The 19th International Conference on Strangeness in Quark Matter
May 17-21, 2021, sponsored by Brookhaven National Laboratory, Upton, New York
SQM 2021

清華大学
Tsinghua University

Charmonium in strong electromagnetic and vorticity fields

Jiaxing Zhao(Tsinghua University)

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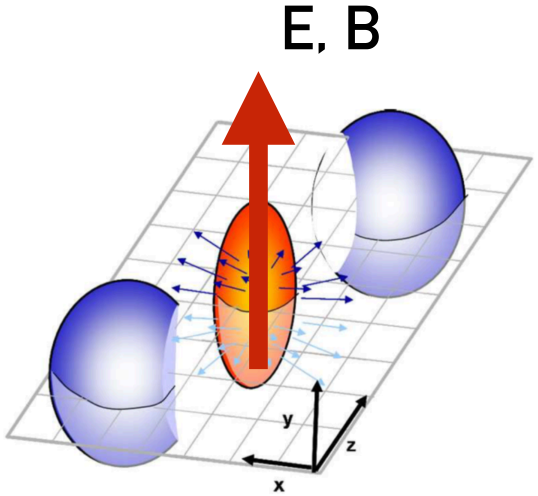
In collaboration with : Shile Chen and Prof. Pengfei Zhuang

S. Chen, J. Zhao, and P. Zhuang, Phys.Rev.C 103 (2021) 3, L031902.

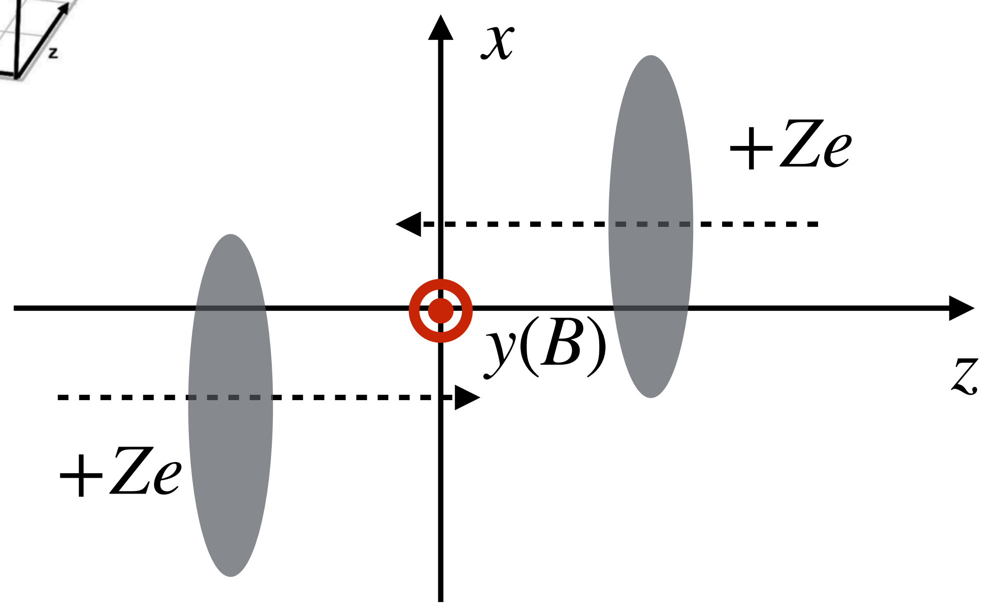
Outline

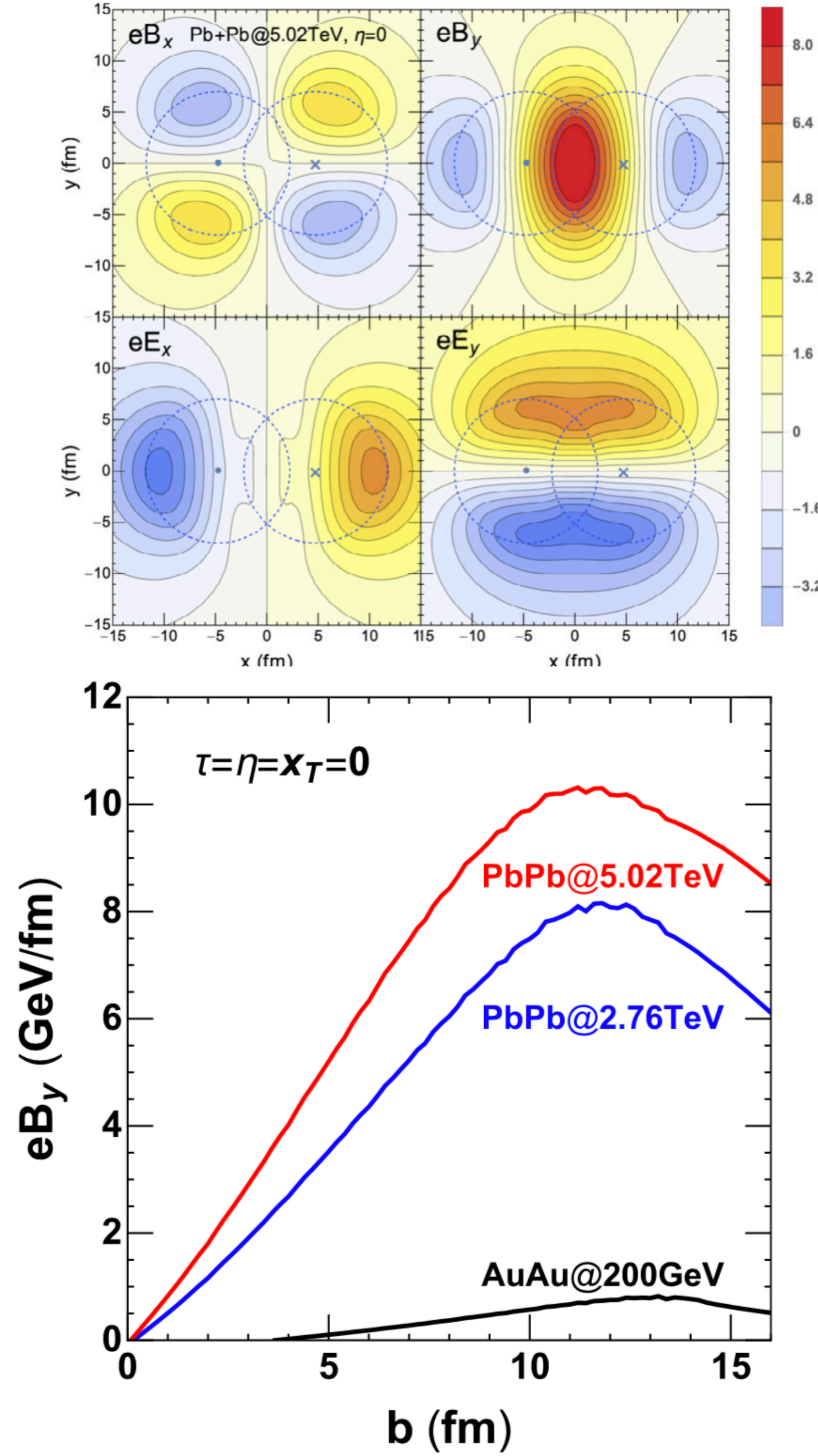
- *Brief introduction about the electromagnetic, vorticity fields and heavy flavor observations in heavy ion collisions*
- *Study the static properties of charmonium states in electromagnetic field, vorticity field, and both two fields.*
- *Summary and outlook*

Electromagnetic in HIC



$$eB_y \sim \gamma\beta\frac{Z\alpha}{R_A^2} \sim m_\pi^2$$

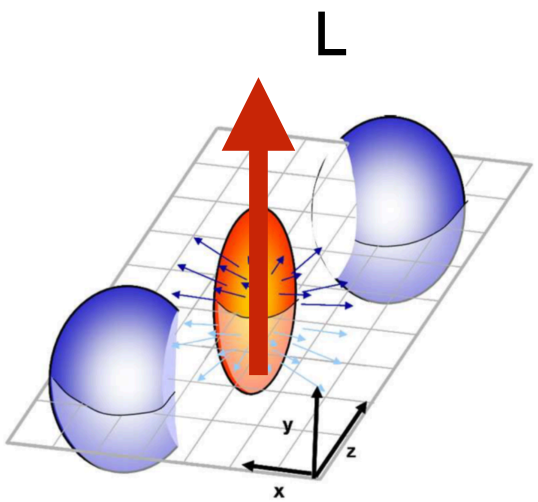




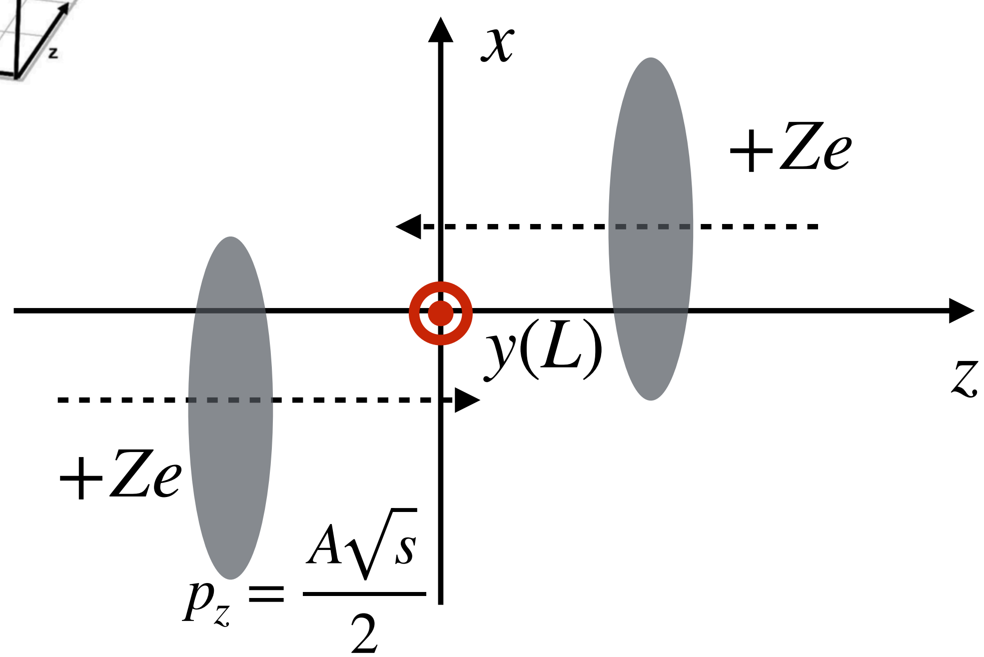
Strongest B field and E field generated by spectators and participant.
(~70m_π^2 at LHC and ~5m_π^2 at RHIC).

