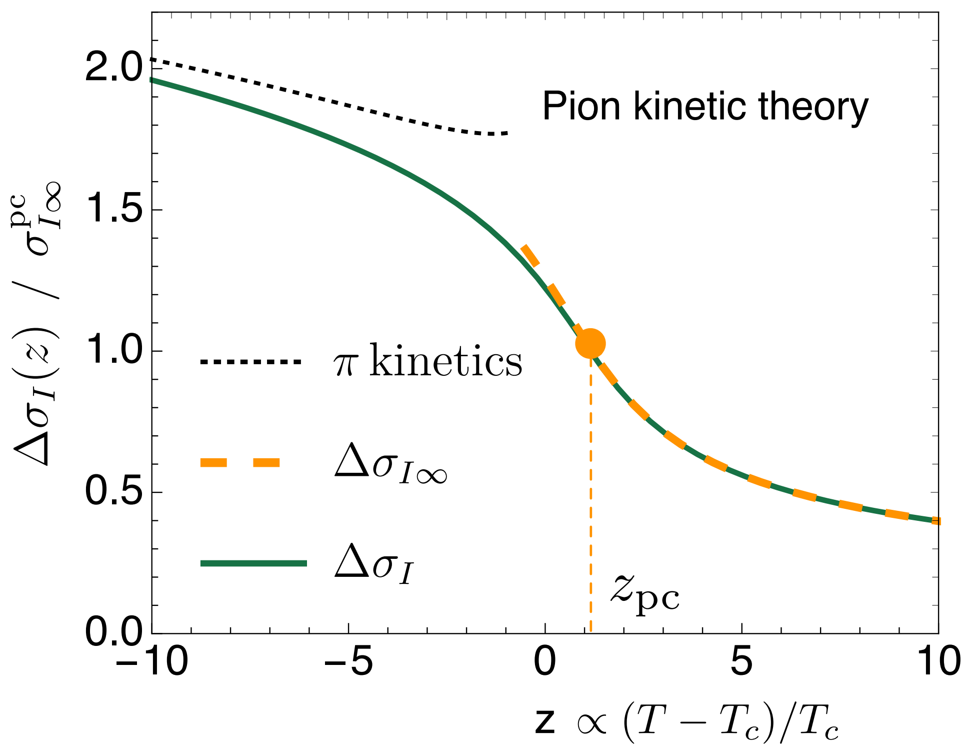
$$(\sigma_V^2)_{\rm dyn}^{\rm max} \approx 0.75\,(\sigma_V^2)_{\rm eq}^{\rm max}$$
$$((\kappa\sigma^2)_V)_{\rm dyn}^{\rm min} \approx 0.5\,(\kappa\sigma_V^2)_{\rm eq}^{\rm min}$$

- expected behavior with varying D and c_s^2 (expansion rate)

MN, M. Bluhm, T. Schaefer, S. Bass, PRD99 (2019), MN, M. Bluhm, PRD102 (2020)

The conductivity through T_c

$$\sigma_I = \sigma_{I\text{reg}} + \underbrace{\Delta\sigma_I}_{\text{critical part}}$$

