Overview of recent lattice QCD results: phase diagram, fluctuations and strangeness

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May 17st 2021

Aix*Marseille université

Not in this talk

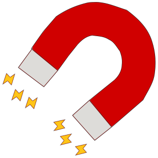
LQCD with magnetic field:

Review:

- [Guenther:2021lrv]

Recent works:

- [Bali:2020bcn]
- [Ding:2020inp]
- [Tomiya:2019nym]
- [Xu:2020yag]
- [Endrodi:2019zrl]
- [DElia:2018xwo]
- ...



LQCD with unphysical quark masses/Columbia plot:

Review:

- [Philipsen:2019rjq]
- [Guenther:2021lrv]

Recent works:

- [Philipsen:2020exx]
- [Kuramashi:2020meg]
- [Bazavov:2017xul]
- [Cuteri:2017gci]
- [Clarke:2020htu]
- ...

The QCD Lagrangian

$$\mathcal{L}_{QCD} = -\frac{1}{4} F^a_{\mu\nu} F^{a,\mu\nu} + \overline{\psi} \left(\mathrm{i}\gamma_\mu D^\mu - m \right) \psi$$

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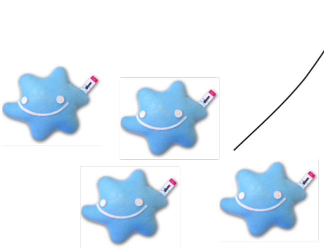

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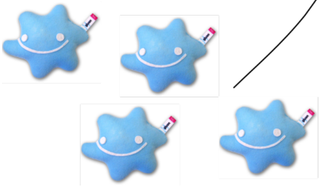

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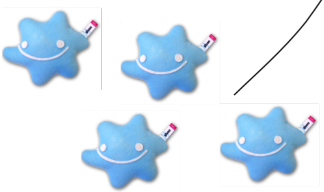
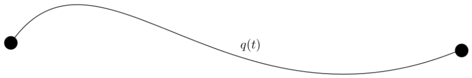


The path integral quantization: from M to QM to QFT

Mechanics:



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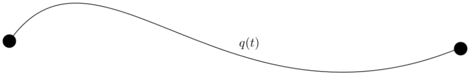
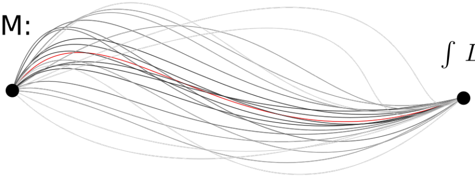


QM:



$$\int Dq(t)e^{\mathrm{i}\int \mathrm{d}t\, L}$$

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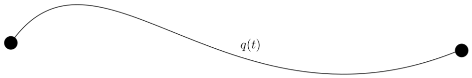

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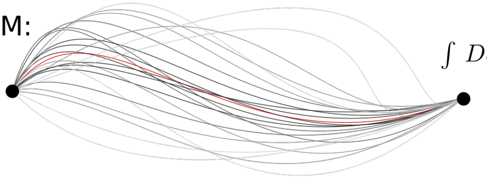

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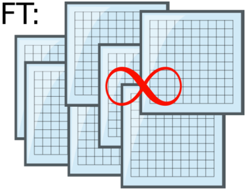
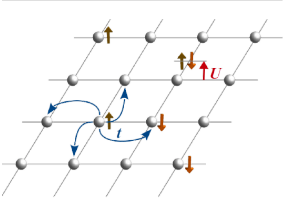
QFT:

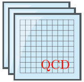


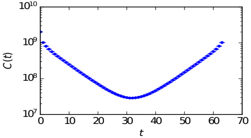
$$\int \mathcal{D}\phi(x) e^{\mathrm{i}\int \mathrm{d}^4x \mathcal{L}}$$

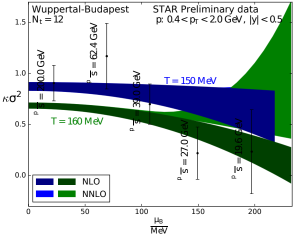


The work flow

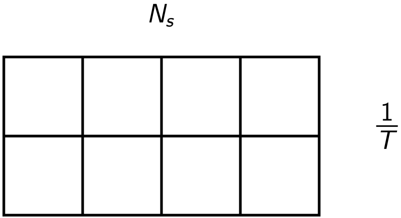
simulation parameters

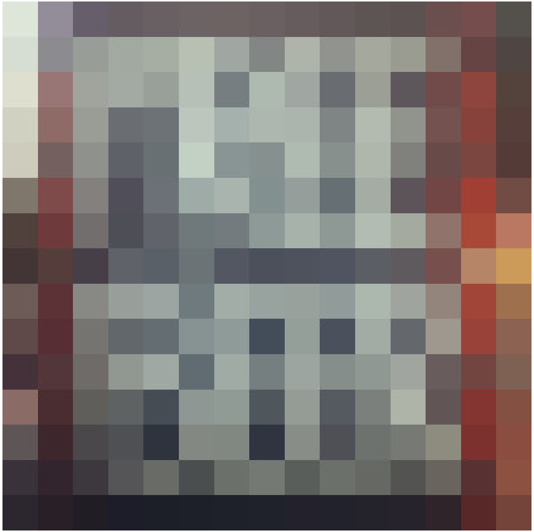






The continuum limit





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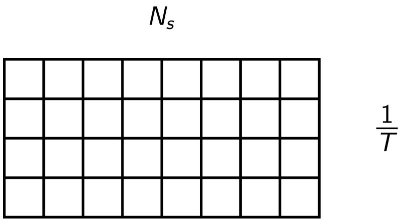

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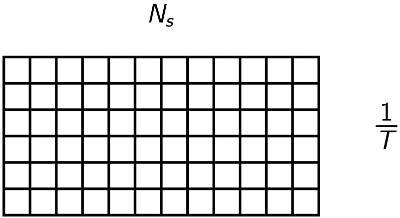

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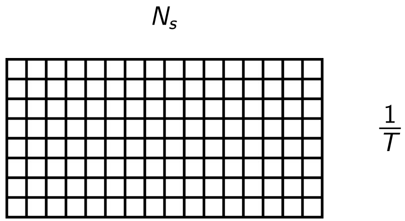
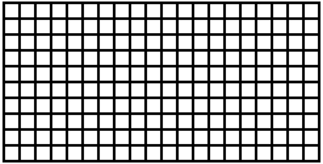


The continuum limit

N_s



$\frac{1}{T}$

The continuum limit

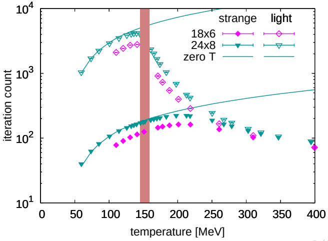
N_s

$\frac{1}{T}$

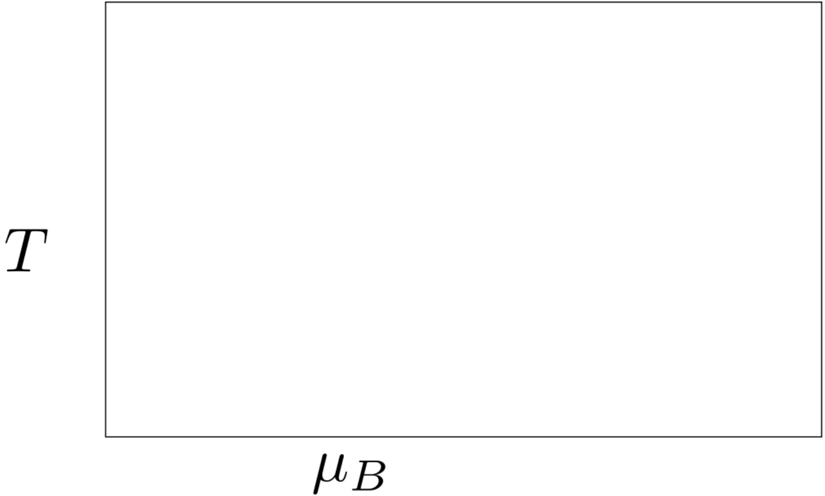
Why aren't we finished yet?

