High-energy showers inside the QGP:

weak vs. strong couping, the LPM effect, & overlapping sequential bremsstrahlung

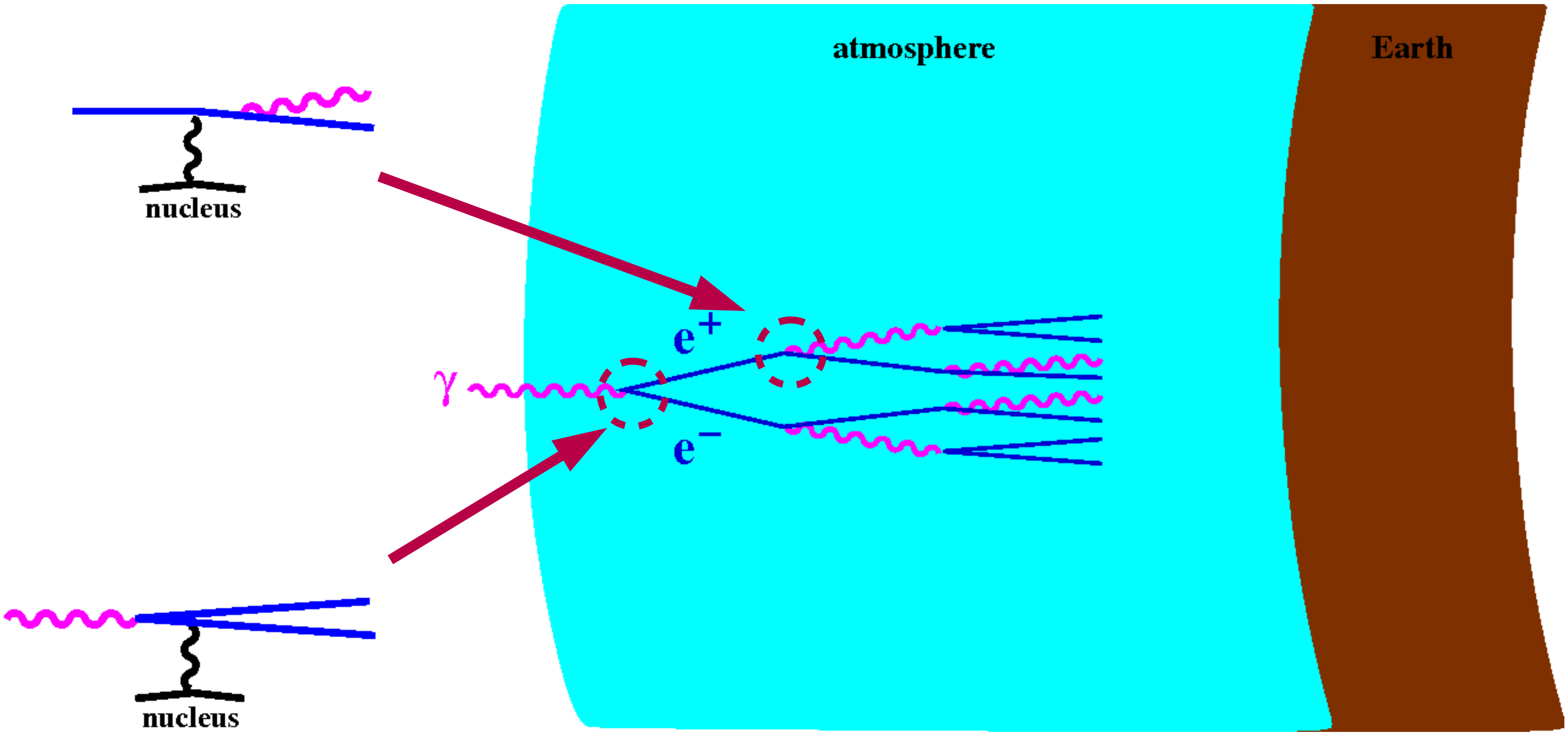
Peter Arnold
University of Virginia

Reporting (eventually) on work with

Shahin Iqbal
Central China Normal University;
National Centre for Physics, Pakistan

Tyler Gorda
University of Virginia;
Technische Universität Darmstadt

High energy particles traveling through matter lose energy via successive bremsstrahlung and pair production:



[Oversimplification: Only electromagnetic shower shown.]

Part 1

THE LPM EFFECT IN QED

[LPM = Landau, Pomeranchuk, Migdal]

Review of high-energy bremsstrahlung

Collisions with the medium

generate chances for bremsstrahlung

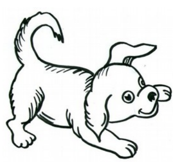
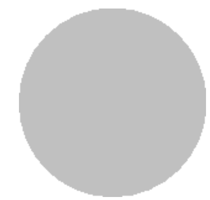
Naively,

prob of emission ~ α per collision

BUT

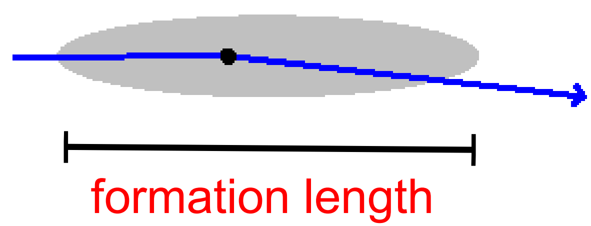
Light can't resolve features on small scales.

Non-relativistic:

 and  both look like  if $\lambda >> d$.

Extremely relativistic, nearly-collinear motion:

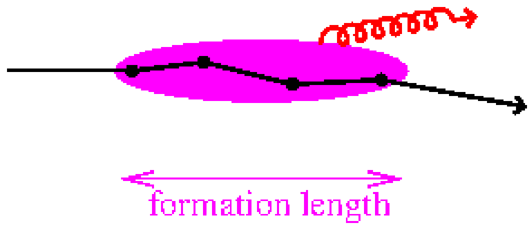
Similar effect, but size of fuzziness stretched out!



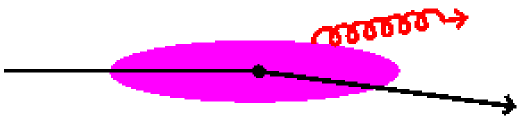
$$l_{\text{form}} \propto \sqrt{E} \qquad \text{(for fixed } x\text{)}$$

Qualitative point for later:

The less collinear the bremsstrahlung, the shorter the formation length.



indistinguishable from



So

prob of emission ~ α per formation length $l_{\text{form}} \propto \sqrt{E}$

Calculated quantitatively by

LPM for QED (1950s)
BDMPS-Z for QCD (1990s)

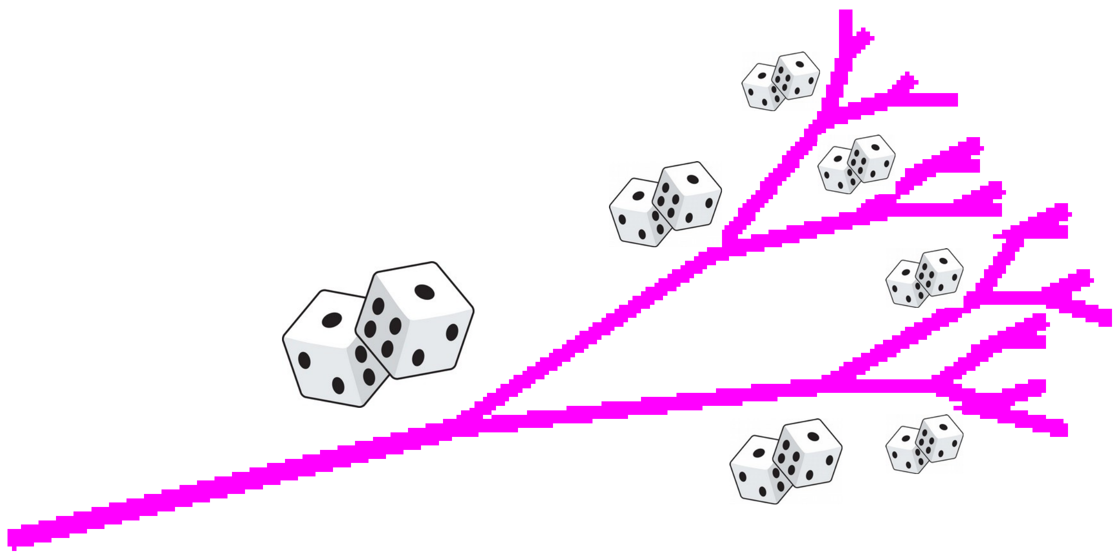
and investigated in many ways by many people since.

Consequence: At high enough energy, the effective bremsstrahlung rate in medium is reduced by factor $\propto \sqrt{E}$

Part 2

A new puzzle for LPM calculations in the 2nd Millenium

An idealized Monte Carlo picture of in-medium evolution



As time passes,

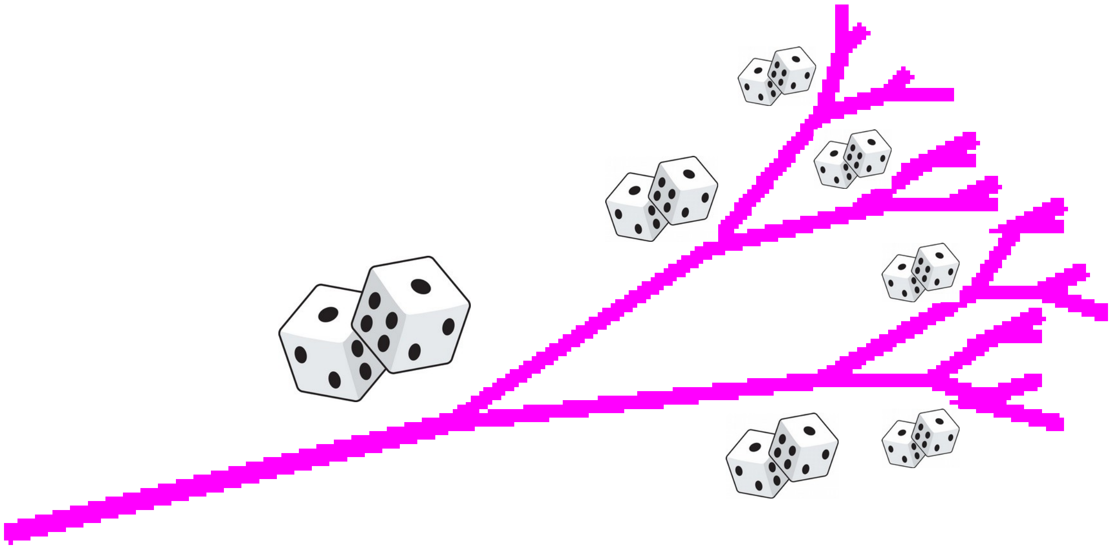
roll classical dice for probability of each splitting

weighted by the quantum calculation of the single splitting rate

$$\frac{d\Gamma_{\rm brem}}{dx}$$ for each vertex  shown above.

LPM effect included in this rate!

