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Heavy quark momentum diffusion from the lattice

using gradient flow

Bielefeld U. L. Altenkort, O. Kaczmarek, L. Mazur, H.-T. Shu
and **TU Darmstadt** A. M. Eller, G. D. Moore

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Luis Altenkort
Bielefeld University, Faculty of Physics

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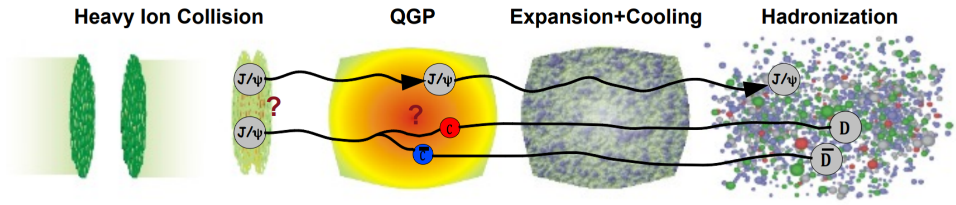
Heavy quarkonia in heavy ion collisions I

What happens to heavy quarkonia in HICs?



- Heavy quarkonia mainly produced in early hard collisions ($M \gg T$)
  - some remain as bound states ($J/\Psi, \Upsilon$)
  - some melt into constituents
    - travel through medium, thermalize to some extent via **diffusion**
    - form $D\bar{D}$ or $B\bar{B}$ meson pairs, decay into dileptons
- Evidence of in-medium interactions from strong modification of heavy hadron p_T distributions
  - ⇒ probes for transport properties of QGP

J/Ψ	Υ
$c\bar{c}$	$b\bar{b}$
$3.1\,\mathrm{GeV}$	$9.5\,\mathrm{GeV}$

cf. $T \sim \mathcal{O}(100)\,\mathrm{MeV}$

Figure: Steffen Bass, mod. by O. Kaczmarek

How much do heavy quarks thermalize in the hot medium formed in HICs?

- Naive picture: full thermalization needs many collisions, not possible within $5\text{-}10\,\mathrm{fm}/c$
  - ⇒ kinetic equilibration time (hydrodynamics): $\tau_{\text{kin}}^{\text{heavy}} \simeq \frac{M}{T}\tau_{\text{kin}}^{\text{light}} \gg \tau_{\text{kin}}^{\text{light}}$
- But: significant collective motion (v_2)! ⇒ $\tau_{\text{kin}}^{\text{heavy}} \overset{?}{\approx} \tau_{\text{kin}}^{\text{light}}$

Can we calculate τ_{kin} from first principles?

- Consider non-relativistic limit ($M \gg \pi T$):

$$\tau_{\text{kin}} = \eta_D^{-1}$$

$$\eta_D = \frac{\kappa}{2M_{\text{kin}}T}\left(1 + \mathcal{O}\left(\frac{\alpha_s^{3/2}T}{M_{\text{kin}}}\right)\right)$$

$$D = 2T^2/\kappa$$

- **Problem:** perturbative series for D or κ ill-behaved!
  - ⇒ need for **non-perturbative ab-initio** calculation
    - ⇒ **lattice QCD**

How to calculate diffusion coefficients from the lattice?

- Linear response theory: diffusion physics encoded in **in-equilibrium** spectral function (SPF)
  - ⇒ on the lattice: reconstruct SPF from Euclidean correlation functions

Two approaches on the lattice:

A. Spatial diffusion from hadronic correlators

- Starting point: SPF of HQ vector current

$$\boldsymbol{\rho}^{ii}(\omega) = \int_{-\infty}^{\infty} \mathrm{d}t\, \mathrm{e}^{i\omega t} \int \mathrm{d}^3\mathbf{x} \left\langle \frac{1}{2} \left[\hat{\mathcal{J}}^i(\mathbf{x},t), \hat{\mathcal{J}}^i(\mathbf{0},0) \right] \right\rangle$$

encodes spatial diffusion coeff. D through Kubo-formula:

$$D = \frac{1}{3\chi^{00}} \lim_{\omega\to 0} \sum_{i=1}^{3} \frac{\boldsymbol{\rho}^{ii}(\omega)}{\omega}$$

