QCD/Quark equation of state for neutron stars

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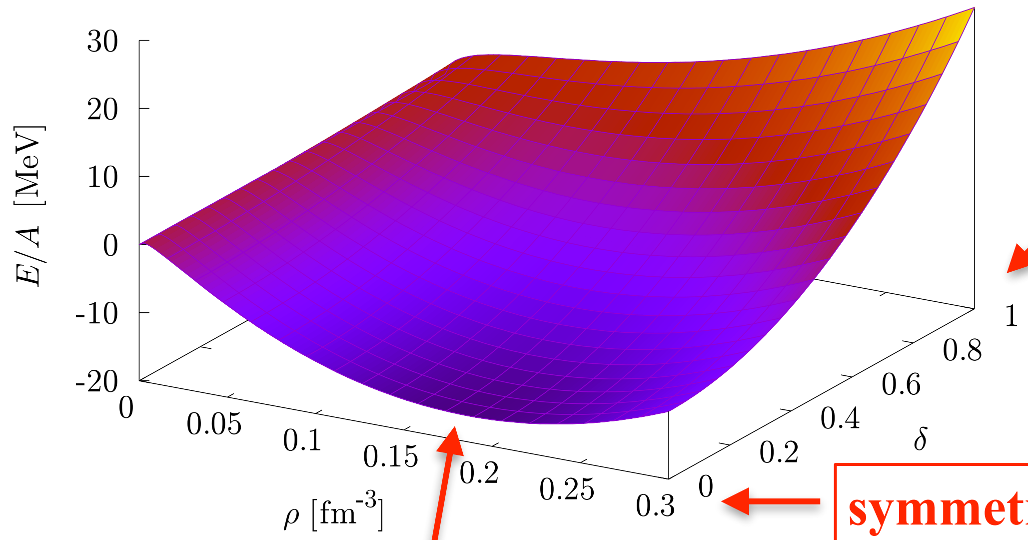
Difference (I)

Neutron Star

Dominated by *n*
(large isospin)

Heavy-Ion Collision

Isospin fixed by *p*/*n* ratio
(small isospin)



neutron matter

symmetric nuclear matter (HIC)

saturation density
(1st-order transition)

$$\delta = \frac{\rho_n - \rho_p}{\rho_n + \rho_p}$$

$$\rho = \rho_n + \rho_p$$

Phase structures may be very different.

Difference (II)

Neutron Star

β equilibrium

$$\mu_s = 0$$

High baryon density should involve hyperons (Λ, Σ, etc)

EoS too soft? / Cooling too fast?

Hyperon Puzzle

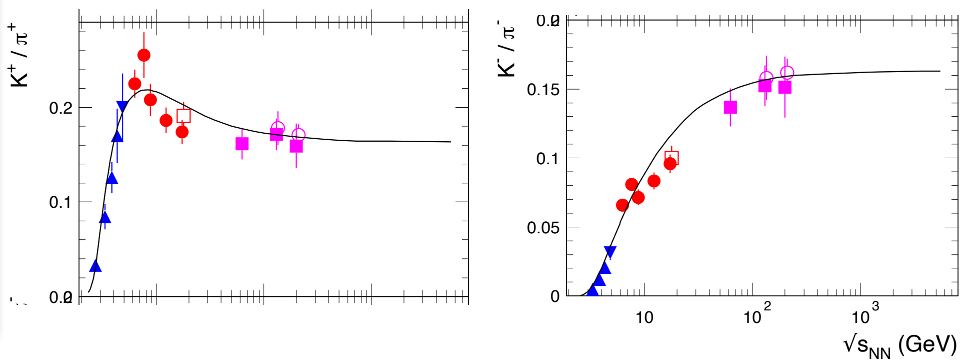
* Interactions may suppress hyperons (3-body forces YNN)
* Interactions may make EoS stiff (repulsive forces at high density)

Heavy-Ion Collision

Zero net strangeness

$$n_s = 0 \qquad (\mu_s \sim \frac{1}{3}\mu_B)$$

Hyperon strangeness is canceled by strange mesons ($\bar{s}$ in mesons sensitive to μ_B)

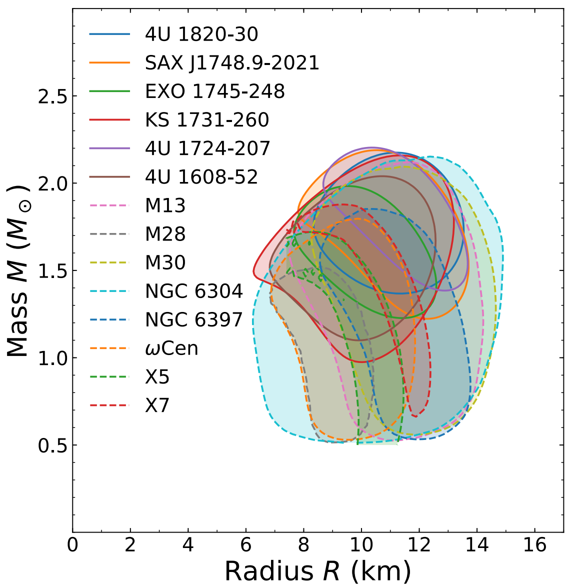


Difference (III)

Neutron Star

Probabilistic data with limited statistics

TOV equations are parameter free (with small corrections)



QCD-based predictions are limited.

Heavy-Ion Collision

Fluctuating and convoluted data with high statistics

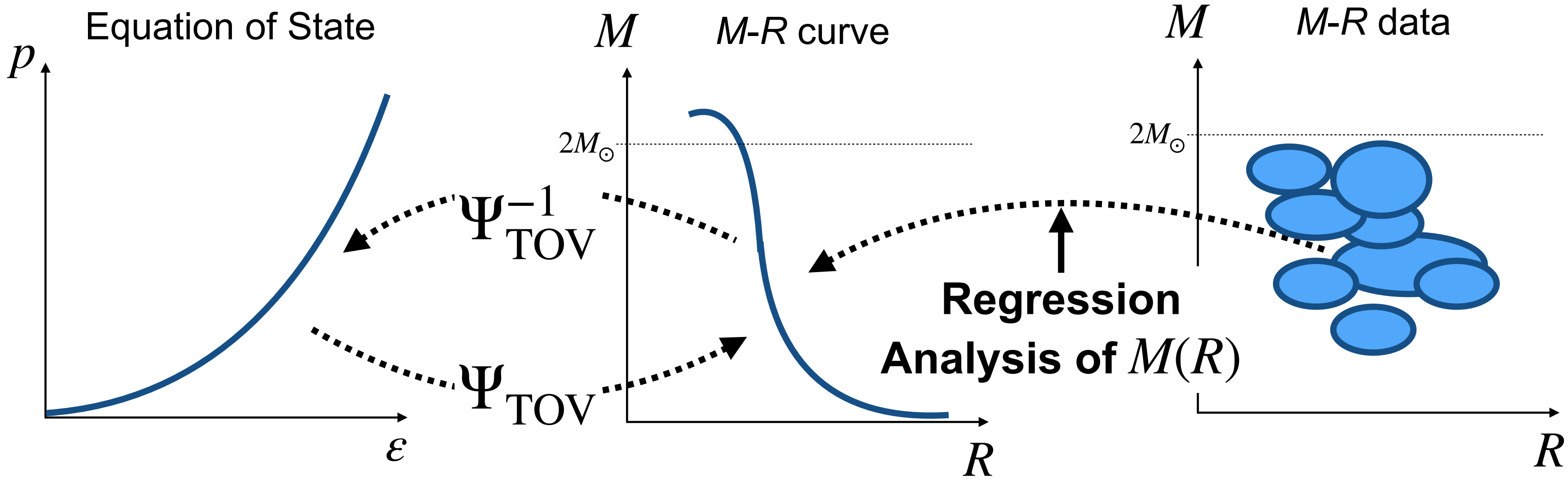
Hydro model involves many (T- and μ-dependent) parameters.