An idealized Monte Carlo picture of in-medium evolution



Built-in assumption:

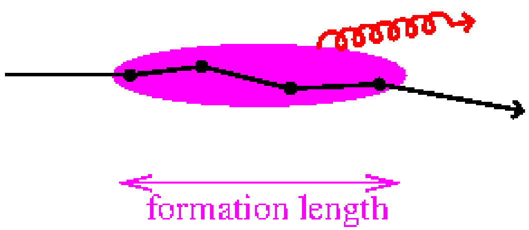
Consecutive splittings are quantum-mechanically independent.

(Are they ?)

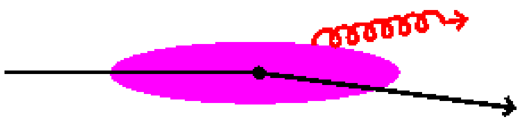
Remember from previous discussion:

Chance of brem ~ α per formation time

because



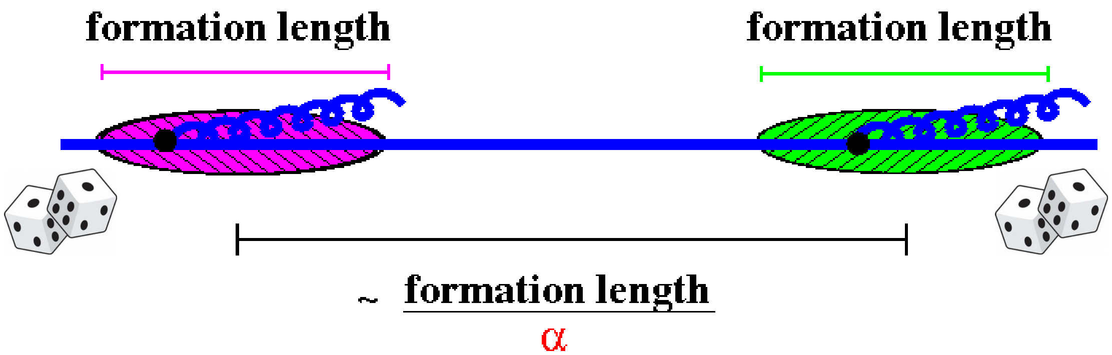
indistinguishable from



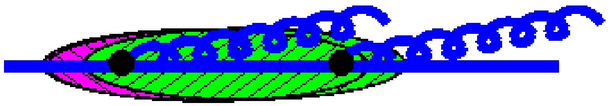
Consecutive emissions

Chance of brem ~ α per formation time

means that two consecutive splittings will typically look like



So chance of overlap (i.e. "rolling dice separately" breaking down) is



$\propto \alpha$

How big is "α" ??

How big is α_s?

Nothing to do with whether medium is

sQGP / perfect liquid [$\alpha_s(T)$ big]	vs.	weakly-coupled QGP [$\alpha_s(T)$ small]

α_s on previous slide associated with emission vertex:

costs roughly $\alpha_s(Q_\perp)$ with $Q_\perp \sim (\hat{q}E)^{1/4} \lesssim$ a few GeV

panic and/or fool around with AdS/CFT energy loss [$\alpha_s(Q_\perp)$ big]	vs.	LPM-based analysis [$\alpha_s(Q_\perp)$ small]

Does the wisdom of the ages tell us if α_s(few GeV) is small?

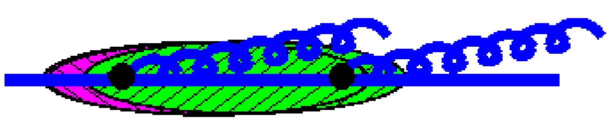
Particle physics in vacuum:

Small for some things, like matching lattice calculations to continuum MS-bar α_s

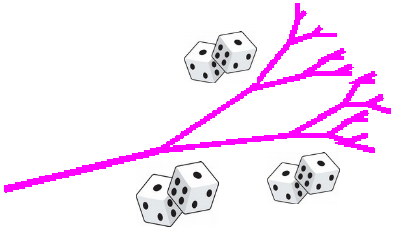
High-temperature physics:

Bad news (except possibly if one does sophisticated resummations of perturbation series)

Overlapping formation times effects on cascade:

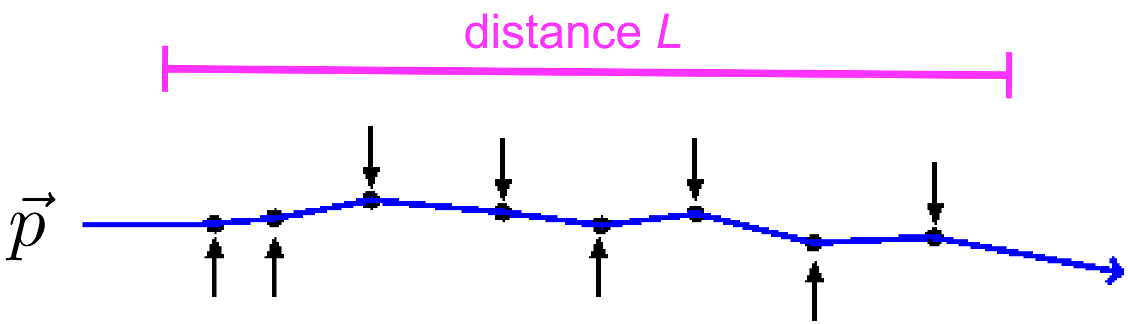


$\propto$ α effect on

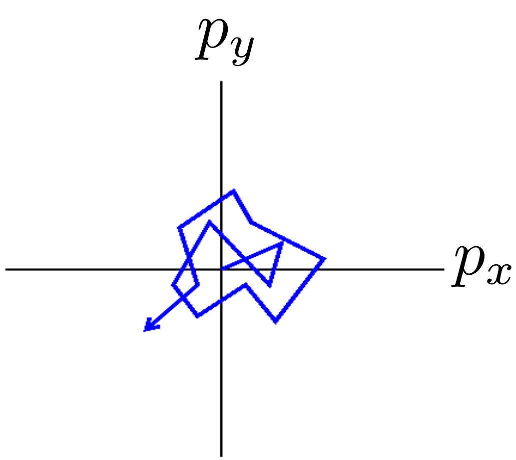


We should calculate it and see.

Characterizing the medium: $\hat{q}$



Random kicks from medium change p_T by tiny amounts << E



→ Random walk in transverse momentum plane:

$$(p_\perp)_{\rm rms} \propto \sqrt{N_{\rm kicks}} \propto \sqrt{L}$$

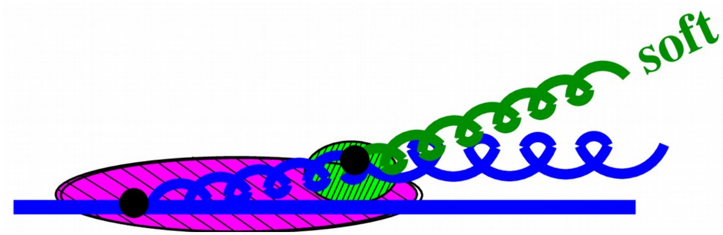
$$\langle p_\perp^2 \rangle = \hat{q} L$$

$\hat{q}$ defined as this proportionality constant

It's the only characteristic of the medium that matters for the problem under discussion.

Soft emission

Soft emissions are generally enhanced by logs.
Path-breaking authors found small-*x*-like double logs in this case,



$$\propto \alpha_s \ln^2 \left(\frac{E}{\hat{q}\tau_{\rm mfp}} \right)$$

Blaizot & Mehtar-Tani; Iancu; Wu (2014)

This is a BIG effect for large *E*.
But they found soft emission effects could be absorbed into the medium parameter

$$\hat{q} \to \hat{q}_{\rm eff}(E) \propto E^{\#\sqrt{\alpha_s}}$$

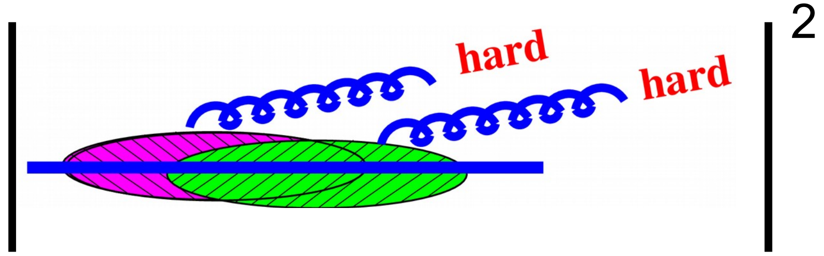
following Liou, Mueller, Wu (2013)

Refined question

What about overlap effects that *can't* be absorbed into $\hat{q}$?

Our program

Compute the effect of the overlap for hard emissions

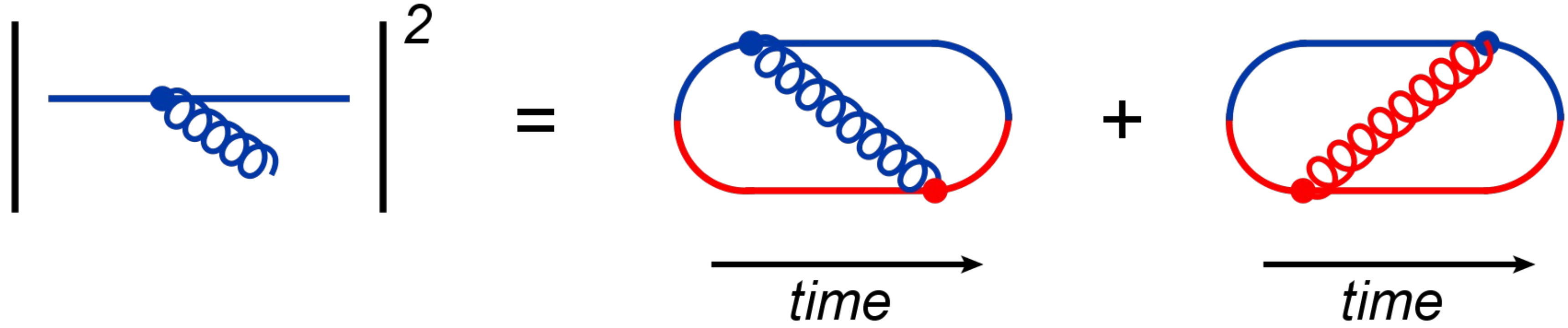


⇒ relative $O(\alpha_s)$ correction due to overlap effects

In broad brush: interesting and fun field theory problem.