- difficult to resolve transport peak at $\omega \to 0$

⇒ see yesterday's talk by H.-T. Shu

B. Momentum diffusion from gluonic correlators

- Same starting point, but: utilize nonrelativistic limit $M \gg \pi T$
  - ⇒ can construct "color-electric correlator" whose SPF encodes momentum diff. coeff.

$$\boldsymbol{\kappa} = \lim_{\omega\to 0} 2T \frac{\boldsymbol{\rho}(\omega)}{\omega}$$

- smooth $\omega \to 0$ limit expected: no transport peak!
- needs sophisticated noise reduction method

⇒ **this talk**

κ and the color-electric (EE) correlator

Where does κ hide on the lattice for $M \gg \pi T$?

- nonrel. limit enables Foldy-Wouthuysen transformation of S_{QCD} (Heavy Quark Effective Theory)
  - ⇛ decouples quarks and anti-quarks (up to $\mathcal{O}(1/M^2)$)
- starting from spectral function of HQ current, derive/construct Euclidean correlator Caron-Huot et al. 2009
  - ⇛ $G(\tau)$: purely gluonic "color-electric correlator" or "EE correlator"

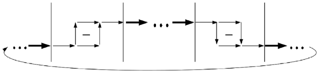
$$G(\tau) \equiv -\frac{1}{3}\sum_{i=1}^{3} \frac{\langle \mathrm{Re}\,[\mathrm{tr}\,[\; U_{(\beta,\tau)} \;\; gE_i(\mathbf{0},\tau) \;\; U_{(\tau,0)} \;\; gE_i(\mathbf{0},0) \;]]\rangle}{\langle \mathrm{Re}\,[\mathrm{tr}[\; U_{(\beta,0)} \;]]\rangle}$$

$$= \int_0^\infty \mathrm{d}\omega \, \frac{\cosh(\omega(\tau-\beta/2))}{\sinh(\omega\beta/2)} \, \boldsymbol{\rho}(\omega)$$

- encodes transport physics of **static heavy quark** in thermalized medium

$$\Rightarrow \kappa = \lim_{\omega\to 0} 2T \frac{\boldsymbol{\rho}(\omega)}{\omega}$$ ← no transport peak, smooth limit expected

- lattice discretization:



- needs sophisticated noise reduction method!

Gradient flow for EE correlator

- **Problem:** transport physics encoded in long-distance (low frequency) correlation
  - ⇒ high freq. gauge field fluctuations overshadow low freq. ones ⇒ poor signal-to-noise ratio!
  - need noise reduction method that's valid in full dynamical QCD
- **Solution: gradient flow** 🔗 Lüscher 2010
  - introduces "flow time" $\tau_\mathrm{F} := ta^2$ with dimensionless parameter t

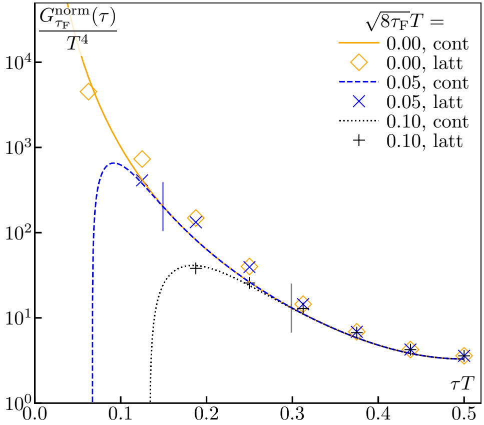
$$\frac{\mathrm{d}A_{\mu(x,\tau_\mathrm{F})}}{\mathrm{d}\tau_\mathrm{F}} \sim \frac{-\,\delta S_{G[A_\mu(x,\tau_\mathrm{F})]}}{\delta A_{\mu(x,\tau_\mathrm{F})}}, \qquad A_{\mu(x,\boldsymbol{\tau_\mathrm{F}=0})} = A_{\mu(x)}$$