First-principles EoS available from lattice as long as $\mu/T < 1$.

EoS Reconstruction

Conventional Model Approach

Model → Solving TOV → *M-R* Curve → Observation



Initial condition: $p(r \simeq 0) = p(\rho_{\max}), \quad \varepsilon(r \simeq 0) = \varepsilon(\rho_{\max})$

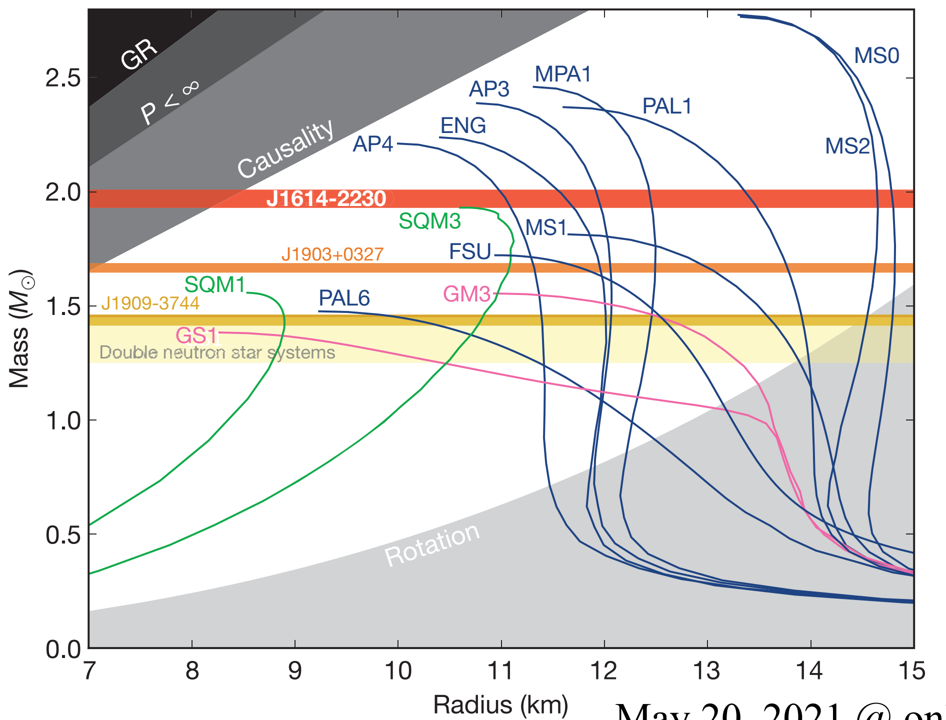
***M-R*:** $p(r = R) = 0, \quad M = \int^R d^3x\, \varepsilon(r)$

EoS Reconstruction

Conventional Model Approach

Model → Solving TOV → *M-R* Curve → Observation

[Very Famous Example]



Demorest et al. (2010-2016)
1.928(17) $M_{\rm sun}$ (J1614-2230)

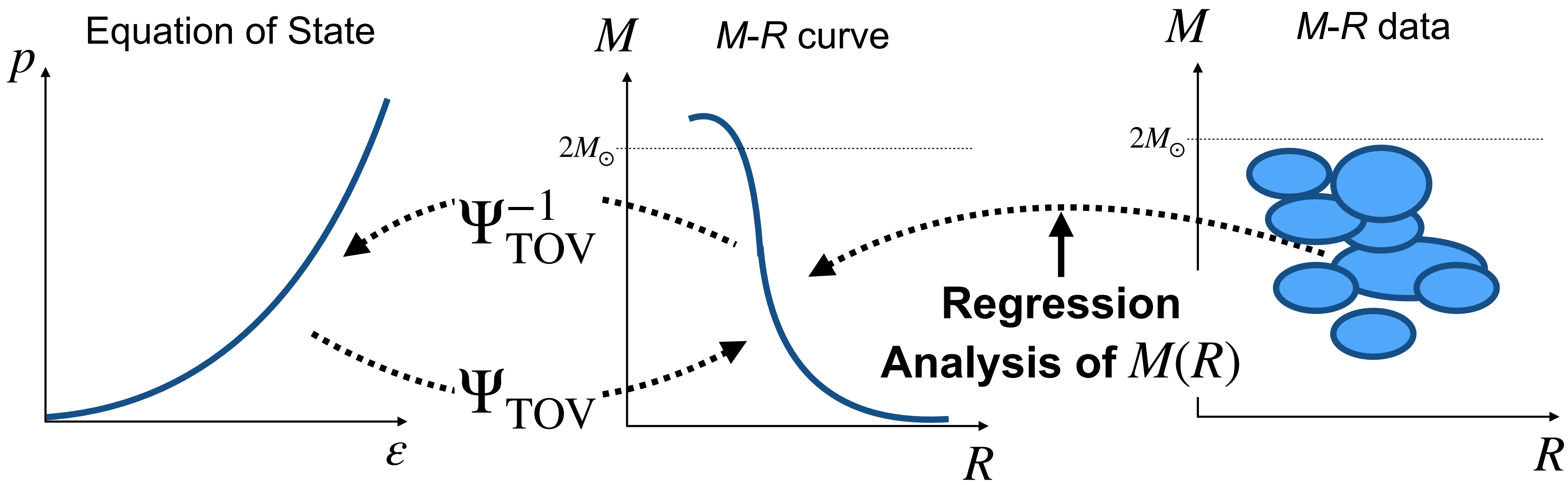
Some models excluded from observations

Even more massive NSs have been discovered later.

EoS Reconstruction

Model Independent Approach

EoS ← Solving TOV ← *M-R* Curve ← Observation



Once one *M-R* curve is identified, one EoS is concluded.

The best we can do is to find the "likely" *M-R* curve.