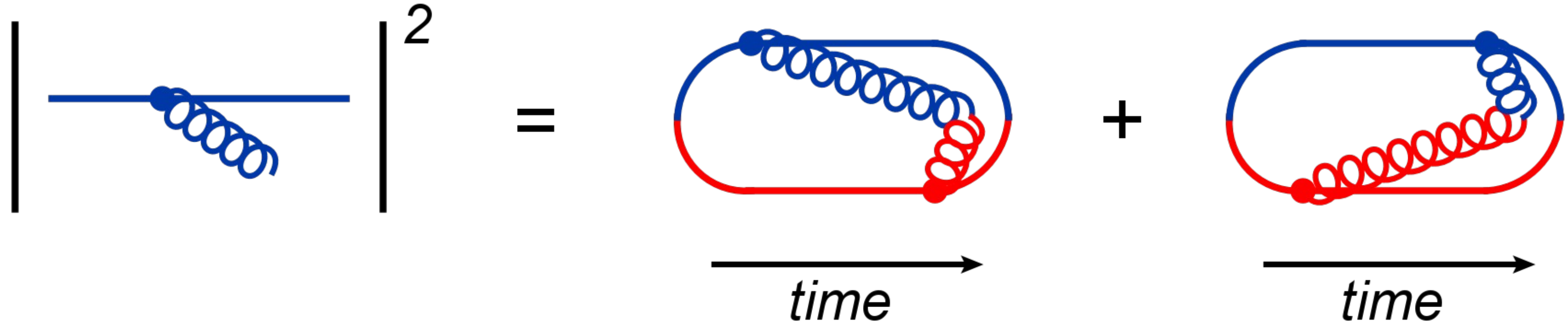
In calculational detail: a pain in the ass.

First: How we draw diagrams



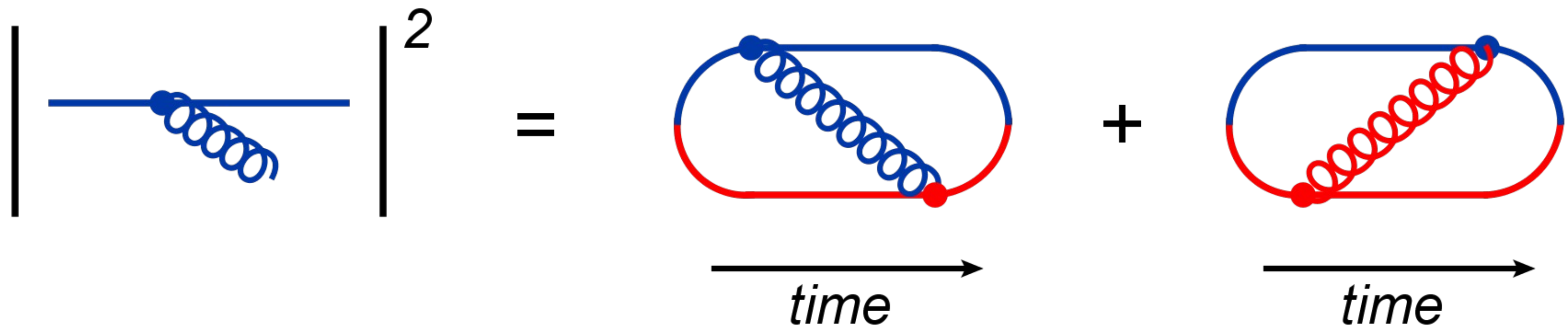
[In this paper, non-curly propagators can be quarks OR gluons, and will usually be gluons!]

First: How we draw diagrams



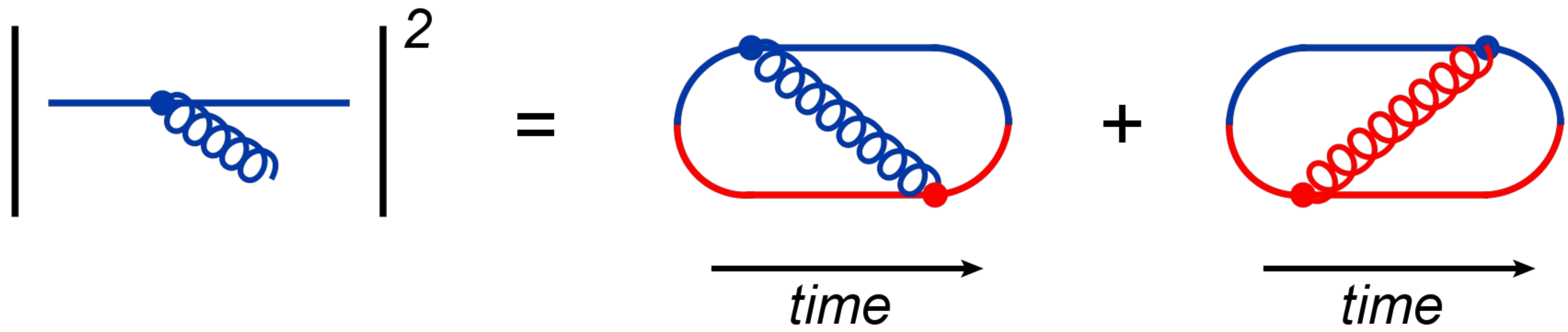
[In this paper, non-curly propagators can be quarks OR gluons, and will usually be gluons!]

First: How we draw diagrams

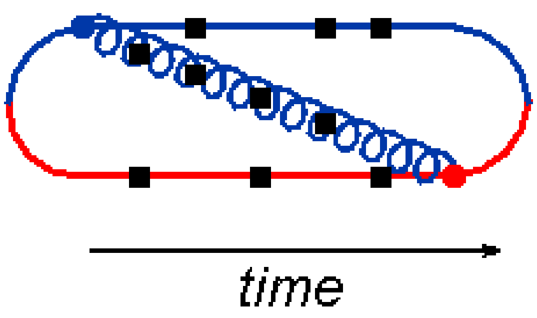


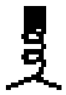
[In this paper, non-curly propagators can be quarks OR gluons, and will usually be gluons!]

First: How we draw diagrams

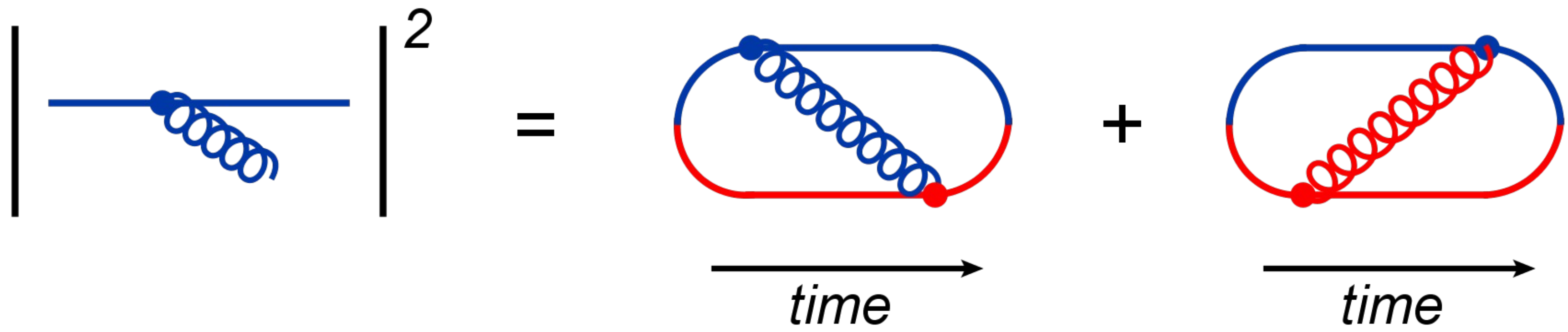


implicitly including interactions with the medium (in invisible ink above):

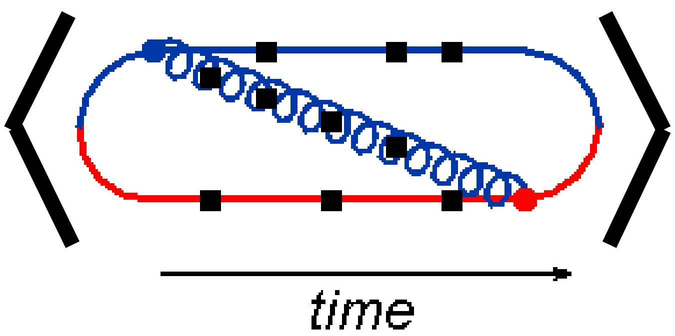


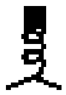
■ = interaction with medium = 

First: How we draw diagrams



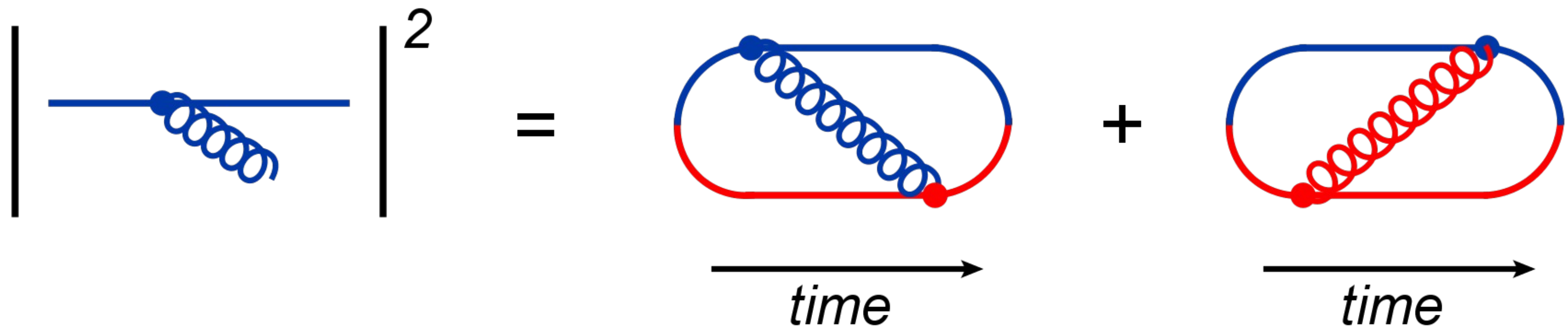
implicitly including interactions with the medium (in invisible ink above):



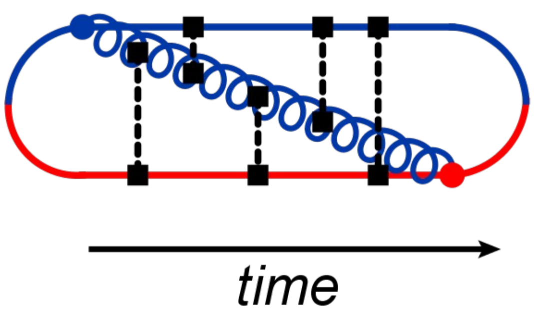
■ = interaction with medium = 

$\langle \cdots \rangle$ = medium average

First: How we draw diagrams



implicitly including interactions with the medium (in invisible ink above):



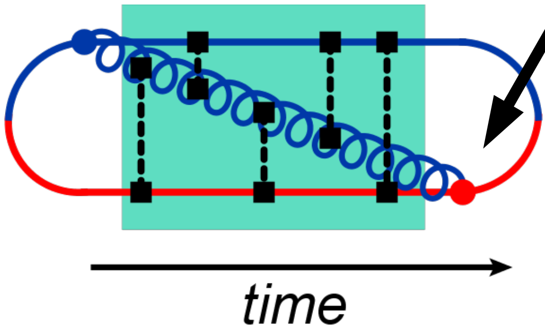
■ = interaction with medium

- - - - = correlations in medium
(relatively localized in time)
taken from

- perturbation theory
- AdS/CFT [Liu, Rajagopal, Weidemann '06]
- or phenomenological fit to $\hat{q}$

Medium-averaged evolution can be treated (at high energy) as (non-Hermitian) 2-dim quantum mechanics problem in transverse plane.

