Net-particle number fluctuations in a hydrodynamic description of heavy-ion collisions

Volodymyr Vovchenko (LBNL)

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V.V., C. Shen, V. Koch, *to appear*

V.V., V. Koch, Phys. Rev. C 103, 044903 (2021)

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Study of the QCD phase diagram with heavy-ion collisions

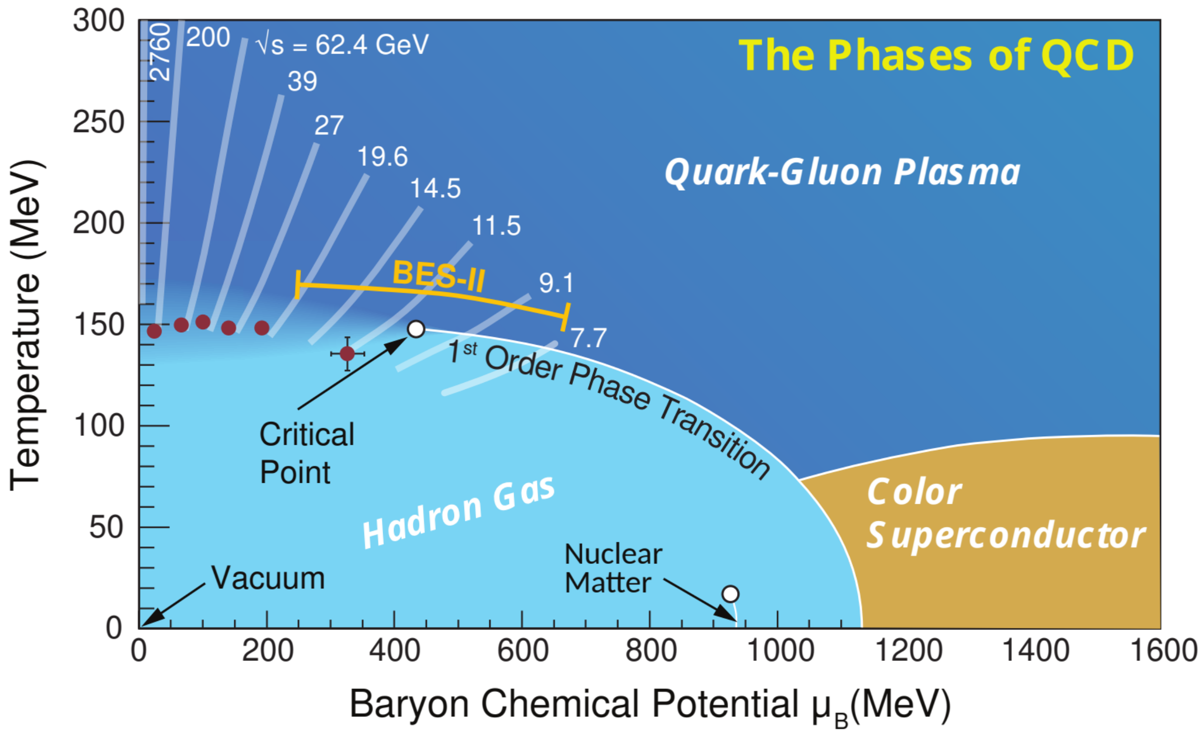
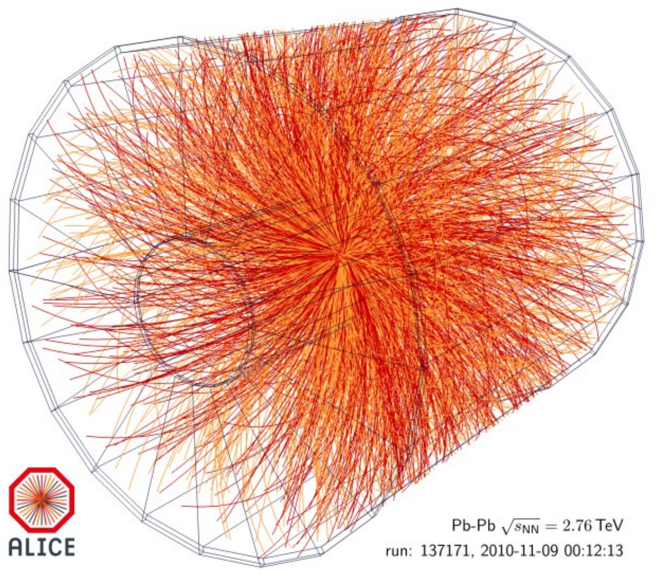


Figure from Bzdak et al., Phys. Rept. '20



ALICE event display

Thousands of particles created in relativistic heavy-ion collisions

↓

Apply concepts of statistical mechanics

Event-by-event fluctuations and statistical mechanics

Cumulant generating function

$$K_N(t) = \ln\langle e^{tN}\rangle = \sum_{n=1}^{\infty} \kappa_n \frac{t^n}{n!}$$

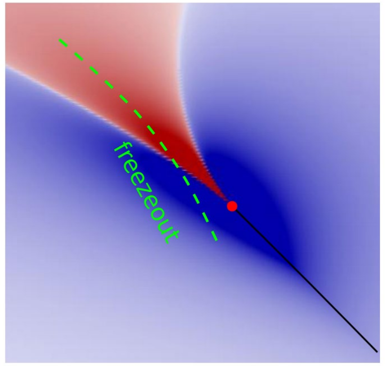
Grand partition function

$$\ln Z^{\rm gce}(T, V, \mu) = \ln\left[\sum_N e^{\mu N/T} Z^{\rm ce}(T, V, N)\right]$$

$$\kappa_n \propto \frac{\partial^n(\ln Z^{\rm gce})}{\partial\mu^n}$$

Cumulants measure chemical potential derivatives of the (QCD) equation of state

- QCD critical point



M. Stephanov, PRL '09
Energy scans at RHIC (STAR)
and CERN-SPS (NA61/SHINE)

- Test of (lattice) QCD at $\mu_B \approx 0$

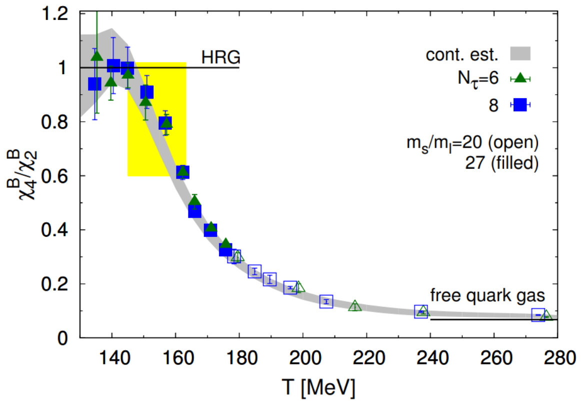
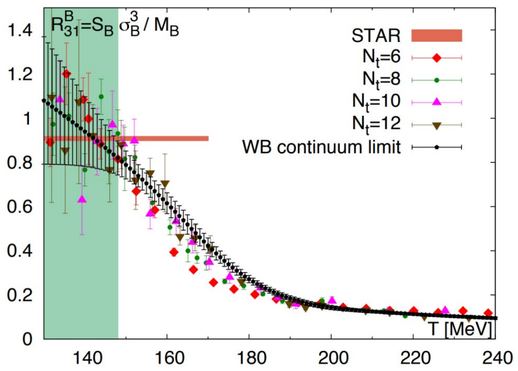