  - evolves gauge fields $A_{\mu(x)}$ towards minimum of the action
  - smears them over spherical extent with "flow radius" $\simeq \sqrt{8\tau_\mathrm{F}}$

$$A^{\mathrm{LO}}_{\mu\,(x,\tau_\mathrm{F})} = \int \mathrm{d}y \, \left(\sqrt{2\pi}\sqrt{8\tau_\mathrm{F}}/2\right)^{-4} \exp\left(\frac{-(x-y)^2}{\sqrt{8\tau_\mathrm{F}}^{\,2}/2}\right) A_{\mu(y)}$$

  - **improves signal & produces renormalized fields...**
    - ...but destroys EE correlator $G(\tau)$ for $\sqrt{8\tau_\mathrm{F}}T \gtrsim \tau T/3$ according to LO pert. theory 🔗 Eller, Moore 2018
    - ⇒ flow "a little", extrapolate back to $\tau_\mathrm{F} = 0$ after continuum extrapolation

Leading-order perturbative EE correlator under Wilson flow



- continuum from Eller, Moore 2018 , lattice from Eller, Moore, LA et al. 2021
- “flow limit:” cont. correlator deviates $< 1\%$ for $\sqrt{8\tau_F}T \lesssim \tau T/3$ ⇒ vertical lines Eller, Moore 2018

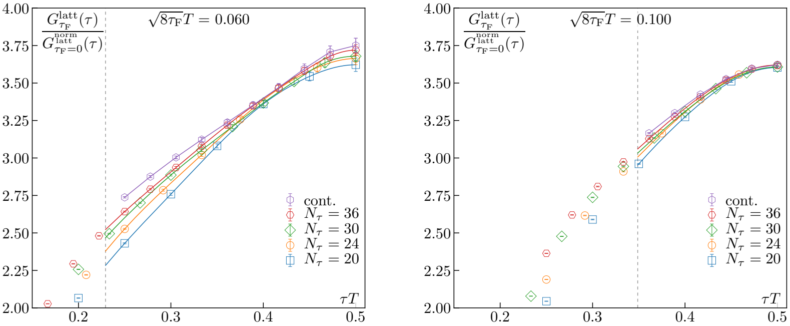
Use this to enhance non-pert. lattice results:

- filter out dominant τ^{-4} behavior of $G^{\text{latt}}_{\text{non-pert.}}$ using $G^{\text{latt}}_{\text{non-pert.}}/G^{\text{norm}}$
  - ⇒ increases visibility of details
- approx. remove tree-level discretization errors from comparison of LO cont. and LO latt. correlators

EE correlator, non-pert. lattice, $T \approx 1.5\, T_c$

$N_\sigma^3 \times N_\tau$	a [fm]
$80^3 \times 20$	0.0213
$96^3 \times 24$	0.0176
$120^3 \times 30$	0.0139
$144^3 \times 36$	0.0116

- 10000 quenched conf. each
- well-separated:
  500 sweeps of (1 HB, 4 OR)
- $\mathcal{O}(a^2)$-improved "Zeuthen flow"
- 3rd-order RK with adaptive stepsize

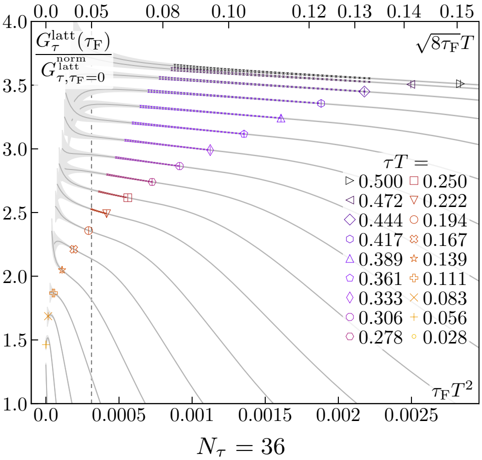


- divided by pert. correlator: dominant τ^{-4} behavior filtered out, tree-level improvement
- interpolation through cubic splines (no smoothing)
- flow limit $\sqrt{8\tau_F}T \approx \tau T/3 \Rightarrow$ dashed line