EoS Reconstruction

Model Independent Approach

EoS ← Solving TOV ← *M-R* Curve ← Observation

$\boxed{\text{Bayesian Analysis}}$ **Ozel et al., Steiner et al. (2015~)**

A : EoS Parameters B : M-R Observation

(Bayes' theorem)

Normalization

$$\underbrace{P(A|B)}_{\text{Want to know}}\cancel{P(B)} = \underbrace{P(B|A)}_{\text{Likelihood}}\underbrace{P(A)}_{\text{prior}}$$

Model

EoS Reconstruction

Model Independent Approach

EoS ← Solving TOV ← *M-R* Curve ← Observation

Machine Learning Inference **Fujimoto-Fukushima-Murase (2018,19,20)**

Several *M-R* observation points with errors

Nonlinear Mapping

Several parameters to characterize EoS

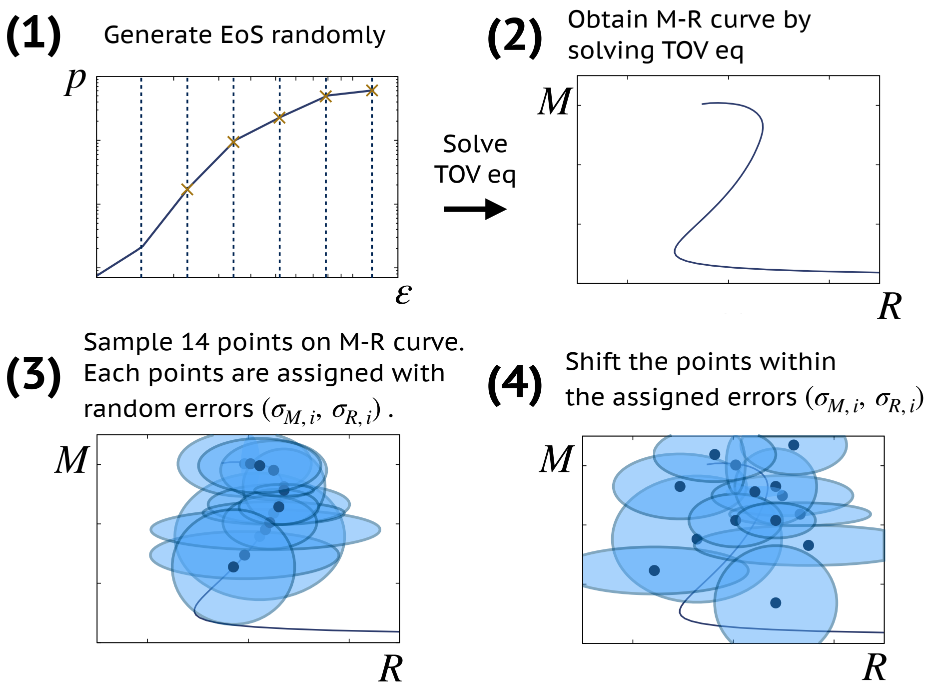
$$\{M_i, R_i\} \qquad \{P_i\} = F(\{M_i, R_i\}) \qquad \{P_i\}$$

EoS Reconstruction

Model Independent Approach

EoS ← Solving TOV ← *M-R* Curve ← Observation

Machine Learning Inference **Fujimoto-Fukushima-Murase (2018,19,21)**



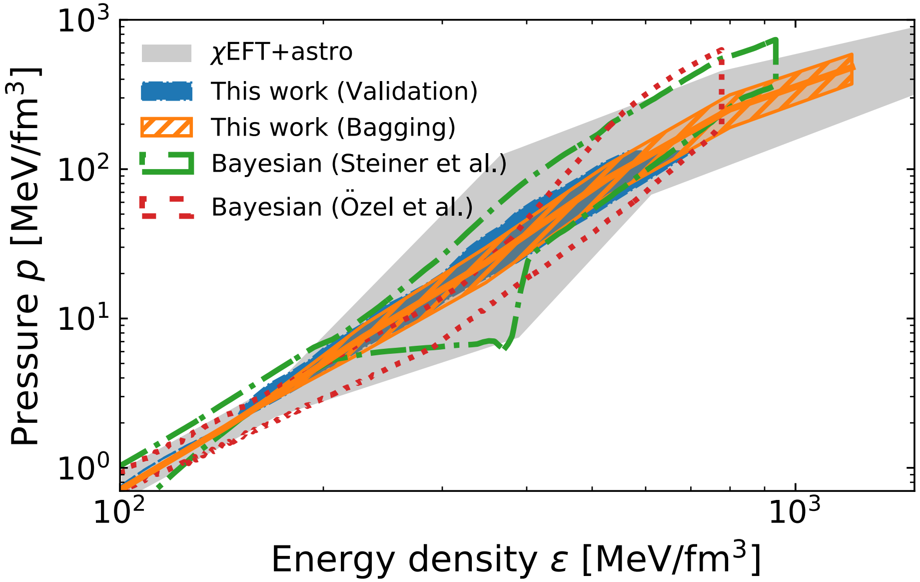
EoS parametrized by speed of sound c_s^2

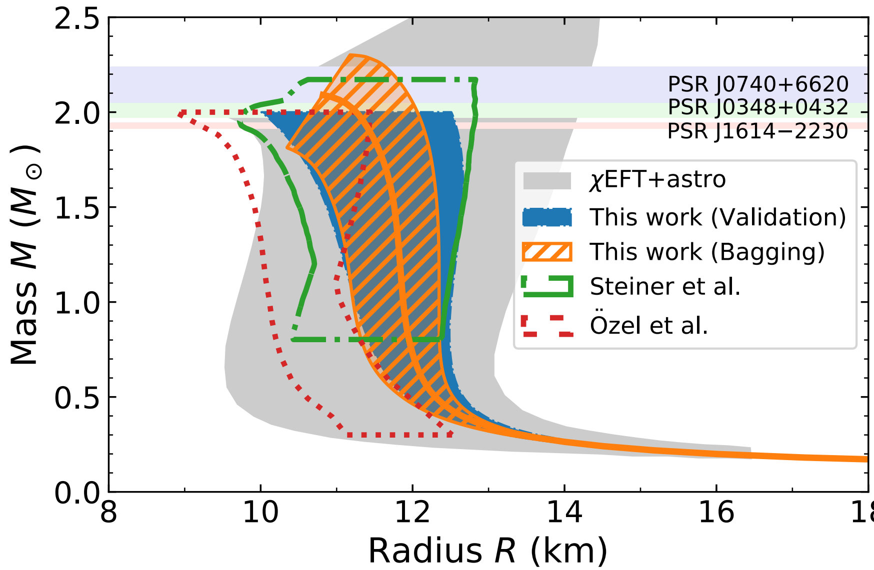
Convoluted with error bands (**Data Augmentation**)