Figure from Bazavov et al. PRD 95, 054504 (2017)
Probed by LHC and top RHIC

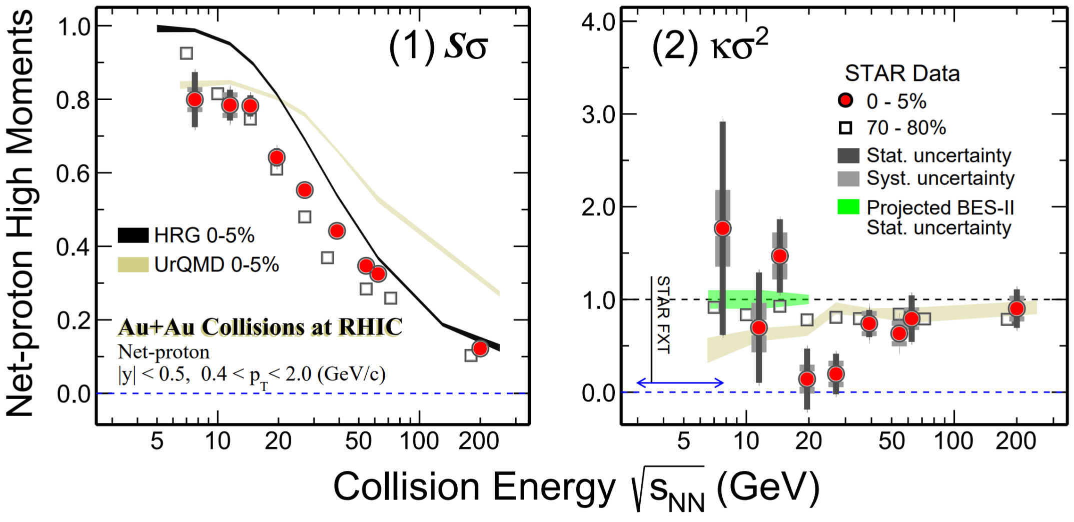
- Freeze-out from fluctuations



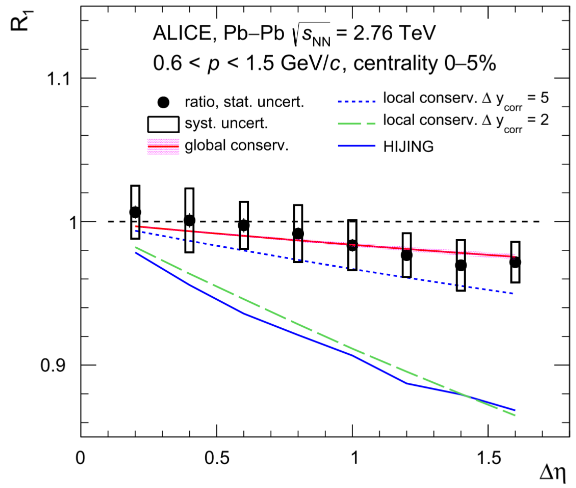
Borsanyi et al. PRL 113, 052301 (2014)
Bazavov et al. PRL 109, 192302 (2012)
...

Experimental measurements

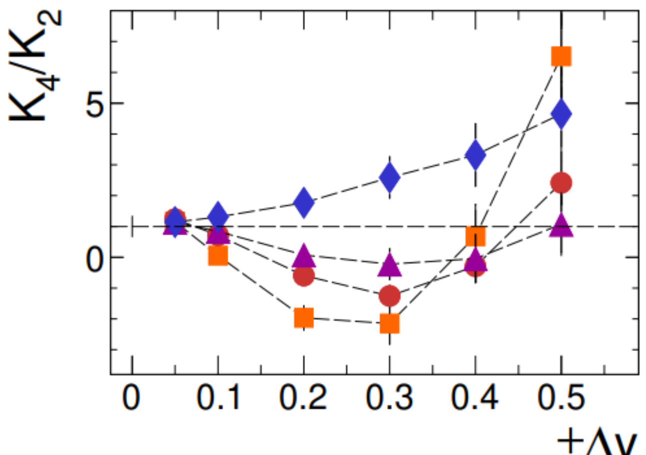
STAR Collaboration, PRL 126, 092301 (2021)



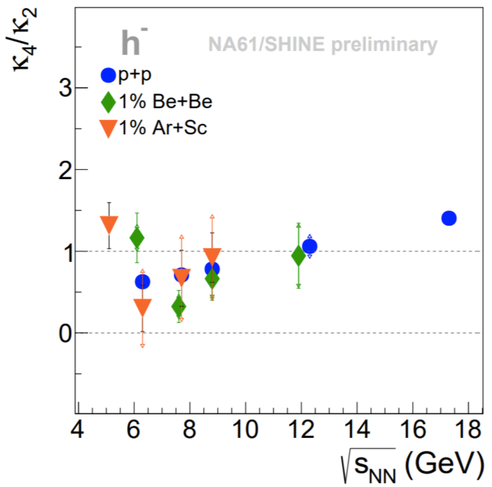
ALICE Collaboration, PLB 807, 135564 (2020)



HADES Collaboration, PRC 102, 024914 (2020)



NA61/SHINE Collaboration, SQM2021



Theory vs experiment: Caveats

- **accuracy of the grand-canonical ensemble (global conservation laws)**
  - **subensemble acceptance method (SAM)**

  VV, Savchuk, Poberezhnyuk, Gorenstein, Koch, PLB 811, 135868 (2020)

- **coordinate vs momentum space (thermal smearing)**

  Ling, Stephanov, PRC 93, 034915 (2016); Ohnishi, Kitazawa, Asakawa, PRC 94, 044905 (2016)

- **proxy observables in experiment (net-proton, net-kaon) vs actual conserved charges in QCD (net-baryon, net-strangeness)**

  Kitazawa, Asakawa, PRC 85, 021901 (2012); VV, Jiang, Gorenstein, Stoecker, PRC 98, 024910 (2018)

- volume fluctuations

  Gorenstein, Gazdzicki, PRC 84, 014904 (2011); Skokov, Friman, Redlich, PRC 88, 034911 (2013)
  X. Luo, J. Xu, B. Mohanty, JPG 40, 105104 (2013); Braun-Munzinger, Rustamov, Stachel, NPA 960, 114 (2017)

- non-equilibrium (memory) effects

  Mukherjee, Venugopalan, Yin, PRC 92, 034912 (2015)

- hadronic phase

  Steinheimer, VV, Aichelin, Bleicher, Stoecker, PLB 776, 32 (2018)

Need for ***dynamical description***

Hydrodynamic description

- Collision geometry based 3D initial state [Shen, Alzhrani, PRC '20]
  - Constrained to net proton distributions
- Viscous hydrodynamics evolution – MUSIC-3.0
  - Energy-momentum and baryon number conservation
  - NEOS-BSQ equation of state [Monnai, Schenke, Shen, PRC '19]
  - Shear viscosity via IS-type equation

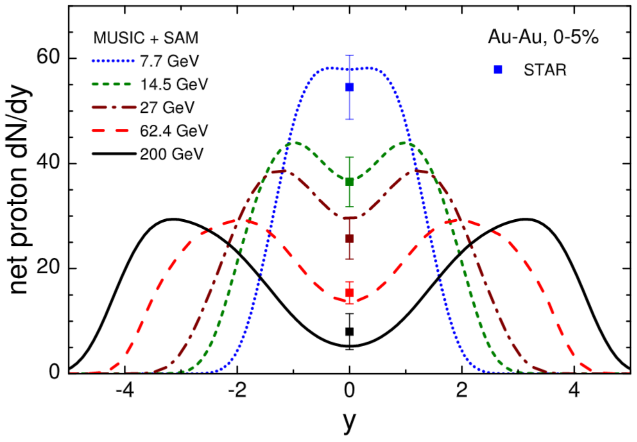
- Cooper-Frye particlization at $\epsilon_{sw} = 0.26$ GeV/fm^3

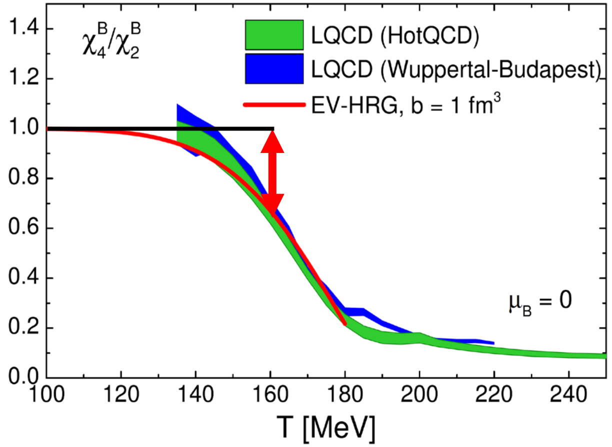
$$\omega_p \frac{dN_j}{d^3p} = \int_{\sigma(x)} d\sigma_\mu(x)\, p^\mu \, \frac{d_j \, \lambda_j^{\mathrm{ev}}(x)}{(2\pi)^3} \, \exp\left[\frac{\mu_j(x) - u^\mu(x) p_\mu}{T(x)}\right].$$