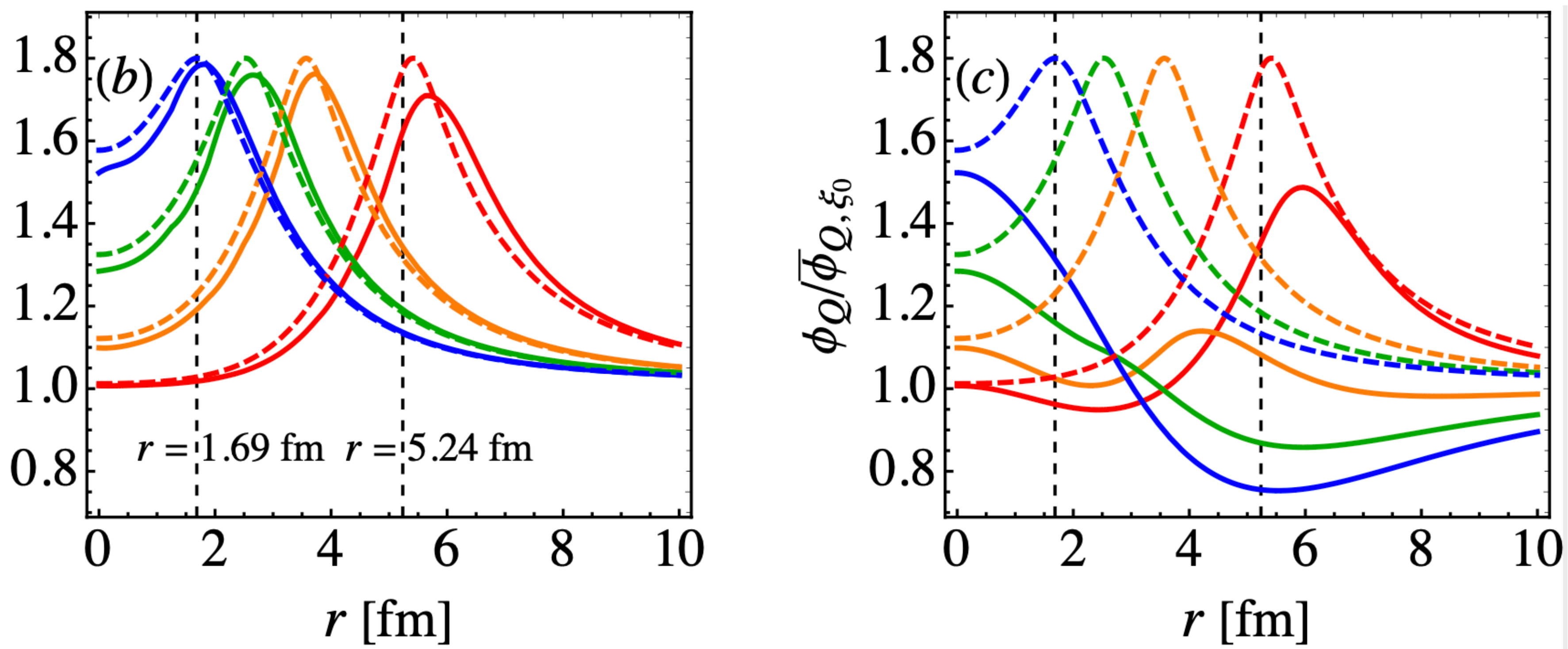


$$\sigma^{\rm pc}_{I\infty} \equiv \left.\frac{T}{16\pi D_m m}\right|_{z=z_{\rm pc}}$$

Estimate of the absolute magnitude for $\sigma^{\rm pc}_{I\infty}$ with $T_{\rm pc} \simeq 155\,\text{MeV}$

$$\Delta D_I = \left(\frac{\Delta\sigma_I}{\chi}\right) = \frac{0.50}{2\pi T} \times \left(\frac{1.3}{m_{\rm pc}/T}\right)\left(\frac{0.4}{\chi_Q/T^2}\right)\left(\frac{3.0}{2\pi T D_m}\right)$$