- Simulations take a lot of computer time
- Not everything can be calculated directly. For example:
    - Only thermal equilibrium
    - Only simulations at $\mu_B = 0 \Rightarrow \langle n_B \rangle = 0$ heavy ion collision experiments

1000 configurations on a $64^3 \times 16$ lattice cost about 1 million core hours



The (T, μ_B)-phase diagram of QCD



Our observables: T_c, Equation of state, Fluctuations

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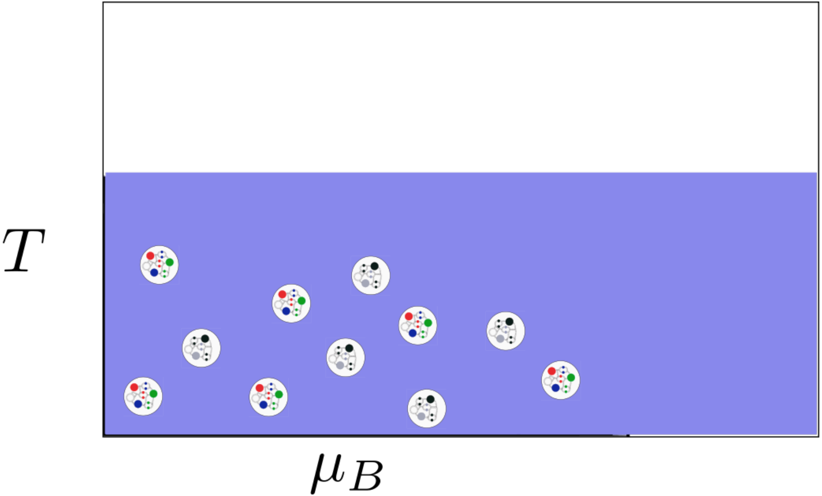

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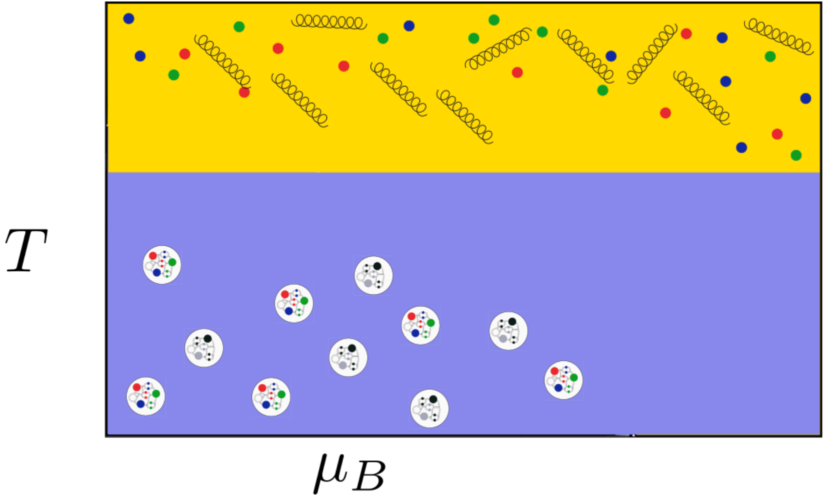

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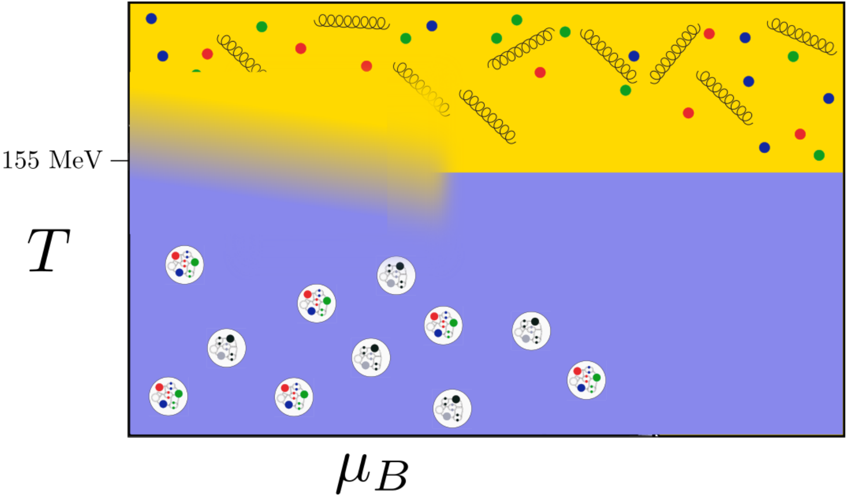

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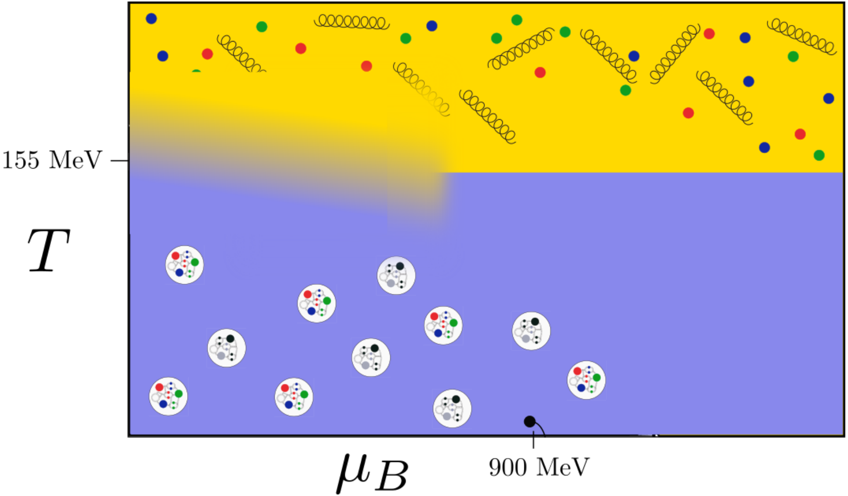

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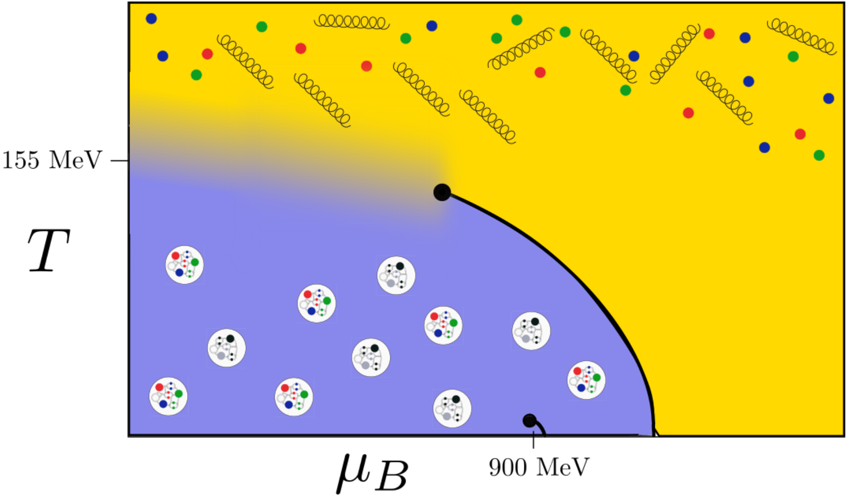

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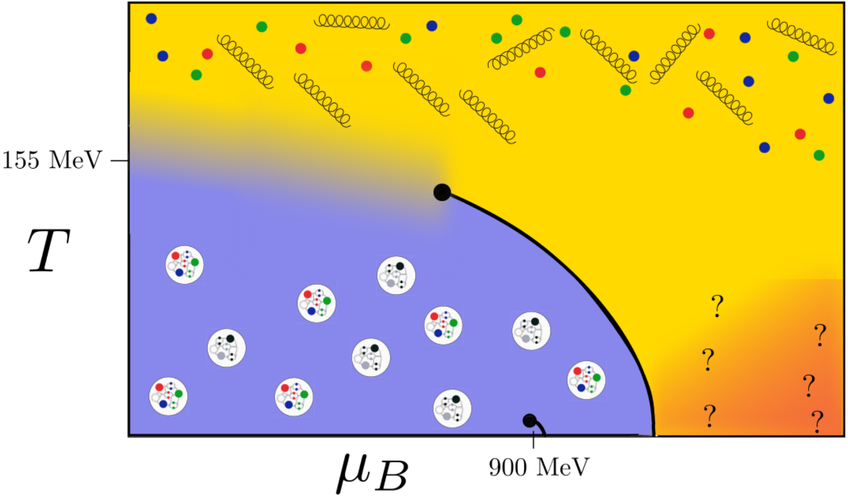

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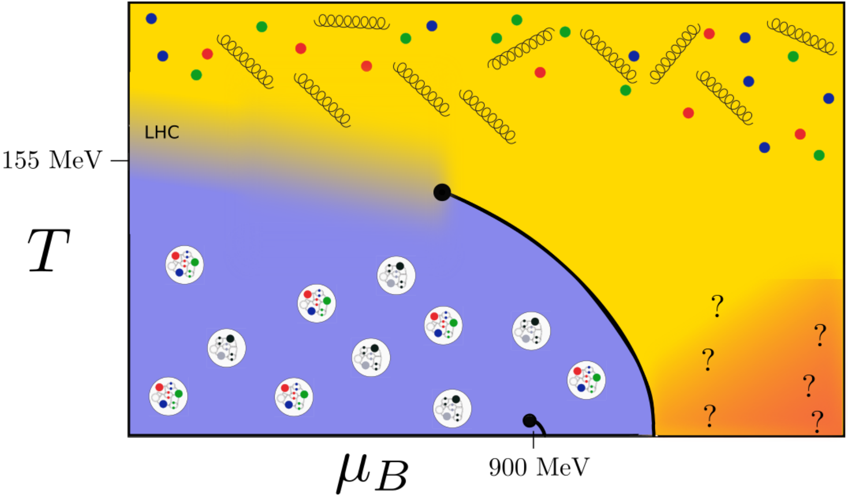