High-energy splitting vertices can be taken from QFT (DGLAP splitting amplitudes).



■ = interaction with medium

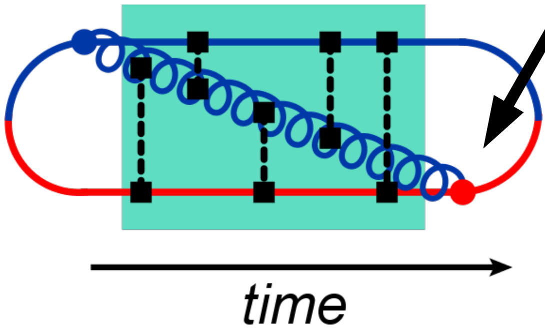
- - - - = correlations in medium
(relatively localized in time)
taken from

- perturbation theory
- AdS/CFT [Liu, Rajagopal, Weidemann '06]
- or phenomenological fit to $\hat{q}$

$$\epsilon_p = \sqrt{p_z^2 + p_\perp^2} \simeq p_z + \frac{p_\perp^2}{2p_z} \qquad \text{is like} \qquad \text{const} + \frac{p_\perp^2}{2M}$$

Medium-averaged evolution can be treated (at high energy) as (non-Hermitian) 2-dim quantum mechanics problem in transverse plane.

High-energy splitting vertices can be taken from QFT (DGLAP splitting amplitudes).



■ = interaction with medium

- - - - = correlations in medium (relatively localized in time) taken from

- perturbation theory
- AdS/CFT [Liu, Rajagopal, Weidemann '06]
- or phenomenological fit to $\hat{q}$

Simplifying assumptions in this talk

"infinite" medium

i.e. homogenous on scale of formation time

Medium correlations (our QM "potential") characterized by $\hat{q}$

Formally justified in high-energy limit*

→ (non-Hermitian) harmonic oscillator approximation to the QM "potential"

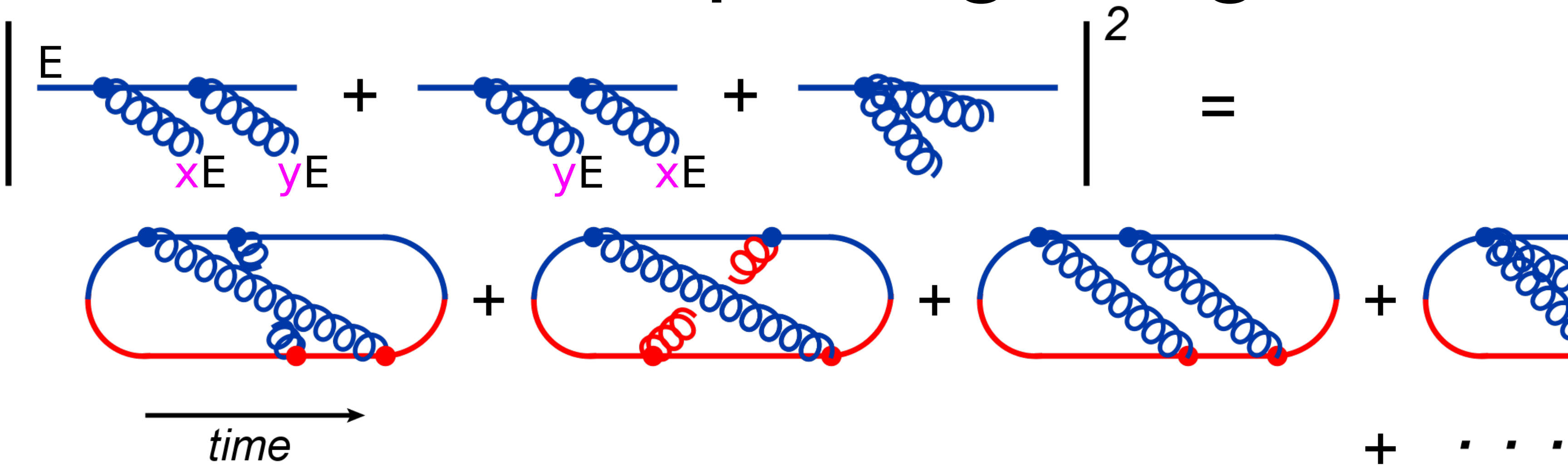
* Won't discuss today caveats and counter caveats!

And for what we'll be doing next (overlapping formation times),

Large-N_c limit

We know in principle how to do N_c=3, but much harder calculation (which would require much harder numerics).

Double Splitting Diagrams



[calculated with Shahin Iqbal and Han-Chih Chang]

UV Issue:

Above is a tree-level process, but *individual time-ordered* diagrams are UV divergent! UV divergences cancel at end of the day, but must be careful to consistently regularize each diagram. Yecch! (We use dimensional regularization.)

Infrared Issue:

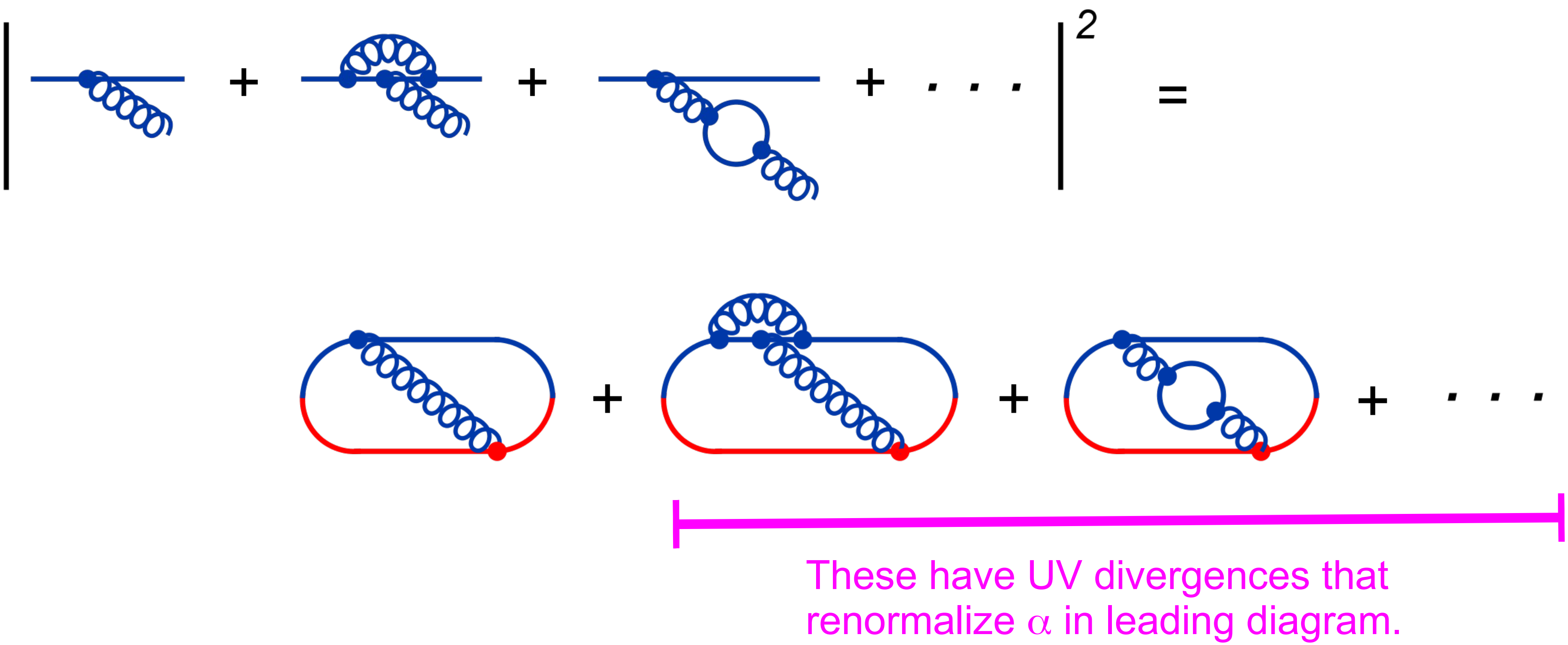
$$\frac{d\Gamma}{dx\,dy} \sim \frac{\alpha_s^2}{xy^{3/2}} \sqrt{\frac{\hat{q}}{E}} \qquad (\text{for } y \lesssim x),$$

giving power-law IR-divergent contributions to energy loss, etc.

Part 3

VIRTUAL CORRECTIONS

Need virtual corrections to single splitting

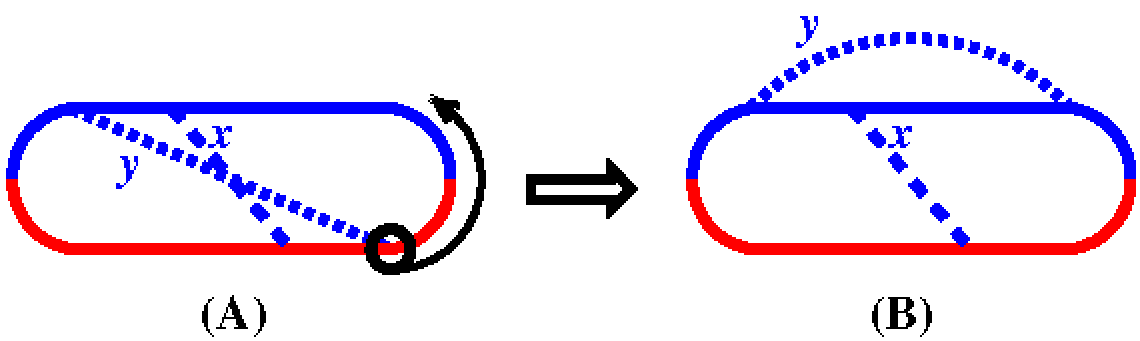


There are a *lot* of UV-regularized time-ordered loop diagrams to compute.

Fortunately, there are tricks to get *almost* all of them from previous calculations...

“Back-end” tansformations

Move the *latest*-time vertex from amplitude to conjugate amplitude or visa versa:



Then

$$\left[\frac{d\Gamma}{dx}\right]_{(B)} = -\int_0^{1-x} dy \left[\frac{d\Gamma}{dx\,dy}\right]_{(A)}$$

which could be notationally summarized as essentially just a minus sign:

$$\text{“}\left[\frac{d\Gamma}{dx\,dy}\right]_{(B)}\text{”} = -\left[\frac{d\Gamma}{dx\,dy}\right]_{(A)}$$

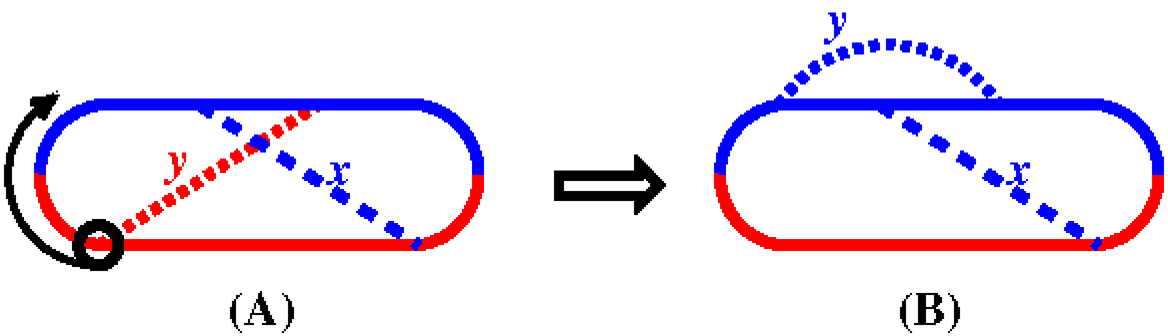
Handwaving reason: Related to conservation of probability.