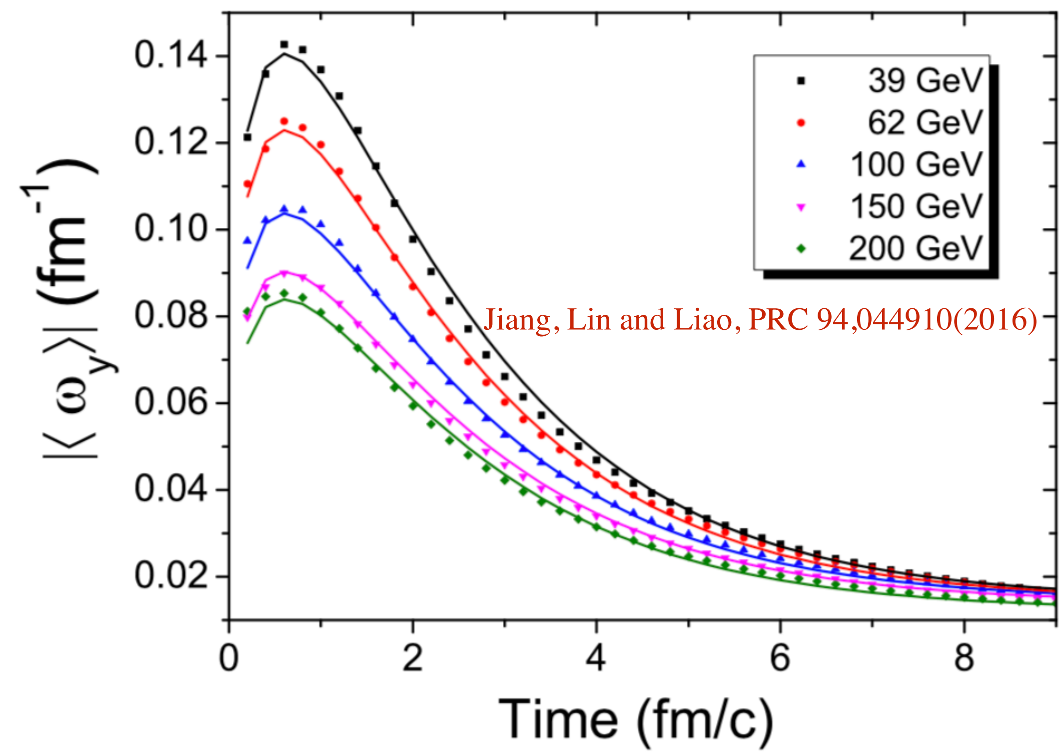
D. E. Kharzeev, L. D. McLerran, and H. J. Warringa, Nucl.Phys.A 803 (2008) 227-253
V. Skokov, A. Y. Illarionov and V. Toneev, Int. J. Mod. Phys. A 24, 5925-5932 (2009).
V. Voronyuk, and S. A. Voloshin. et al ,Phys.Rev.C 83 (2011) 054911 .
W. T. Deng and X. G. Huang, Phys. Rev. C 85, 044907 (2012).
K. Tuchin, Adv. High Energy Phys. 2013, 490495 (2013).
……

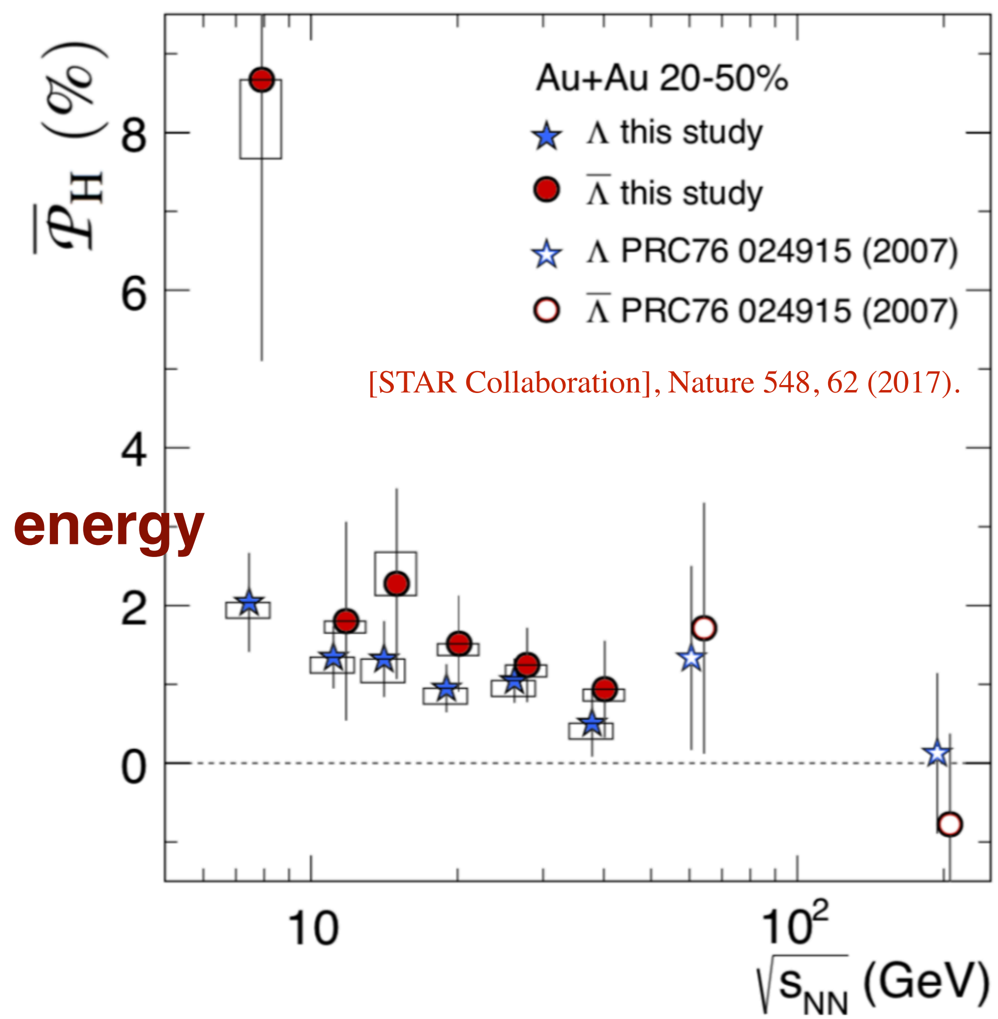
Vorticity in HIC



$$L_y = \frac{Ab\sqrt{s}}{2} \sim 10^{4\sim5}\hbar$$





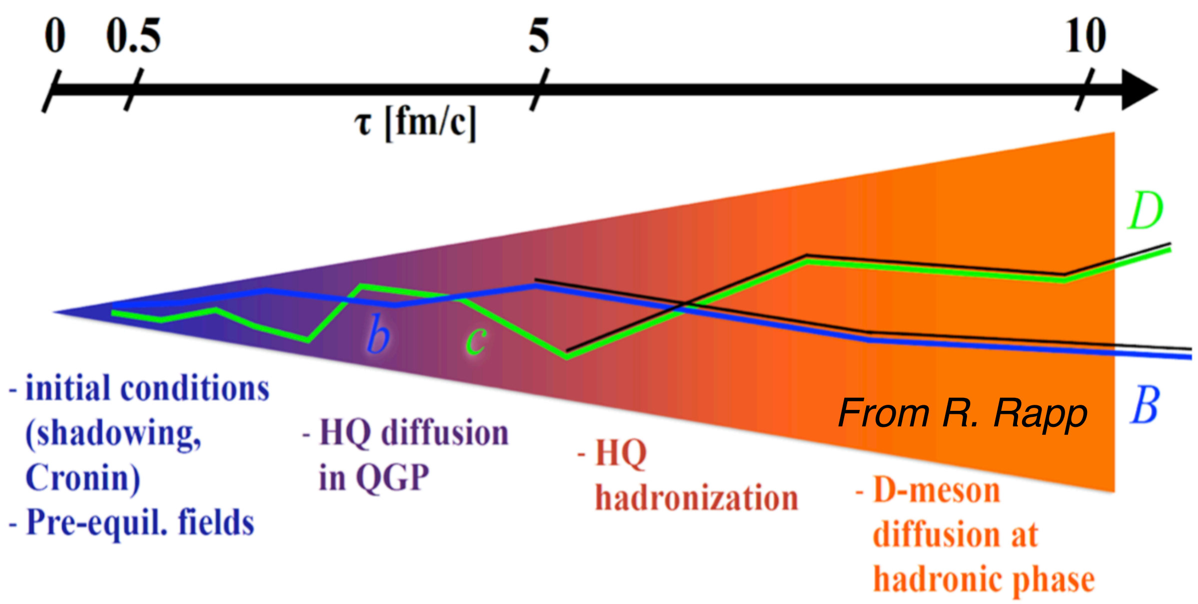


- **Huge angular momentum in non-central HIC.**
- **What fraction stays in QGP depends on the colliding energy**
- **The most vortical fluid !** $\omega \approx (9 \pm 1) \times 10^{21} s^{-1}$

Z. T. Liang and X. N. Wang, Phys. Rev. Lett. 94, 102301 (2005).
F. Becattini, et al. Phys. Rev. C 77, 024906 (2008); Eur. Phys. J. C 75 (2015) 9, 406.
Y. Jiang, Z. Lin, and J. Liao, Phys.Rev.C 94, 044910 (2016).
V. Skokov, A. Y. Illarionov and V. Toneev, Int. J. Mod. Phys. A 24, 5925-5932 (2009).
W.T.Deng and X.G.Huang, Phys. Rev. C 93 (2016) 064907.
Y.B. Ivanov and A.A. Soldatov, Phys. Rev. C 95 (2017) 054915.
Y.Xie, G.Chen, and L .P .Csernai. Eur. Phys. J. C 81 (2021) 1, 12. ...

Heavy Flavor: a sensitive probe to EM and vorticity fields

- *Mc, Mb >>* Λ_{QCD}, *produced by initial hard scattering and can be described by pQCD.*
- *Mass not change in QGP medium, number conserved. strong interaction with the hot medium.*
- *Heavy quarks produced at very early stage can feel strongest EM and vorticity fields*



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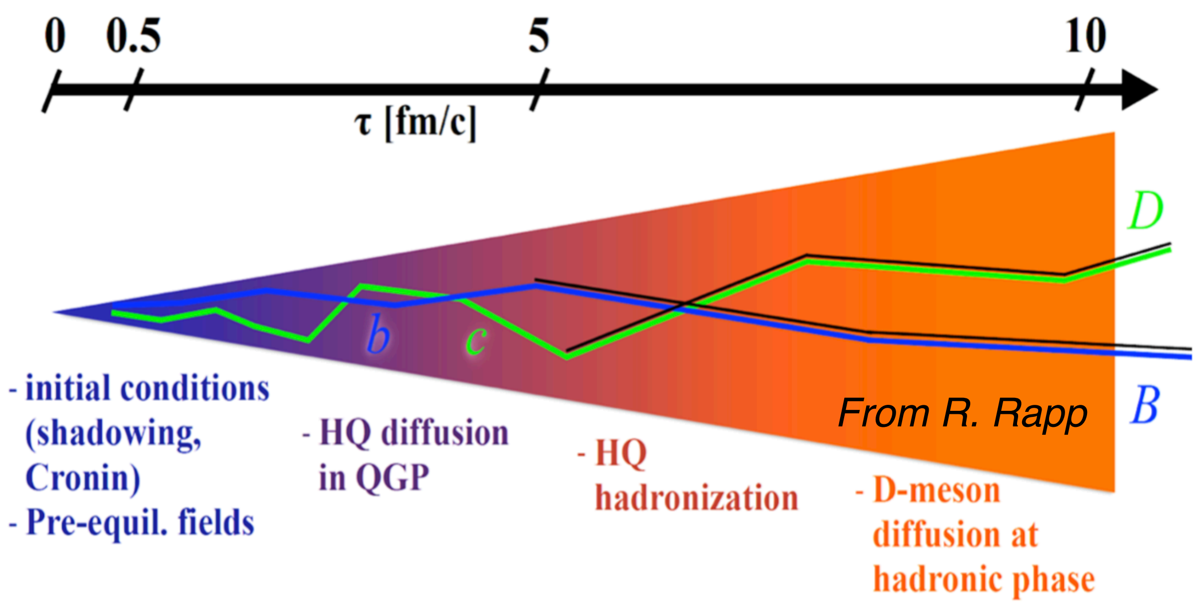

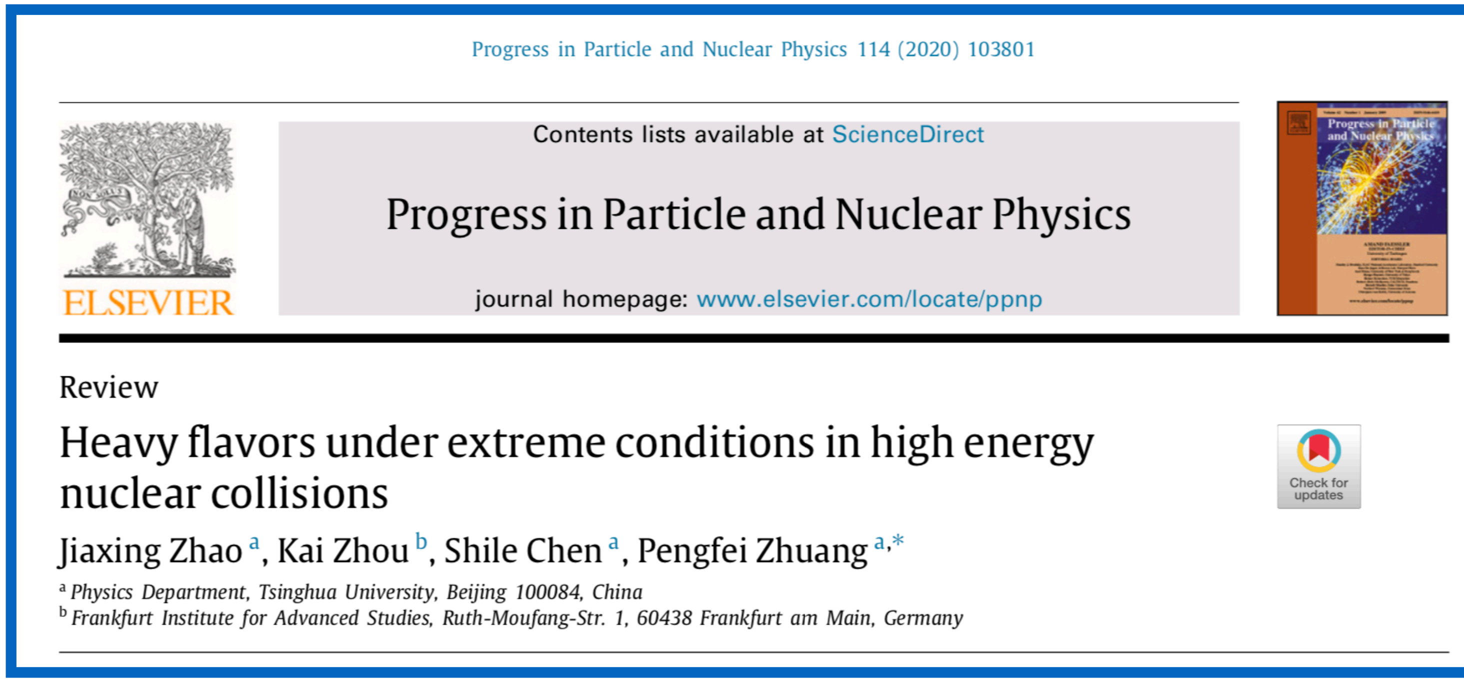

Progress in Particle and Nuclear Physics 114 (2020) 103801

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Heavy flavors under extreme conditions in high energy nuclear collisions