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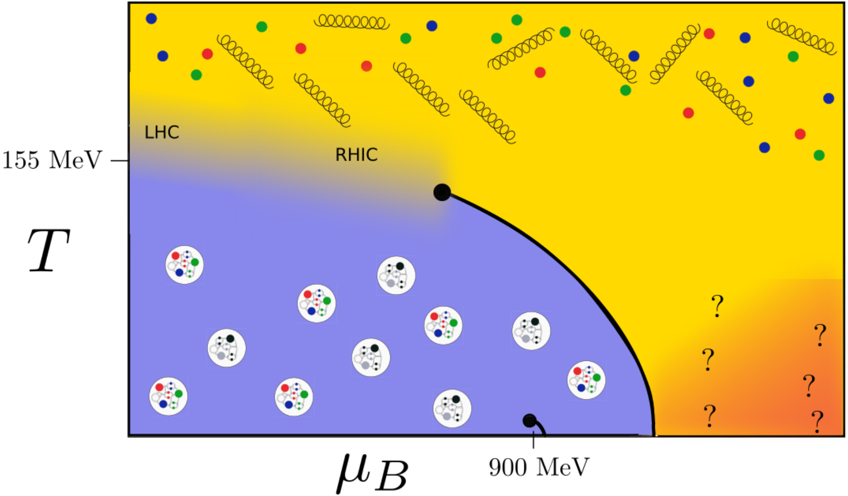

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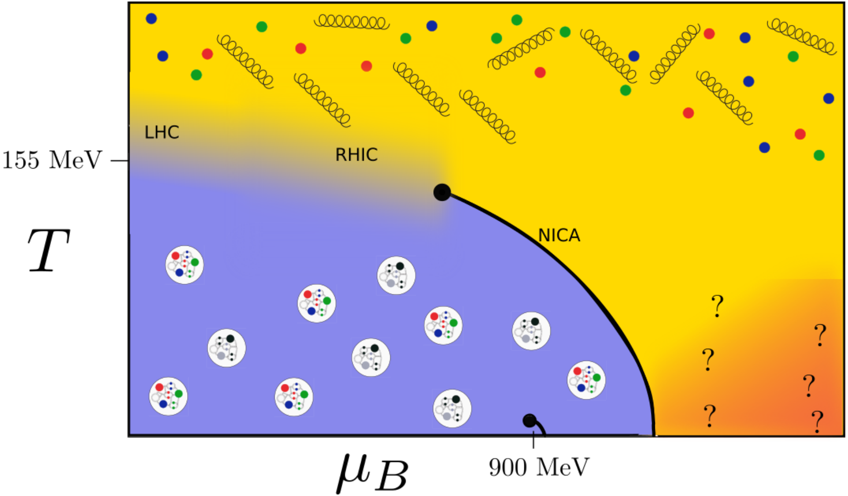

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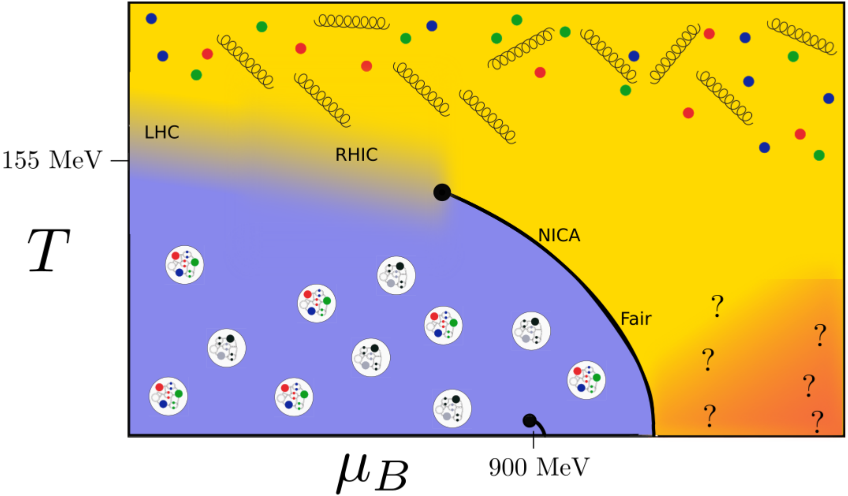

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Dealing with the sign problem

- Reweighting techniques
- Canonical ensemble
- Lefshetz Thimble
- Density of state methods
- Dual variables
- Complex Langevin
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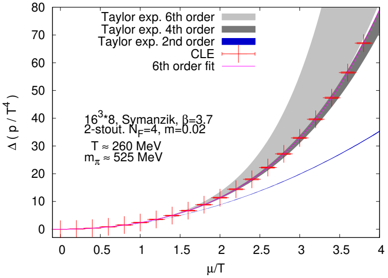

[Sexty:2019vqx]

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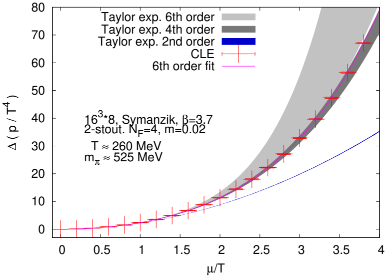
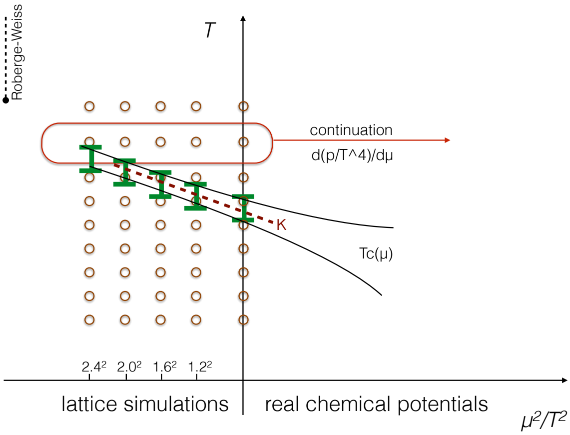


[Sexty:2019vqx]

- (Taylor) expansion ⟶ J. Goswami: Tue 11:50, D. Bollweg: Thu. 9:50
- Imaginary μ ⟶ P. Parotto: Thu. 9:30

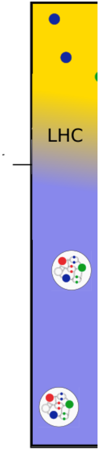
Analytic continuation from imaginary chemical potential



Common technique:

- [deForcrand:2002hgr]
- [Bonati:2015bha]
- [Cea:2015cya]
- [DElia:2016jqh]
- [Bonati:2018nut]
- . . .

Expansion from $\mu = 0$



Taylor expansion

$$\frac{p}{T^4} = \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} \frac{1}{j!k!} \chi_{jk}^{BS} \hat{\mu}_B^j \hat{\mu}_S^k$$

with $\hat{\mu} = \frac{\mu}{T}$

- rapid convergence in Stephan-Boltzmann ($T = \infty$) limit
- expansion coefficients are convenient lattice observables

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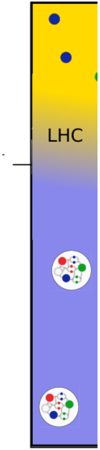

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Fugacity expansion/sector method

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- information about particle content

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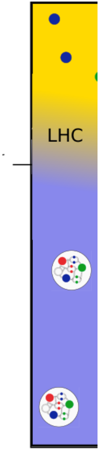

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- often the expansion is done for a specific choice of μ_S

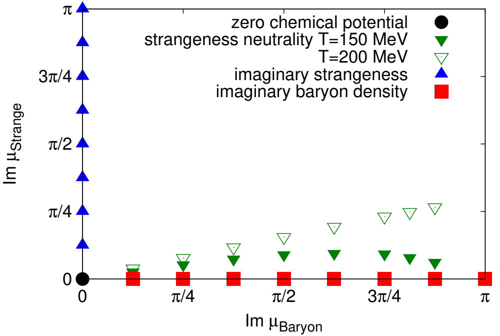
Two common choices for μ_S

$\mu_S = 0$

- less terms in expansion
- less parameters in simulations

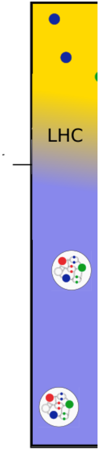
$\langle n_S \rangle = 0$

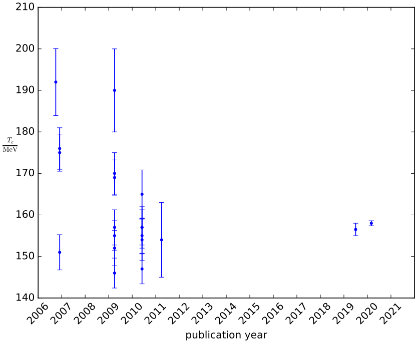
- matches heavy ion collision experiments



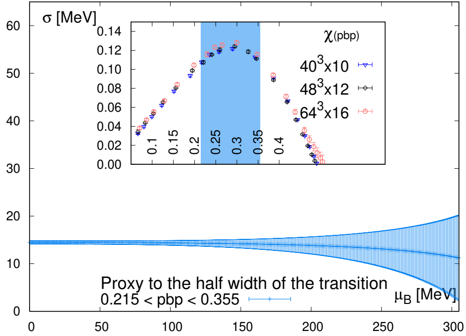
Simulation parameters of the WB collaboration