Amazing consistency check: The physically-irrelevant UV divergence of the time-ordered diagram (A) for a tree-level process reproduces the physically-relevant UV divergence of (B).

Technical note: Limits on above integral not (-∞,+∞) because we organize our calculation using Light Cone Perturbation Theory.

“Front-end” tansformations

Move the *earliest*-time vertex from amplitude to conjugate amplitude or visa versa:



Then

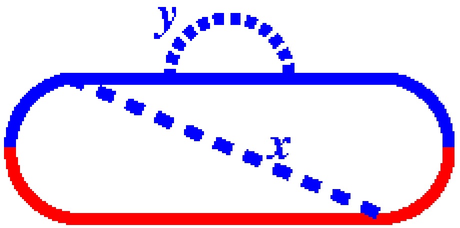
$$\left[\frac{d\Gamma}{dx}\right]_{(B)} = -\int_0^{1-x} dy \left\{ \left[\frac{d\Gamma}{dx\,dy}\right]_{(A)} \text{ with } (x,y,E) \longrightarrow \left(\tfrac{x}{1-y}, \tfrac{-y}{1-y}, (1-y)E\right) \right\}$$

Equivalent to	$xE \longrightarrow xE$		(unchanged)
	$yE \longrightarrow -yE$		(changed from red line to blue line)
	$(1-y)E \longrightarrow E$		(initial particle line changed)

Technical note: also needs overall factor of $(1-y)^{-\varepsilon}$ in $4-\varepsilon$ dimensions.

What Remained?

The *only* time-ordered diagram we can't get from another diagram is (can be taken to be)



Remember: All time evolution is in medium background, statistically averaged over medium fluctuations.

Took a lot of work to do in dimensional regularization, but we did –

first in large-N_f QED [Arnold,Iqbal '19]

then converted to large-N_c QCD [Arnold,Gorda,Iqbal '00]

Our final formula…

(drum roll please)

a. Crossed Diagrams

Here we collect the result for the crossed diagrams [21] as corrected by ref. [23]. A brief summary of the interpretation of each piece below can be found in section VIII of ref. [21].

$$\left[\frac{d\Gamma}{dx\,dy}\right]_{\rm crossed} = A(x,y) + A(z,y) + A(x,z) \tag{A10}$$

$$A(x,y) = A^{\rm pole}(x,y) + \int_0^\infty d(\Delta t)\, 2\,{\rm Re}\big[B(x,y,\Delta t) + B(y,x,\Delta t)\big] \tag{A11}$$

$$\begin{aligned} B(x,y,\Delta t) &= C(\{\hat{x}_i\},\alpha,\beta,\gamma,\Delta t) + C(\{x'_i\},\beta,\alpha,\gamma,\Delta t) + C(\{\tilde{x}_i\},\gamma,\alpha,\beta,\Delta t) \\ &= C(-1,y,z,x,\alpha,\beta,\gamma,\Delta t) + C\big(-(1{-}y),-y,1{-}x,x,\beta,\alpha,\gamma,\Delta t\big) \\ &\quad + C\big(-y,-(1{-}y),x,1{-}x,\gamma,\alpha,\beta,\Delta t\big) \end{aligned} \tag{A12}$$

$$C = D - \lim_{\bar{q}\to 0} D \tag{A13}$$

$$\begin{aligned} &D(x_1,x_2,x_3,x_4,\alpha,\beta,\gamma,\Delta t) = \\ &\quad \frac{C_{\rm A}^2\alpha_{\rm s}^2 M_{\rm i} M_{\rm f}}{32\pi^4 E^2}(-x_1x_2x_3x_4)\Omega_+\Omega_-\csc(\Omega_+\Delta t)\csc(\Omega_-\Delta t) \\ &\quad \times \Big\{(\beta Y_{\rm y} Y_{\bar{\rm y}} + \alpha \overline{Y}_{{\rm y}\bar{\rm y}} Y_{{\rm y}\bar{\rm y}}) I_0 + (\alpha+\beta+2\gamma) Z_{{\rm y}\bar{\rm y}} I_1 \\ &\quad + \big[(\alpha+\gamma) Y_{\rm y} Y_{\bar{\rm y}} + (\beta+\gamma)\overline{Y}_{{\rm y}\bar{\rm y}} Y_{{\rm y}\bar{\rm y}}\big] I_2 - (\alpha+\beta+\gamma)(\overline{Y}_{{\rm y}\bar{\rm y}} Y_{\bar{\rm y}} I_3 + Y_{\rm y} Y_{{\rm y}\bar{\rm y}} I_4)\Big\} \end{aligned} \tag{A14}$$

$$\begin{aligned} A^{\rm pole}(x,y) = \frac{C_{\rm A}^2\alpha_{\rm s}^2}{8\pi^2}\, xyz(1{-}x)(1{-}y)\,{\rm Re}\Big(&-i\big(\Omega_{-1,1-x,x} + \Omega_{-(1-y),z,x} + \Omega_{-1,1-y,y} + \Omega_{-(1-x),z,y}\big) \\ &\times \Big\{\big((\alpha{+}\beta)z(1{-}x)(1{-}y) + (\alpha{+}\gamma)xyz\big)\Big[\ln\Big(\tfrac{z}{(1-x)(1-y)}\Big) - i\pi\Big] \\ &+ 2(\alpha{+}\beta{+}\gamma)xyz\Big\}\Big) \end{aligned} \tag{A15}$$

$$I_0 = \frac{4\pi^2}{(X_{\rm y} X_{\bar{\rm y}} - X_{{\rm y}\bar{\rm y}}^2)} \tag{A16a}$$

$$I_1 = -\frac{2\pi^2}{X_{{\rm y}\bar{\rm y}}}\ln\left(1 - \frac{X_{{\rm y}\bar{\rm y}}^2}{X_{\rm y} X_{\bar{\rm y}}}\right) \tag{A16b}$$

$$I_2 = \frac{2\pi^2}{X_{{\rm y}\bar{\rm y}}^2}\ln\left(1 - \frac{X_{{\rm y}\bar{\rm y}}^2}{X_{\rm y} X_{\bar{\rm y}}}\right) + \frac{4\pi^2}{(X_{\rm y} X_{\bar{\rm y}} - X_{{\rm y}\bar{\rm y}}^2)} \tag{A16c}$$

$$I_3 = \frac{4\pi^2 X_{{\rm y}\bar{\rm y}}}{X_{\bar{\rm y}}(X_{\rm y} X_{\bar{\rm y}} - X_{{\rm y}\bar{\rm y}}^2)} \tag{A16d}$$

$$I_4 = \frac{4\pi^2 X_{{\rm y}\bar{\rm y}}}{X_{\rm y}(X_{\rm y} X_{\bar{\rm y}} - X_{{\rm y}\bar{\rm y}}^2)} \tag{A16e}$$

$$\begin{pmatrix} X_{\rm y} & Y_{\rm y} \\ Y_{\rm y} & Z_{\rm y} \end{pmatrix} \equiv \begin{pmatrix} |M_{\rm i}|\Omega_{\rm i} & 0 \\ 0 & 0 \end{pmatrix} - i a_{\rm y}^{-1\top}\underline{\Omega}\cot(\underline{\Omega}\,\Delta t)\, a_{\rm y}^{-1} \tag{A17a}$$

$$\begin{pmatrix} X_{\bar{\rm y}} & Y_{\bar{\rm y}} \\ Y_{\bar{\rm y}} & Z_{\bar{\rm y}} \end{pmatrix} \equiv \begin{pmatrix} |M_{\rm f}|\Omega_{\rm f} & 0 \\ 0 & 0 \end{pmatrix} - i a_{\bar{\rm y}}^{-1\top}\underline{\Omega}\cot(\underline{\Omega}\,\Delta t)\, a_{\bar{\rm y}}^{-1} \tag{A17b}$$

$$\begin{pmatrix} X_{{\rm y}\bar{\rm y}} & Y_{{\rm y}\bar{\rm y}} \\ \overline{Y}_{{\rm y}\bar{\rm y}} & Z_{{\rm y}\bar{\rm y}} \end{pmatrix} \equiv -i a_{\rm y}^{-1\top}\underline{\Omega}\csc(\underline{\Omega}\,\Delta t)\, a_{\bar{\rm y}}^{-1} \tag{A17c}$$

$$\underline{\Omega} \equiv \begin{pmatrix} \Omega_+ & \\ & \Omega_- \end{pmatrix} \tag{A18}$$

$$M_{\rm i} = x_1 x_4 (x_1{+}x_4) E, \qquad M_{\rm f} = x_3 x_4 (x_3{+}x_4) E \tag{A19a}$$

$$\Omega_{\rm i} = \sqrt{-\frac{i\hat{q}_{\rm A}}{2E}\left(\frac{1}{x_1} + \frac{1}{x_4} - \frac{1}{x_1{+}x_4}\right)}, \qquad \Omega_{\rm f} = \sqrt{-\frac{i\hat{q}_{\rm A}}{2E}\left(\frac{1}{x_3} + \frac{1}{x_4} - \frac{1}{x_3 + x_4}\right)} \tag{A19b}$$

$$\Omega_{\xi_1,\xi_2,\xi_3} = \sqrt{-\frac{i\hat{q}_{\rm A}}{2E}\left(\frac{1}{\xi_1} + \frac{1}{\xi_2} + \frac{1}{\xi_3}\right)} \tag{A20}$$

$$a_{\bar{\rm y}} = \begin{pmatrix} C_{34}^+ & C_{34}^- \\ C_{12}^+ & C_{12}^- \end{pmatrix} \tag{A21}$$

$$a_{\rm y} = \frac{1}{(x_1 + x_4)}\begin{pmatrix} -x_3 & -x_2 \\ x_4 & x_1 \end{pmatrix} a_{\bar{\rm y}} \tag{A22}$$

$$\begin{aligned} \begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = \;& \begin{pmatrix} - \\ + \\ + \end{pmatrix}\Bigg[\left|\frac{x}{y^3 z(1{-}x)^3(1{-}y)^3}\right| + \left|\frac{y}{x^3 z(1{-}x)^3(1{-}y)^3}\right| \\ &\qquad + \left|\frac{1{-}x}{x^3y^3z(1{-}y)^3}\right| + \left|\frac{1{-}y}{x^3y^3z(1{-}x)^3}\right|\Bigg] \\ &+ \begin{pmatrix} + \\ - \\ + \end{pmatrix}\Bigg[\left|\frac{x}{y^3z^3(1{-}x)(1{-}y)}\right| + \left|\frac{y}{x^3z^3(1{-}x)(1{-}y)}\right| \\ &\qquad + \left|\frac{z}{x^3y^3(1{-}x)(1{-}y)}\right| + \left|\frac{1}{x^3y^3z^3(1{-}x)(1{-}y)}\right|\Bigg] \\ &+ \begin{pmatrix} + \\ + \\ - \end{pmatrix}\Bigg[\left|\frac{1{-}x}{xyz^3(1{-}y)^3}\right| + \left|\frac{1{-}y}{xyz^3(1{-}x)^3}\right| \\ &\qquad + \left|\frac{z}{xy(1{-}x)^3(1{-}y)^3}\right| + \left|\frac{1}{xyz^3(1{-}x)^3(1{-}y)^3}\right|\Bigg] \end{aligned} \tag{A23}$$

Note that the (α,β,γ) used in the definition (A12) of B are implicitly functions (A46) of the *arguments* x and y of $B(x,y,\Delta t)$ [with $z \equiv 1{-}x{-}y$]. This is important in formulas such as (A11), where in some terms those local arguments are replaced by other variables.

plus 3 more unreadable slides, as intricate as this one.