- Particlization includes QCD-based baryon number distribution
  - Here incorporated via baryon excluded volume

[VV, Pasztor, Fodor, Katz, Stoecker, PLB 775, 71 (2017)]

VV, C. Shen, V. Koch, in preparation





Calculating cumulants at particlization

- Strategy:
  1. Calculate proton cumulants in experimental acceptance in the grand-canonical limit*
  2. Apply correction for exact baryon number conservation

First step:

- Sum contributions from each fluid element x_i
  - Cumulants of joint (anti)proton/(anti)baryon distribution
  - Assumes small correlation length $\xi \to 0$

$$\kappa_{n,m}^{B^\pm,p^\pm,\text{gce}}(\Delta p_{\text{acc}}) = \sum_{i\in\sigma} \delta\kappa_{n,m}^{B^\pm,p^\pm,\text{gce}}(x_i;\Delta p_{\text{acc}})$$

- To compute each contribution
  - Grand-canonical susceptibilities $\chi^{B^\pm}(x_i)$ of (anti)baryon number
  - Each baryon ends up in acceptance Δp_{acc} with binomial probability
  - Each baryon is a proton with probability $q(x_i) = \langle N_p(x_i)\rangle / \langle N_B(x_i)\rangle$

[Kitazawa, Asakawa, Phys. Rev. C 85 (2012) 021901]

$$p_{\text{acc}}(x_i;\Delta p_{\text{acc}}) = \frac{\int_{p\in\Delta p_{\text{acc}}} \frac{d^3p}{\omega_p}\delta\sigma_\mu(x_i)\, p^\mu\, f[u^\mu(x_i)p_\mu; T(x_i),\mu_j(x_i)]}{\int \frac{d^3p}{\omega_p}\delta\sigma_\mu(x_i)\, p^\mu\, f[u^\mu(x_i)p_\mu; T(x_i),\mu_j(x_i)]}.$$

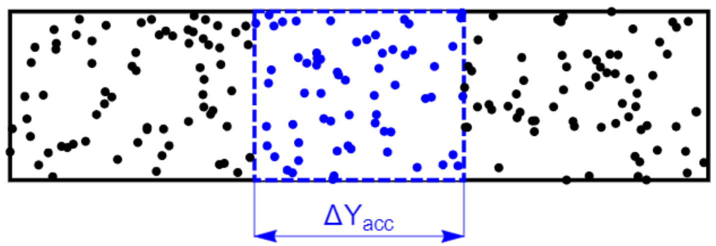
*For similar calculations of critical fluctuations see Ling, Stephanov, 1512.09125 and Jiang, Li, Song, 1512.06164

Correcting for baryon number conservation

- Subensemble acceptance method (SAM)
  - Corrects *any* equation of state for global charge conservation
  - Canonical ensemble cumulants in terms of grand-canonical ones

VV, Savchuk, Poberezhnyuk, Gorenstein, Koch, Phys. Lett. B 811, 135868 (2020) [arXiv:2003.13905]

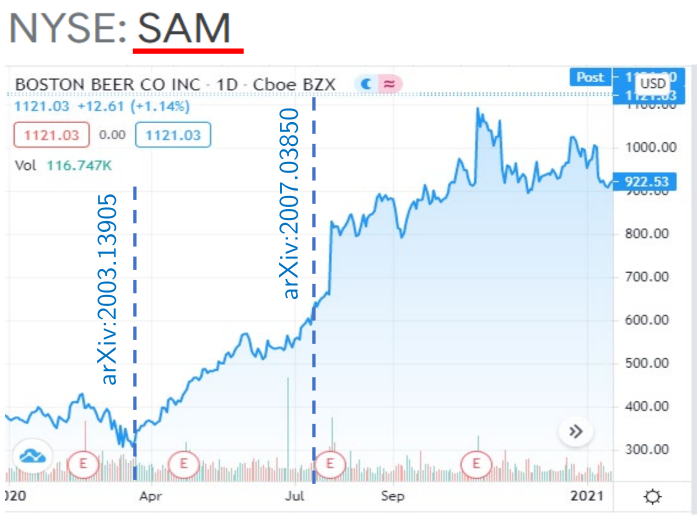
VV, Poberezhnyuk, Koch, JHEP 10, 089 (2020) [arXiv:2007.03850]



- SAM-2.0
  - Non-conserved quantities (e.g. proton number)
  - Spatially inhomogeneous systems
  - Momentum space
  - Map "grand-canonical" cumulants inside and outside the acceptance to the "canonical" cumulants inside the acceptance

$$\kappa_{p,B}^{\text{in,ce}} = \text{SAM}\left[\kappa_{p,B}^{\text{in,gce}}, \kappa_{p,B}^{\text{out,gce}}\right]$$

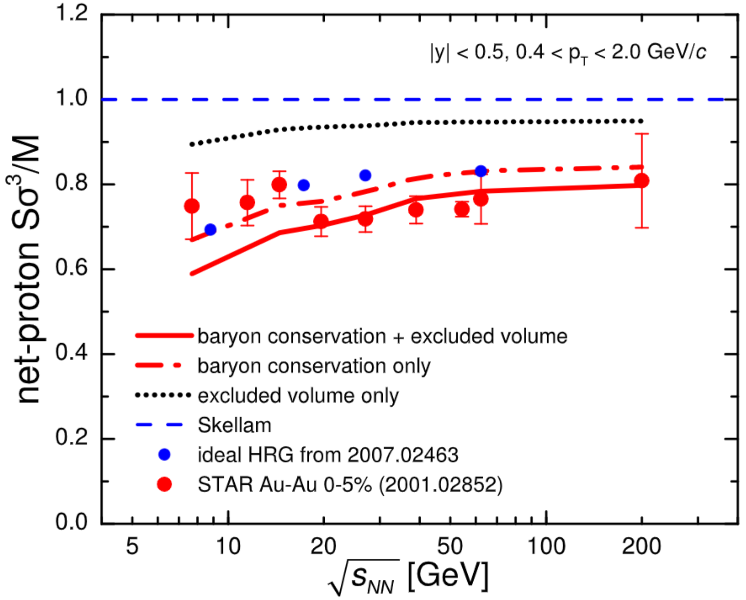
VV, *to appear*



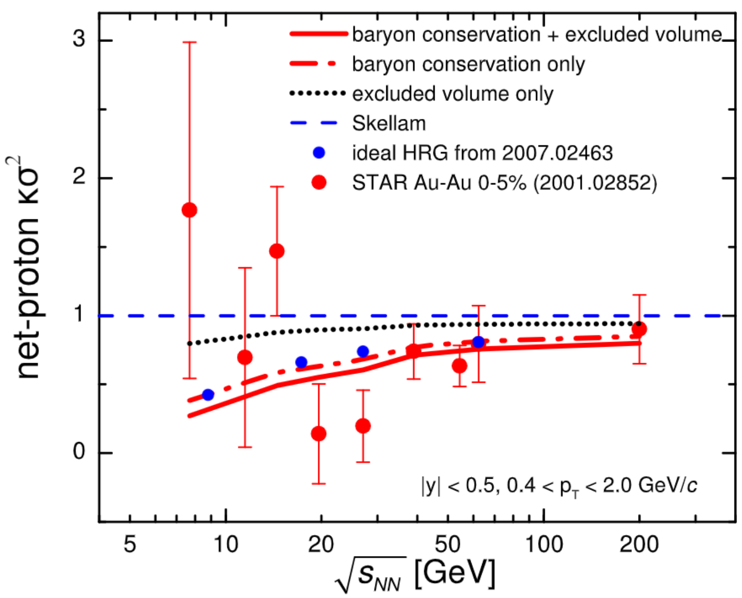
Implementation in the (extended) **Thermal-FIST** package https://github.com/vlvovch/Thermal-FIST