EoS Reconstruction

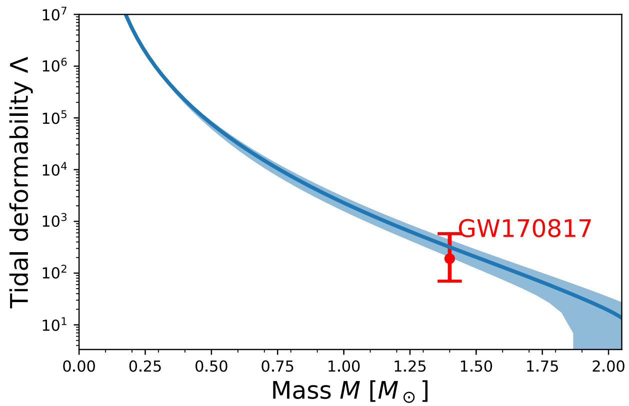
Most Likely EoS from ML

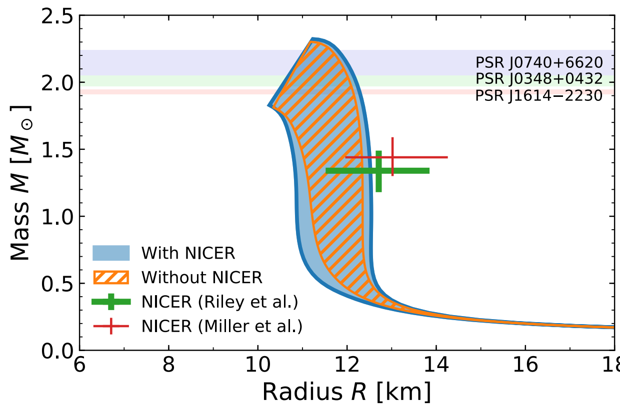
Fujimoto-Fukushima-Murase (2018,19,21)





Consistency Check



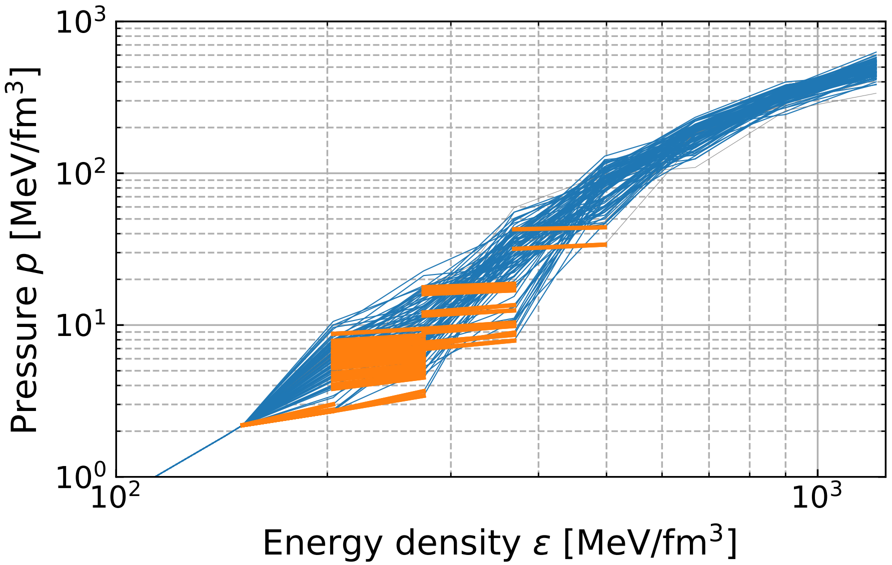


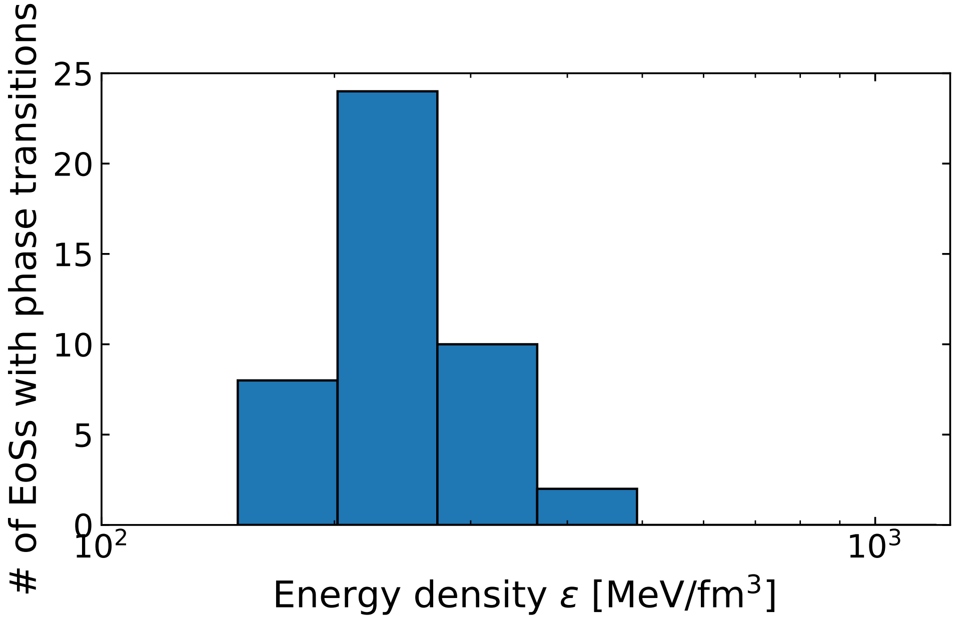
EoS Reconstruction

Most Likely EoS from ML **Fujimoto-Fukushima-Murase (2018,19,21)**

Independently learned NNs lead to acceptable EoSs

Some among 100NNs contain a 1st-order transition



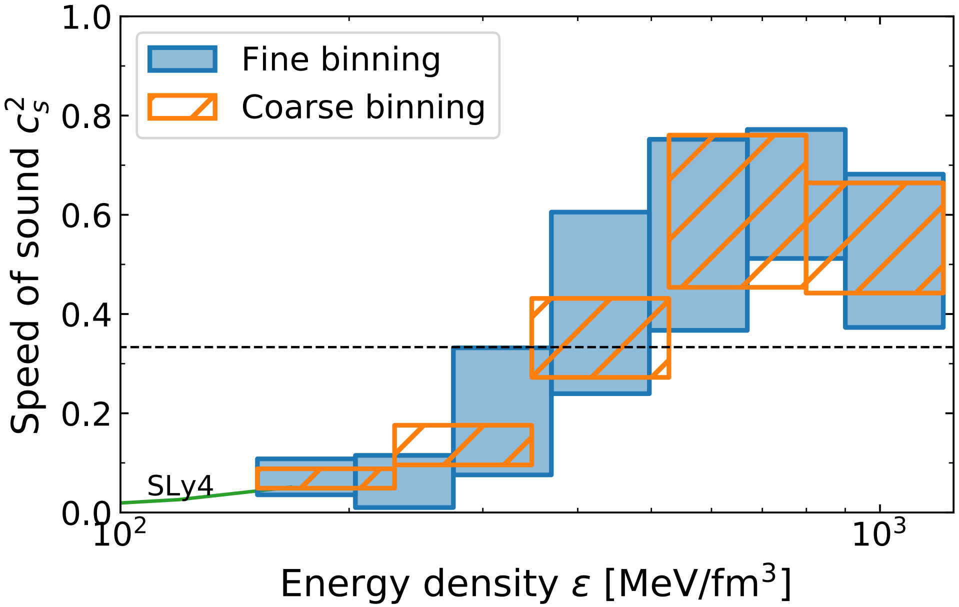


1st-order transition not necessarily excluded !