The transition temperature



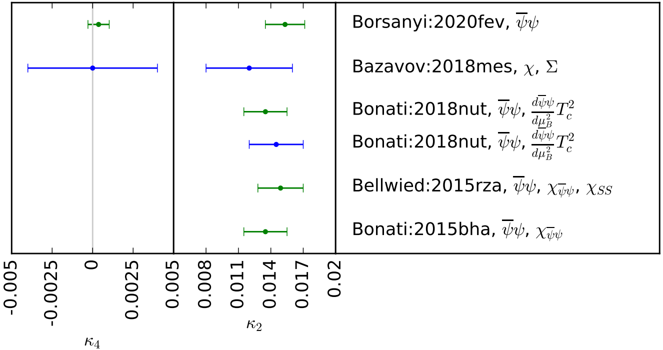


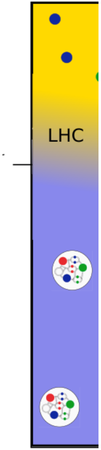
[Cheng:2006qk], [Aoki:2006br], [Aoki:2009sc], [Bazavov:2009zn], [Borsanyi:2010bp], [Bazavov:2011nk], [Bazavov:2018mes], [Borsanyi:2020fev]



[Borsanyi:2020fev]

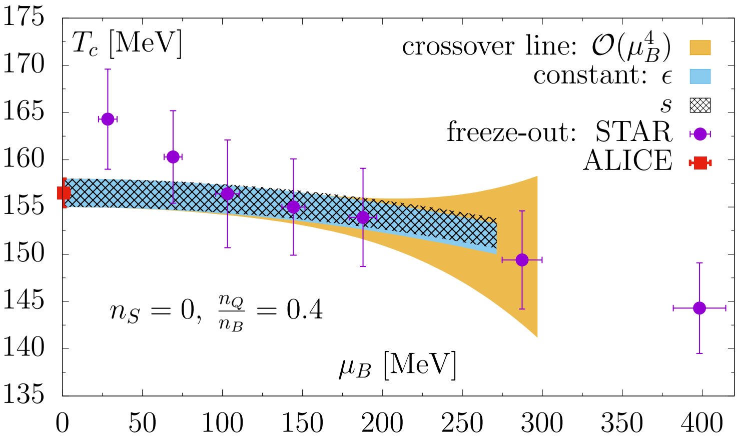
Curvature of the transition temperature



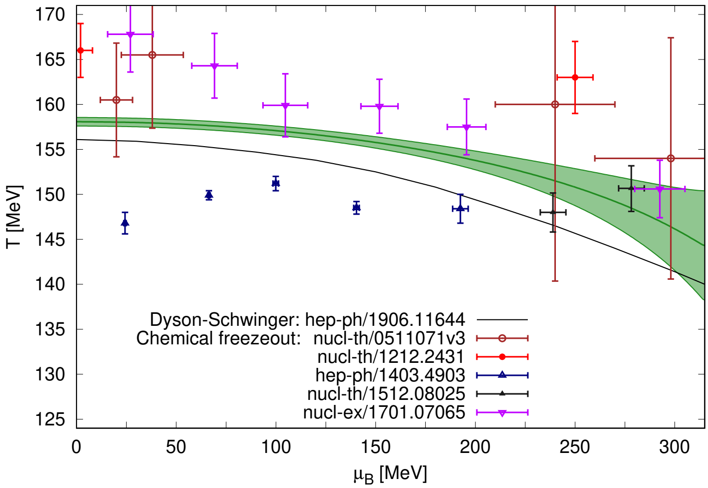


$$\frac{T_c(\mu_B)}{T_c(0)} = 1 - \kappa_2 \left(\frac{\mu_B}{T_c}\right)^2 - \kappa_4 \left(\frac{\mu_B}{T_c}\right)^4 + \mathcal{O}(\mu_B^6)$$

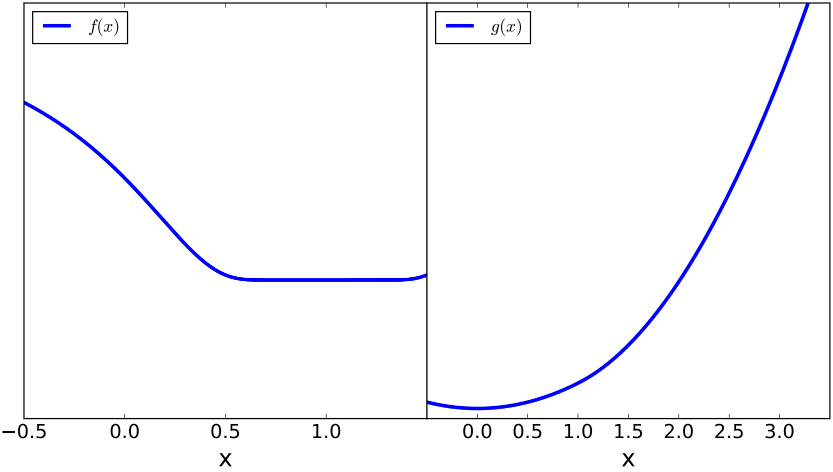
Extrapolation Bazavov:2018mes



Extrapolation Borsanyi:2020fev



Does that mean there is no critical endpoint?



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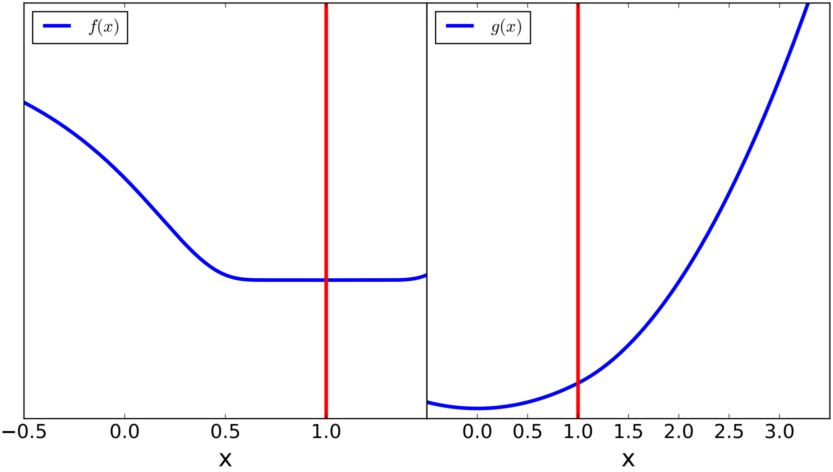

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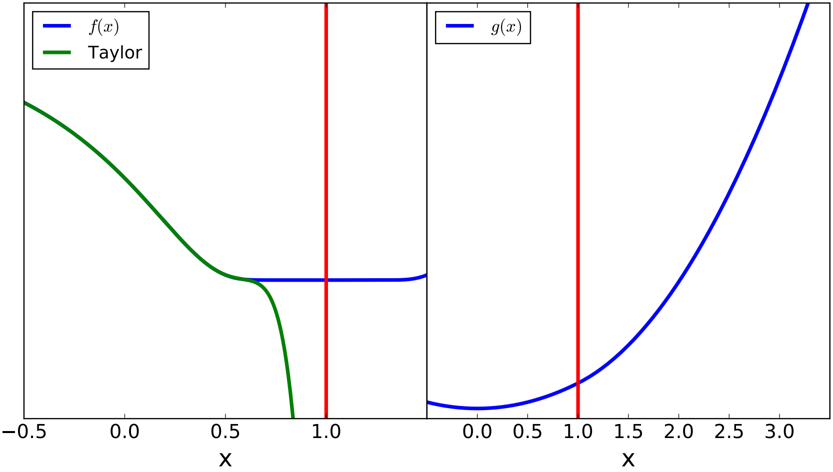

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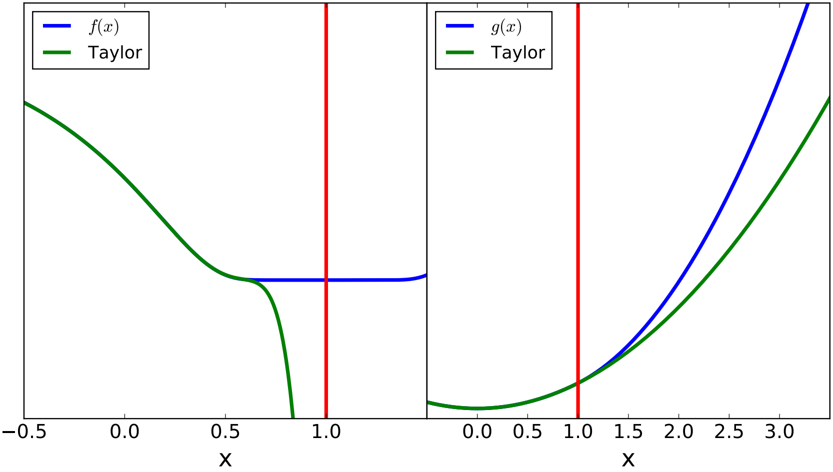
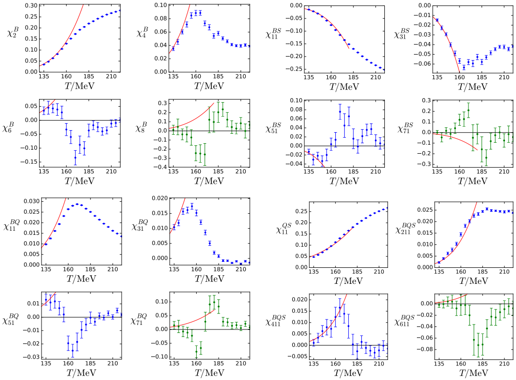