Net proton cumulant ratios

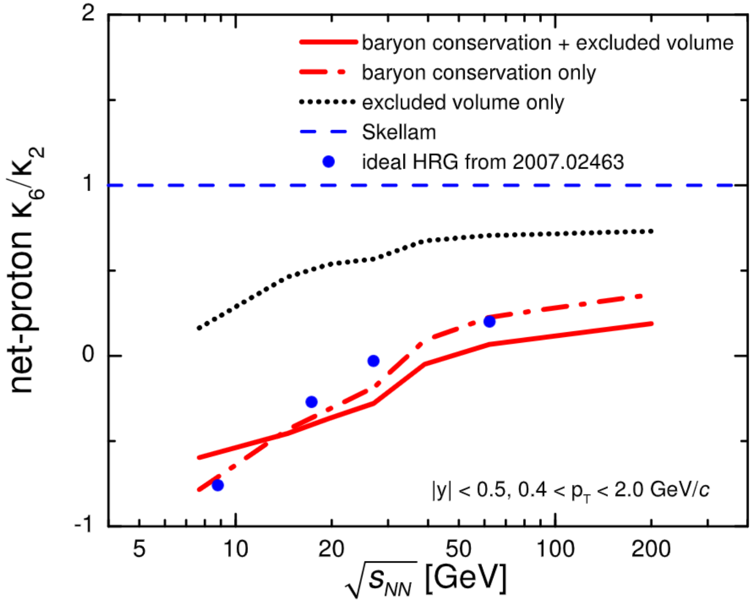
κ_3/κ_1



κ_4/κ_2



κ_6/κ_2



- Both the baryon conservation and repulsion needed to describe data at $\sqrt{s_{NN}} \geq 20$ GeV quantitatively
- Effect from baryon conservation is larger than from repulsion
- Canonical ideal HRG limit is consistent with the data-driven study of [Braun-Munzinger et al., 2007.02463]
- κ_6/κ_2 turns negative at $\sqrt{s_{NN}}$~50 GeV

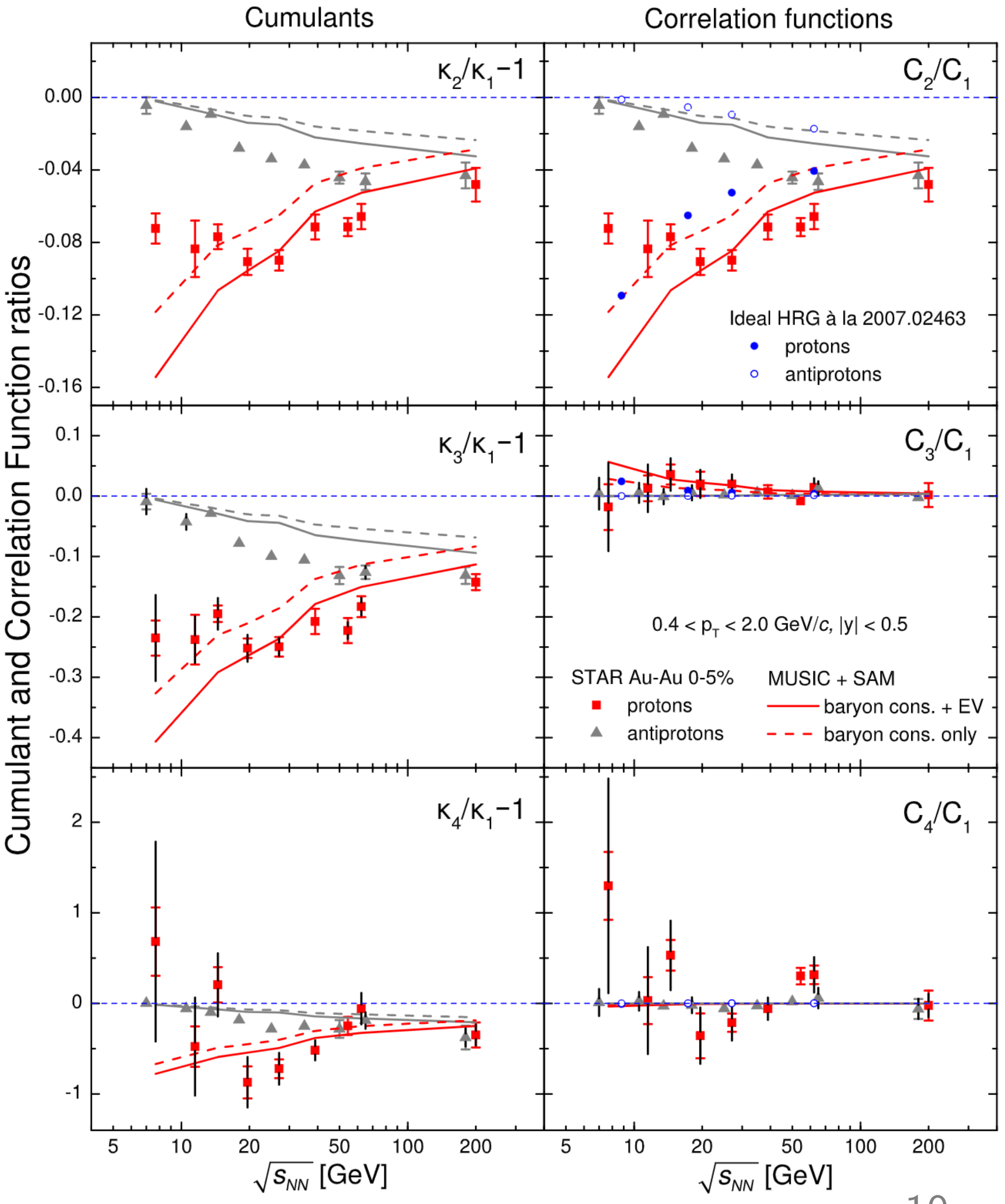
Cumulants vs Correlation Functions

- Analyze genuine multi-particle correlations via **factorial cumulants** [Bzdak, Koch, Strodthoff, PRC '17]

$$\hat{C}_1 = \kappa_1, \qquad \hat{C}_3 = 2\kappa_1 - 3\kappa_2 + \kappa_3,$$
$$\hat{C}_2 = -\kappa_1 + \kappa_2, \quad \hat{C}_4 = -6\kappa_1 + 11\kappa_2 - 6\kappa_3 + \kappa_4.$$

- Three- and four-particle correlations are small
  - Higher-order cumulants are driven by two-particle correlations
  - Small positive $\hat{C}_3/\hat{C}_1$ in the data is explained by baryon conservation + excluded volume
  - Strong multi-particle correlations would be expected near the critical point [Ling, Stephanov, 1512.09125]