EoS Reconstruction

Most Likely EoS from ML **Fujimoto-Fukushima-Murase (2018,19,21)**

Speed of sound may exceed the conformal limit (=1/3)



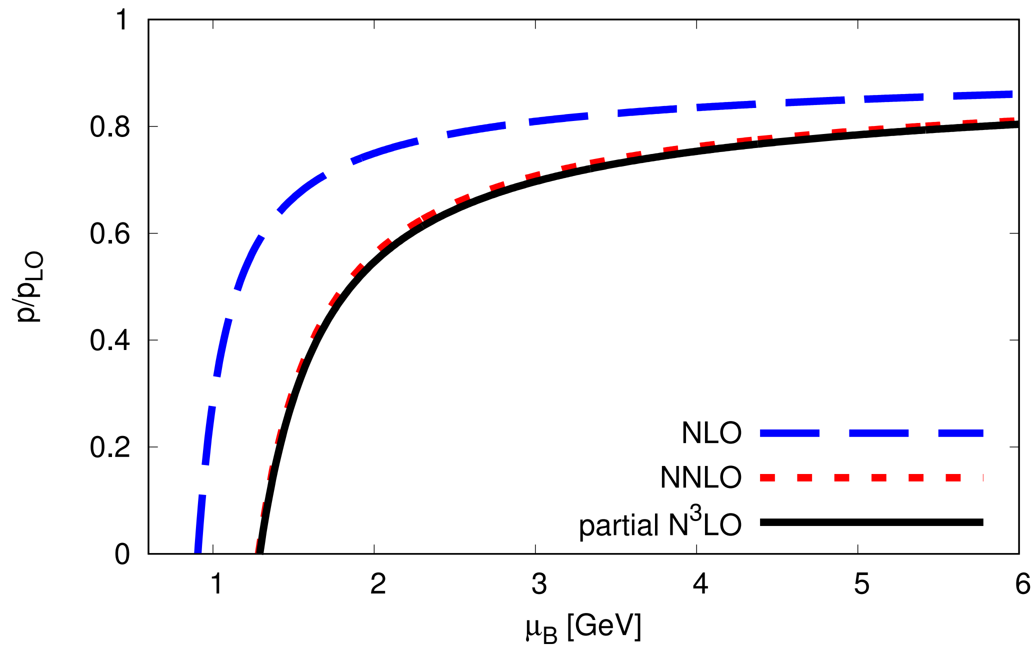
Is this a hint for the presence of quark matter ?

ML tends to favor the most conservative prediction.

QCD Approach — Current Status

At high density perturbation theory should work.

Gorda-Kurkela-Romatschke-Sappi-Vuorinen (2018)



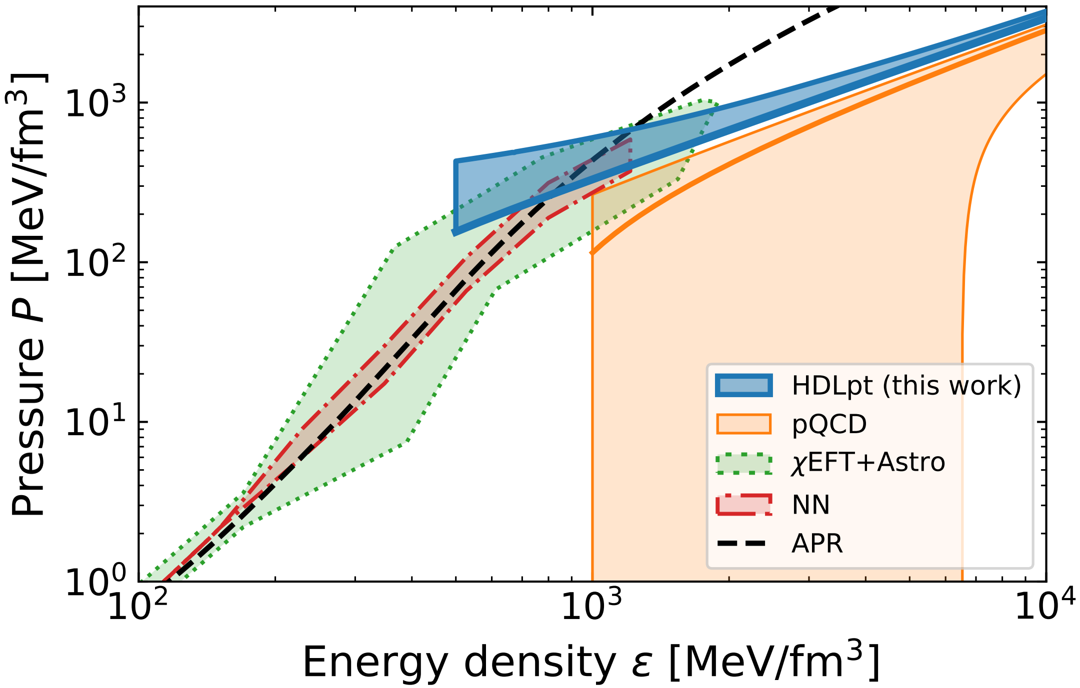
NNLO was known from Freedoman-McLerran (1977)

Convergence is much better than high-*T* perturbation.

No oscillatory divergences of asymptotic series…

QCD Approach

$$\alpha_s(\bar{\Lambda}) = \frac{4\pi}{\beta_0 \ln(\bar{\Lambda}^2/\Lambda_{\overline{\rm MS}}^2)}\left[1 - \frac{2\beta_1}{\beta_0^2}\frac{\ln\ln(\bar{\Lambda}^2/\Lambda_{\overline{\rm MS}}^2)}{\ln(\bar{\Lambda}^2/\Lambda_{\overline{\rm MS}}^2)})\right]$$



Fujimoto-Fukushima (2020)

$\bar{\Lambda}$ is changed as μ, 2μ, 4μ

Orange band represents NNLO + strange mass

pQCD works for $\bar{\Lambda} > 2\mu$ reasonably!

Band is widened by a single line of $\bar{\Lambda} = \mu$

QCD Approach

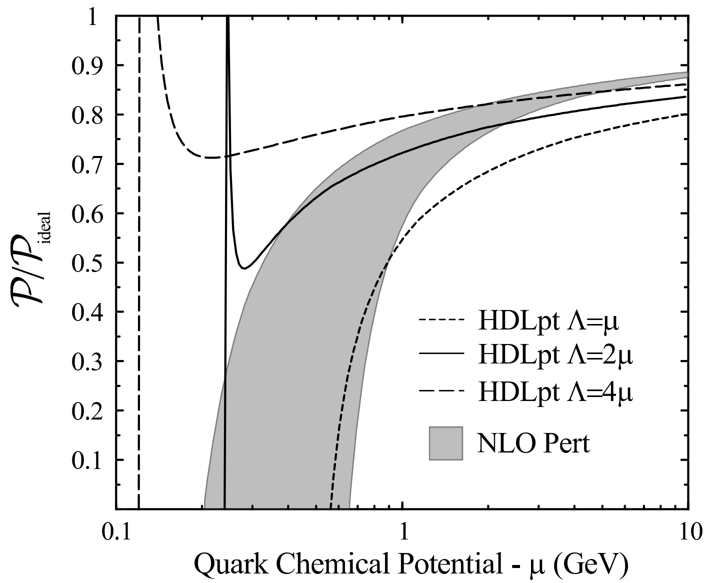
Approach with resummation (one-loop of resummed prop.)