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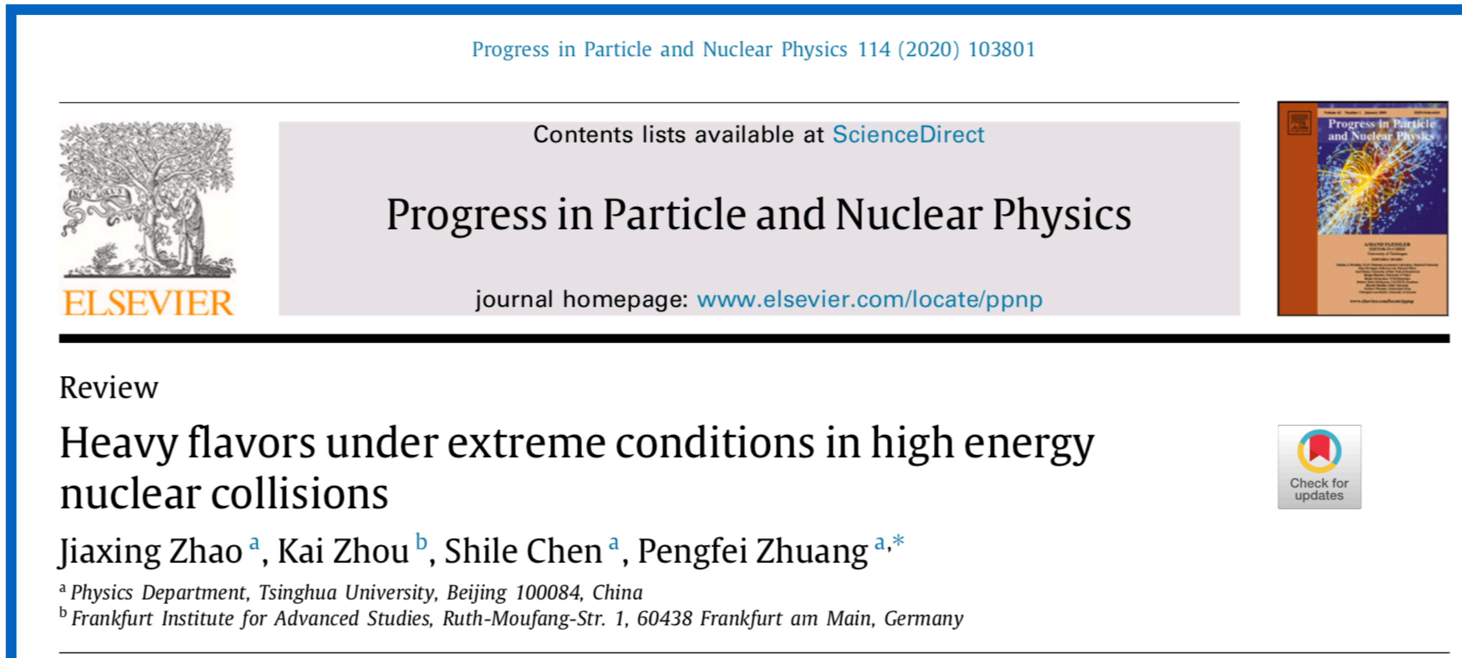
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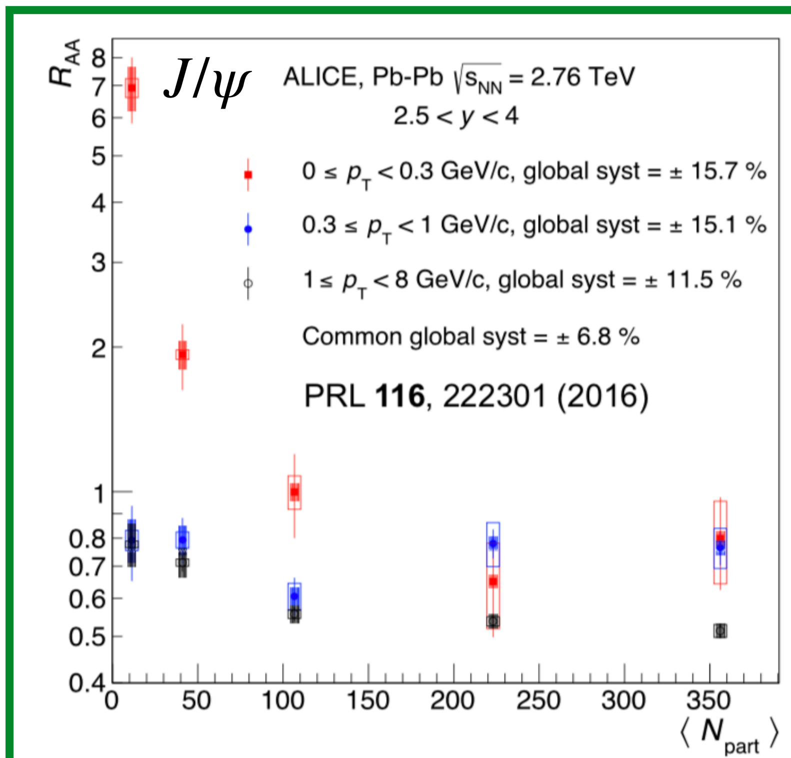

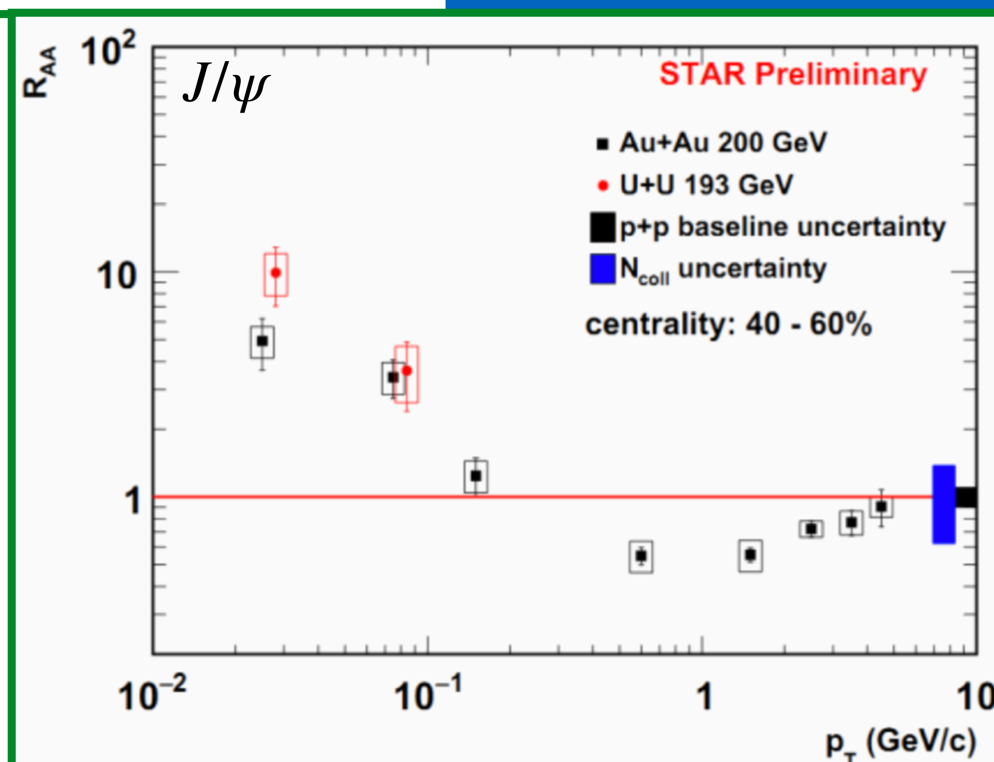

See also:

***W. Zha. Z. Tang, Y. Zhang. L. Ruan. Z. Xu. et al.** NPPP. 289-290, PLB 789 (2019) 238-242 ;*

***B. Chen. et al** (PLB. 777(2018)). …*

See the photoproduction in LHCb: J. Sun talk, Mon. Plenary

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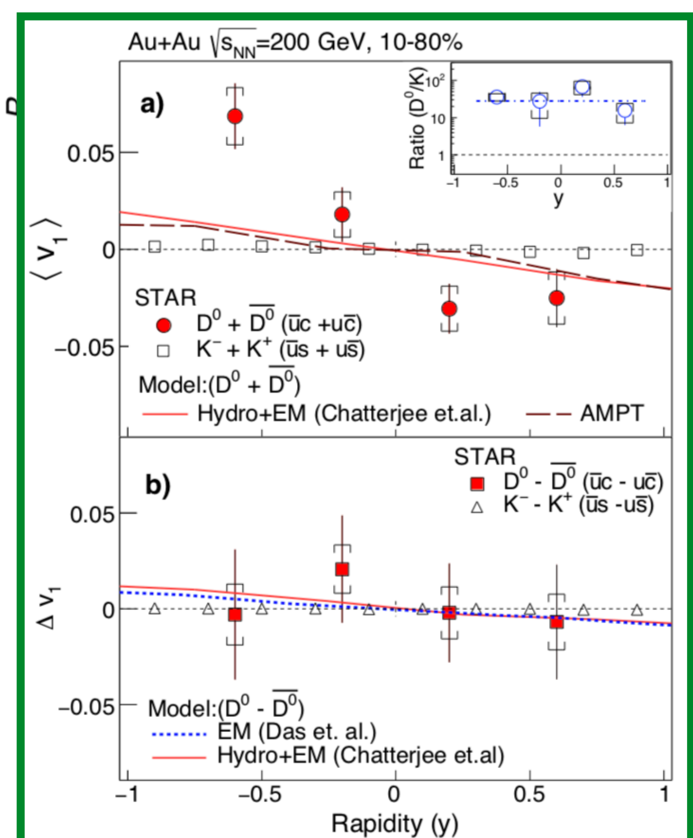

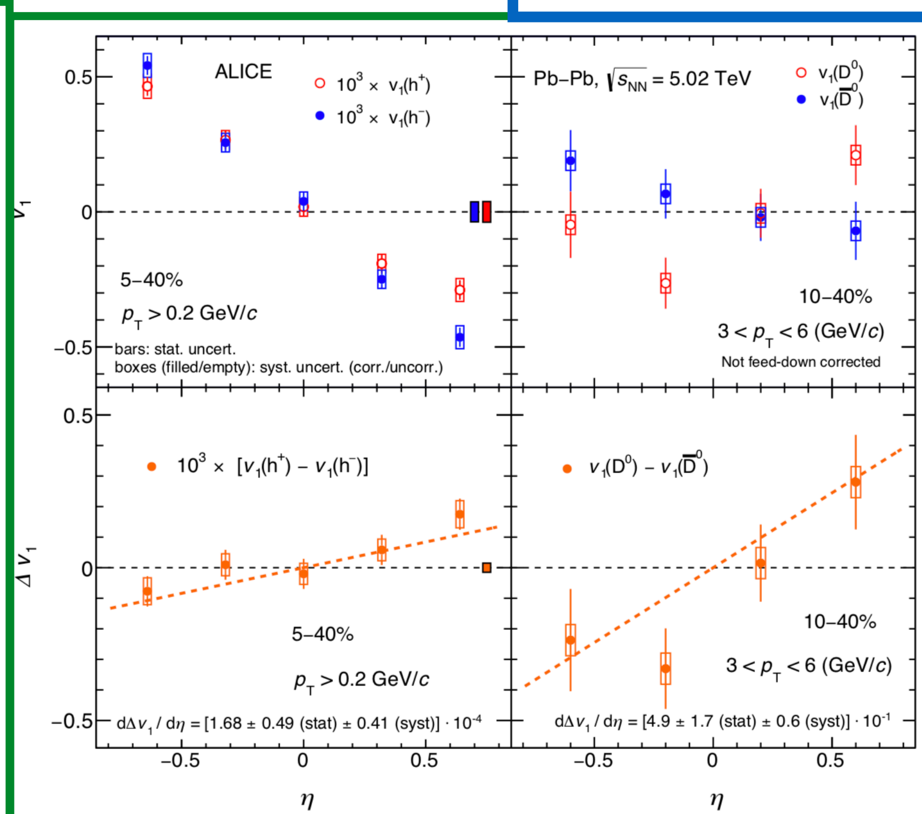

See also:

S. Chatterjee and P. Bozek.

PRL120(2018)192301;

Y. Sun, S. Das, S. Plumari, V. Greco. et al.

PLB768(2017) 260-264. PLB 816 (2021) 136271.