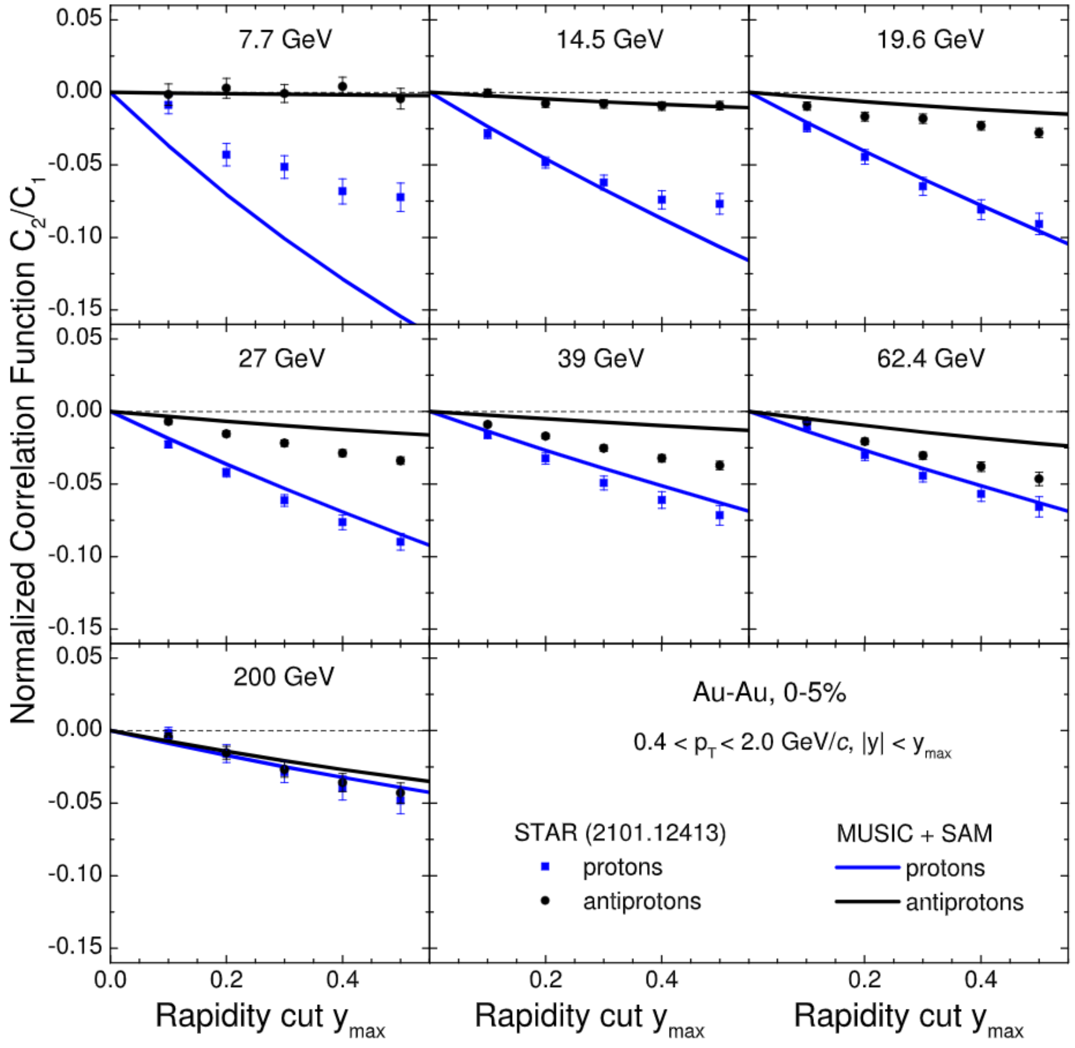
- Two-particle correlations are negative
  - Protons at $\sqrt{s_{NN}} \leq 14.5$ GeV overestimated
  - Antiprotons at $19.6 \leq \sqrt{s_{NN}} \leq 62.4$ GeV underestimated



*We use the notation for (factorial) cumulants from Bzdak et al., Phys. Rept. '20. This is different from STAR's 2101.12413 where it is reversed

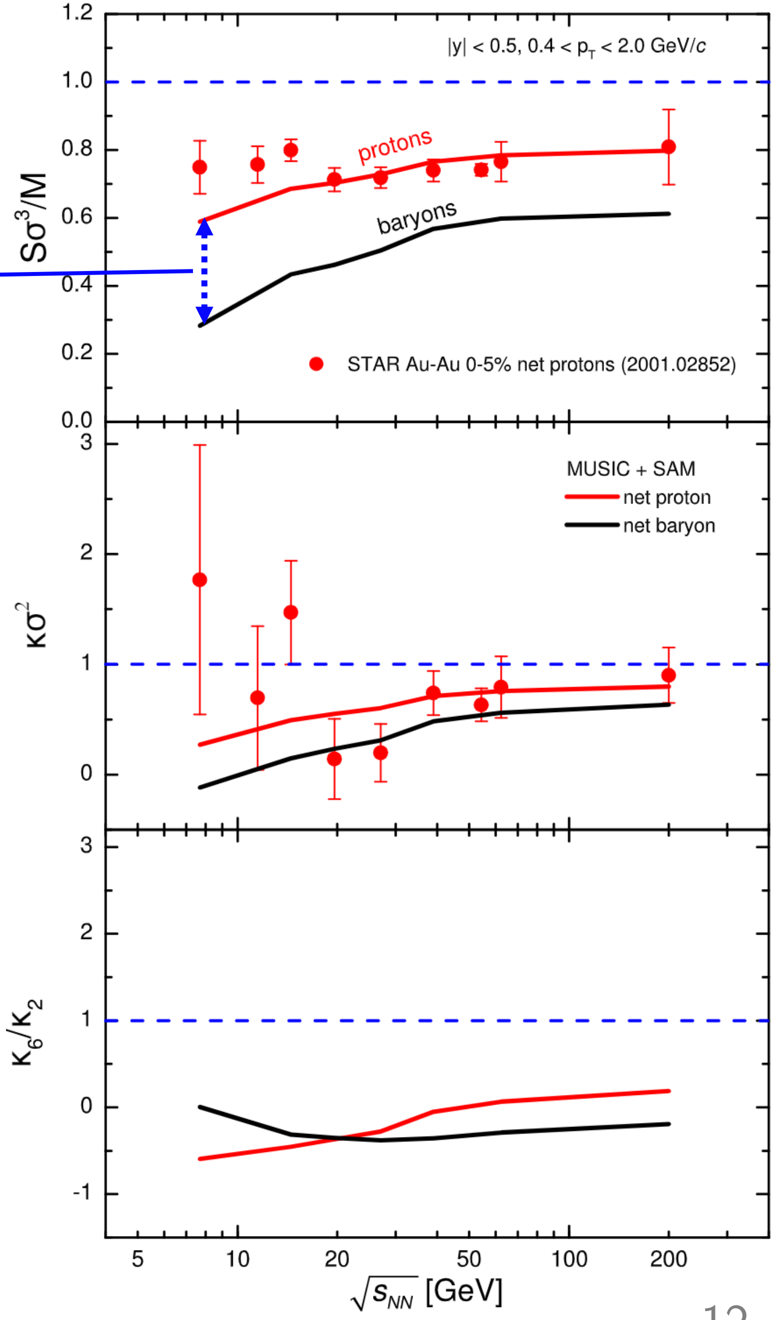
Acceptance dependence of two-particle correlations

- Qualitative agreement with the STAR data
- Data indicate a changing y_{max} slope at $\sqrt{s_{NN}} \leq 14.5$ GeV
- Volume fluctuations? [Skokov, Friman, Redlich, PRC '13]
  - Can improve low energies but spoil high energies?
- Exact electric charge conservation?
  - Worsens the agreement at $\sqrt{s_{NN}} \leq 14.5$, higher energies virtually unaffected (see backup)
- Attractive interactions?
  - Could work if baryon repulsion switches to attraction in the high-μ_B regime