Baier-Redlich (1999)

Zero temperature limit of HTL resummation (HDL)

Andersen-Strickland (2002)

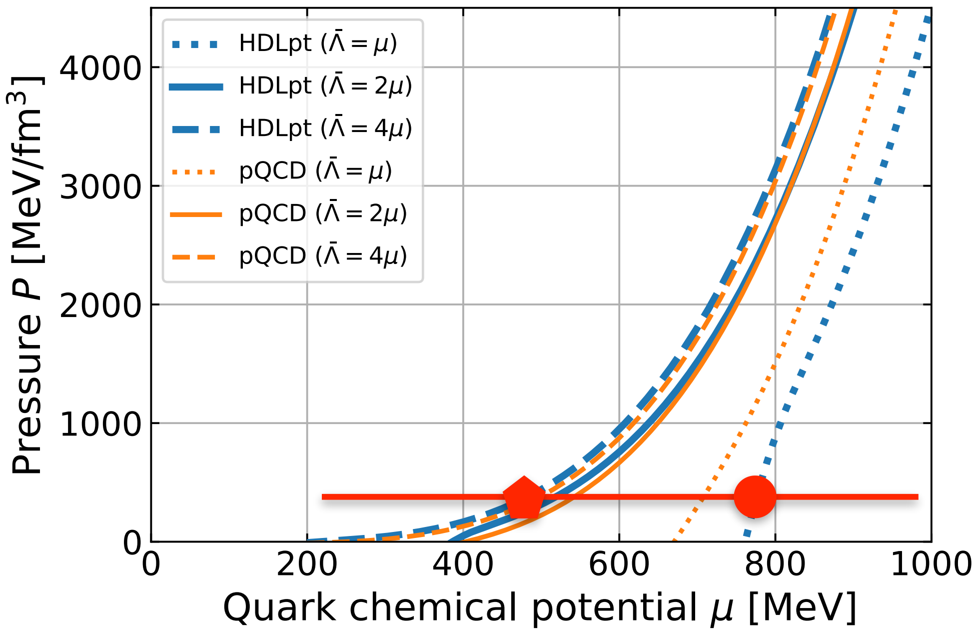
HDL EoS to solve dense quark stars

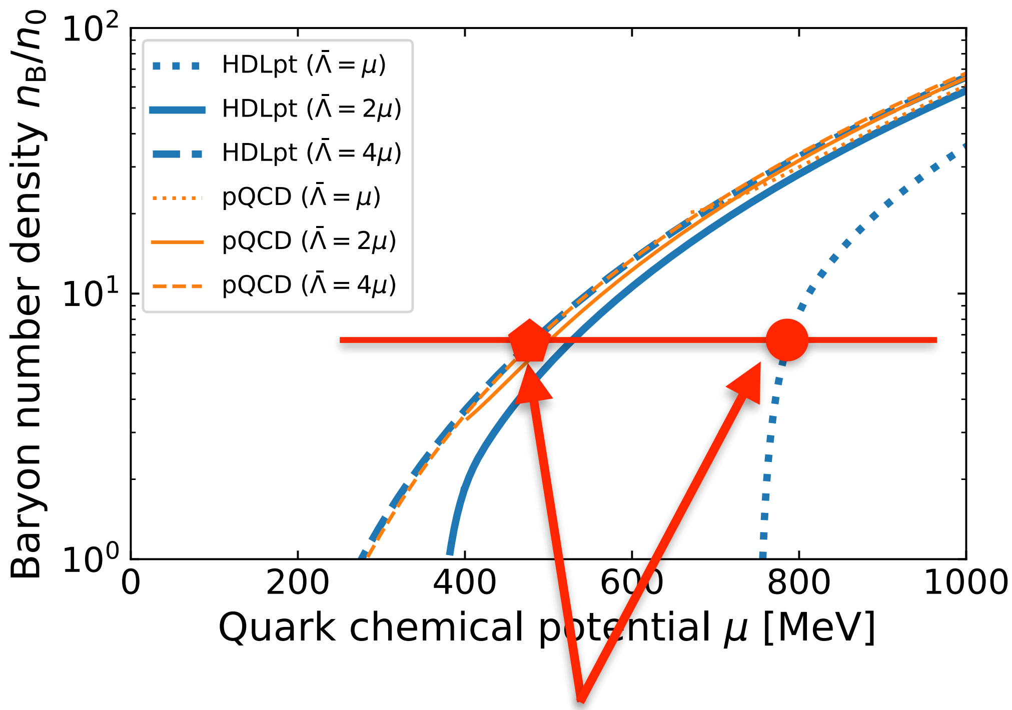


They say that uncertainty is large, but you see that it is driven by $\bar{\Lambda} = \mu$ (dotted) line.

QCD Approach

Fujimoto-Fukushima (2020)





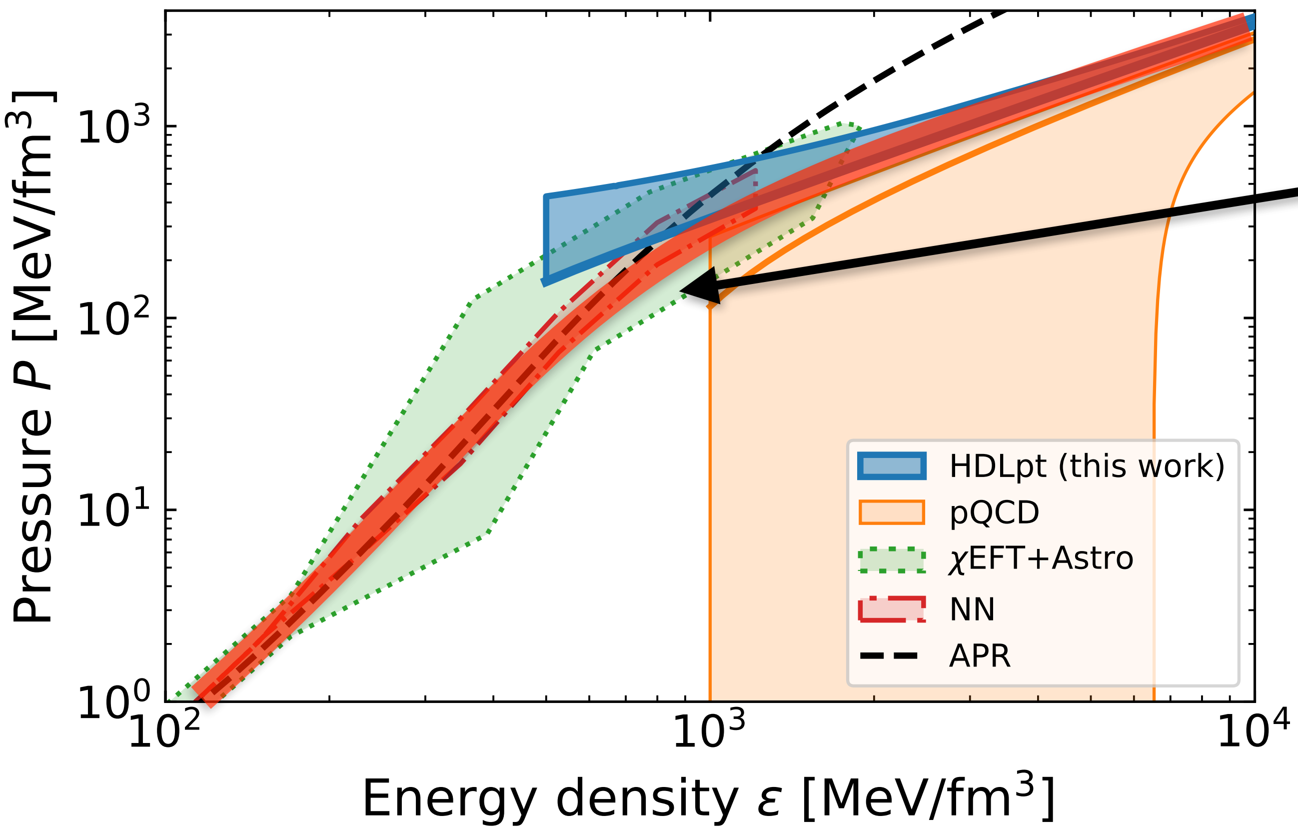
For a given density the corresponding μ is pushed up by the resummation