See: S. Plumari talk, Wed. Room C

M. Kurian talk, Tue. Room D

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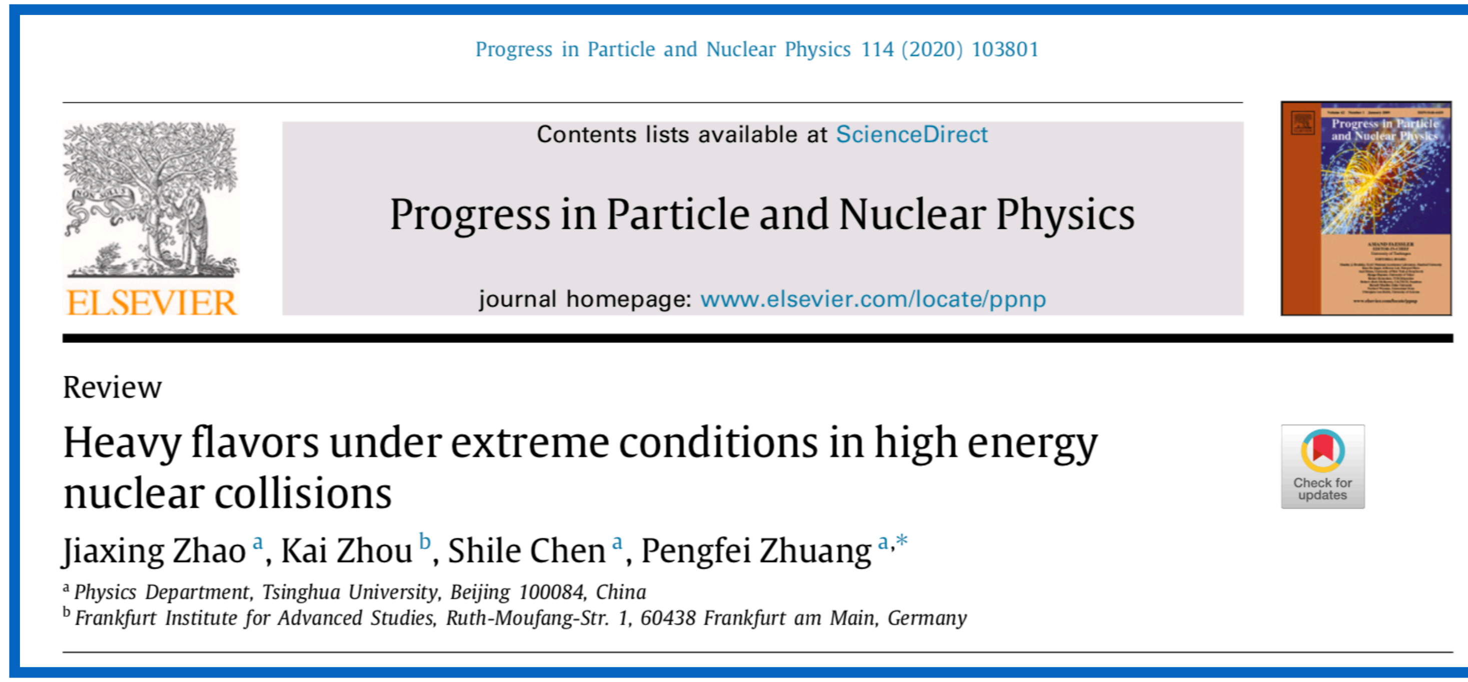
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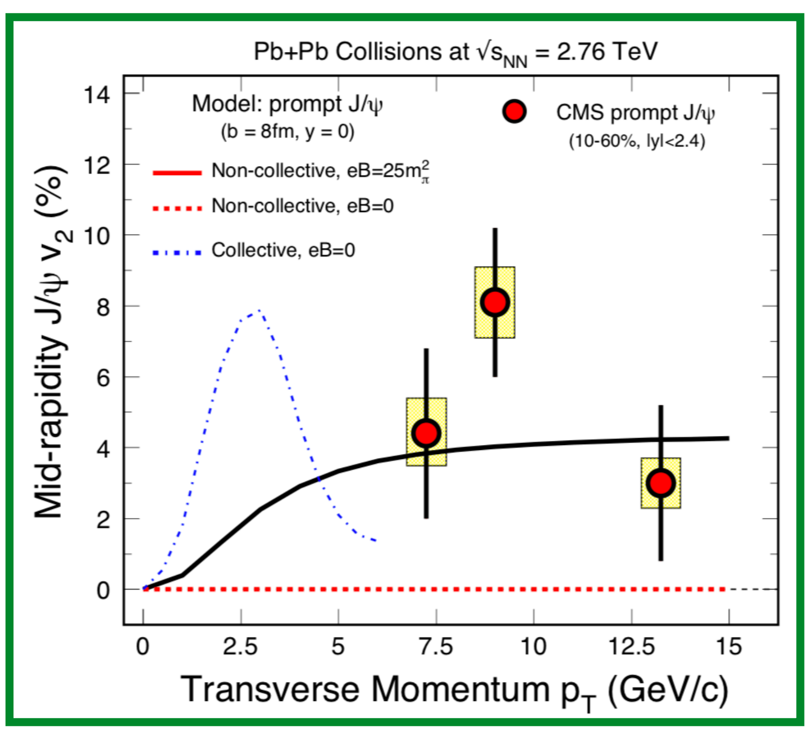
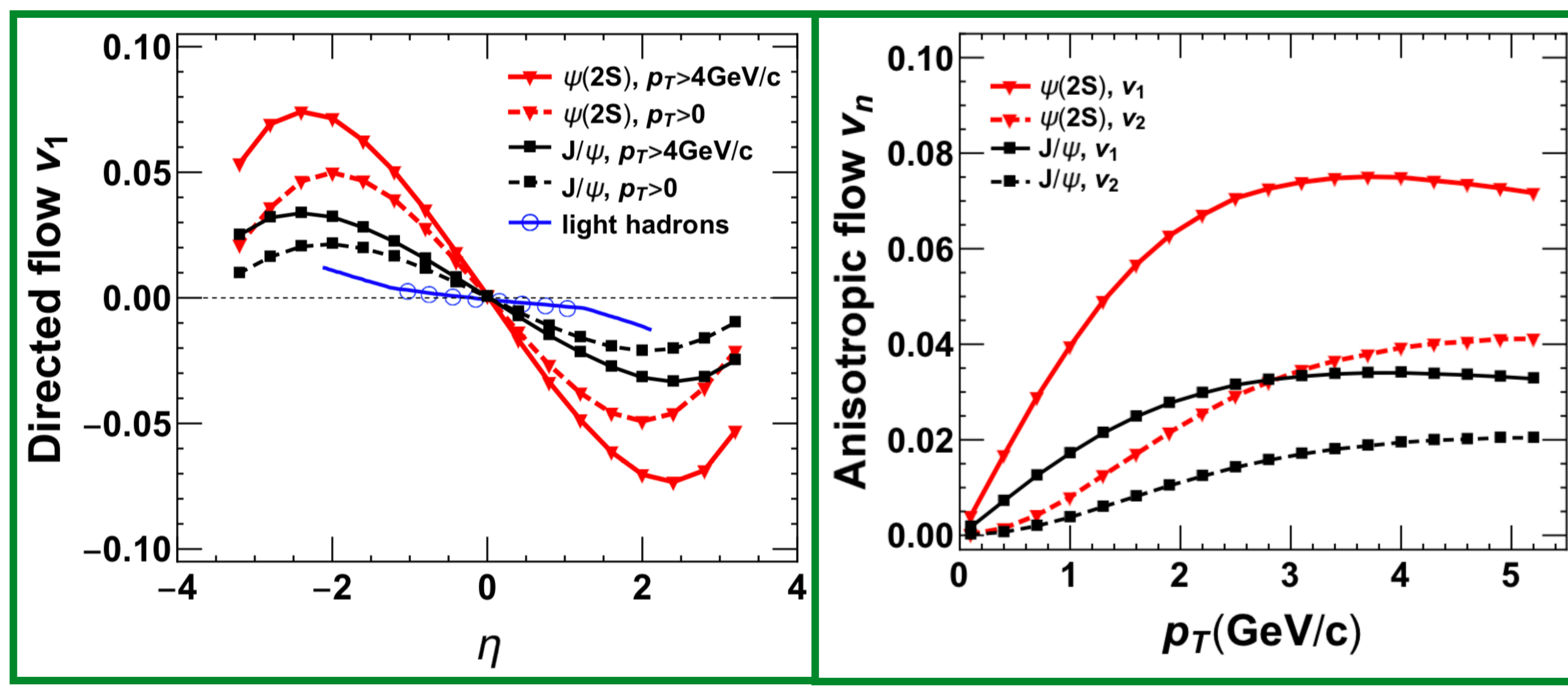


See detail: X. Guo, S. Shi, N. Xu, and P. Zhuang, *PLB751 (2015) 215-219.*



See detail: B. Chen, M. Hu, H. Zhang, and J. Zhao, *PLB802 (2020) 135271.*

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- **Change the static properties of heavy flavor hadrons**