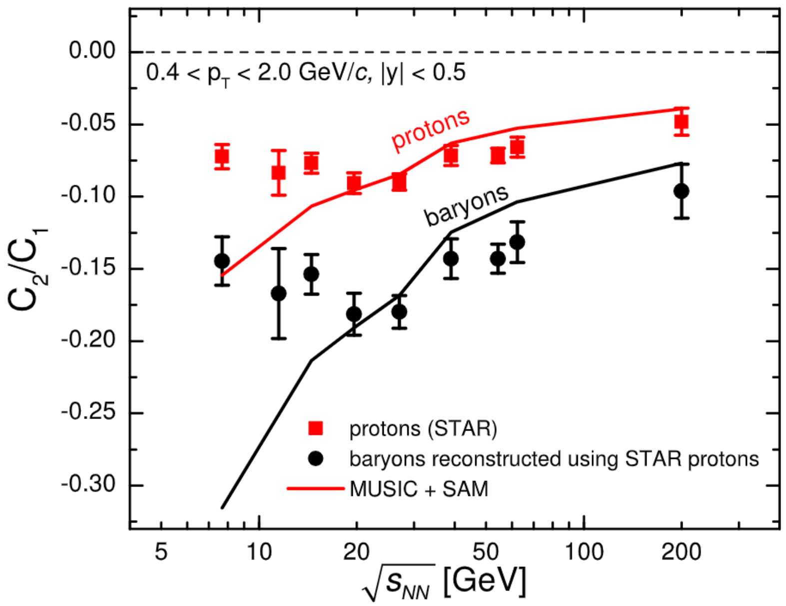


Net baryon vs net proton

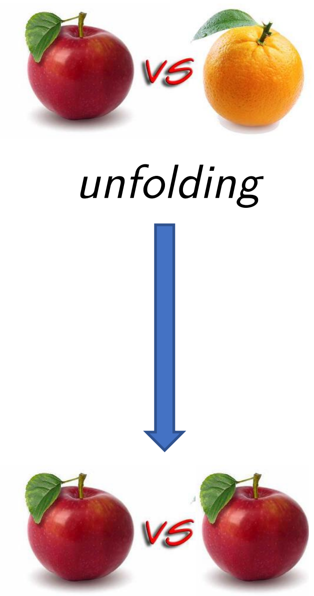
- **net baryon $\neq$ net proton**



- Baryon cumulants can be reconstructed from proton cumulants via binomial (un)folding based on isospin randomization [Kitazawa, Asakawa, Phys. Rev. C 85 (2012) 021901]
  - Requires the use of joint factorial moments, only experiment can do it model-independently

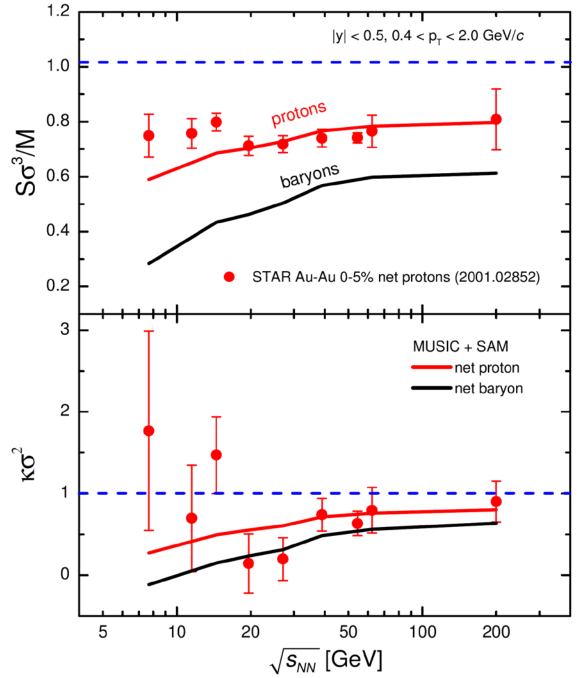


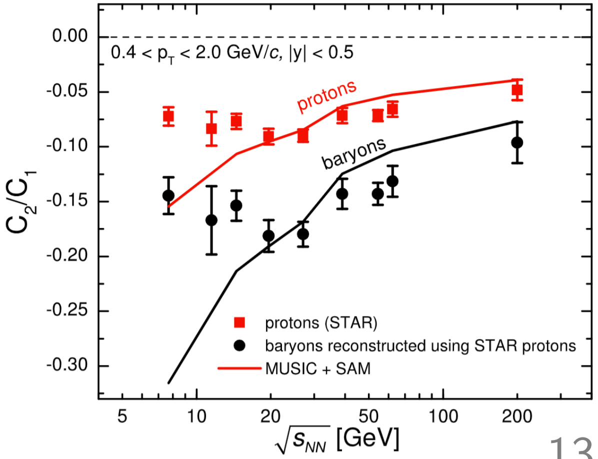
$$\frac{\hat{C}_2^B}{\hat{C}_1^B} \approx 2\,\frac{\hat{C}_2^p}{\hat{C}_1^p}$$



Summary

- (Net-)(anti-) proton cumulants calculated in a hydro description
  - true momentum space acceptance instead of coordinate space
  - simultaneous effects of baryon conservation and repulsive interactions
- Quantitative analysis of Au-Au collisions at $\sqrt{s_{NN}}$=7.7-200 GeV
  - STAR protons are described quantitatively at $\sqrt{s_{NN}} \geq 20$ GeV
  - Significant difference between protons and baryons
- Factorial cumulants carry rich information
  - Small three- and four-particle correlations in absence of critical point effects
  - Possible evidence for attractive proton interactions at $\sqrt{s_{NN}} \leq 14.5$ GeV
  - No quantitative description of antiprotons at $19.6 \leq \sqrt{s_{NN}} \leq 62.4$ GeV