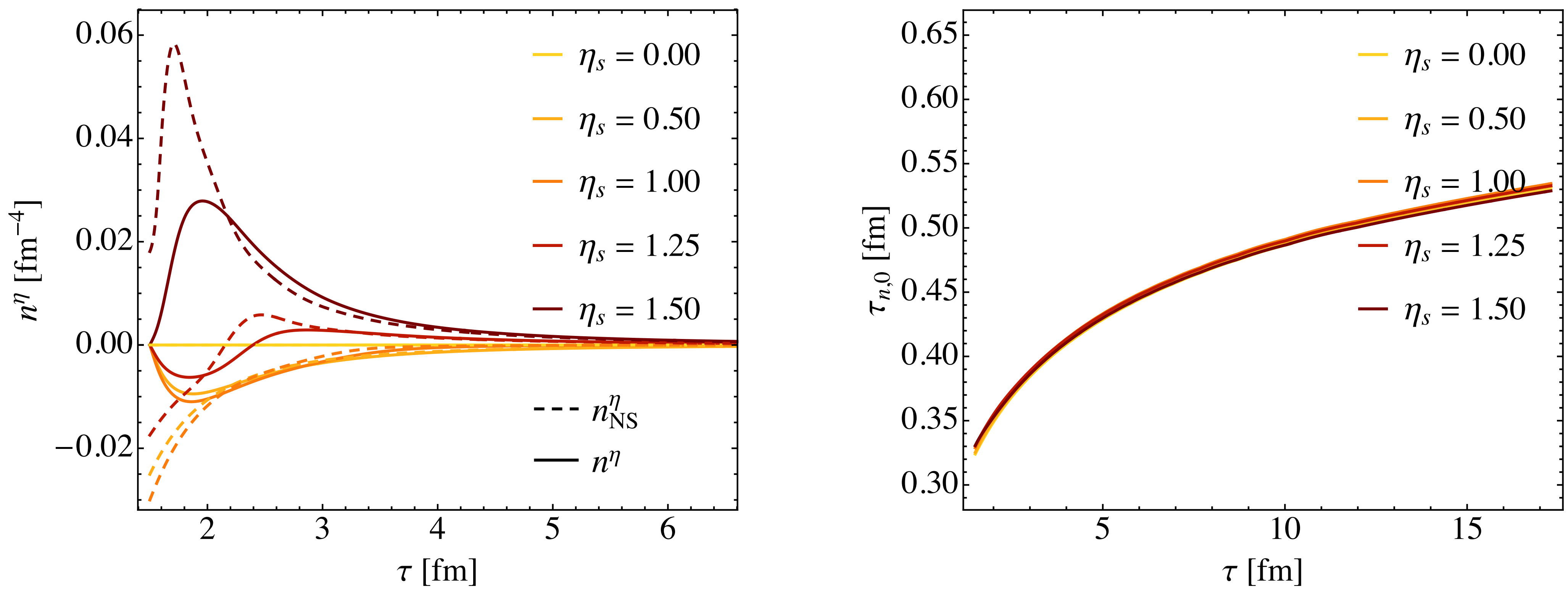
Critical slowing down and advection



Turning off radial flow

Turning on radial flow

Time evolution of baryon diffusion current



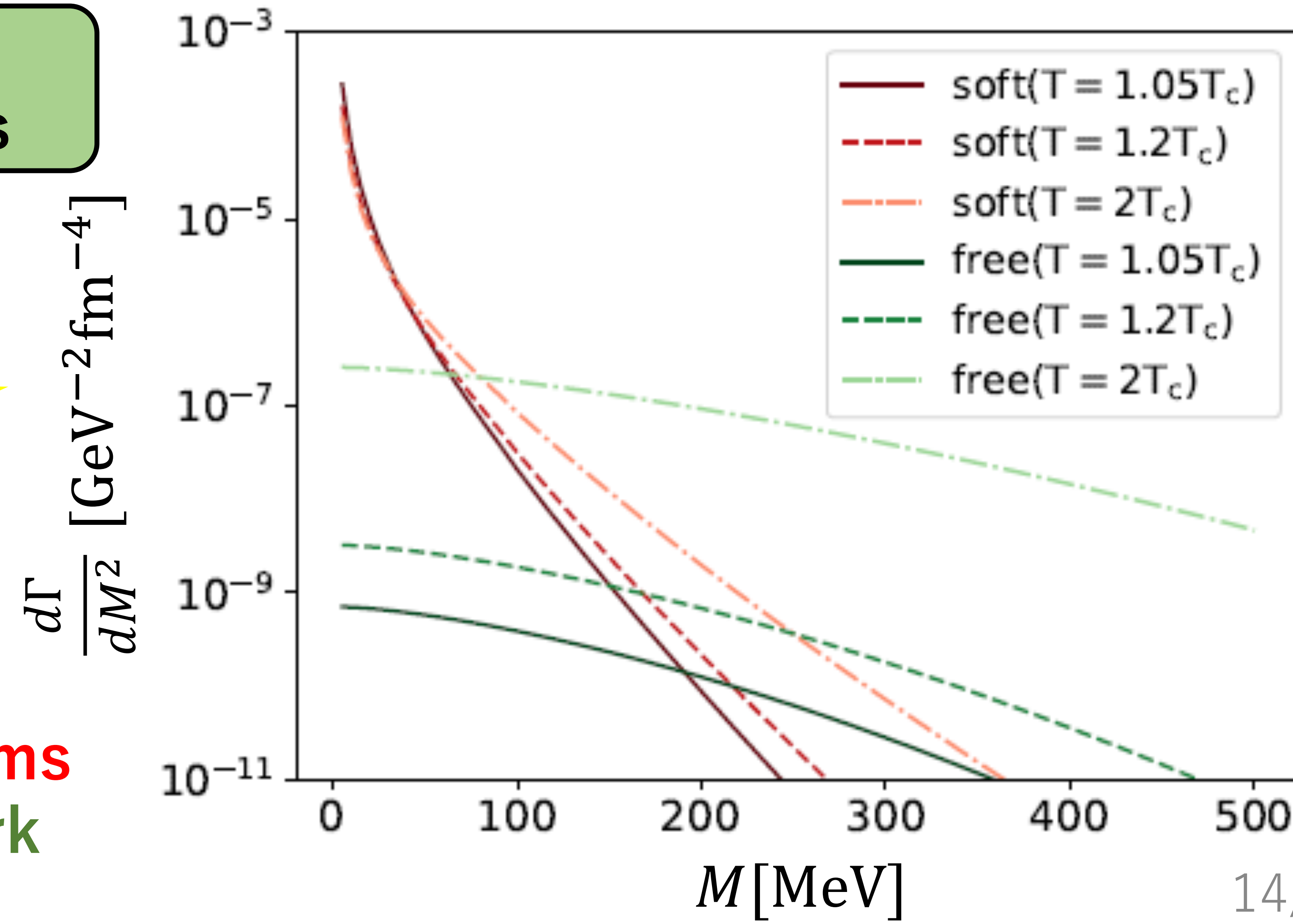
- With the fireball cooling down, the driving force of diffusion current ($\kappa_n \nabla(\mu/T)$) decreases:
  - Two reasons: (a) gradient $\nabla(\mu/T)$ gets smoothened; (b) κ_n decreases.
- Response to the driving force also gets slower, because of the growing relaxation time;
- Critical slowing-down ($\tau_n \simeq 6\, \tau_{n,0}$) would help n^η to stay at (almost) zero, even if $\kappa_n \nabla(\mu/T)$ got affected by the critical point.