Uncertainty canceled out?

QCD Approach

Fujimoto-Fukushima (2020)

Smooth continuation from the nuclear side to the quark side could be possible now!

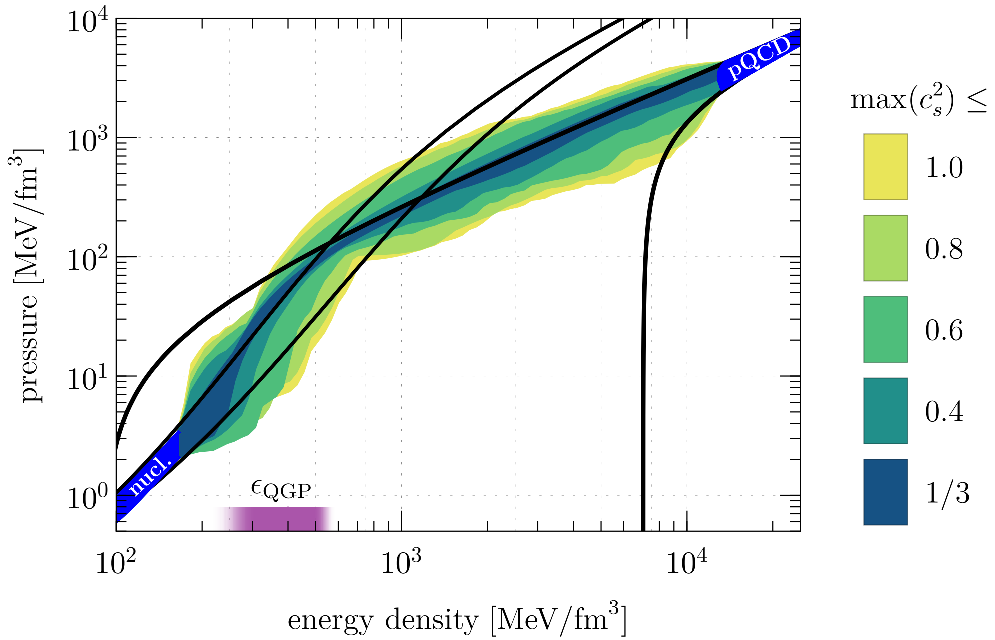


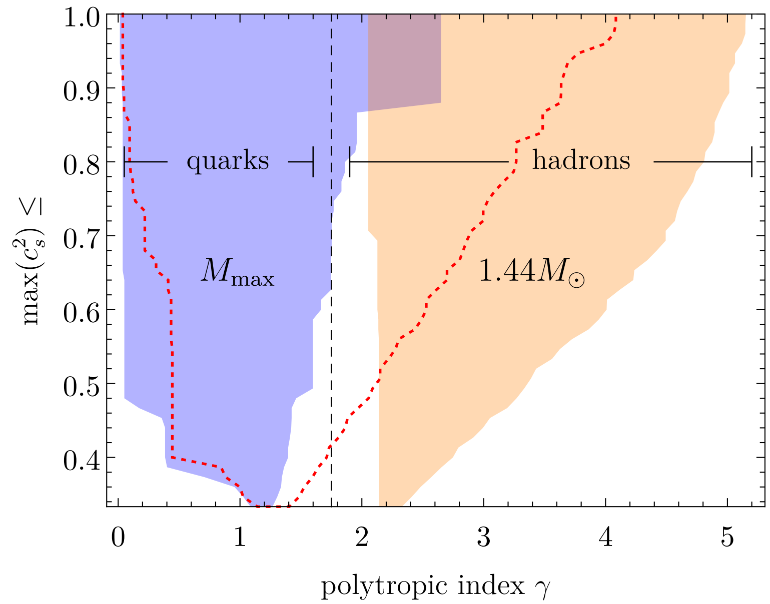
Crossover around here?

QCD Approach

This bending behavior had been phenomenologically found.

Annala-Gorda-Kurkela-Nattila-Vuorinen (2019)





Phenomenological characterization of crossover to quark matter

$$\gamma = \frac{d \ln p}{d \ln \epsilon}$$

QCD Approach

Neutron star EoS has been established within reasonable theoretical uncertainty: good enough to find a crossover to quark matter!

[Caveats]

Physical contents of EoS inferred from observation are unknown (no clue for the hyperon puzzle).

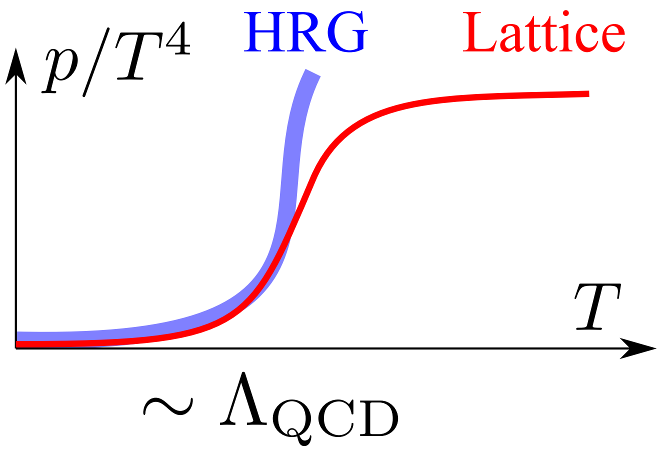
Strangeness contents are quantified in QCD calculations (which imply a significant amount of strangeness).