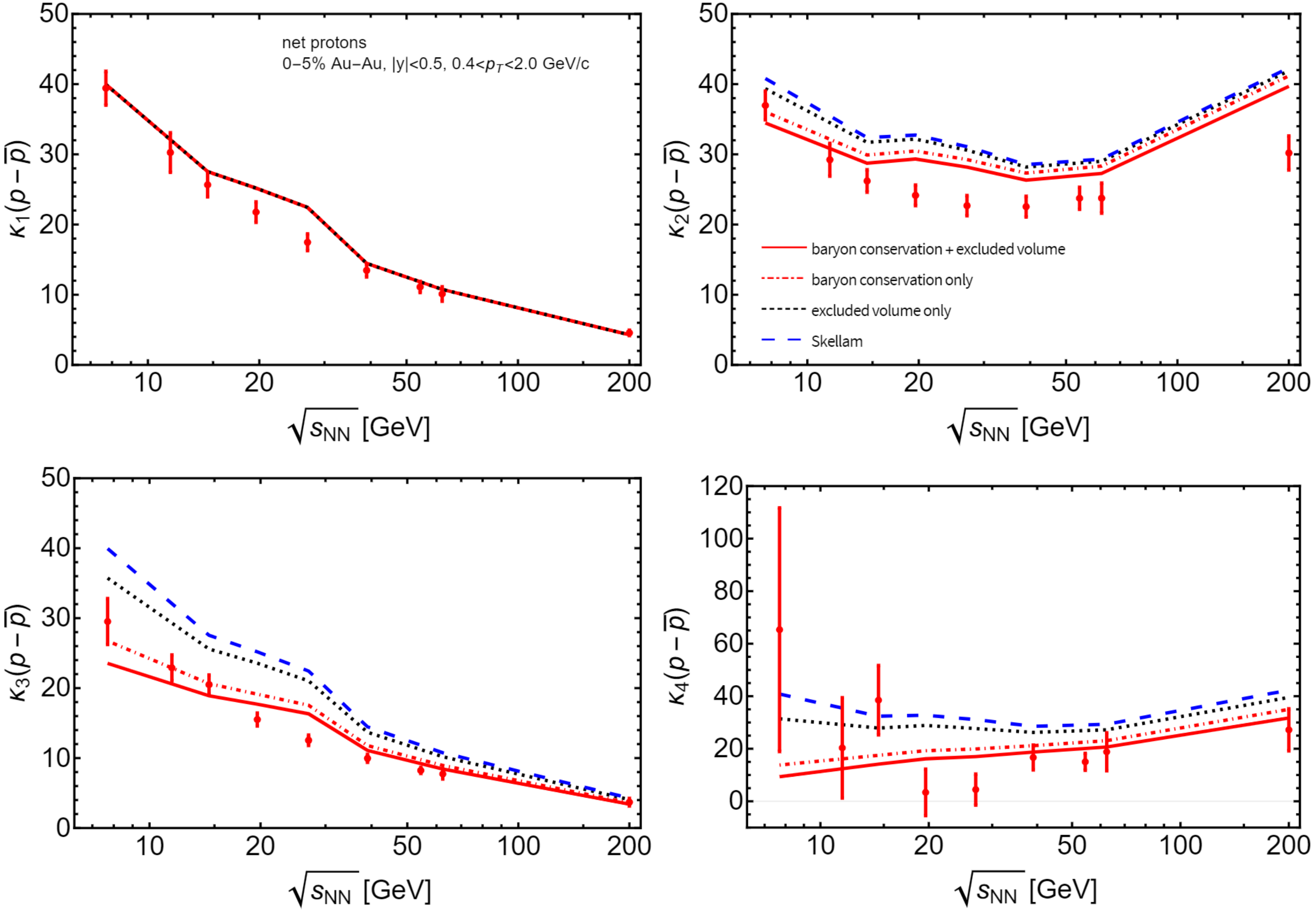




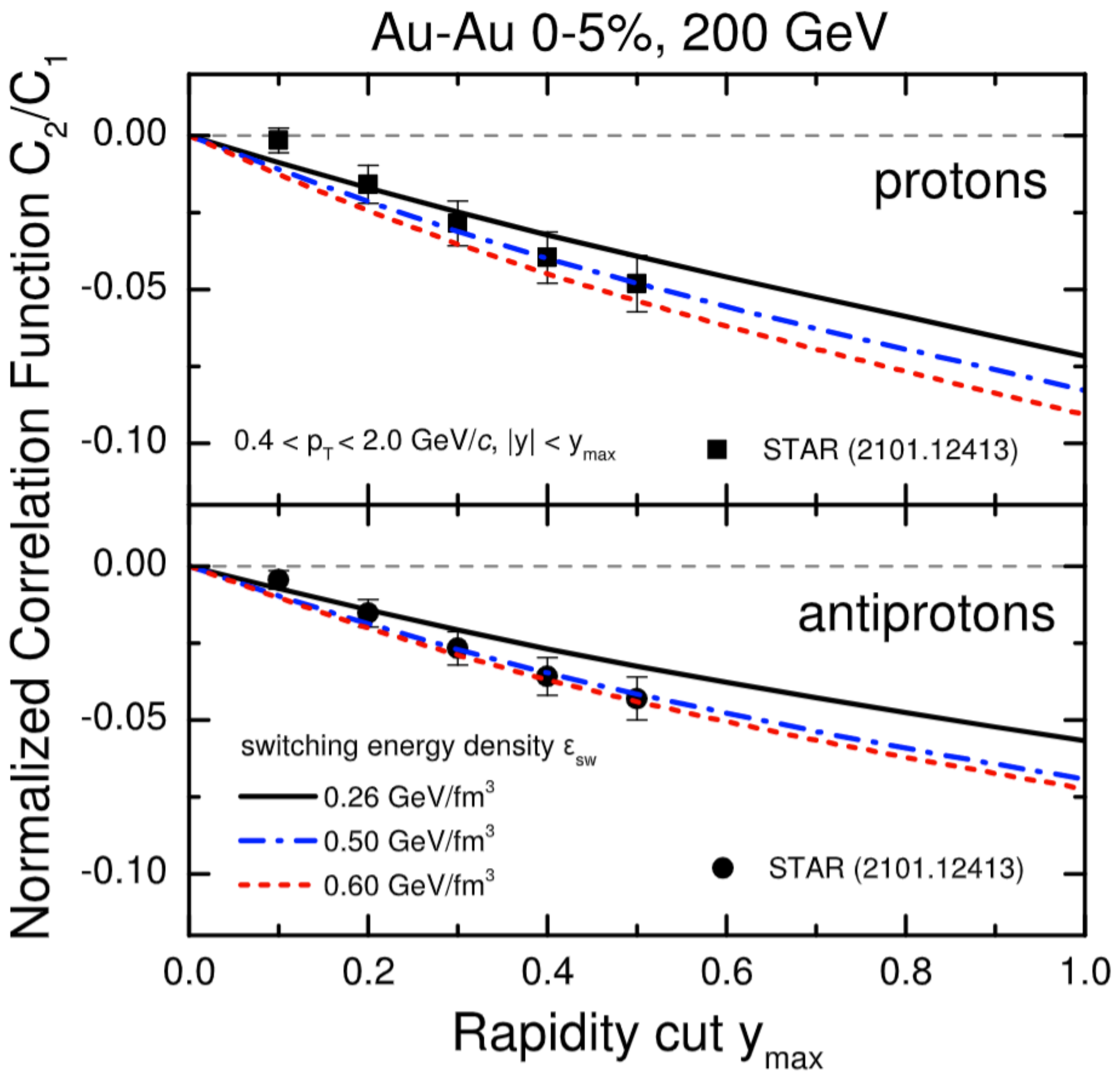
Thanks for your attention!