Fluctuations on the lattice

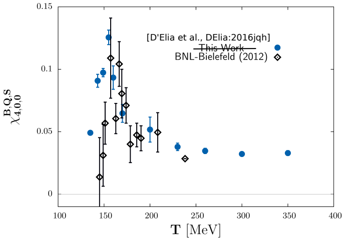
$$\chi^{B,Q,S}_{i,j,k} = \frac{\partial^{i+j+k}(p/T^4)}{(\partial\hat{\mu}_B)^i(\partial\hat{\mu}_Q)^j(\partial\hat{\mu}_S)^k}\,,\quad \hat{\mu}_i = \frac{\mu}{T}$$

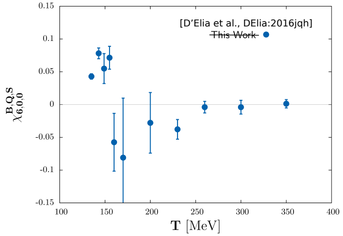
- can be calculated on the lattice
- can be compared to various models
- can be compared to experiment
- can be used as building blocks for various observables

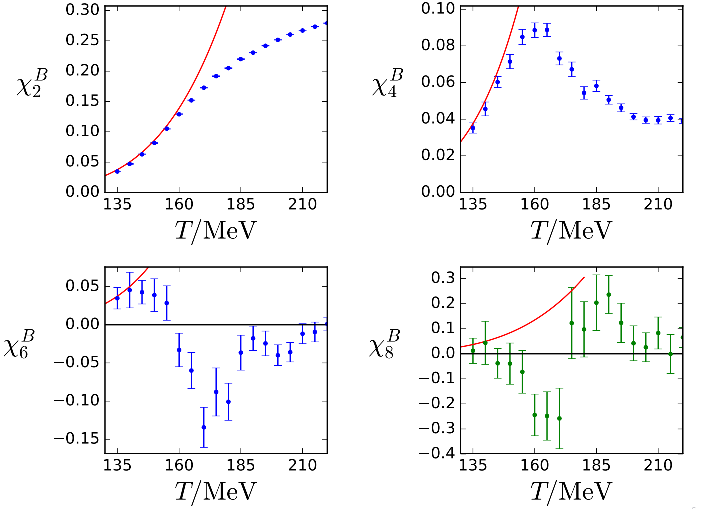


[Borsanyi:2018grb]

χ_2^B, χ_4^B, χ_6^B and χ_8^B on finite lattices

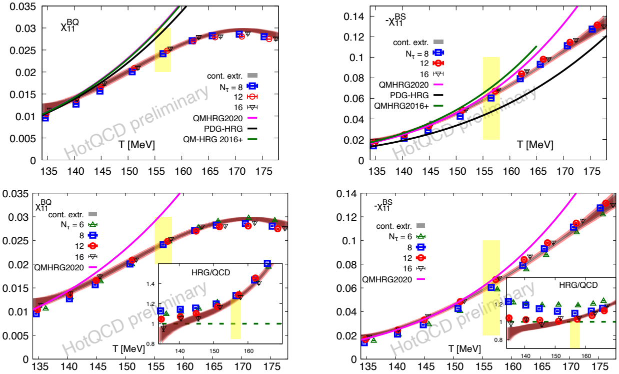






[DElia:2016jqh], [Borsanyi:2018grb], see also [Bazavov:2017dus]

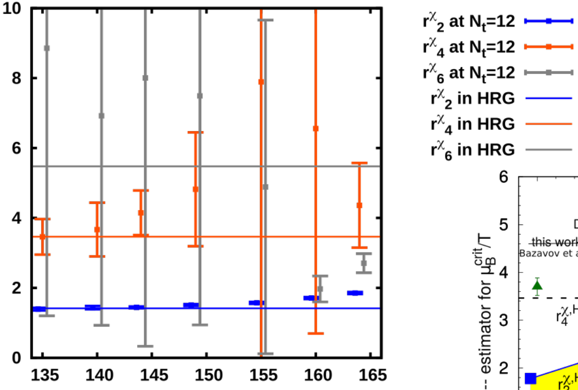
Update on 2nd order fluctuations



- continuum from $N_t = 6 - 16$
- good agreement with HRG at $T < 150$ MeV
- addition of QM states without double counting
- comparison with excluded volume model and viral expansion/S-matrix calculations

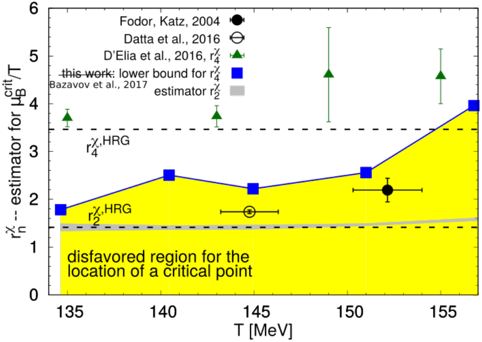
More details by J. Goswami on Tue at 11:50.

Ratios for the radius of convergence



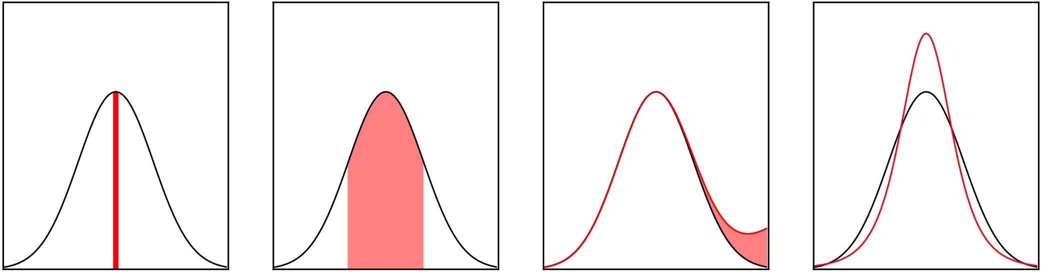
[Fodor:2018wul]

[Bazavov:2017dus]



Observables

Cumulants of the net baryon number distributions:

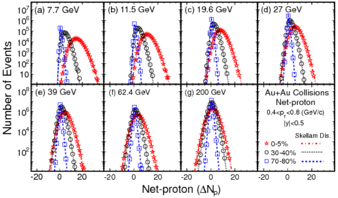


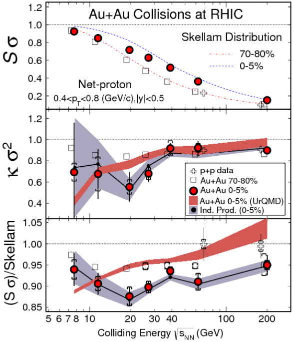
- mean M_B
- variance σ_B^2
- skewness S_B: asymmetry of the distribution
- kurtosis κ_B: "tailedness" of the distribution

Calculating observables

We have derivatives with respect to $\hat{\mu}_B$, $\hat{\mu}_Q$ and $\hat{\mu}_S$ of the pressure at $\mu_S = \mu_Q = 0$. Notation:

$$\chi^{B,Q,S}_{i,j,k} = \frac{\partial^{i+j+k}(p/T^4)}{(\partial\hat{\mu}_B)^i(\partial\hat{\mu}_Q)^j(\partial\hat{\mu}_S)^k}, \quad \hat{\mu}_i = \frac{\mu}{T}$$





We want ratios of the cumulants that are approximately independent of the volume at $\mu_B > 0$, $\langle n_S \rangle = 0$ and $\langle n_Q \rangle = 0.4 \langle n_B \rangle$:

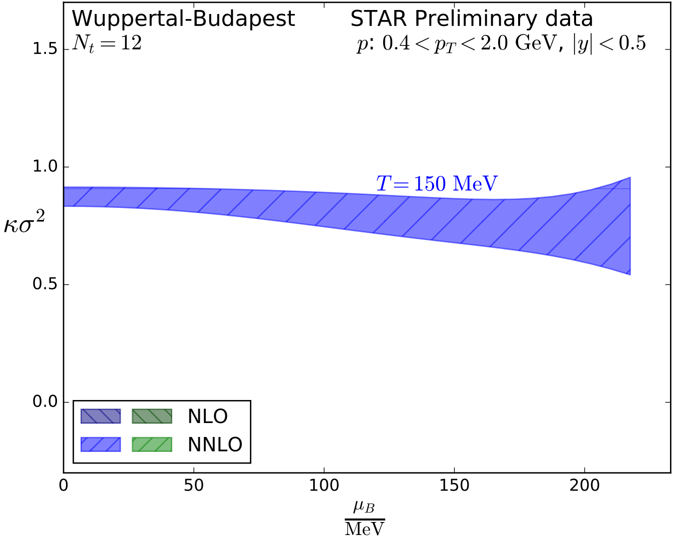
$$\frac{M_B}{\sigma_B^2} = \frac{\chi_1^B(T,\hat{\mu}_B)}{\chi_2^B(T,\hat{\mu}_B)} = \hat{\mu}_B r_{12}^{B,1} + \hat{\mu}_B^3 r_{12}^{B,3} + \ldots$$

$$\frac{S_B\sigma_B^3}{M_B} = \frac{\chi_3^B(T,\hat{\mu}_B)}{\chi_1^B(T,\hat{\mu}_B)} = r_{31}^{B,0} + \hat{\mu}_B^2 r_{31}^{B,2} + \ldots$$

$$\kappa_B\sigma_B^2 = \frac{\chi_4^B(T,\hat{\mu}_B)}{\chi_2^B(T,\hat{\mu}_B)} = r_{42}^{B,0} + \hat{\mu}_B^2 r_{42}^{B,2} + \hat{\mu}_B^4 r_{42}^{B,4} + \ldots$$

[Bazavov:2017dus], [Karsch:2017zzw], figs: [STAR, Adamczyk:2013dal]

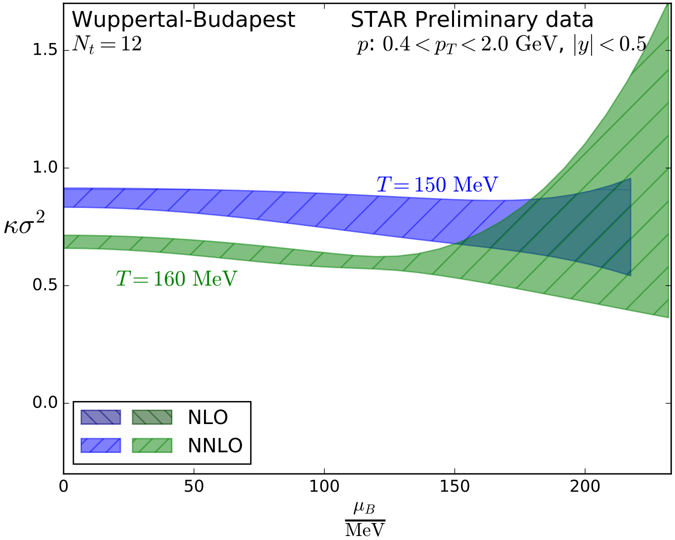
Extrapolation $\kappa\sigma^2$



[Borsanyi:2018grb]

$$\kappa_B\sigma_B^2 = \frac{\chi_4^B(T,\hat{\mu}_B)}{\chi_2^B(T,\hat{\mu}_B)}$$
$$= r_{42}^{B,0} + \hat{\mu}_B^2 r_{42}^{B,2} + \hat{\mu}_B^4 r_{42}^{B,4} + \ldots$$

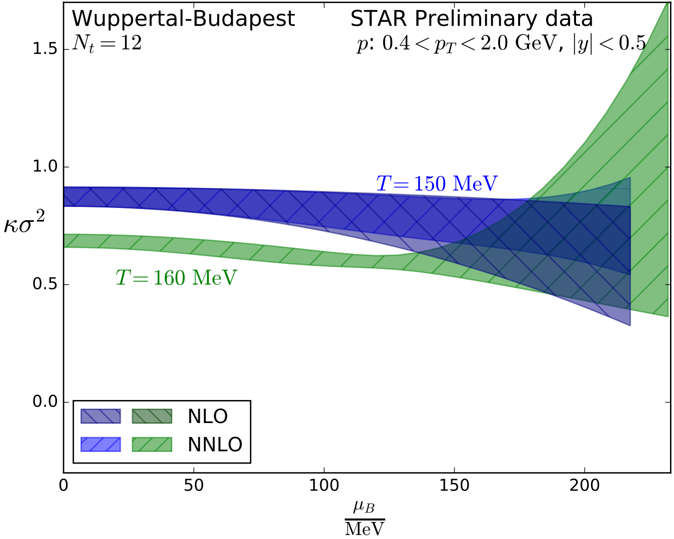
Extrapolation $\kappa\sigma^2$



[Borsanyi:2018grb]

$$\kappa_B \sigma_B^2 = \frac{\chi_4^B(T, \hat{\mu}_B)}{\chi_2^B(T, \hat{\mu}_B)} = r_{42}^{B,0} + \hat{\mu}_B^2 r_{42}^{B,2} + \hat{\mu}_B^4 r_{42}^{B,4} + \ldots$$

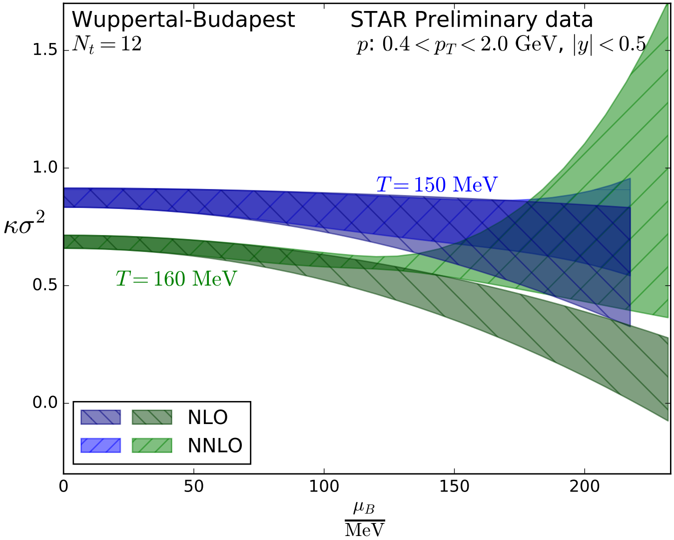
Extrapolation $\kappa\sigma^2$



[Borsanyi:2018grb]

$$\kappa_B\sigma_B^2 = \frac{\chi_4^B(T,\hat\mu_B)}{\chi_2^B(T,\hat\mu_B)}$$
$$= r_{42}^{B,0} + \hat\mu_B^2 r_{42}^{B,2} + \hat\mu_B^4 r_{42}^{B,4}$$
$$+ \ldots$$

Extrapolation $\kappa\sigma^2$

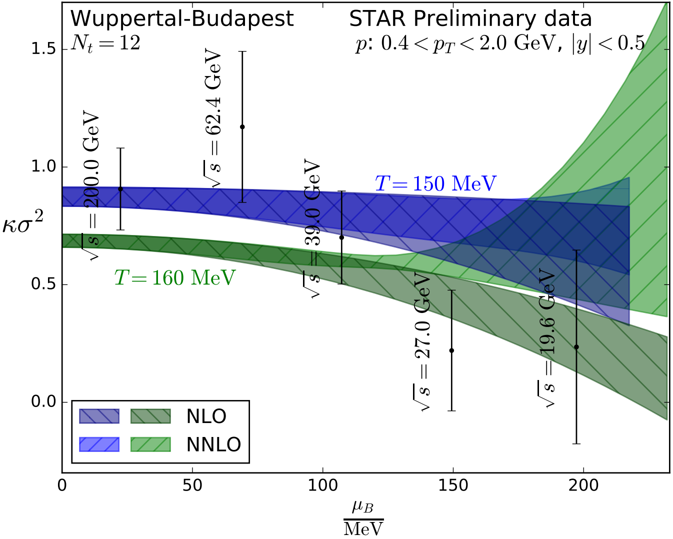


[Borsanyi:2018grb]

$$\kappa_B\sigma_B^2 = \frac{\chi_4^B(T,\hat{\mu}_B)}{\chi_2^B(T,\hat{\mu}_B)}$$

$$= r_{42}^{B,0} + \hat{\mu}_B^2 r_{42}^{B,2} + \hat{\mu}_B^4 r_{42}^{B,4} + \ldots$$

Extrapolation $\kappa\sigma^2$

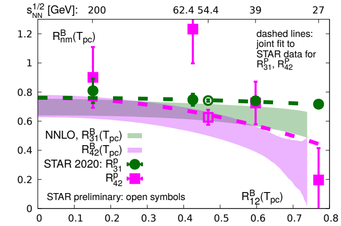


[Borsanyi:2018grb]

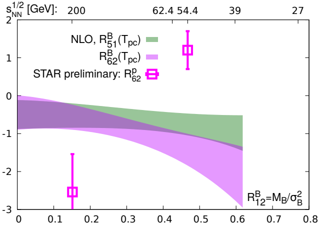
$$\kappa_B\sigma_B^2 = \frac{\chi_4^B(T,\hat{\mu}_B)}{\chi_2^B(T,\hat{\mu}_B)} = r_{42}^{B,0} + \hat{\mu}_B^2 r_{42}^{B,2} + \hat{\mu}_B^4 r_{42}^{B,4} + \ldots$$

Results from [Bazavov:2020bjn]

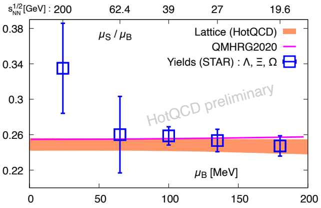
Continuum estimate from $N_t = 8, 12$

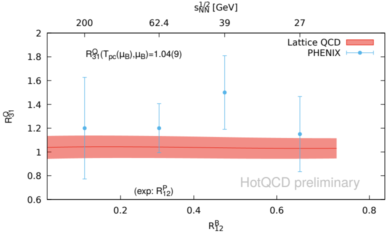


$N_t = 8$



Here the extrapolation is done along the transition line.