■Invariant mass spectrum

$$\frac{d\Gamma}{dM^2} = \frac{\alpha}{6\pi^3 M^2}\int dk \frac{k^2}{\omega}\frac{\mathrm{Im}\Pi^{\mu}{}_{\mu}{}^{R}(\boldsymbol{k},\omega)}{e^{\beta\omega}-1}$$

Comparable with experiments

Enhancement in low-invariant mass region

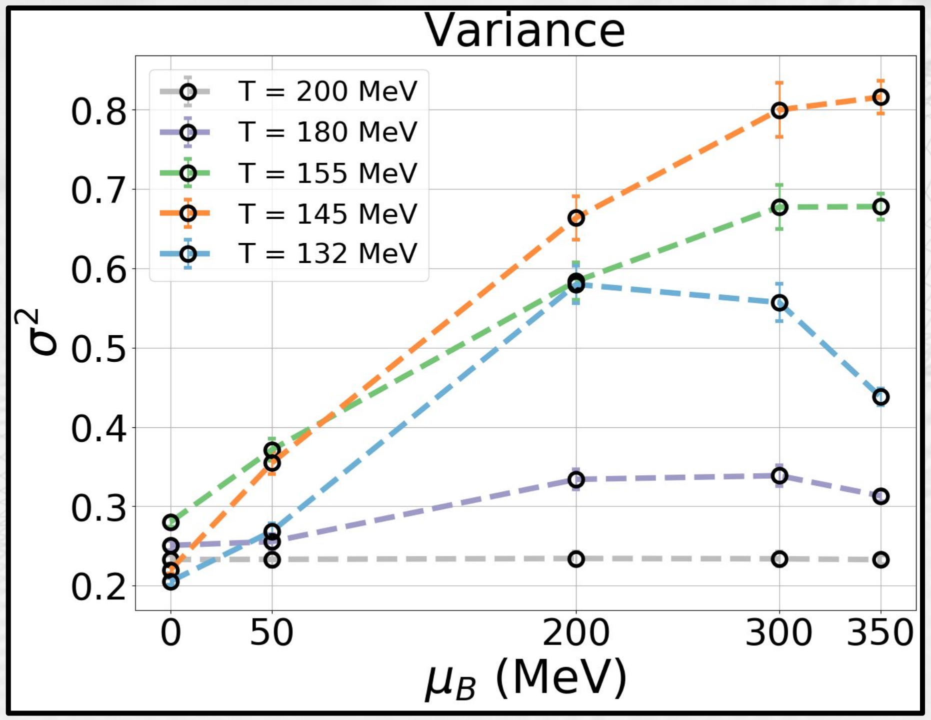


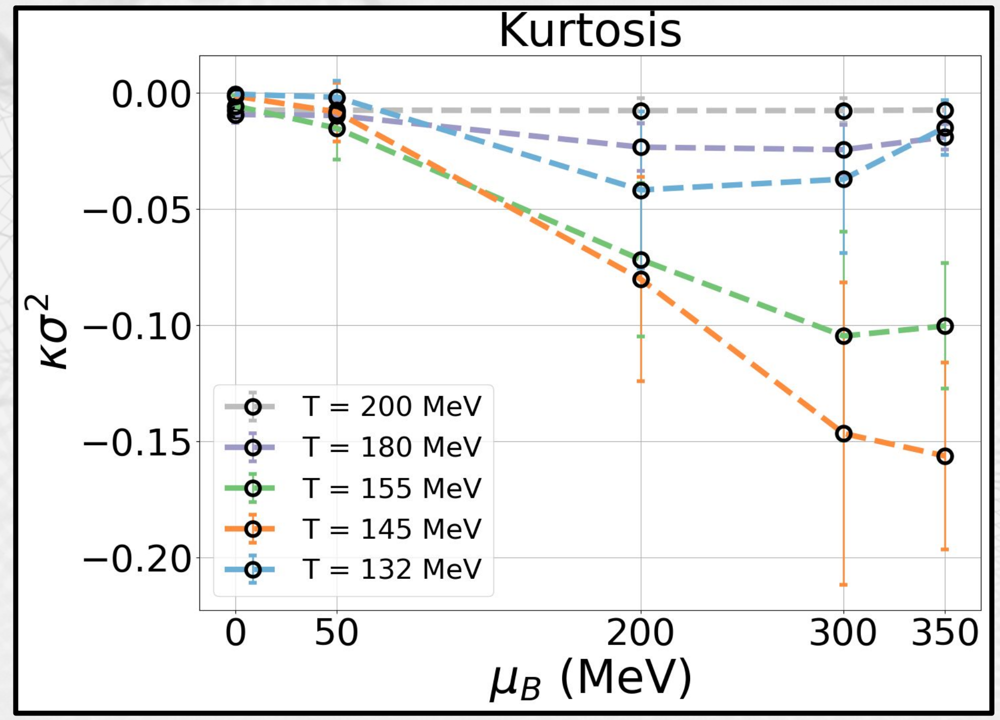
Red：AL, MT terms

Green：free quark

Including non-linear coupling : Variance and Kurtosis

$$F(T, \delta n_B) = T \int \left[\frac{m^2_{\text{eff}}(T)}{2 n_c^2} \delta n_B^2 + \frac{K}{2 n_c^2} (\nabla \delta n_B)^2 + \frac{\lambda_{3,\text{eff}}(T)}{2 n_c^3} \delta n_B^3 + \frac{\lambda_{4,\text{eff}}(T)}{2 n_c^4} \delta n_B^4 + \frac{\lambda_{6,\text{eff}}}{2 n_c^6} \delta n_B^6 \right] d\vec{x}$$