  $\Rightarrow$ more flow = smaller window of noncontaminated data

LA et al. 2021

EE correlator as a function of flow time

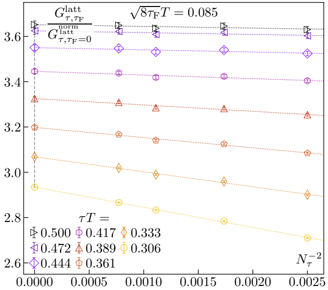


- flow limit $\sqrt{8\tau_F}T \approx \tau T/3$ ⇛ vertical lines
- for large τT: modest flow dependence

  ⇛ extrapolation to $\tau_F = 0$
- need some flow to get signal, but too much contaminates the physics
- initial rising behavior (visible for $\tau T = 0.056$): discretization-induced tadpole renormalization effect also found in pert. NLO lattice QED

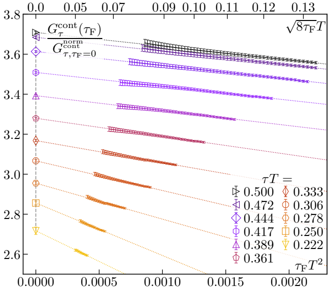
LA et al. 2021

- 1. Continuum extrapolation (linear in N_τ^{-2})



- taken separately for each flow time
- removes a^2/τ^2-type discretization errors
- ansatz motivated by gauge action discretization

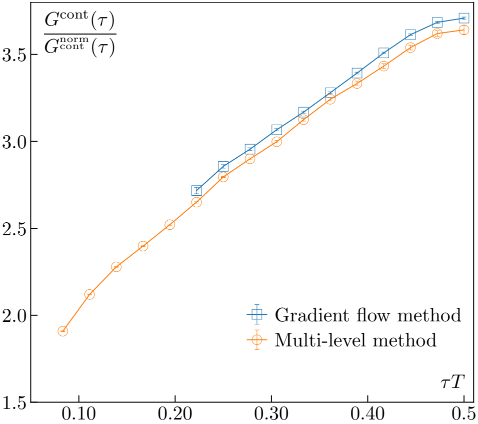
- 2. Flow-time-to-zero extrapolation (linear in τ_F)



- ansatz motivated by NLO pert. theory Eller 2021
- removes τ_F/τ^2-type effects
- flow time window depends on:
  - signal-to-noise ratio
  - $\sqrt{8\tau_F} \gtrsim a$ (renormalization, suppression of latt. artifacts)
  - $\sqrt{8\tau_F} \lesssim \tau/3$ (flow limit)

⇒ a^2/τ_F-type errors only vanish if **continuum limit** is taken **first!** LA et al. 2021

Renormalized continuum EE correlator



- Nonpert.-renormalized continuum EE correlator after $a \to 0$ and $\tau_F \to 0$ extrapolations
- shape consistent with previous (only pert. renorm.) results
  Francis et al. 2015 , Christensen, Laine 2016
- overall shift due to
  - nonperturbative renormalization
  - difference in statistical power of gauge conf.
  - systematic uncertainty introduced by flow extrapolation
- only infrared part of correlator can be obtained
  ⇒ not a problem for transport physics!