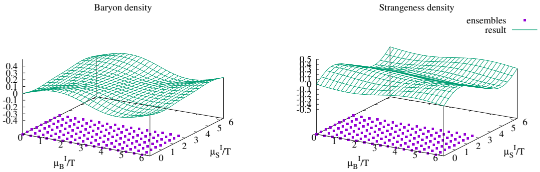


More details and update with finer lattices and smaller T by D. Bollweg on Thu. at 9:50.

2d-Extrapolation: [Bellwied:2021nrt]

144 ensembles for each temperature and lattice

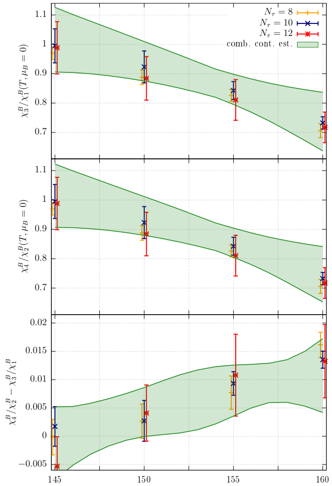
Example at $T = 155$ MeV:



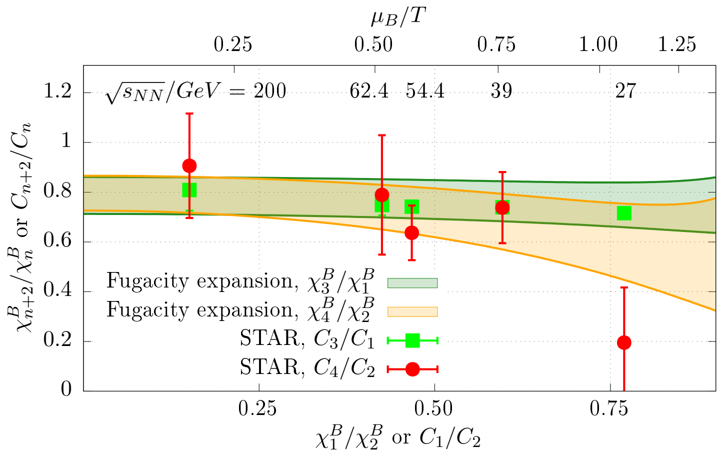
$$P(T, \hat{\mu}^I_B, \hat{\mu}^I_S) = \sum_{j,k} P^{BS}_{jk}(T) \cos(j\hat{\mu}^I_B - k\hat{\mu}^I_S) \ .$$

$$-S = -1, 0, 1, 2, 3; \quad B = 0, 1, 2, 3$$

A surface is fitted on the baryon and strangeness densities, as well as on their susceptibilities.



Comparisons

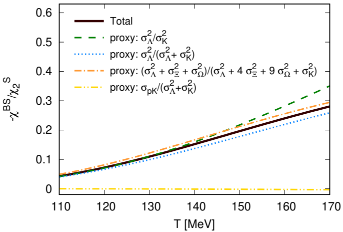


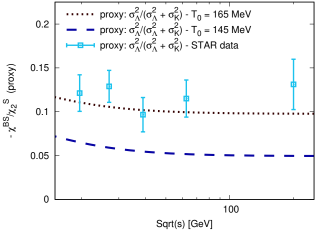
[Bellwied:2021nrt]

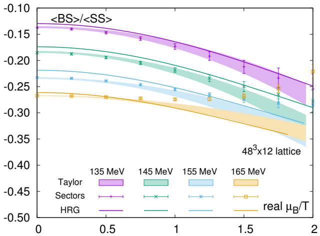
The extrapolation is done along the transition line.

Proxy for strange particle

Using the HRG to estimate how well an observable can be measured in experiment [Bellwied:2019pxh]:







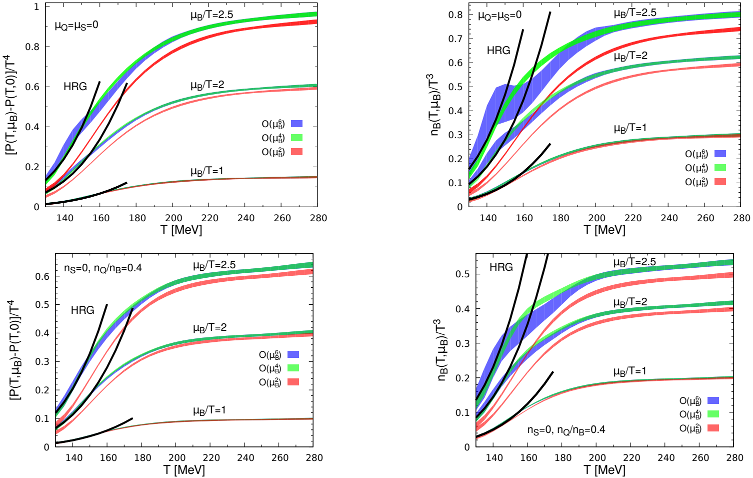
Equation of state [Bazavov:2017dus]

$\mu_S = 0$

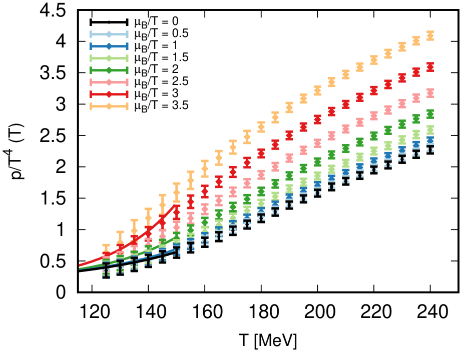
Continuum limit for 2nd order ($N_t = 6, 8, 12, (16)$)

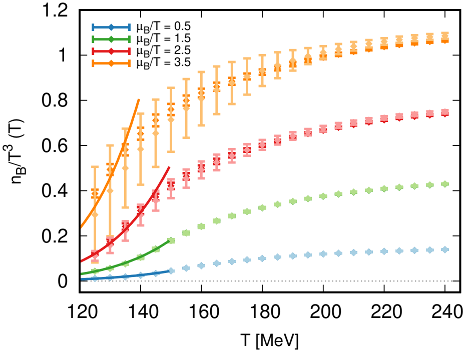
Continuum estimate for 4th and 6th order ($N_t = 6, 8$)

$\langle n_S \rangle = 0$



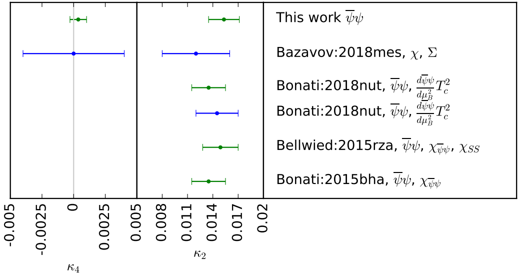
Equation of State [Borsanyi:2021sxv]

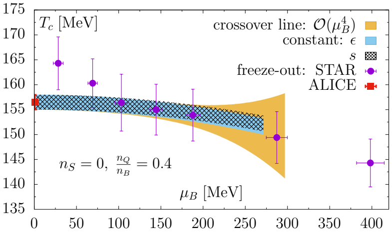


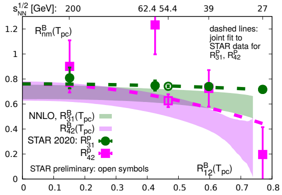


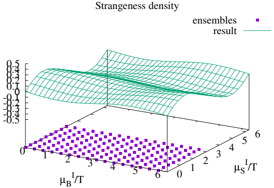
More details in the talk of P. Parotto at Thu. 9:30.