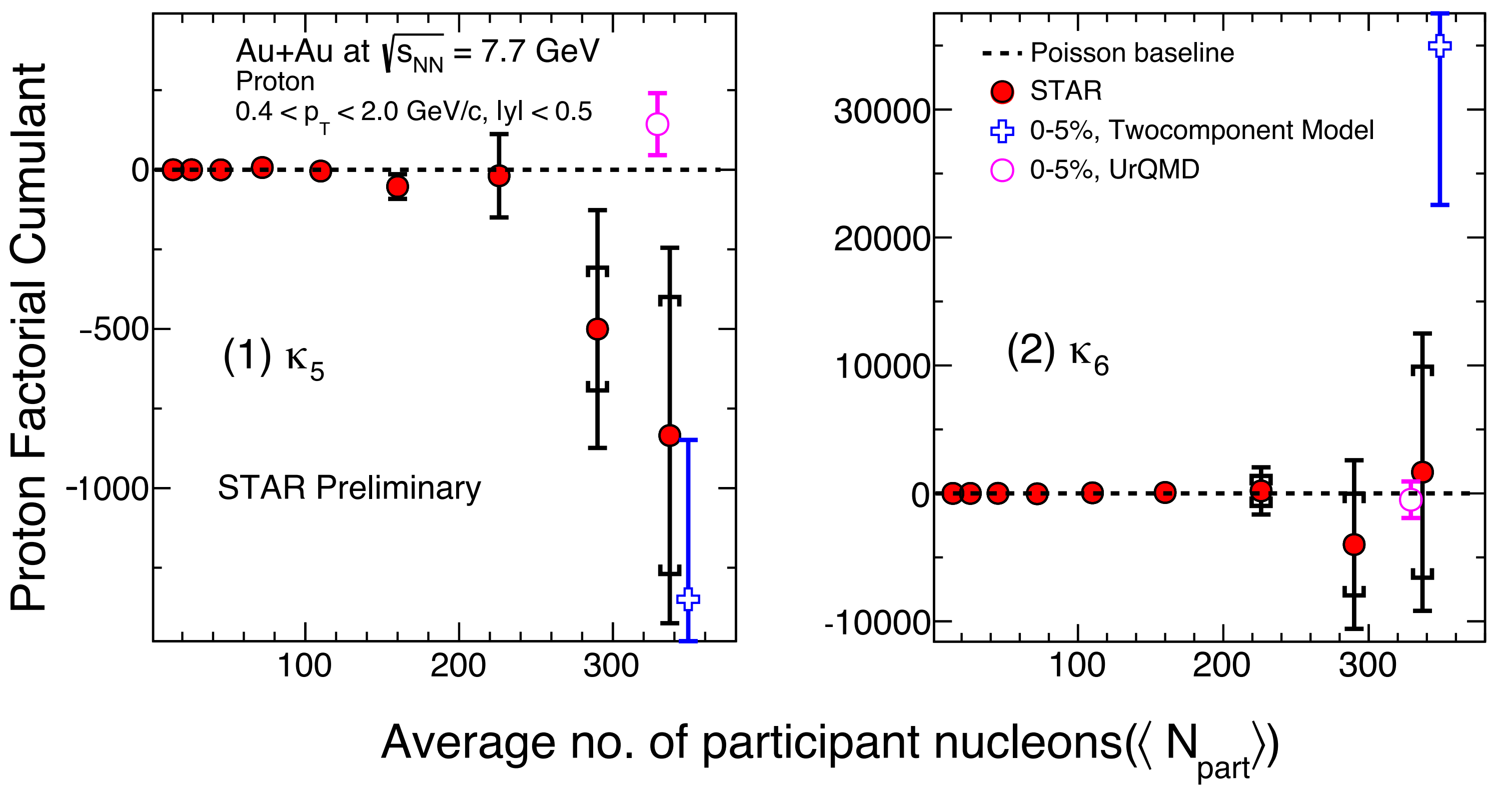


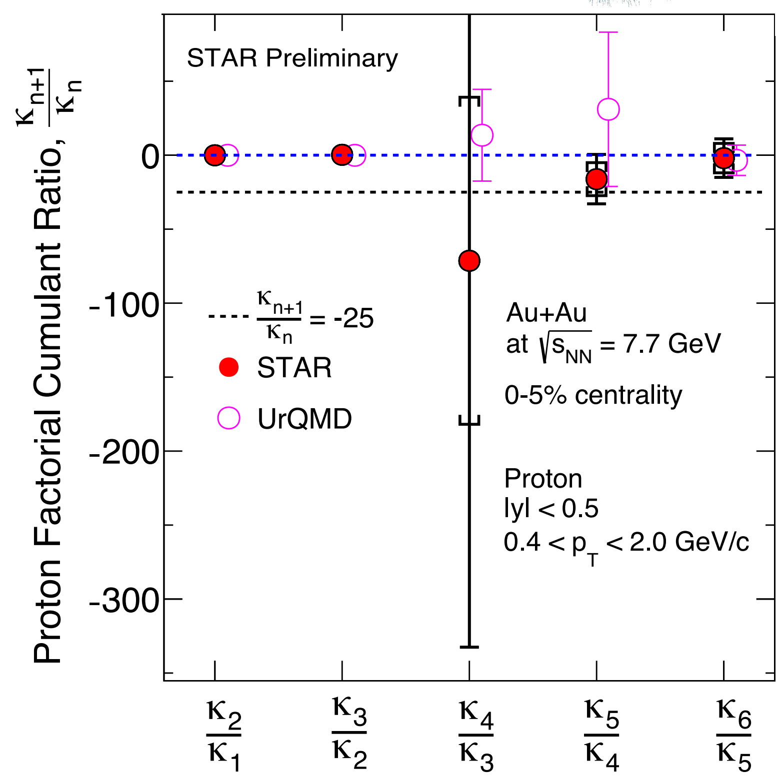


The growth of cumulants near the critical point can be seen at freeze-out

Very sensitive to the freeze-out temperature

Proton Factorial Cumulant κ_5 and κ_6 at 7.7 GeV

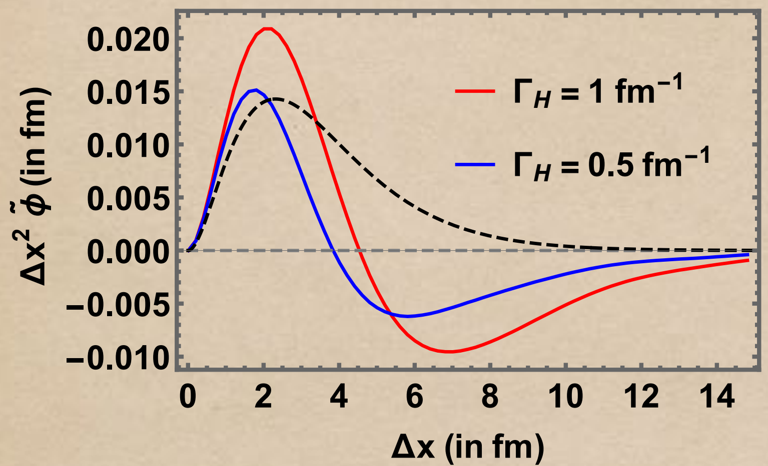