LA et al. 2021

Spectral reconstruction through pert. model fits

- valid only at $\tau_F = 0$: Eller 2021

$$G(\tau) = \int_0^\infty \mathrm{d}\omega \, \frac{\cosh(\omega(\tau - \beta/2))}{\sinh(\omega\beta/2)} \, \rho(\omega),$$

$$\kappa = \lim_{\omega \to 0} 2T \frac{\rho(\omega)}{\omega}$$

⇒ ill-posed integral inversion problem

- strategy: constrain allowed form of $\rho(\omega)$ to

$$\rho^{(\mu,i)}_{\mathrm{model}}(\omega) \equiv \left[1 + \sum_{n=1}^{n_{\max}} c_n e_n^{(\mu)}(y)\right] \sqrt{\left[\phi_{\mathrm{IR}}(\omega)\right]^2 + \left[\phi^{(i)}_{\mathrm{UV}}(\omega)\right]^2}$$

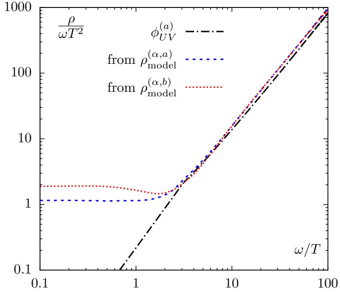
using IR and UV asymptotics:

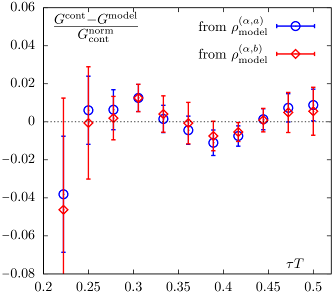
$$\phi_{\mathrm{IR}}(\omega) \equiv \frac{\kappa\omega}{2T}, \quad \phi^{(a)}_{\mathrm{UV}}(\omega) \equiv \frac{g^2(\bar{\mu}_\omega) C_F \omega^3}{6\pi}, \quad \ldots$$

⇒ well-defined fit with parameters κ/T^3 and c_n via

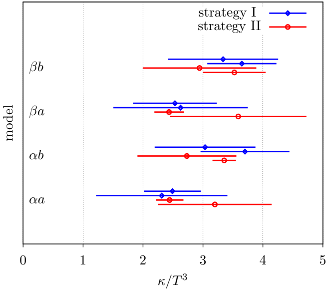
$$\chi^2 \equiv \sum_\tau \left[\frac{G^{\mathrm{cont}}(\tau) - G^{\mathrm{model}}(\tau)}{\delta G^{\mathrm{cont}}(\tau)}\right]^2$$

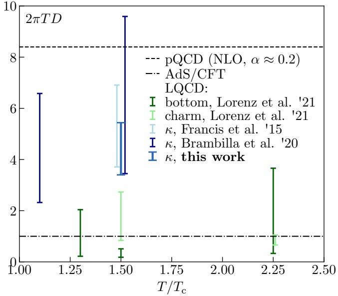
- for details see LA et al. 2021





HQ momentum diffusion coefficient κ at $T = 1.5\,T_c$





- We find

$$\boxed{\kappa/T^3 = 2.31\ \ldots\ 3.70}$$

and (for $M \gg \pi T$ using $D = 2T^2/\kappa$):

$$\boxed{2\pi TD = 3.40\ \ldots\ 5.44}$$

⇒ kinetic equilibration time:

$$\tau_{\mathrm{kin}} = \eta_D^{-1} = (1.63\ \ldots\ 2.61)\left(\frac{T_c}{T}\right)^2\left(\frac{M}{1.5\mathrm{GeV}}\right)\mathrm{fm/c}$$

- κ/T^3-value similar / slightly larger compared to previous study Francis et al. 2015 (using quenched-only multi-level method)

LA et al. 2021

What do we want?

- a first-principles nonperturbative estimate from dynamical QCD for the **heavy quark momentum diffusion coefficient** κ (or in turn D, τ_{kin})

Why?

- phenomenologically interesting: explain experimental data for heavy quarks
- crucial input for transport simulations

What did we achieve so far?

- proof-of-concept for gradient flow method using quenched QCD
  - no restrictions for application to dynamical QCD!
  - high-precision data (nonperturbatively renormalized) for relevant infrared part of EE correlator
  - consistent results for κ from reconstructed spectral function (pert. model fits)

What to do next?

- perform analysis on a set of dynamical QCD lattices (HISQ) **[in progress]**
- determine finite mass correction from color-magnetic (BB) correlator Bouttefeux, Laine 2021 **[in progress]**