Schematic form of results

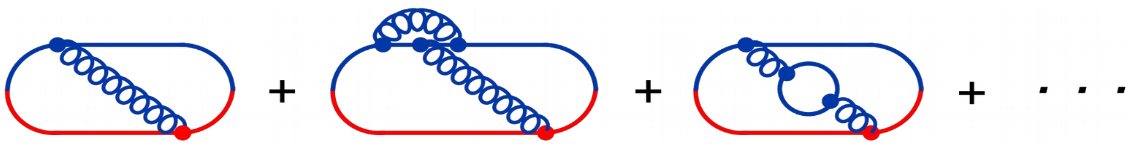
$$\frac{d\Gamma}{dx\,dy} = \text{stuff} + \int_0^\infty d(\Delta t)\ [\text{complicated formula}]$$

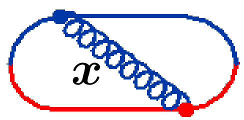
one (convergent) time integral that must be done numerically

In applications, will appear in integrals where *x* and *y* must also be integrated (numerically) and IR divergences must be subtracted and organized.

Sanity Checks

1. Total UV divergence of virtual diagrams correctly renormalizes coupling of 1990's LPM single-splitting calculation (BDMPS-Z):



= 

$$\times \left[1 + \alpha_s\left(-\frac{\beta_0}{\epsilon} + \text{UV-finite}(x)\right)\right]$$ ✓

2. Power-law IR divergences cancel between real and virtual processes. ✓

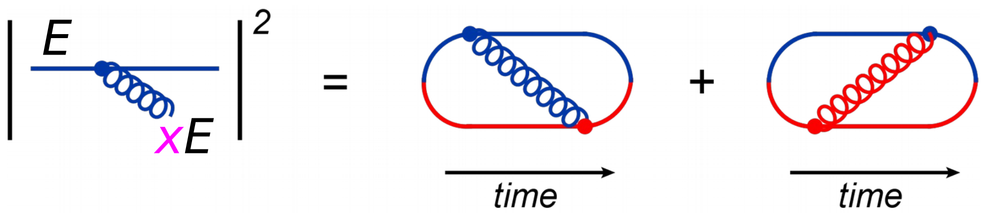
3. Double-log IR divergences match known leading-log results. ✓

[And we now have the first calculation, so far numerical, of sub-leading single-log IR divergences. Analytic results coming in the future.]

Part 4

MORE ON POWER-LAW IR DIVERGENCES

IR behavior of *single* splitting

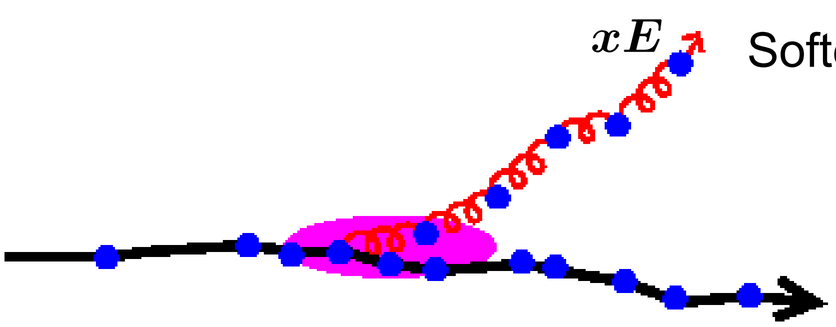


Result from 1990's [BDMPS] has IR behavior $\frac{d\Gamma}{dx} \sim \frac{\alpha_s}{x^{3/2}} \sqrt{\frac{\hat{q}}{E}}$

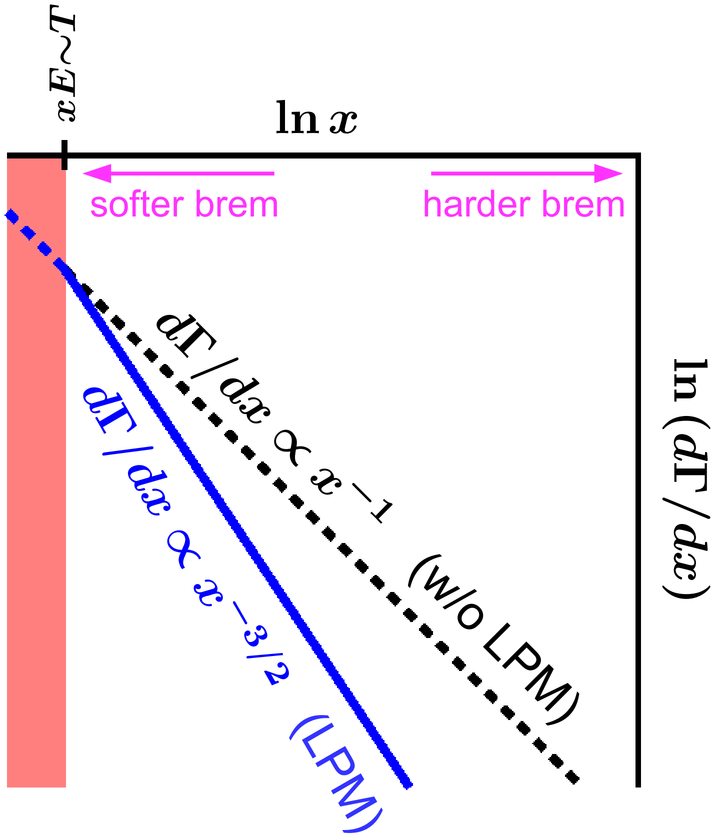
Integrate over x → total bremsstrahlung rate Γ has *power-law* IR divergence in QCD!

Q: Why more divergent than usual logarithmic IR divergence for brem in vacuum?
A: Because in a medium, there is *less* LPM rate suppression for *softer* gluon brem...

Why? Remember: LPM suppression depends on collinearity of the splitting.



Softer gluon emission
→ gluon direction more easily changed by scattering from medium
→ less collinear
→ less LPM suppression!



is where all high-energy approxs break down:

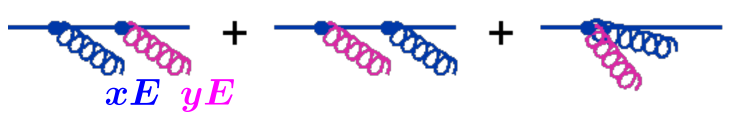
- no LPM effect
- $\hat{q}$ approximation invalid
- 2d QM description invalid

Fortunately, this IR divergence is still mild enough to not generate a divergence in *energy* loss!

e.g. $\int dx\, xE\, \frac{d\Gamma}{dx}$ converges.

IR behavior of *double* splitting

But the same effect is a potential problem for (overlapping) double splitting:



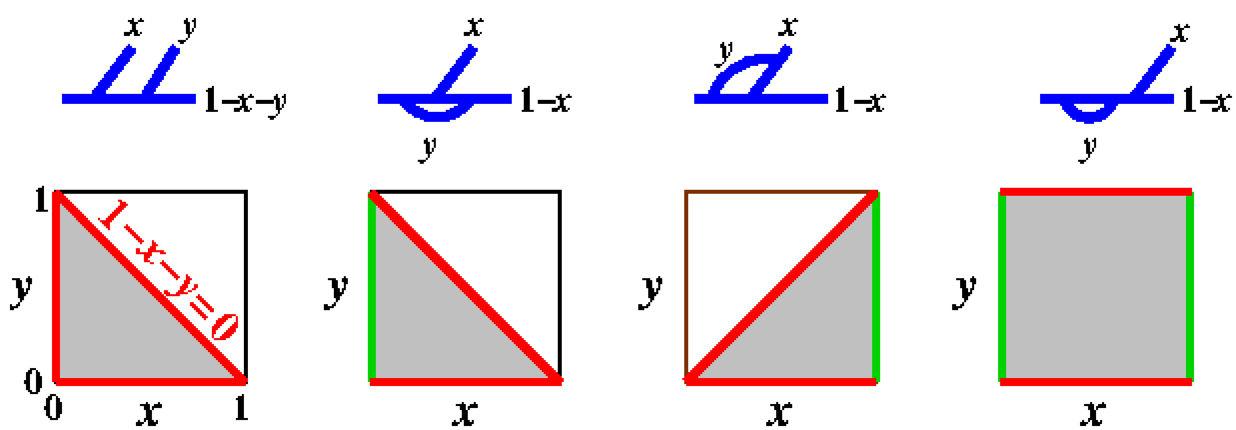
$$\frac{d\Gamma}{dx\,dy} \sim \frac{\alpha_s^2}{xy^{3/2}} \sqrt{\frac{\hat{q}}{E}} \qquad (\text{for } y \lesssim x)$$

gives a power-law divergent contribution to energy loss!

e.g. $\int dx\,dy\,(xE + yE)\,\frac{d\Gamma}{dx\,dy}$ diverges due to $y{\to}0$ behavior.

As mentioned before: power-law divergences cancel with virtual diagrams.
But the organization of that cancellation is complicated!