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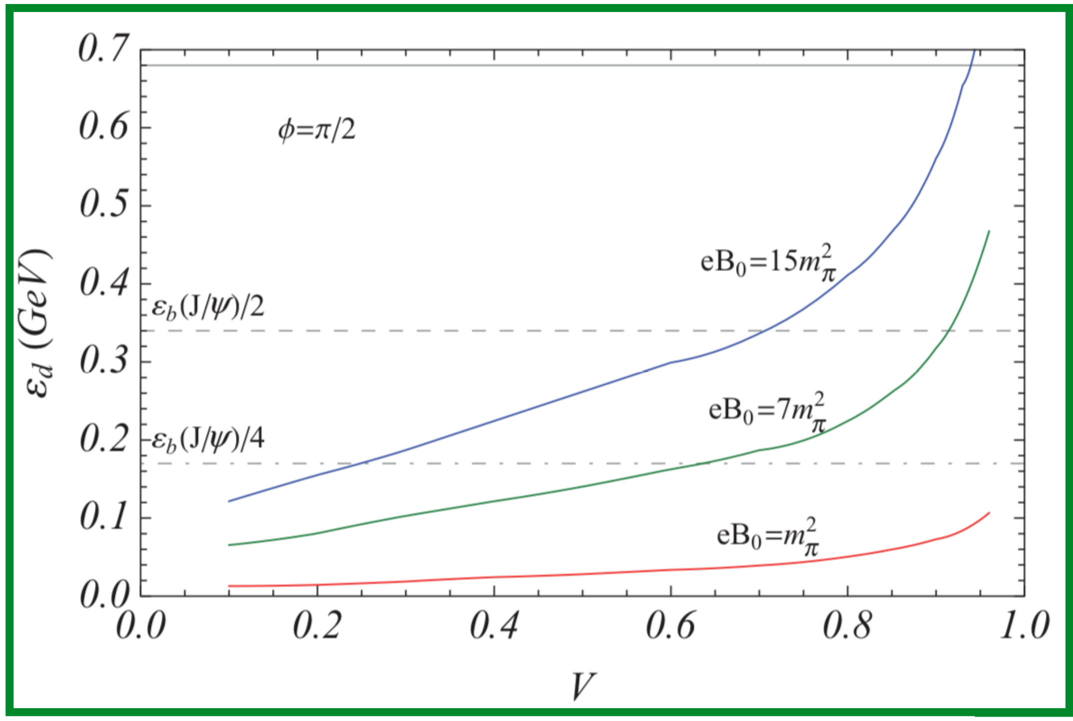
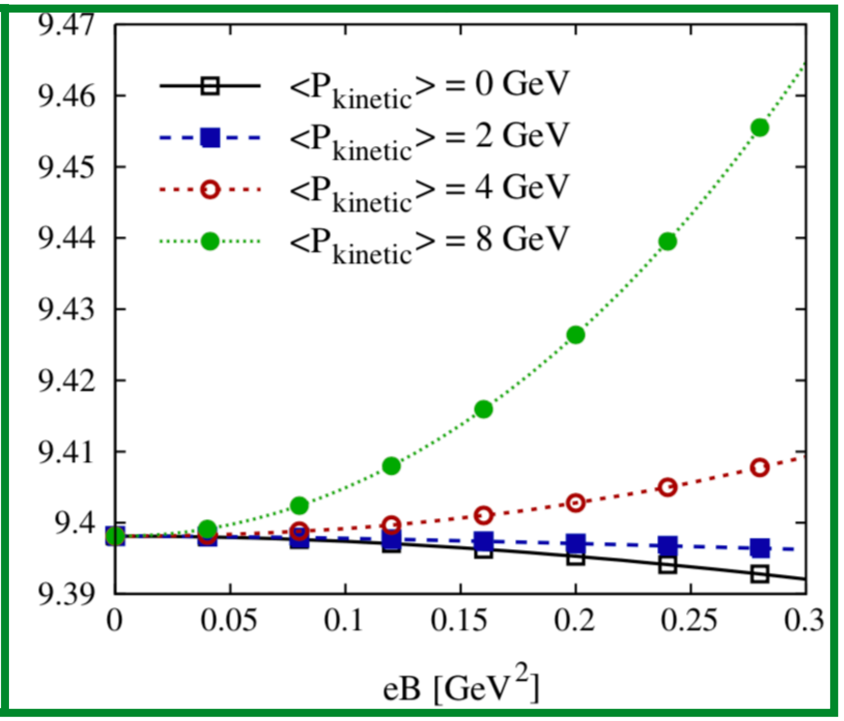




See detail:

K. Marasinghe and K. Tuchin,
PRC 84 (2011) 044908.

J.Alford and M.Strickland,
PRD88, 105017(2013).

S.Cho, K.Hattori, S.Lee, K.Morita and S.Ozaki,
PRL. 113, 172301(2014).

T.Yoshida and K.Suzuki,
PRD94, 074043(2016).

S. Iwasaki, M. Oka, K. Suzuki,
arXiv:2104.13990[A review]

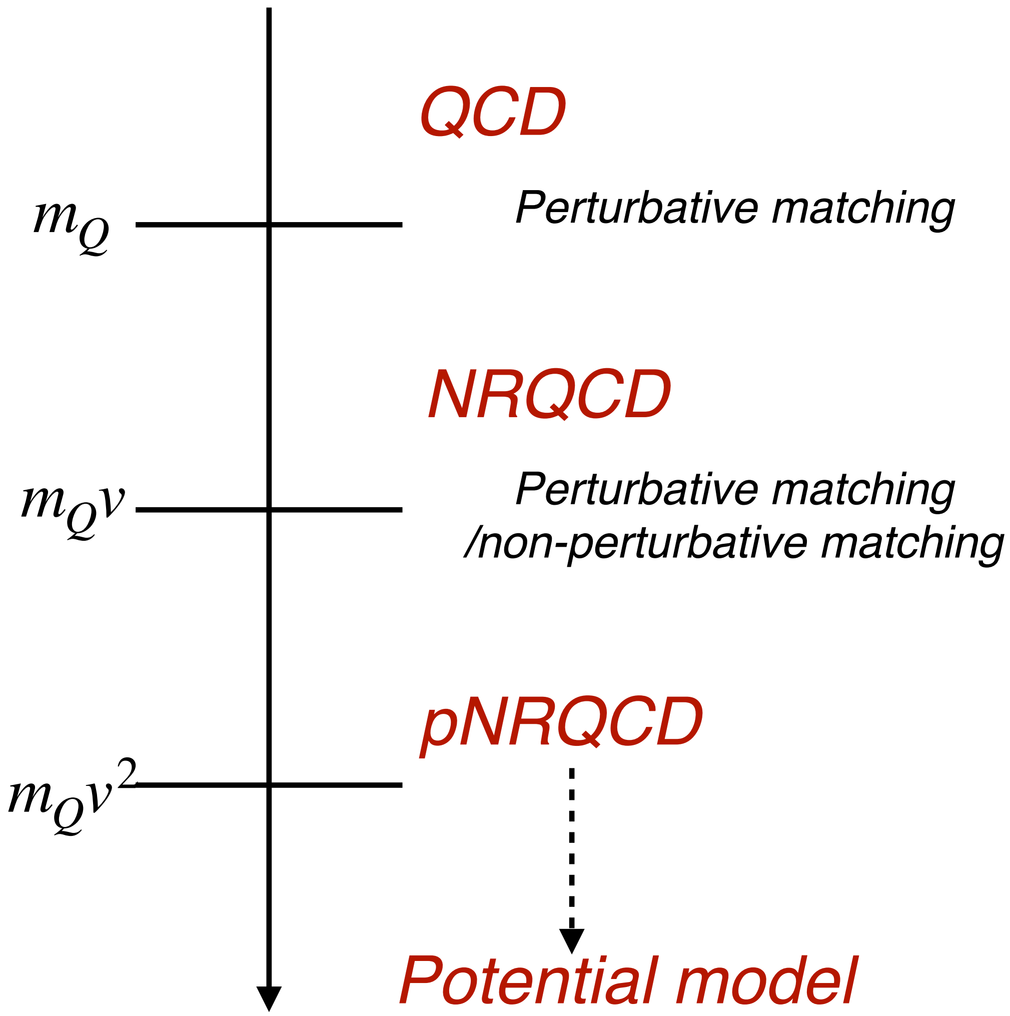
...

Heavy Flavor Effective Theory

$$m_c \sim 1.5 GeV, m_b \sim 4.7 GeV$$

Separation of scales:

$$m_Q \gg m_Q v \gg m_Q v^2$$



“Top - down”

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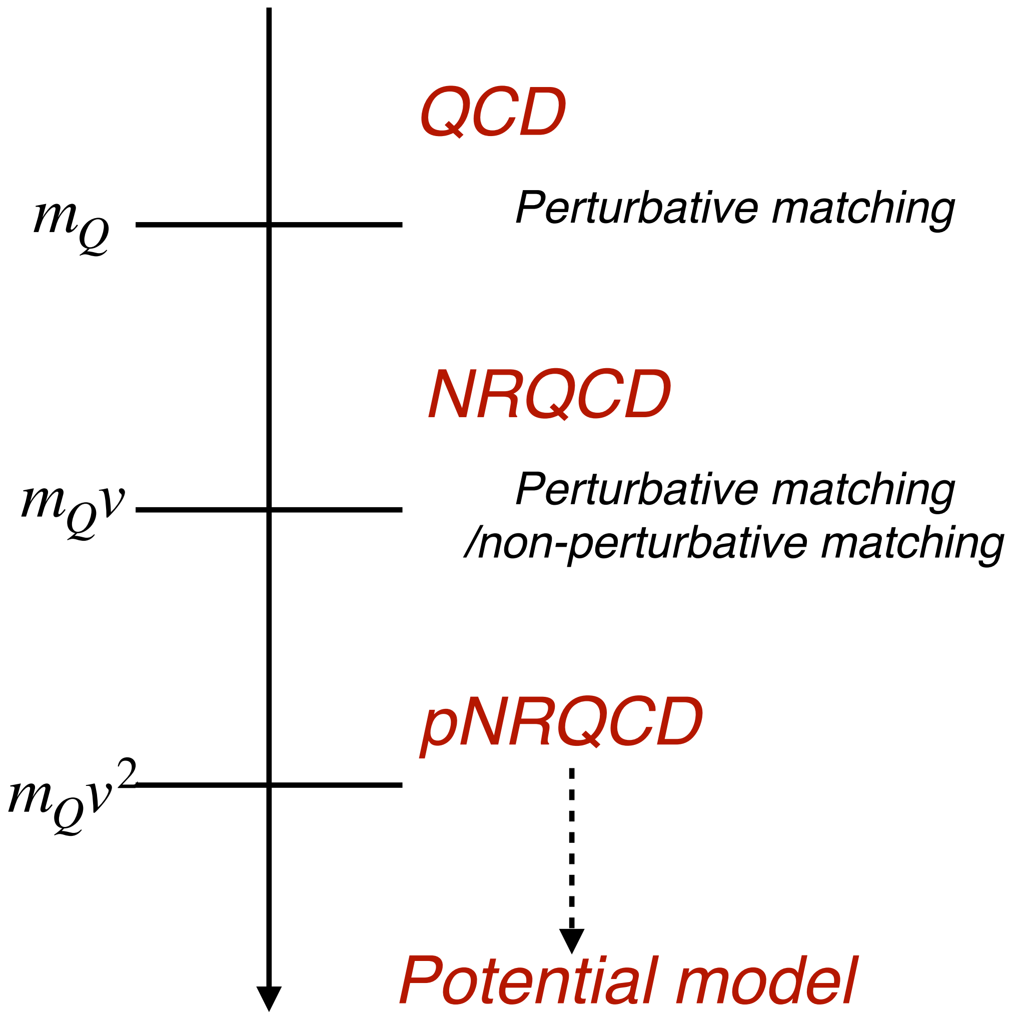

"Top - down"

Two-body Schroedinger equation:

$$\left[\frac{\hat{p}_1^2}{2m_1} + \frac{\hat{p}_2^2}{2m_2} + V(\mathbf{r}_1, \mathbf{r}_2)\right]\psi = E\psi$$

$$V = -\frac{\alpha}{|\mathbf{r}_1 - \mathbf{r}_2|} + \sigma|\mathbf{r}_1 - \mathbf{r}_2| + V_{ss}$$

Cornell potential + Spin-spin interaction

State	$\eta_c(1S)$	$J/\psi(1S)$	$h_c(1P)$	$\chi_c(1P)$	$\eta_c(2S)$	$\psi(2S)$	$h_c(2P)$	$\chi_c(2P)$
M_{Exp} (GeV)	2.981	3.097	3.525	3.556	3.639	3.686	–	3.927
M_{Th} (GeV)	2.967	3.102	3.480	3.500	3.654	3.720	3.990	4.000
$\langle r \rangle$ (fm)	0.365	0.427	0.635	0.655	0.772	0.802	0.961	0.980

State	$\eta_b(1S)$	$\Upsilon(1S)$	$h_b(1P)$	$\chi_b(1P)$	$\eta_b(2S)$	$\Upsilon(2S)$	$h_b(2P)$	$\chi_b(2P)$	$\Upsilon(3S)$
M_{Exp} (GeV)	9.398	9.460	9.898	9.912	9.999	10.023	–	10.269	10.355
M_{Th} (GeV)	9.397	9.459	9.845	9.860	9.957	9.977	10.211	10.221	10.325
$\langle r \rangle$ (fm)	0.200	0.214	0.377	0.387	0.465	0.474	0.597	0.603	0.680

***J. Zhao K. Zhou, S. Chen, P. Zhuang, PPNP*. 114 (2020) 103801.**

Can explain exp. data very well !