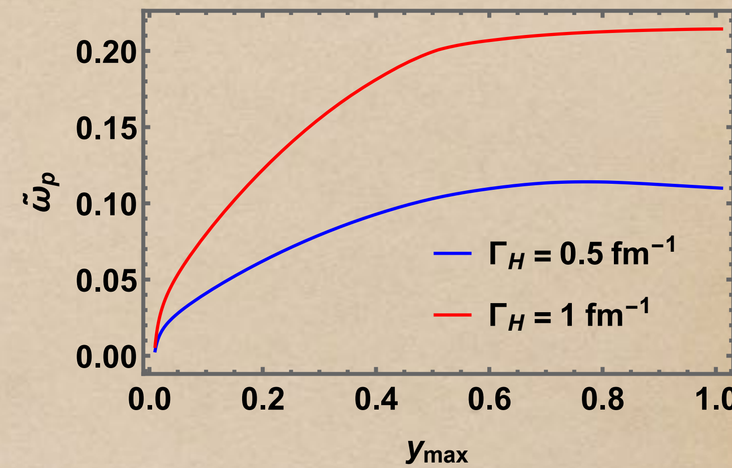


PHYSICAL REVIEW C100, 051902(R) (2019)

κ_5 (0-5%) consistent with two component model expectation within uncertainties while κ_6 (0-5%) remains 1.8σ away. The ratios κ_5/κ_4 and κ_6/κ_5 (0-5%) consistent with zero.