Integration regions for different types of amplitudes



Example – Time evolution of distribution *N* of high-energy particles in a shower:

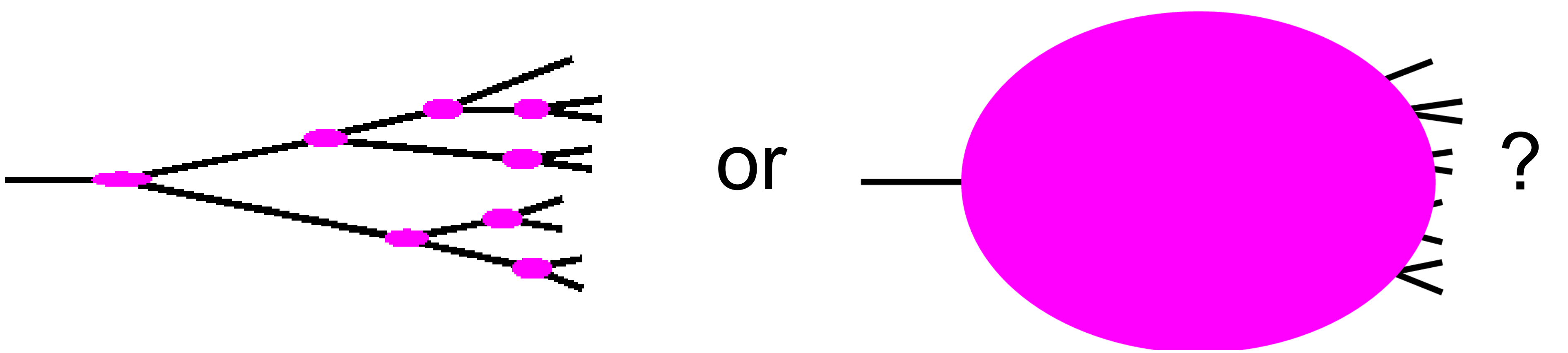
$$\frac{\partial}{\partial t} N(\zeta, E_0, t) = -\Gamma(\zeta E_0)\, N(\zeta, E_0, t) + \int_\zeta^1 \frac{dx}{x} \left[\frac{d\Gamma}{dx}\Big(x, \frac{\zeta E_0}{x}\Big)\right]_{\rm net} N\big(\tfrac{\zeta}{x}, E_0, t\big)$$

$$\text{with} \quad \left[\frac{d\Gamma}{dx}\right]_{\rm net} = \left[\frac{d\Gamma}{dx}\right]_{g\to gg} + \frac{1}{2}\int_0^{1-x} dy \left[\frac{d\Gamma}{dx\,dy}\right]_{g\to ggg}$$

Can't just add together integrands and get power-law divergence free integrals.
But it's possible to reorganize integrations to make cancellation explicit.

Part 5

So what's the answer?

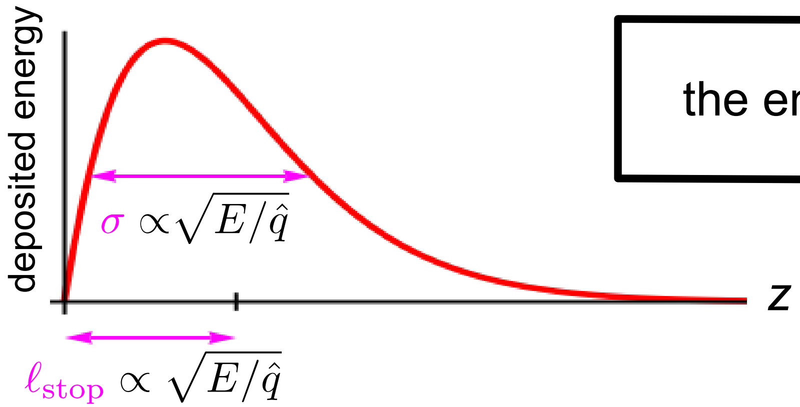


Remember the question

What size are overlap effects that *can't* be absorbed into $\hat{q}$?

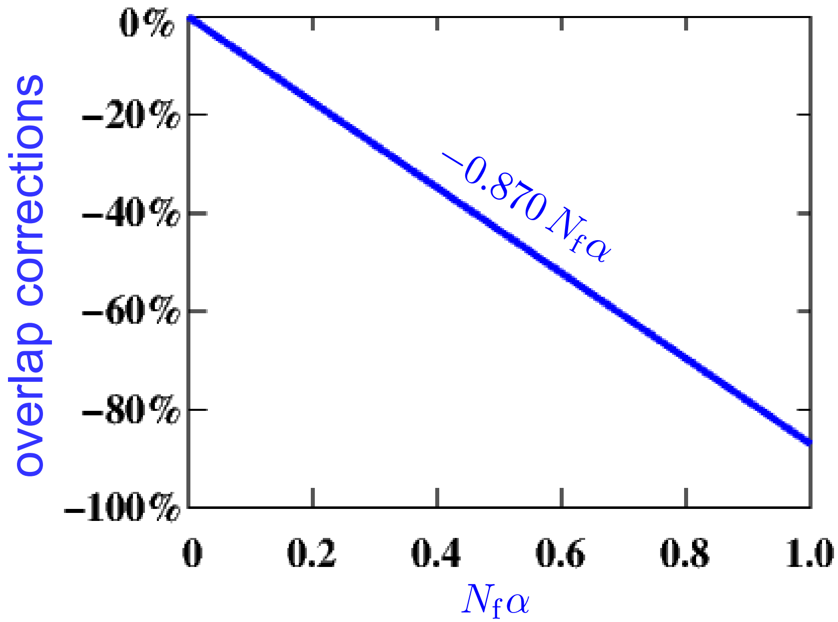
A clean way to answer would be to compute dimensionless characteristics of shower devoplment in which $\hat{q}$ cancels out.

e.g.



the energy deposition "shape" variable $\dfrac{\sigma}{\ell_{\rm stop}}$

We've done this in the case of large-$N_{\rm f}$ QED (in the $\hat{q}$ approximation), with result



[for charge deposition in this case]

What about QCD?

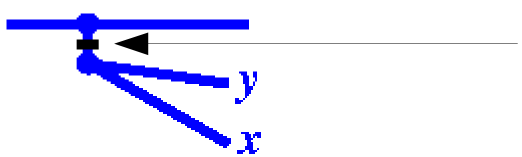
1. The double-log IR divergence causes trouble, despite the fact that it can be absorbed by $\hat{q} \longrightarrow \hat{q}_{\rm eff}(E)$.

   The problem is actually the sub-leading single-log divergences.

   We are currently working on analytic analysis of the single logs.

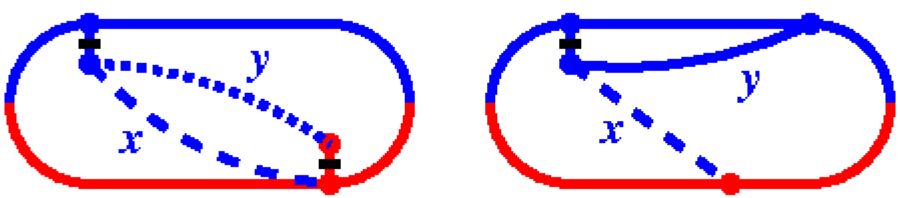
2. There is a class of diagrams we haven't included yet, involving longitudinal gluon exchange.

   e.g.



longitudinal polarized gluon exchange
= an instantaneous interaction in Light Cone Perturbation Theory (LCPT)

and so time-ordered interference diagrams like



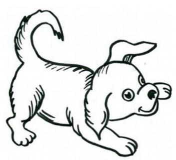
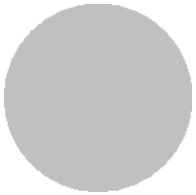
We've done this for large-N_f QED. Work in progress for QCD.

BACKUP 1

Why formation time grows with collinearity

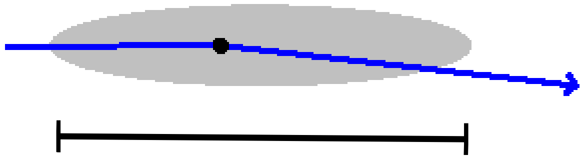
Return to previous slide...

Non-relativistic:

 and  both look like  if $\lambda >> d$.

Extremely relativistic, nearly-collinear motion:

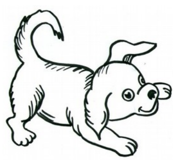
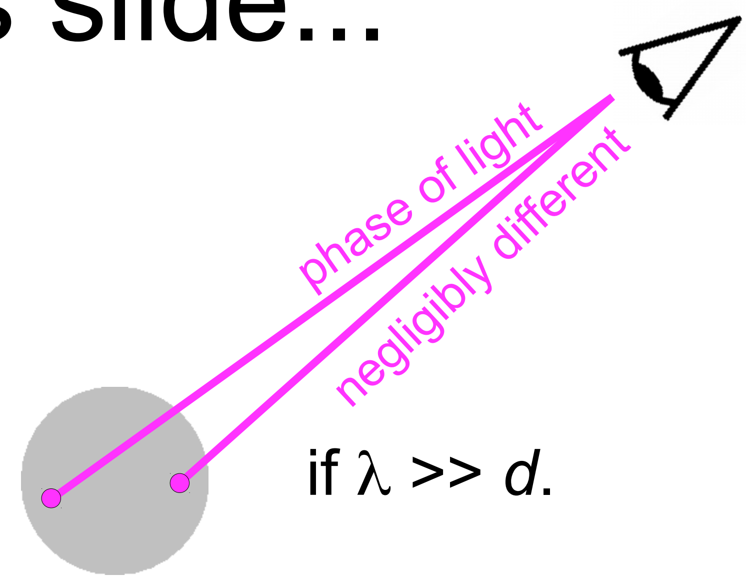
Similar effect, but size of fuzziness stretched out.



$$l_{\rm form} \propto \sqrt{E}$$ (for fixed *x*)

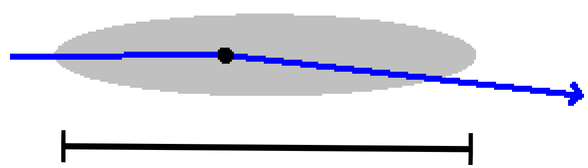
Return to previous slide...

Non-relativistic:

 and  both look like  if $\lambda \gg d$.

Extremely relativistic, nearly-collinear motion:

Similar effect, but size of fuzziness stretched out.



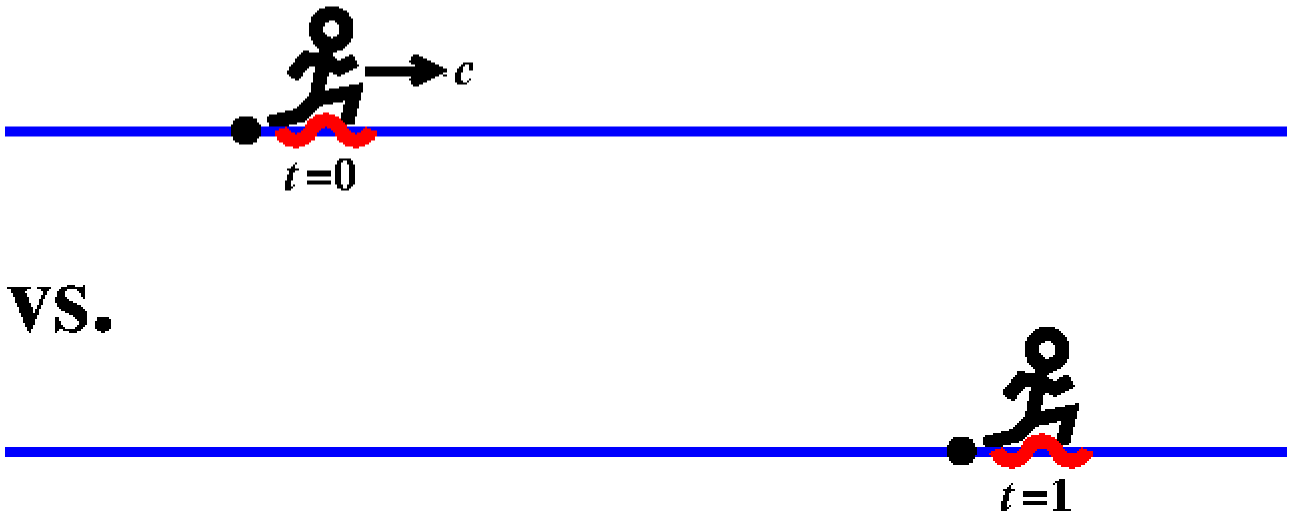
$l_{\rm form} \propto \sqrt{E}$ (for fixed x)

One way to understand stretched version:

Run along with the ultra-relativisitic electon.