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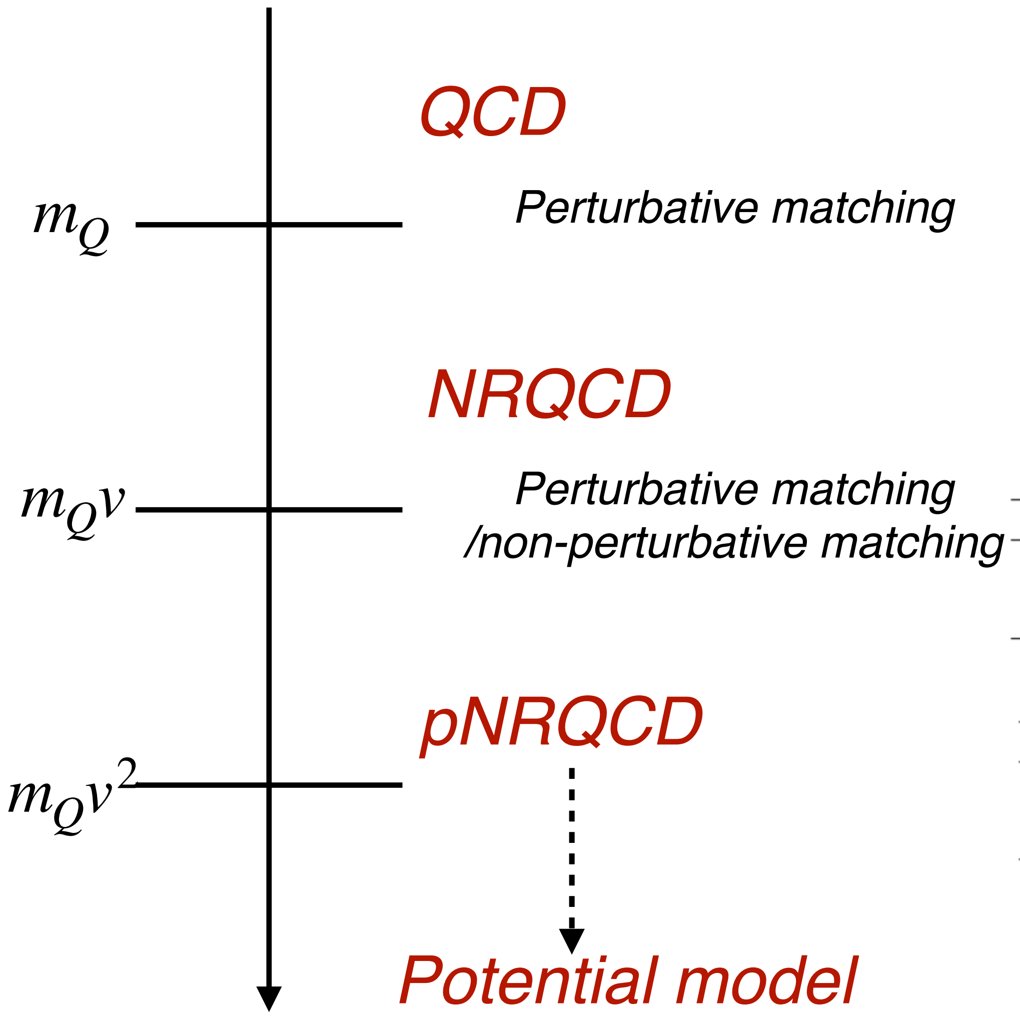

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So, how to consider EM / vorticity field in the framework of two-body SE.?

Two-body Schroedinger equation in EM field

$$\left[\frac{(\mathbf{p}_a - q_a\mathbf{A}_a)^2}{2m} + \frac{(\mathbf{p}_b - q_b\mathbf{A}_b)^2}{2m} - A_a^0 - A_b^0 - \boldsymbol{\mu}\cdot\mathbf{B} + V\right]\Psi(\mathbf{x}_a, \mathbf{x}_b) = E\Psi(\mathbf{x}_a, \mathbf{x}_b).$$

minimal coupling

to center-of-mass and relative coordinations.

$$\boxed{\begin{aligned} &\mathbf{R} \equiv (\mathbf{x}_a + \mathbf{x}_b)/2\,, \qquad \mathbf{P} \equiv (\mathbf{p}_a + \mathbf{p}_b) = -i\hbar\nabla_{\mathbf{R}}\,, \\ &\mathbf{r} \equiv (\mathbf{x}_a - \mathbf{x}_b)\,, \qquad \mathbf{p} \equiv (\mathbf{p}_a - \mathbf{p}_b)/2 = -i\hbar\nabla_{\mathbf{r}}\,. \end{aligned}}$$

$$\begin{aligned} &\frac{(\mathbf{p}_a - q_a\mathbf{A}_a)^2}{2m_q} + \frac{(\mathbf{p}_b - q_b\mathbf{A}_b)^2}{2m_q} \\ = \ &\frac{\mathbf{P}^2 + \frac{1}{4}q^2(\mathbf{B}\times\mathbf{r})^2 - q(\mathbf{B}\times\mathbf{r})\cdot\mathbf{P}}{4m_q} + \frac{\mathbf{p}^2 + \frac{1}{4}q^2(\mathbf{B}\times\mathbf{R})^2 - q(\mathbf{B}\times\mathbf{R})\cdot\mathbf{p}}{m_q} \end{aligned}$$

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$$\frac{(\mathbf{p}_a - q_a\mathbf{A}_a)^2}{2m_q} + \frac{(\mathbf{p}_b - q_b\mathbf{A}_b)^2}{2m_q}$$
$$= \frac{\mathbf{P}^2 + \frac{1}{4}q^2(\mathbf{B}\times\mathbf{r})^2 - q(\mathbf{B}\times\mathbf{r})\cdot\mathbf{P}}{4m_q} + \frac{\mathbf{p}^2 + \frac{1}{4}q^2(\mathbf{B}\times\mathbf{R})^2 - q(\mathbf{B}\times\mathbf{R})\cdot\mathbf{p}}{m_q}$$

$$[\mathbf{P}, H] \neq 0$$

Pseudo-momentum $\mathbf{P}_{ps} \equiv \sum_{i=1}^{2}\left(\mathbf{p}_i + q_i\mathbf{A}_i\right) = \mathbf{P} + \frac{1}{2}q\mathbf{B}\times\mathbf{r}$

$[\mathbf{P}_{ps}, H] = 0$ **Allows us to factorize the total wavefunction as:**

$$\Psi(\mathbf{R}, \mathbf{r}) = e^{i(\mathbf{P}_{ps} - \frac{1}{2}q\mathbf{B}\times\mathbf{r})\cdot\mathbf{R}}\psi(\mathbf{r})$$

See similar strategies:
H. Herold, H. Ruder, and G. Wunner, *J. Phys. B 14, 751(1981)*
J.Alford and M.Strickland, *PRD88, 105017(2013).*
T.Yoshida and K.Suzuki, *PRD94, 074043(2016).*

Two-body Schroedinger equation in EM field

J.Alford and M.Strickland, PRD 88, 105017(2013).

$$\left[\frac{\mathbf{p}^2}{m} - \boldsymbol{\mu}\cdot\mathbf{B} - q\mathbf{E}\cdot\mathbf{r} - \frac{q}{2m}(\mathbf{P}_{ps}\times\mathbf{B})\cdot\mathbf{r} + \frac{q^2}{4m}(\mathbf{B}\times\mathbf{r})^2 + V\right]\psi(\mathbf{r}) = \left[E - \frac{\mathbf{P}_{ps}^2}{4m}\right]\psi(\mathbf{r})$$

Conserved pseudo-momentum

Cornell + spin-spin potential

Harmonic potential which is quadratic in qB

Lorentz potential which derivation is Lorentz force

Electric potential which derivation is electric force

Magnetic field - spin magnetic moment coupling

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J.Alford and M.Strickland, PRD 88, 105017(2013).

$$\left[\frac{\mathbf{p}^2}{m} - \boldsymbol{\mu}\cdot\mathbf{B} - q\mathbf{E}\cdot\mathbf{r} - \frac{q}{2m}(\mathbf{P}_{ps}\times\mathbf{B})\cdot\mathbf{r} + \frac{q^2}{4m}(\mathbf{B}\times\mathbf{r})^2 + V\right]\psi(\mathbf{r}) = \left[E - \frac{\mathbf{P}_{ps}^2}{4m}\right]\psi(\mathbf{r})$$

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Two-body Schroedinger equation in EM field

J.Alford and M.Strickland, PRD 88, 105017(2013).

$$\left[\frac{\mathbf{p}^2}{m} - \boldsymbol{\mu}\cdot\mathbf{B} - q\mathbf{E}\cdot\mathbf{r} \cancel{- \frac{q}{2m}(\mathbf{P}_{ps}\times\mathbf{B})\cdot\mathbf{r}} + \frac{q^2}{4m}(\mathbf{B}\times\mathbf{r})^2 + V\right]\psi(\mathbf{r}) = \left[E - \frac{\mathbf{P}_{ps}^2}{4m}\right]\psi(\mathbf{r})$$

Conserved pseudo-momentum

Cornell + spin-spin potential

Harmonic potential which is quadratic in qB

Lorentz potential which derivation is Lorentz force

Electric potential which derivation is electric force

Magnetic field - spin magnetic moment coupling

$$\left[\frac{\mathbf{p}_a^2}{2m} + \frac{\mathbf{p}_b^2}{2m} + V(r,\theta,B)\right]\Psi(\mathbf{x}_a,\mathbf{x}_b) = E\Psi(\mathbf{x}_a,\mathbf{x}_b)$$

In the static case

$$\boxed{\begin{aligned} V_{eff}(\mathbf{r}) &= V(r) + \frac{q^2(\mathbf{B}\times\mathbf{r})^2}{4m} \\ &= V(r) + \frac{q^2B^2r^2\sin^2\theta}{4m}. \end{aligned}}$$

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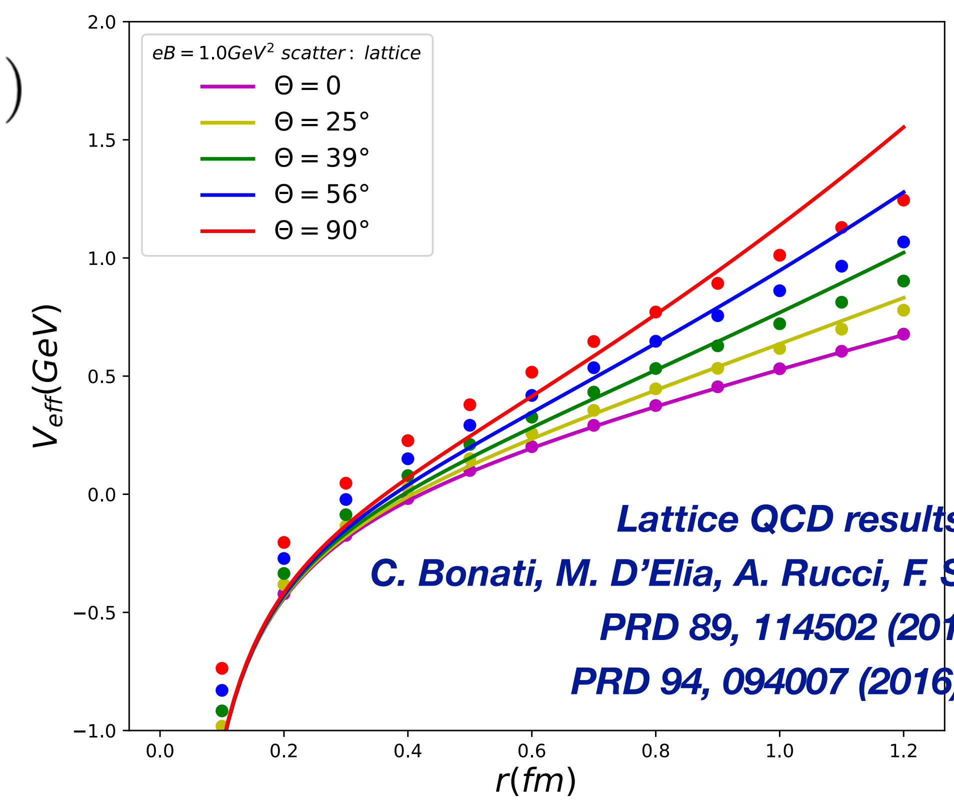

Lattice QCD results:
C. Bonati, M. D'Elia, A. Rucci, F. Sanfilippo, el al.
PRD 89, 114502 (2014)
PRD 94, 094007 (2016). …

Potential becomes anisotropic(B field breaks down the rotational symmetry).
The potential increases as the angle between the $Q\bar{Q}$ separation and B increases.

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We take $\mathbf{B}$ *in the direction of* e_z, P_{ps} *in the transverse plane perpendicular to the* $\mathbf{B}$.

$$\psi(\mathbf{r}, s_c, s_{\bar{c}}) = \psi(\mathbf{r})\,|s, s_z\rangle \qquad |s, s_z\rangle = |0,0\rangle, |1,0\rangle, |1, \pm 1\rangle$$

$$\mathbf{B}\cdot(s_c - s_{\bar{c}})|0,0\rangle = B|1,0\rangle$$
$$\mathbf{B}\cdot(s_c - s_{\bar{c}})|1,0\rangle = B|0,0\rangle$$
$$\mathbf{B}\cdot(s_c - s_{\bar{c}})|1,\pm 1\rangle = 0$$

Orbital angular momentum not conserve anymore.