Interplay of effects in dynamical evolution of QGP



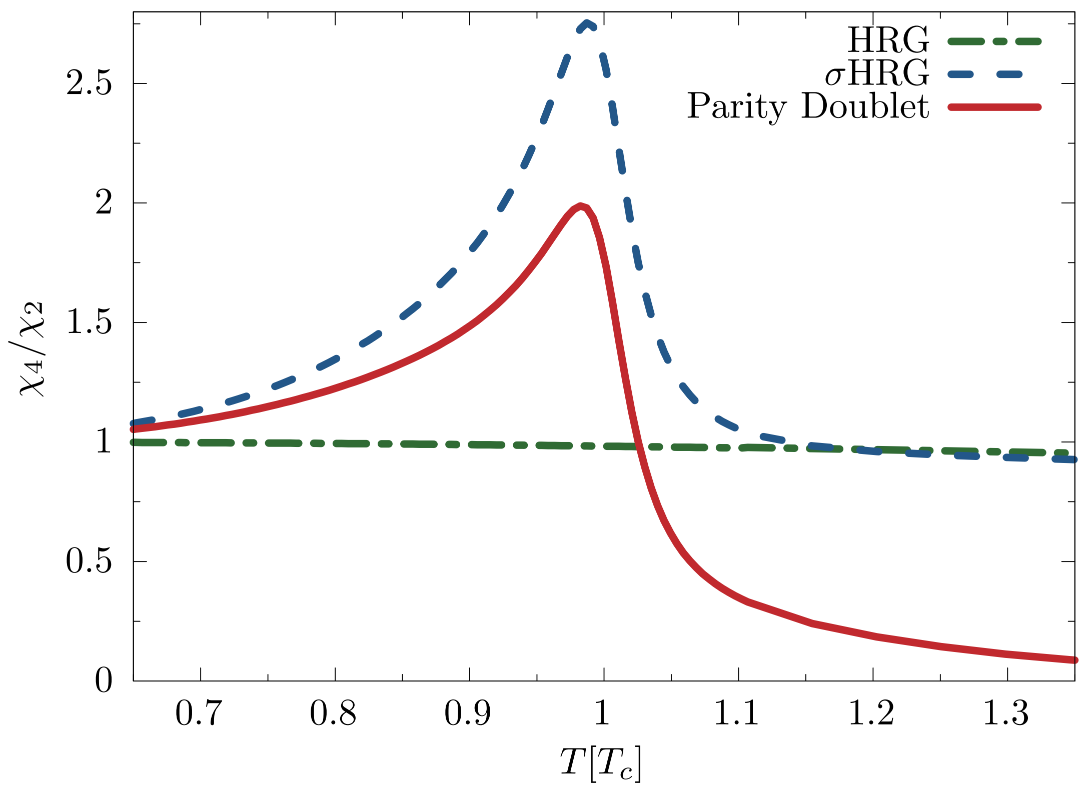


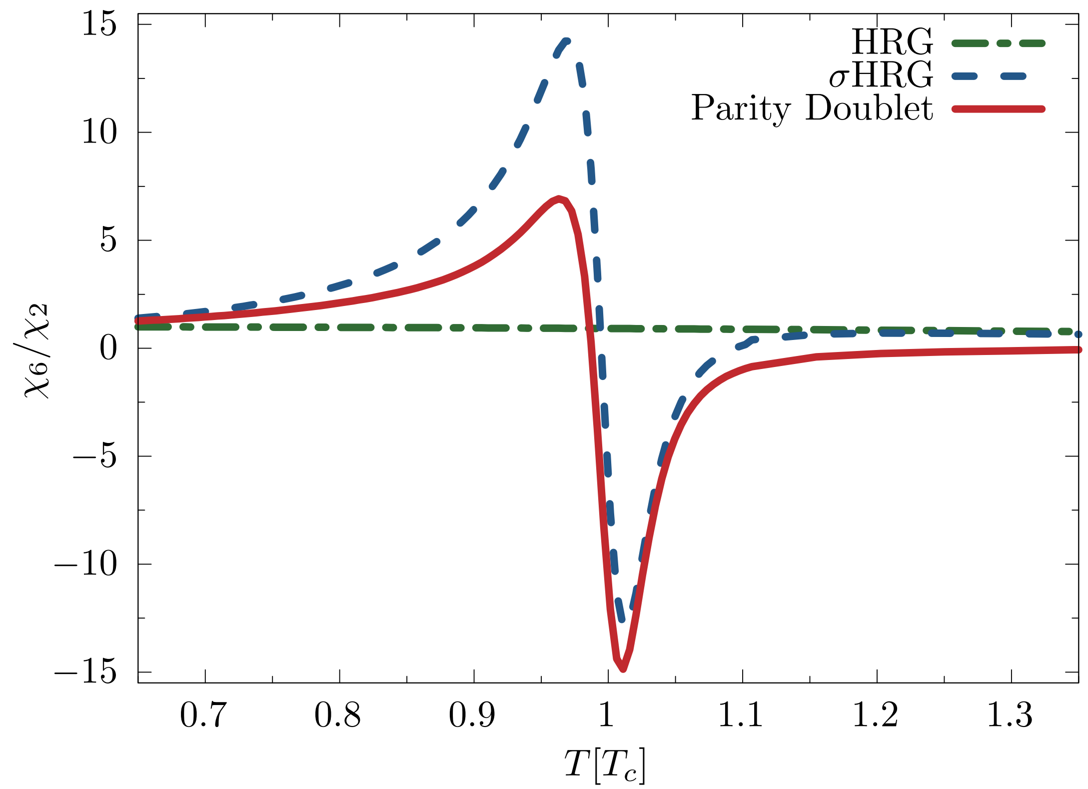
$$\Gamma(x) = \frac{\Gamma_H \xi_0^3}{\xi^3} K(x)\,, K(x) \sim x^2\, \text{for}\, x << 1$$

$$\tilde{\omega}_A(y_{\text{max}}) = \left(\frac{\left\langle \delta N_A^2 \right\rangle_{\sigma,\text{eq}}}{\langle N_A \rangle} \right)^{-1} \frac{\left\langle \delta N_A^2(y_{\text{max}}) \right\rangle_\sigma}{\langle N_A(y_{\text{max}}) \rangle}$$