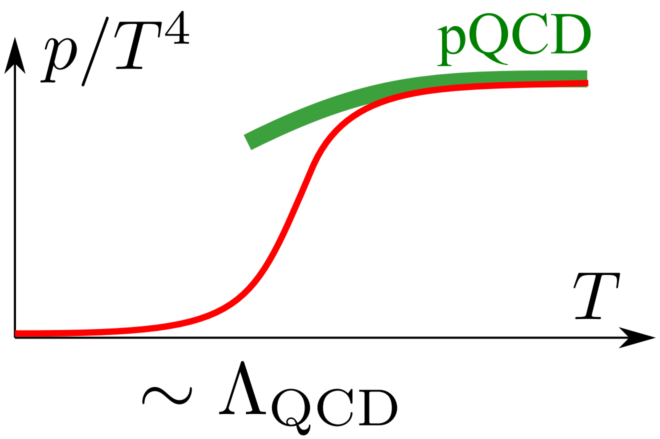
Any more "convenient"approach like a HRG?

This is not easy…

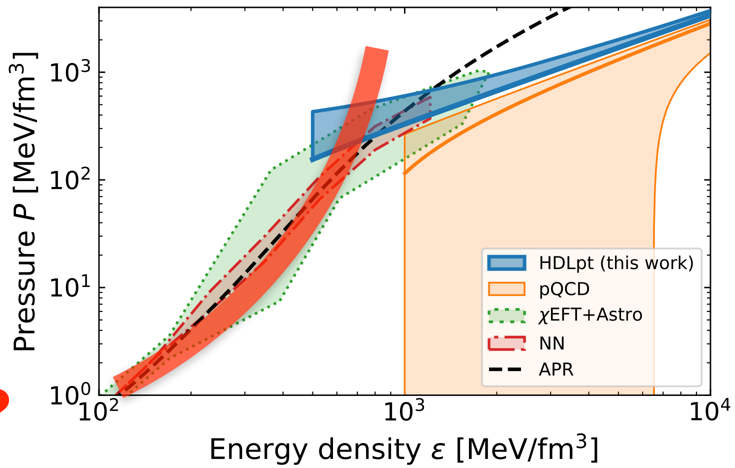
HRG / WdV ?

High-T has been understood by HRG + pQCD





Expectation



pQCD

HRG or WdV?

HRG / WdV ?

Andronic-Braun-Munzinger-Stachel-Winn (2012)

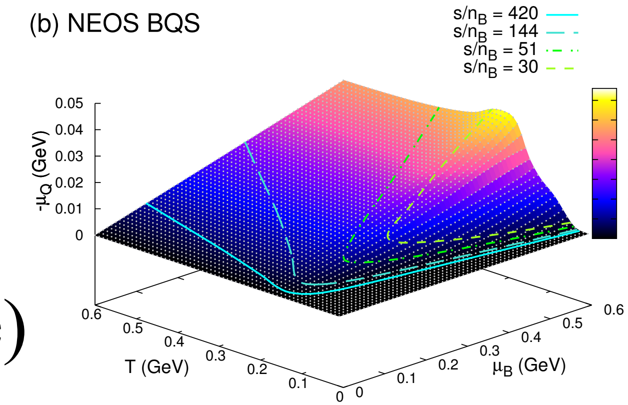
Repulsive interaction through the **excluded volume effect**
Better agreement with lattice

Vovchenko-Anchishkin-Gorenstein (2015)

Van der Waals (**excluded volume effect** + attractive int.)

Monnai-Schenke-Shen (2019)

NEOS — interpolated between lattice and IHRG
(HRG is preferable for Cooper-Frye)

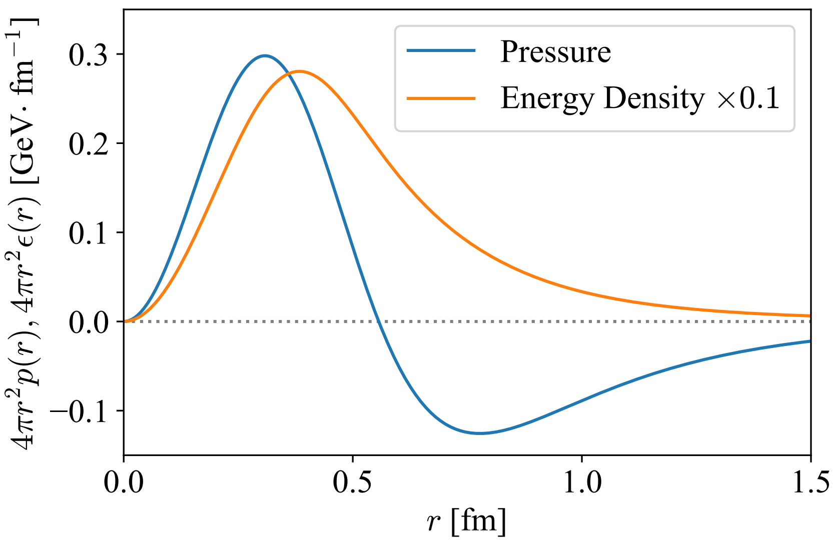


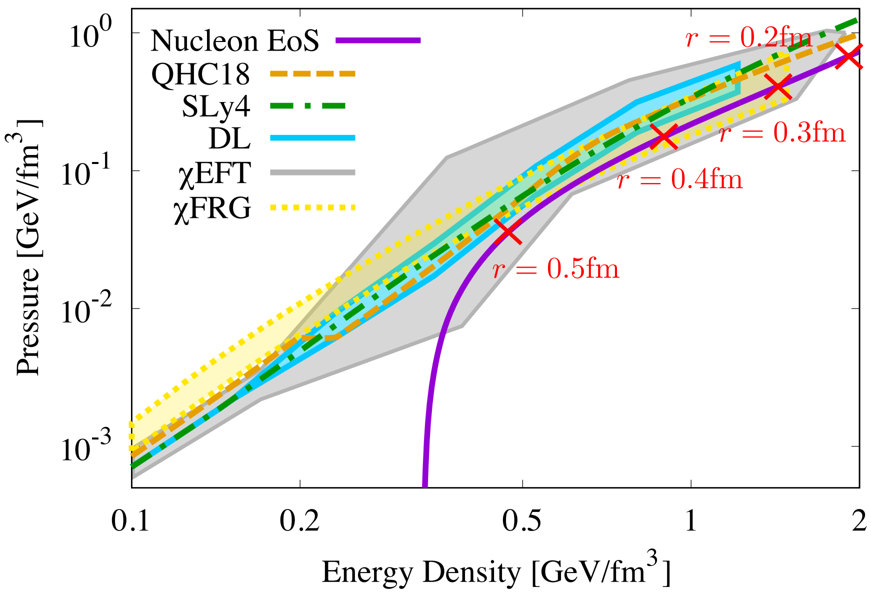
Prospect from EIC

Rajan-Gorda-Liuti-Yagi (2018)

Fukushima-Kojo-Weise (2020)

Energy-momentum tensor of proton can be probed by the gravitational form factor (at EIC).
Deep inside of the proton realizes very dense quark matter.





Model analysis demonstrates good agreement with empirical EoSs!?

Summary

EoS Inference from Observational Data

- □ Bayesian analysis is successful, but still the prior dependence is not under control.
- □ Complementary approach based on Machine Learning works leading to EoS consistent with empirical ones.

Perturbative Approach

- □ Convergence is good enough to identify a crossover region possibly to quark matter.

Extremely High Density

- □ Gravitational form factor may access information on EoS dominated by partons (quarks).