Two-body Schroedinger equation in EM field

J.Alford and M.Strickland, PRD 88, 105017(2013).

$$\left[\frac{\mathbf{p}^2}{m} - \boldsymbol{\mu}\cdot\mathbf{B} - q\mathbf{E}\cdot\mathbf{r} - \frac{q}{2m}(\mathbf{P}_{ps}\times\mathbf{B})\cdot\mathbf{r} + \frac{q^2}{4m}(\mathbf{B}\times\mathbf{r})^2 + V\right]\psi(\mathbf{r}) = \left[E - \frac{\mathbf{P}_{ps}^2}{4m}\right]\psi(\mathbf{r})$$

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$$\mathbf{B}\cdot(s_c - s_{\bar{c}})\,|\,0,0\rangle = B\,|\,1,0\rangle$$
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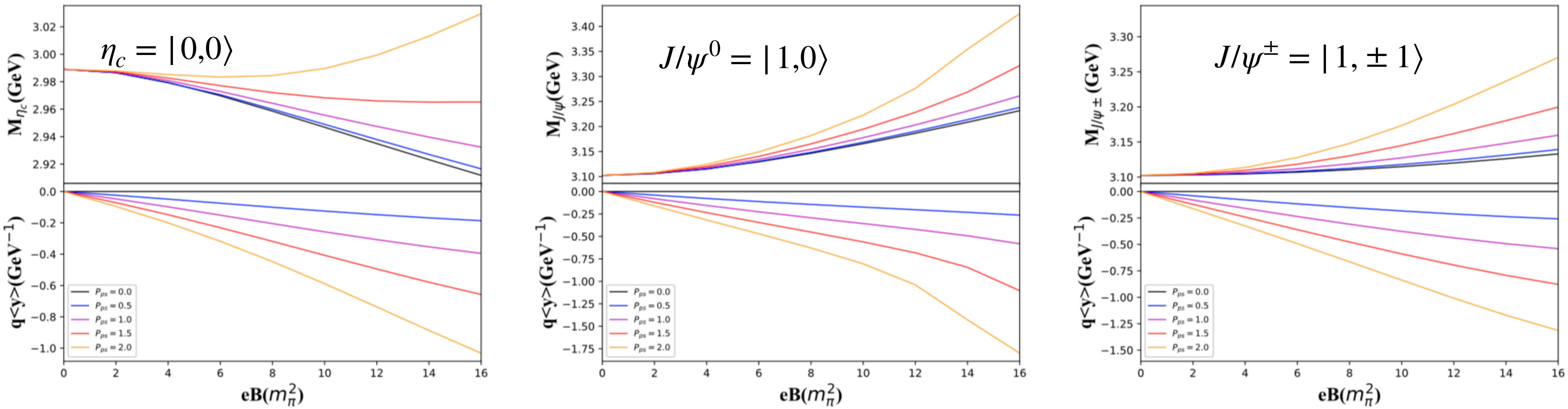
$$r\psi(\mathbf{r},\ 1, \pm 1) = \sum_{lm} a_{lm}u_{lm}(r)Y_{lm}(\theta,\phi)\,|\,1,\pm 1\rangle$$
$$r\psi(\mathbf{r},\ s,\ 0) = \sum_{lm}\left[a_{lm}u_{lm}(r)Y_{lm}(\theta,\phi)\,|\,1,0\rangle + a'_{lm}u'_{lm}(r)Y_{lm}(\theta,\phi)\,|\,0,0\rangle\right]$$

Expand the three-dimensional wave-function in series of spherical harmonics

And solve numerically

Two-body Schroedinger equation in EM field

only B

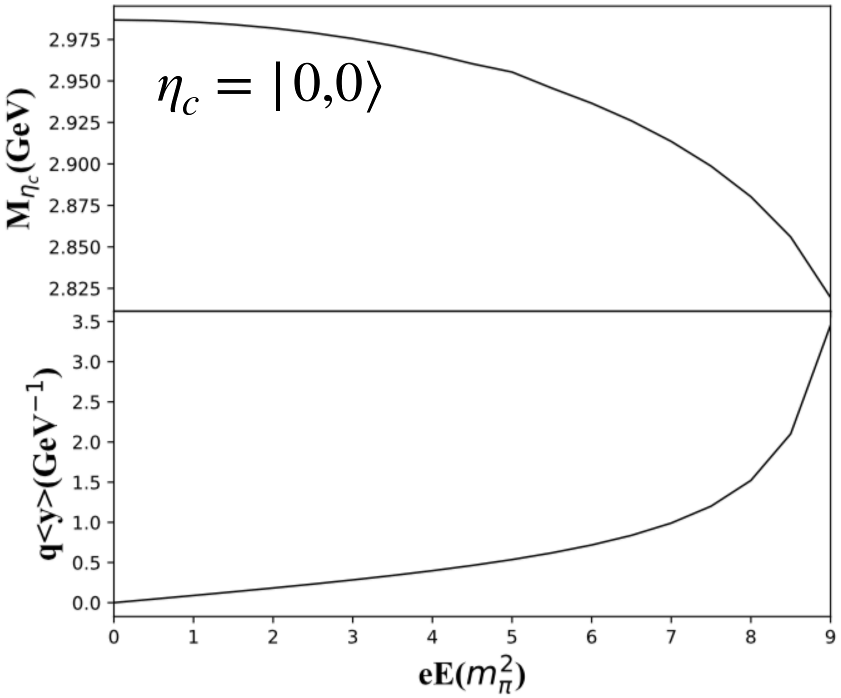


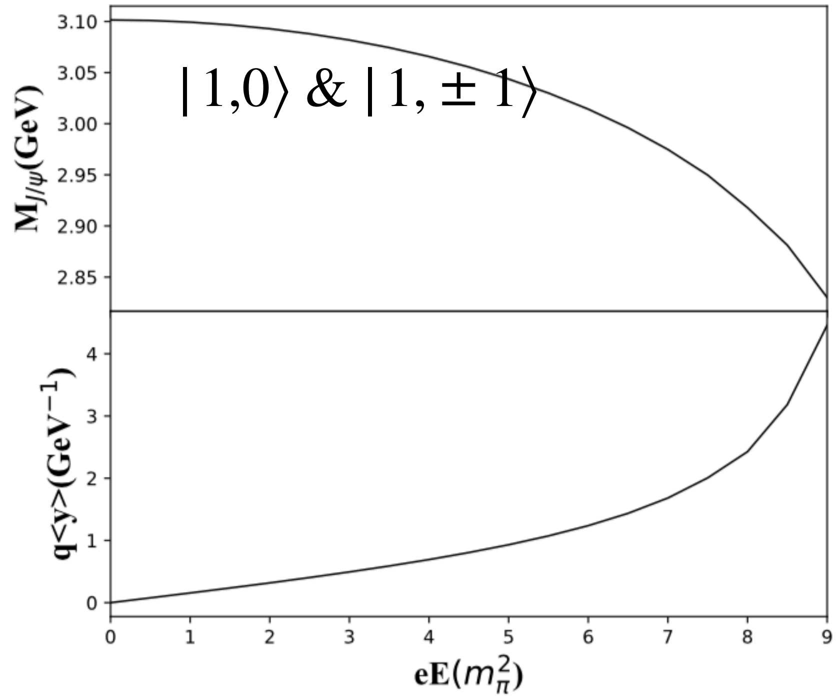
- **The charmonium mass depends on both the strength of B field and pseudo-momentum.**
- **Spin is not conserved, a splitting and coupling of different spin states.**

See also: *[QCD sum rules] S.Cho, K.Hattori, S.Lee, K.Morita and S.Ozaki, PRL. 113, 172301(2014)*
[potential model] X. Guo, S. Shi, N. Xu, and P. Zhuang, PLB751 (2015) 215-219....

Two-body Schroedinger equation in EM field

only E





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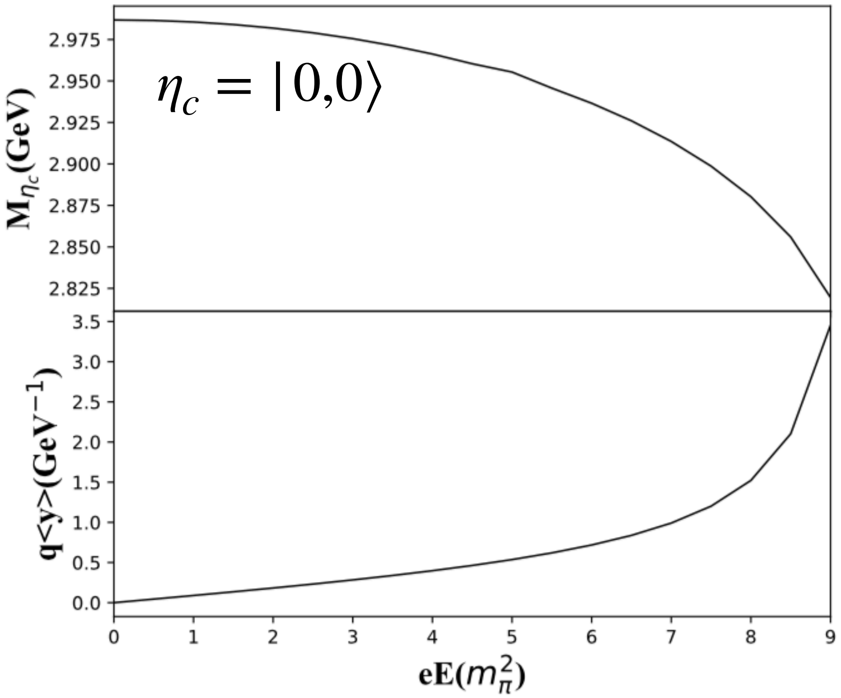
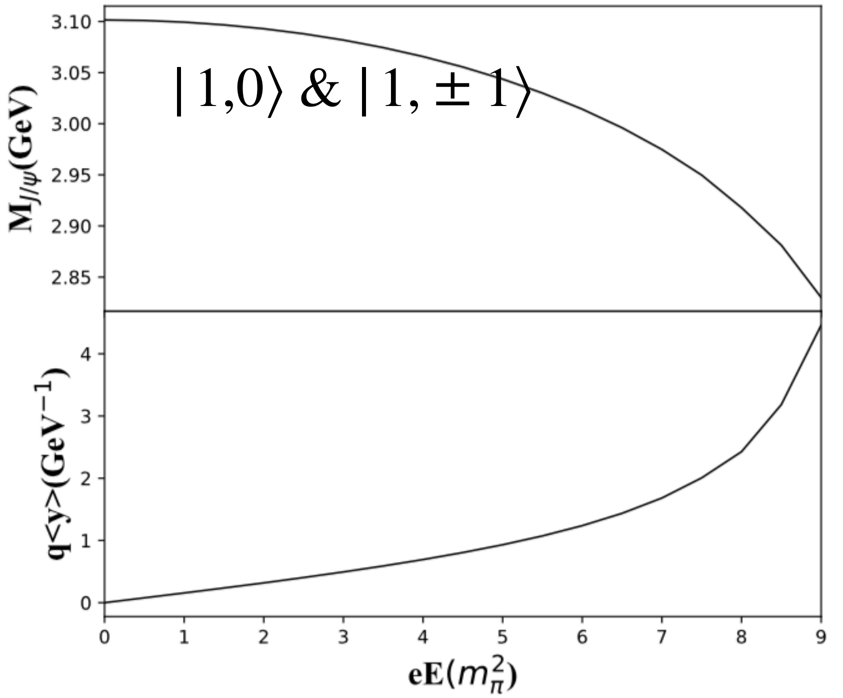
Two-body Schroedinger equation in EM field

only E





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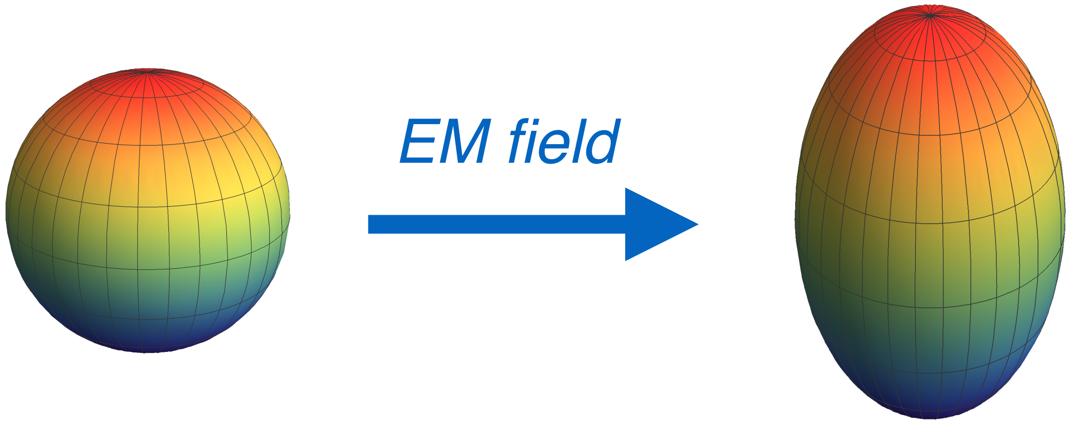

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[potential model] X. Guo, S. Shi, N. Xu, and P. Zhuang, PLB751 (2015) 215-219....