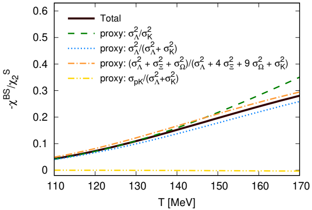
Summary


This work $\overline{\psi}\psi$
Bazavov:2018mes, χ, Σ
Bonati:2018nut, $\overline{\psi}\psi$, $\frac{d\overline{\psi}\psi}{d\mu_B^2}T_c^2$
Bonati:2018nut, $\overline{\psi}\psi$, $\frac{d\overline{\psi}\psi}{d\mu_B^2}T_c^2$
Bellwied:2015rza, $\overline{\psi}\psi$, $\chi_{\overline{\psi}\psi}$, χ_{SS}
Bonati:2015bha, $\overline{\psi}\psi$, $\chi_{\overline{\psi}\psi}$
κ_4
κ_2

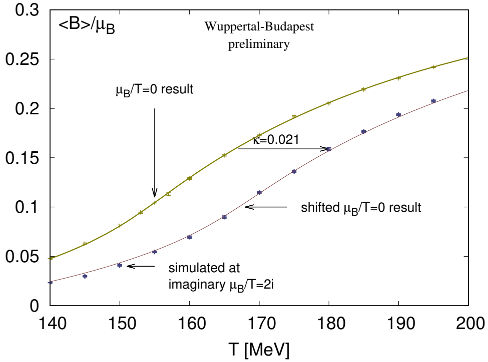

T_c [MeV]
crossover line: $\mathcal{O}(\mu_B^4)$
constant: ϵ
s
freeze-out: STAR
ALICE
$n_S = 0,\ \frac{n_Q}{n_B} = 0.4$
μ_B [MeV]


$s_{NN}^{1/2}$ [GeV]: 200 62.4 54.4 39 27
$R^B_{nm}(T_{pc})$
dashed lines: joint fit to STAR data for R^p_{31}, R^p_{42}
NNLO, $R^B_{31}(T_{pc})$
$R^B_{42}(T_{pc})$
STAR 2020: R^p_{31}
R^p_{42}
STAR preliminary: open symbols
$R^B_{12}(T_{pc})$


Strangeness density
ensembles
result
μ_B^I/T
μ_S^I/T


Total
proxy: $\sigma^2_\Lambda/\sigma^2_K$
proxy: $\sigma^2_\Lambda/(\sigma^2_\Lambda+\sigma^2_K)$
proxy: $(\sigma^2_\Lambda+\sigma^2_\Xi+\sigma^2_\Omega)/(\sigma^2_\Lambda+4\sigma^2_\Xi+9\sigma^2_\Omega+\sigma^2_K)$
proxy: $\sigma_{pK}/(\sigma^2_\Lambda+\sigma^2_K)$
$-\chi^{BS}_{11}/\chi^S_2$
T [MeV]

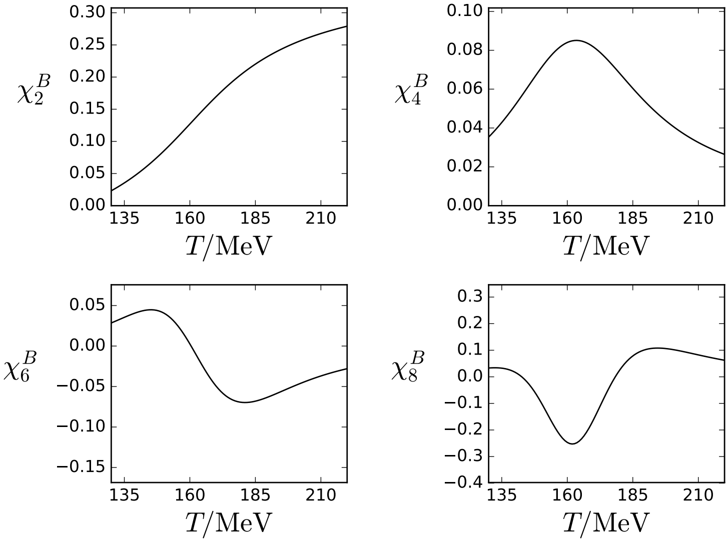
Toy model without critical endpoint



NOTE: The model assumes no criticality

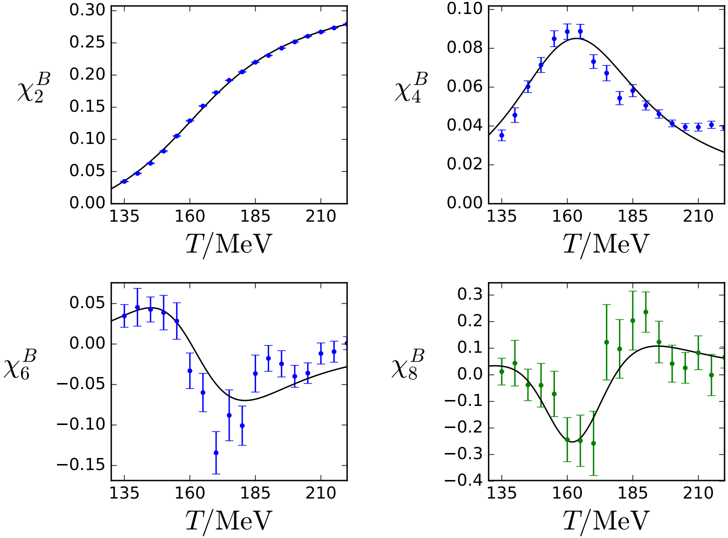
- Start with some parametrization of the curve χ_1^B/μ_B at $\mu = 0$
- Assume that the only difference in the physics at finite μ is a shift in this curve
- The inflection point of this curve is one possible definition of T_c, so shift the curve by using the κ values found in the literature
- You now have a model prediction of χ_1^B for any finite μ, differentiate it a few times at $\mu = 0$ to get estimates of χ_4^B, χ_6^B and χ_8^B

Fluctuations in the toy model



CURVE: The simple model described in the previous slide without criticality

Fluctuations in the toy model



CURVE: The simple model described in the previous slide without criticality

The sign problem

The QCD partition function:

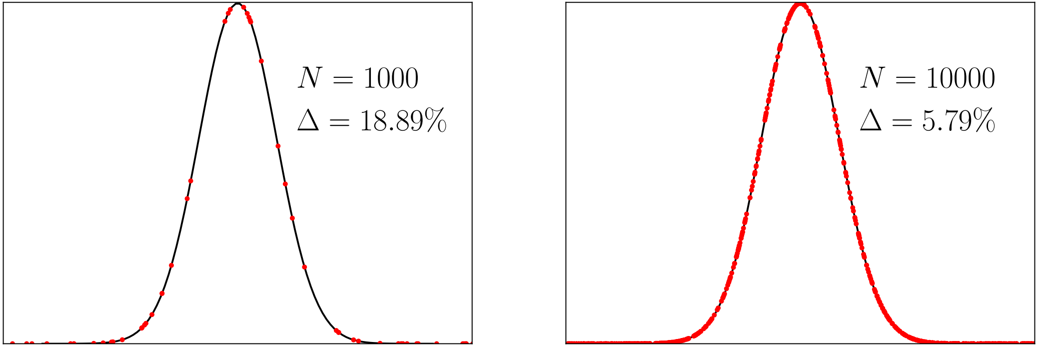
$$Z(V,T,\mu) = \int \mathcal{D}U\mathcal{D}\psi\mathcal{D}\bar{\psi}\; e^{-S_F(U,\psi,\bar{\psi})-\beta S_G(U)}$$
$$= \int \mathcal{D}U \;\det M(U) e^{-\beta S_G(U)}$$

- For Monte Carlo simulations $\det M(U)e^{-\beta S_G(U)}$ is interpreted as Boltzmann weight
- If there is particle-antiparticle-symmetry $\det M(U)$ is real
- If $\mu^2 > 0$ $\det M(U)$ is complex

The sign problem

$$\int_{-\infty}^{\infty} \mathrm{d}x\,(100 - x^2)\frac{e^{-\frac{1}{2}x^2}}{\sqrt{2\pi}} \approx \int_{-100}^{100} \mathrm{d}x\,(100 - x^2)\frac{e^{-\frac{1}{2}x^2}}{\sqrt{2\pi}} = \sum_{i=1}^{N}(100 - x_i^2)\frac{e^{-\frac{1}{2}x_i^2}}{\sqrt{2\pi}} \cdot \frac{200}{N}$$

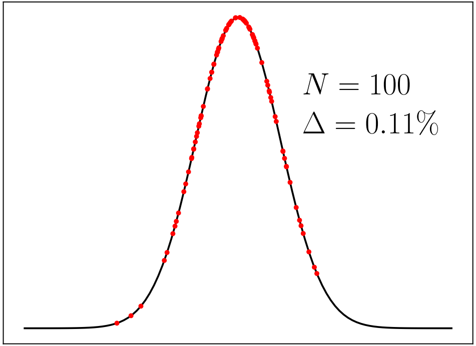
The x_i are drawn from a uniform distribution in the interval $[-100, 100]$

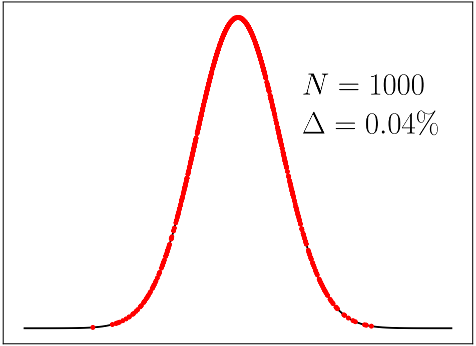


Importance sampling

$$\int_{-\infty}^{\infty} \mathrm{d}x \; (100 - x^2) \frac{e^{-\frac{1}{2}x^2}}{\sqrt{2\pi}} = \sum_{i=1}^{N} (100 - x_i^2) \cdot \frac{1}{N}$$

The x_i are drawn from a normal distribution





The sign problem

$$\int_{-\infty}^{\infty} \mathrm{d}x(100 - x^2)\frac{e^{-\frac{\mathrm{i}}{2}x^2}}{\sqrt{2\pi}}$$