Compare light emitted at two different times.

Focus on collinear limit.

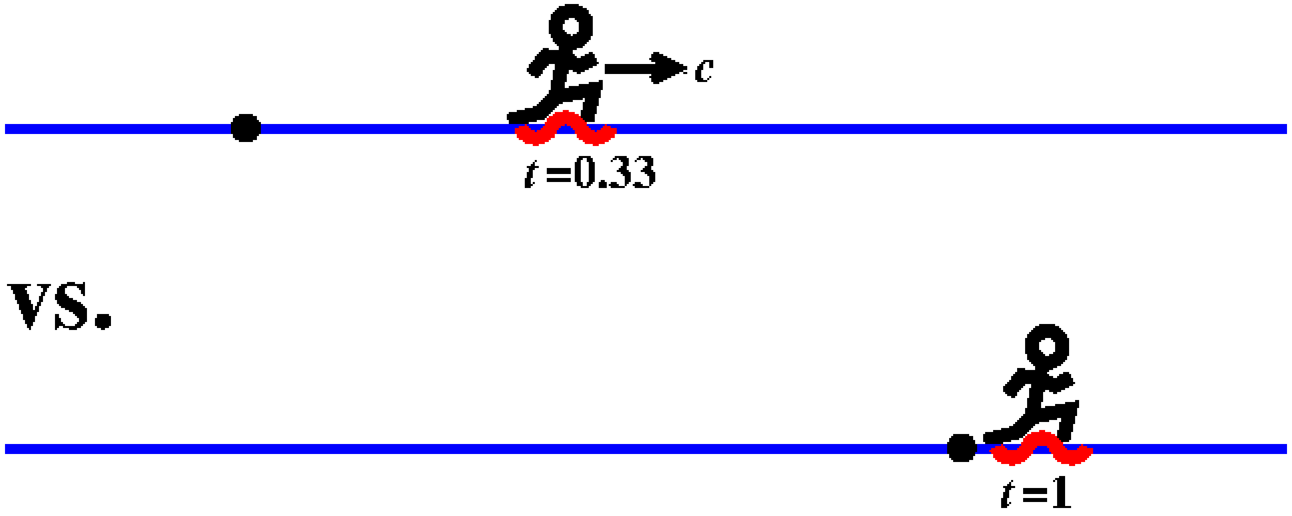


One way to understand it:

Run along with the ultra-relativisitic electon.

Compare light emitted at two different times.

Focus on collinear limit.

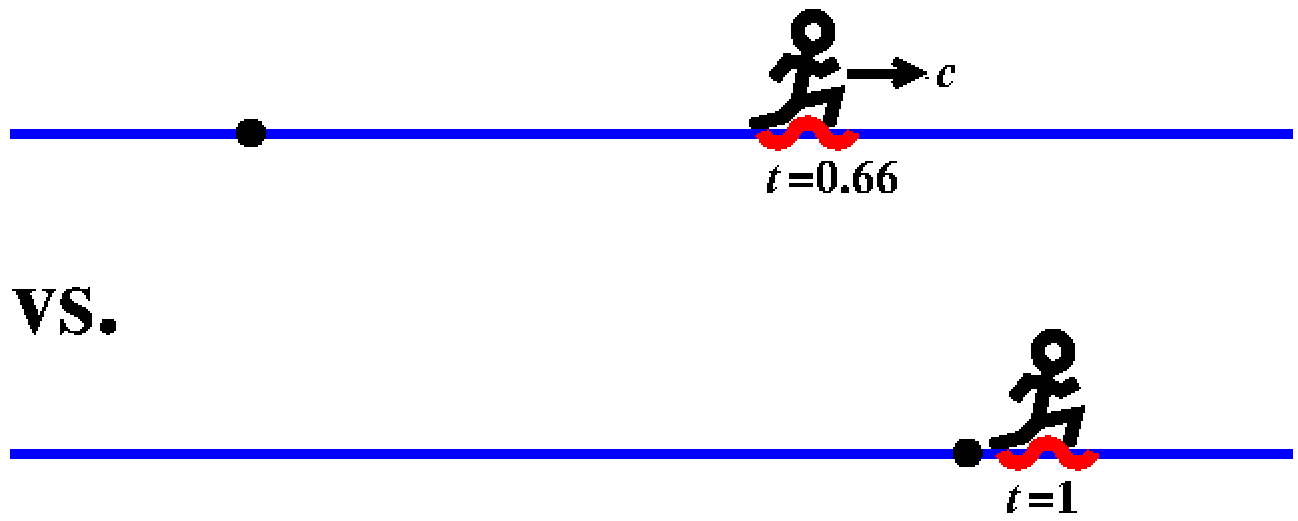


One way to understand it:

Run along with the ultra-relativisitic electon.

Compare light emitted at two different times.

Focus on collinear limit.

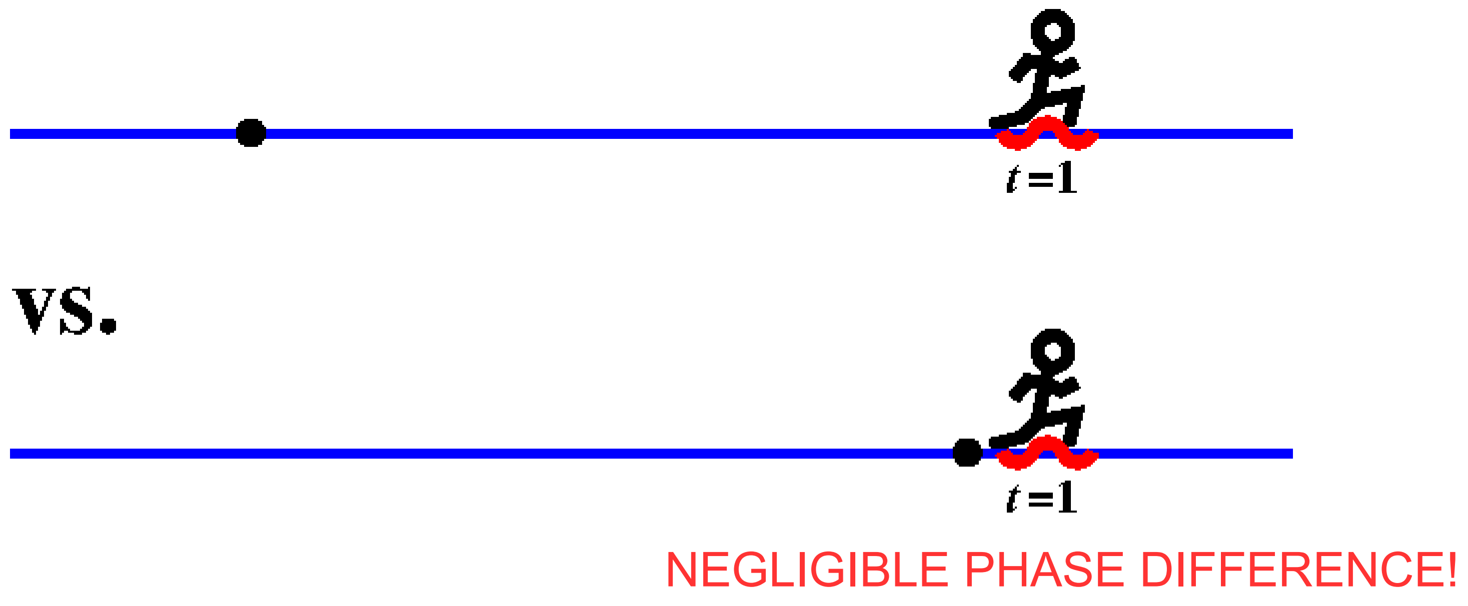


One way to understand it:

Run along with the ultra-relativisitic electon.

Compare light emitted at two different times.

Focus on collinear limit.

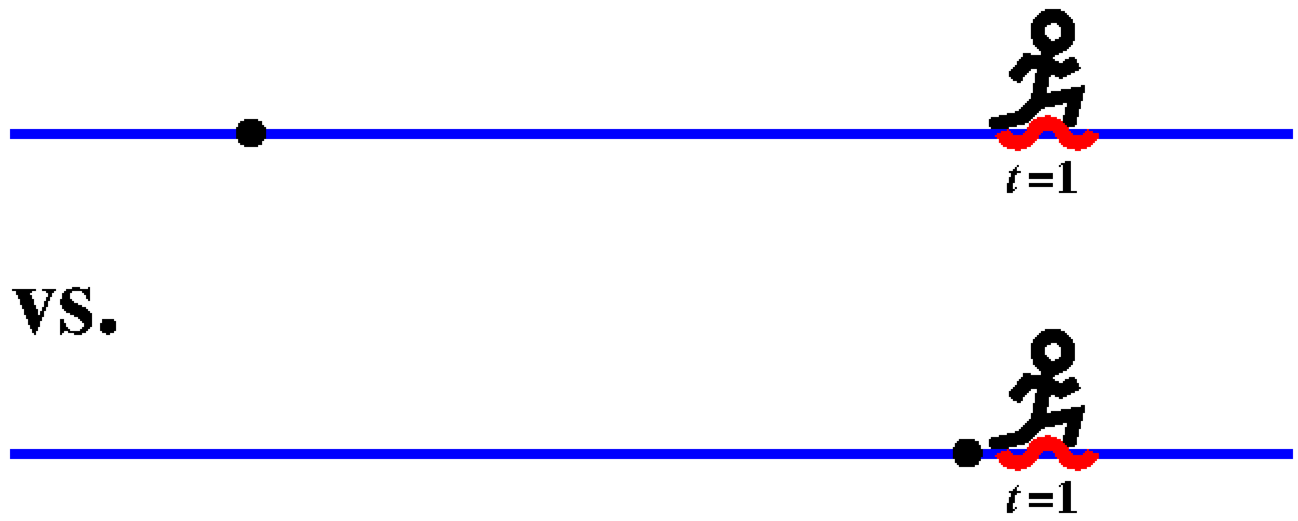


One way to understand it:

Run along with the ultra-relativistic electron.

Compare light emitted at two different times.

Focus on collinear limit.



NEGLIGIBLE PHASE DIFFERENCE!

Take-away

- LPM suppression requires particles have same velocity $(v \simeq c)$
- and process be nearly **collinear**.