Ratios of higher-order cumulants: kurtosis and χ_6/χ_2

interactions $\longrightarrow$ strong deviations from the HRG baseline



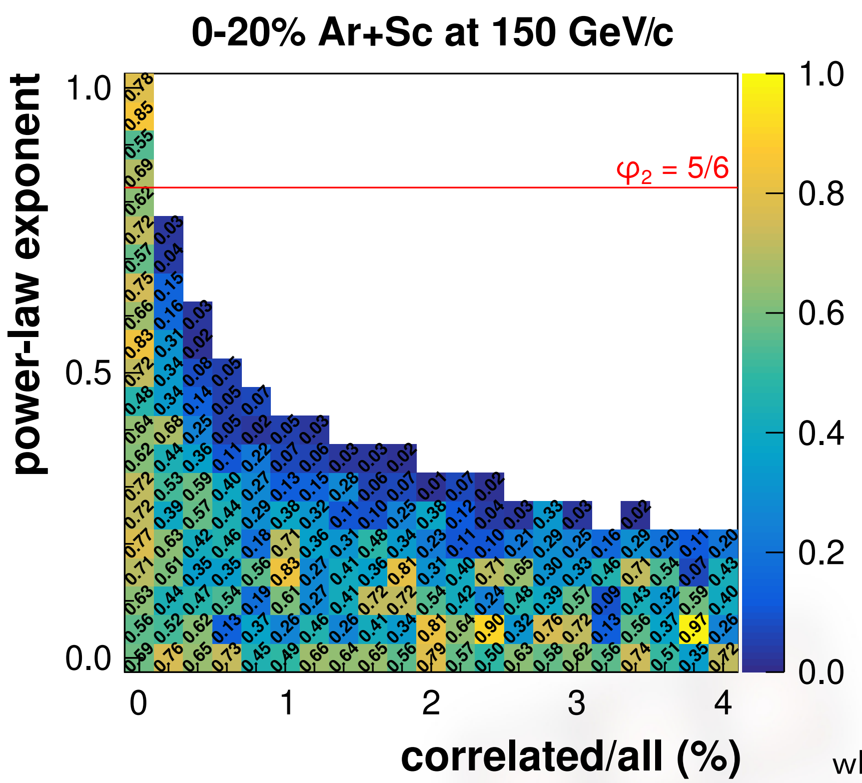


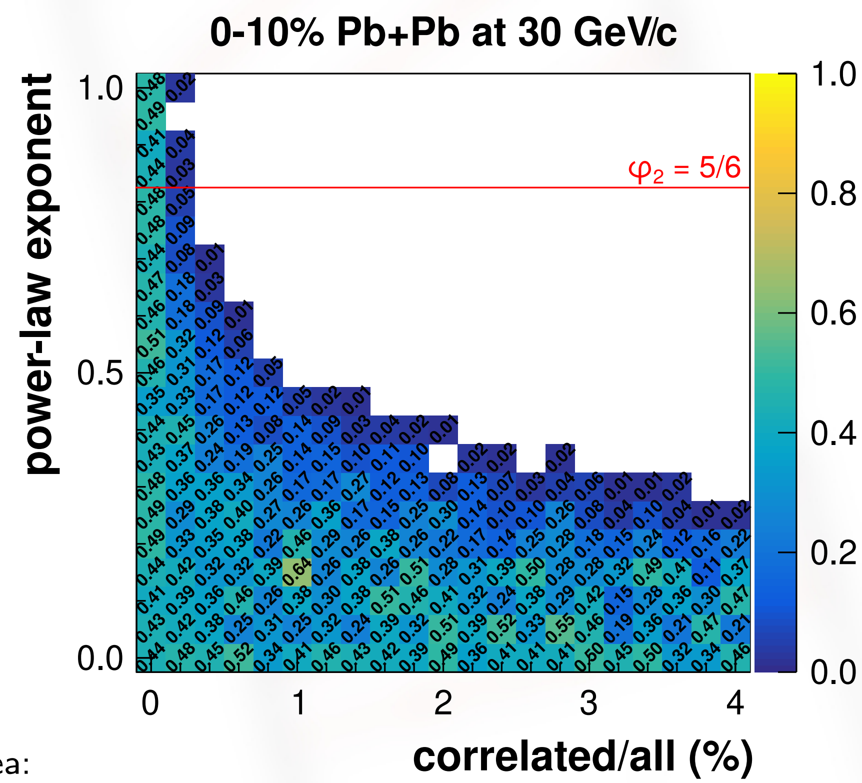
- structure dictated by the chiral symmetry
- no chiral-critical behavior encoded in β

- χ_4/χ_2 and χ_6/χ_2 suppressed by repulsion, but qualitative structure the same

Exclusion plot

Comparison with simple power-law model





white area:
p-value < 0.01

exclusion plots for parameters of simple power-law model

The intermittency index φ_2 for a system freezing out at the QCD critical endpoint is expected to be $\varphi_2 = 5/6$ assuming that the latter belongs to the 3-D Ising universality class.