Backup slides

Net proton cumulants at RHIC

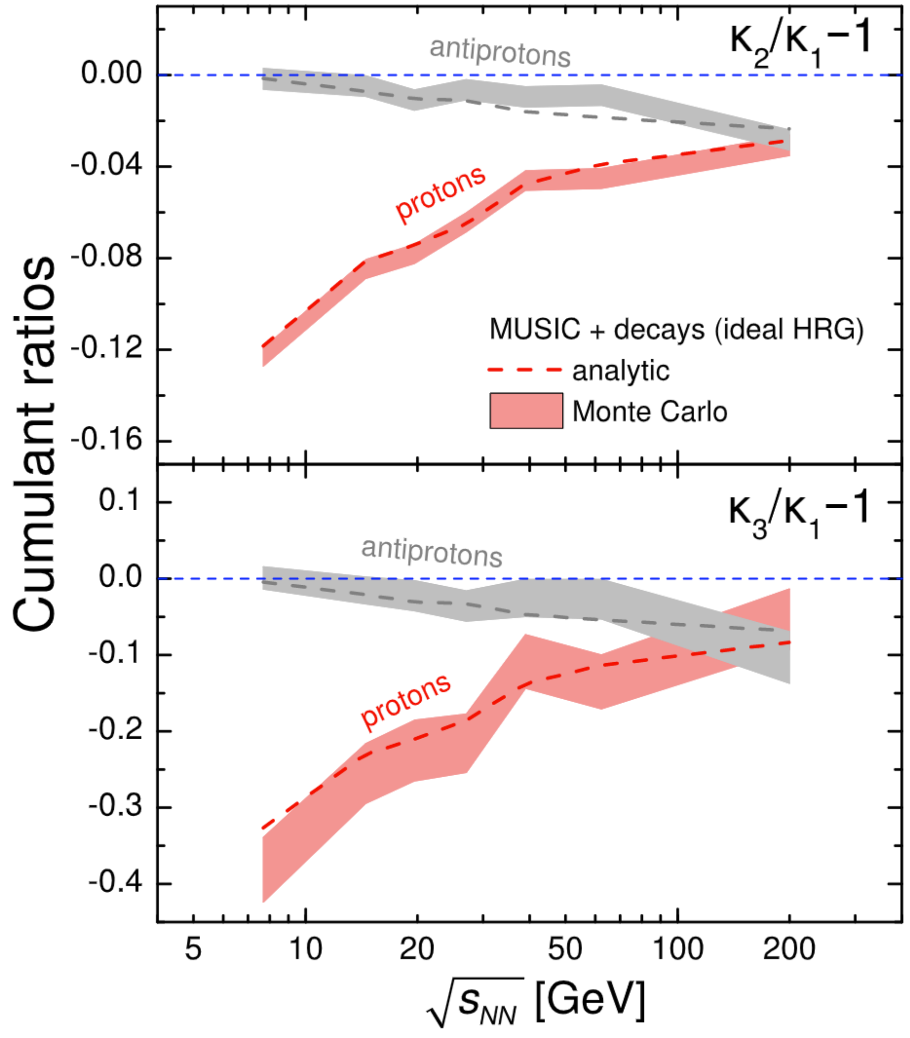


Dependence on the switching energy density



Cross-checking the cumulants with Monte Carlo

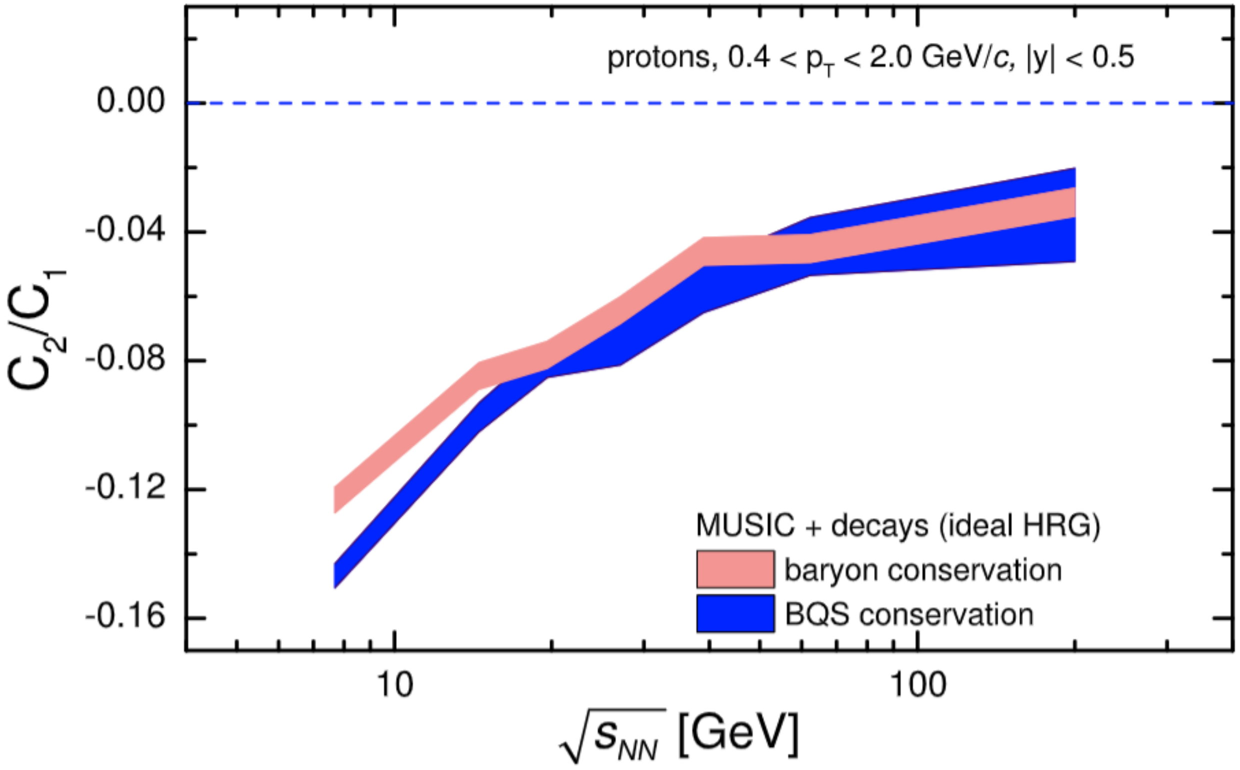
- Sample canonical ideal HRG model at particlization with Thermal-FIST
- Analytic results agree with Monte Carlo within errors



Exact conservation of electric charge

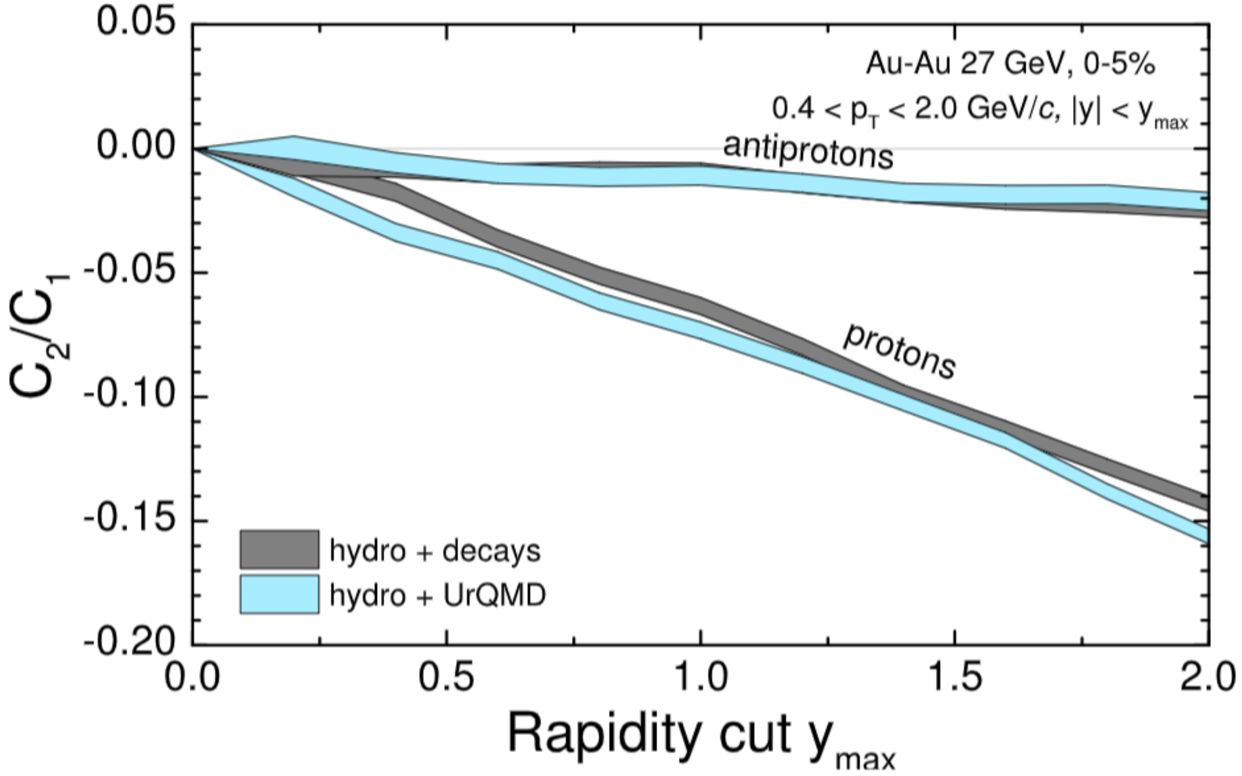
- Sample ideal HRG model at particlization with exact conservation of baryon number, electric charge, and strangeness using Thermal-FIST

- Protons are affected by electric charge conservation at $\sqrt{s_{NN}} \leq 14.5$



Effect of the hadronic phase

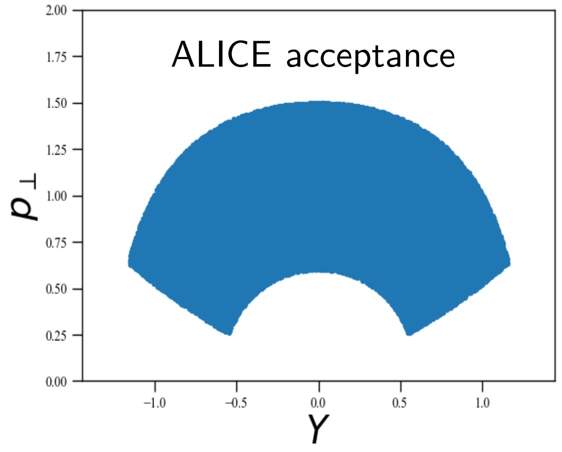
Sample ideal HRG model at particlization with exact conservation of baryon number using Thermal-FIST and run through hadronic afterburner UrQMD



Net-particle fluctuations at the LHC

VV, Koch, Phys. Rev. C 103, 044903 (2021)

- Net protons described within errors but not sensitive to the equation of state for the present experimental acceptance
- Large effect from resonance decays for lighter particles
- Future measurements will require larger acceptance



$0.6 < p < 1.5$ GeV$/c$, $\Delta\eta_{acc} = 1.6$