Two-body Schroedinger equation in vorticity field

For a fermion system in rotating frame:

$$\mathcal{L} = \bar{\psi}\left(i\gamma^\mu\partial_\mu + \gamma_0\boldsymbol{\omega}\cdot\boldsymbol{j} - m\right)\psi$$

Y. Jiang, J. Liao. PRL. 117 (2016) 19, 192302. ...

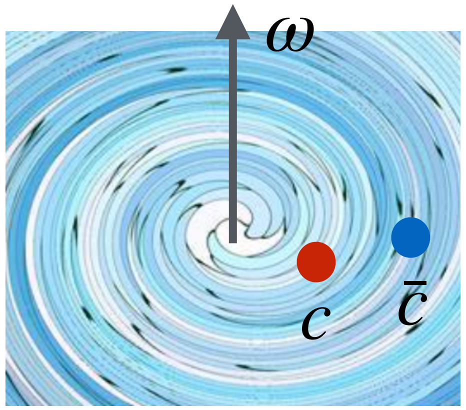
$$\boldsymbol{j} = \boldsymbol{l} + \boldsymbol{s} \qquad \boldsymbol{l} = \boldsymbol{x}\times\boldsymbol{p},\ \boldsymbol{s} = -\gamma_0\gamma_5\boldsymbol{\gamma}/2 = \mathrm{diag}(\boldsymbol{\sigma},\boldsymbol{\sigma})/2$$

Dirac equation in rotating frame:

$$\left(i\gamma^\mu\partial_\mu + \gamma_0\boldsymbol{\omega}\cdot\boldsymbol{j} - m\right)\psi = 0$$

Move to the non-relativistic limit, and to the first order in 1/m :

$$\left(\frac{\boldsymbol{p}^2}{2m} - \boldsymbol{\omega}\cdot\boldsymbol{j}\right)\psi = E\psi$$



Two-body Schroedinger equation for charmonium system:

$$\left[\frac{\mathbf{p}_a^2}{2m} + \frac{\mathbf{p}_b^2}{2m} - \boldsymbol{\omega}\cdot(\mathbf{j}_a + \mathbf{j}_b) + V\right]\Psi(\mathbf{x}_a,\mathbf{x}_b) = E\Psi(\mathbf{x}_a,\mathbf{x}_b)$$

$$\mathbf{R} \equiv (\mathbf{x}_a + \mathbf{x}_b)/2\,, \qquad \mathbf{P} \equiv (\mathbf{p}_a + \mathbf{p}_b) = -i\hbar\nabla_{\mathbf{R}}\,,$$
$$\mathbf{r} \equiv (\mathbf{x}_a - \mathbf{x}_b)\,, \qquad \mathbf{p} \equiv (\mathbf{p}_a - \mathbf{p}_b)/2 = -i\hbar\nabla_{\mathbf{r}}\,.$$

$$\left[\frac{\mathbf{P}^2}{4m} - \boldsymbol{\omega}\cdot(\mathbf{R}\times\mathbf{P}) + \frac{\mathbf{p}^2}{m} - \boldsymbol{\omega}\cdot(\mathbf{r}\times\mathbf{p} + \mathbf{s}) + V\right]\Psi(\mathbf{x}_a,\mathbf{x}_b) = E\Psi(\mathbf{x}_a,\mathbf{x}_b)$$

Two-body Schroedinger equation in vorticity field

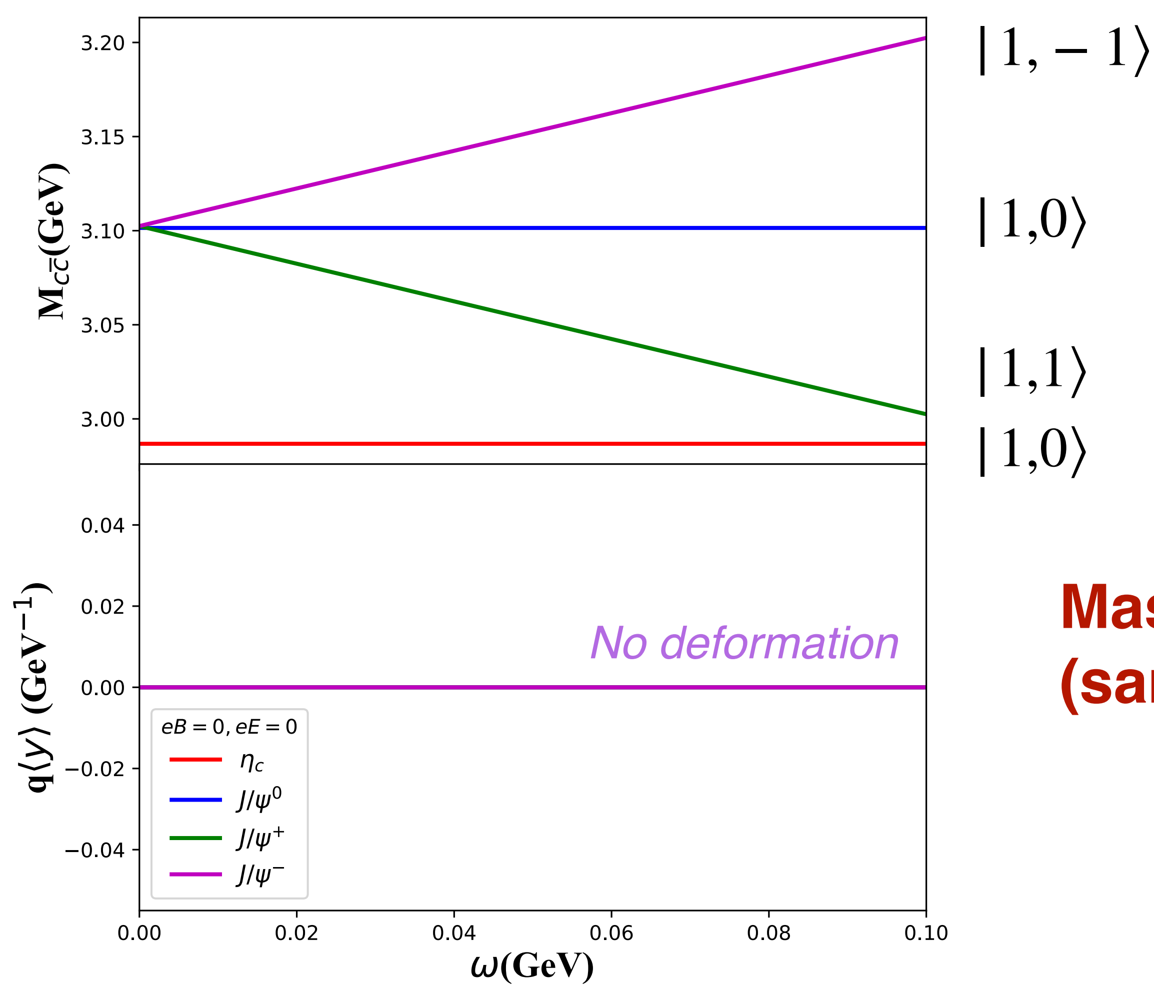
$$\left[\frac{\mathbf{P}^2}{4m} - \boldsymbol{\omega}\cdot(\mathbf{R}\times\mathbf{P}) + \frac{\mathbf{p}^2}{m} - \boldsymbol{\omega}\cdot(\mathbf{r}\times\mathbf{p}+\mathbf{s}) + V\right]\Psi(\mathbf{x}_a,\mathbf{x}_b) = E\Psi(\mathbf{x}_a,\mathbf{x}_b)$$

$$\mathbf{s} = \mathbf{s}_a + \mathbf{s}_b$$

Although total momentum P is not conserved, the total wave function can still be factorized as a center-of-mass and a relative part :

$$\left(\frac{\boldsymbol{p}^2}{m} - \boldsymbol{\omega}\cdot(\boldsymbol{l}+\boldsymbol{s}) + V_c + V_{ss}\right)\psi = \epsilon\psi$$

Corresponds to the Coriolis force $\mathbf{F} = -\boldsymbol{\nabla}(-\boldsymbol{\omega}\cdot\boldsymbol{l}) = -\boldsymbol{\omega}\times\mathbf{p}$



Mass splitting for spin triplet state, shifted (same for p-wave states)

Two-body Schroedinger equation in both EM and vorticity fields

In curved spacetime with EM field:

$$\mathcal{L} = \bar{\psi}\left[i\gamma^{\mu}(\partial_{\mu} + iqA_{\mu} + \Gamma_{\mu}) - m\right]\psi$$

$$\Gamma_{\mu} = -\frac{i}{4}\omega_{\mu ij}\sigma^{ij}\ ,$$
$$\omega_{\mu ij} = g_{\alpha\beta}e_i^{\alpha}(\partial_{\mu}e_j^{\beta} + \Gamma_{\mu\nu}^{\beta}e_j^{\mu})$$
$$\sigma^{ij} = \frac{i}{2}[\gamma^i, \gamma^j]\ .$$

affine connection

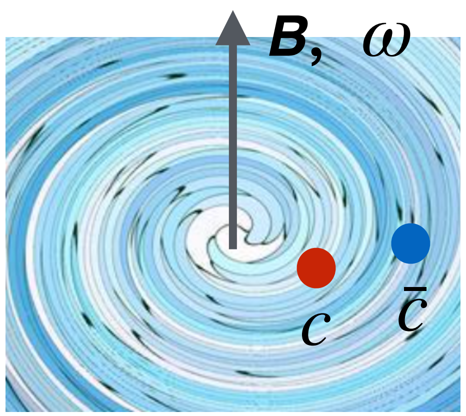
See detail:
Y. Jiang, J. Liao.
PRL. 117 (2016) 19, 192302.
H. Chen, K. Fukushima, X. Huang, and K. Mameda.
PRD 93 (2016) 10, 104052. …

Dirac equation in rotating frame:

$$[i(\gamma^{\mu} - \gamma^0(\boldsymbol{\omega}\times\mathbf{x})^i\delta_i^{\mu})(\partial_{\mu} + ieA_{\mu}) + \gamma^0\mathbf{S}_{4\times4}\cdot\boldsymbol{\omega} - m]\psi = 0$$

- *If the EM field is defined in rotating frame.*

$$A_{\mu} \equiv (A_0, -\frac{1}{2}By, \frac{1}{2}Bx, 0), \quad \boldsymbol{\omega} \equiv (0, 0, \omega)$$



Move to the non-relativistic limit, and to the first order in 1/m :

$$\left[\frac{(\mathbf{p} - q\mathbf{A} - \boldsymbol{\omega}\times\mathbf{x})^2}{2m} - \frac{q}{m}\mathbf{B}\cdot\mathbf{s} - qA_0 - \boldsymbol{\omega}\cdot\mathbf{s} - \frac{1}{2}(\boldsymbol{\omega}\times\mathbf{x})^2\right]\psi = E\psi$$

Clearly, the magnetic field coupled with vorticity field !

See also:
M. Matsuo, J. Ieda, E. Saitoh, and S. Maekawa.
PRL. 106, 076601(2011).
PRB. 84, 104410(2011). …

Two-body Schroedinger equation in both EM and vorticity fields

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$$\mathcal{L} = \bar{\psi}\left[i\gamma^\mu(\partial_\mu + iqA_\mu + \Gamma_\mu) - m\right]\psi$$

affine connection

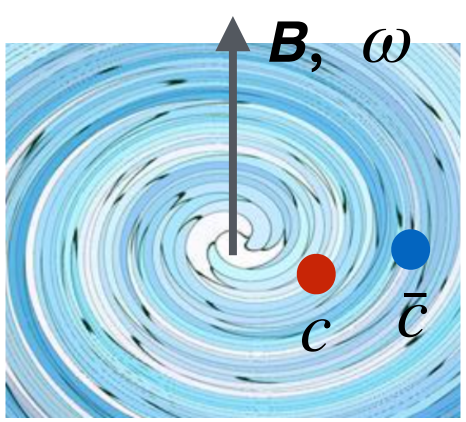
$$\Gamma_\mu = -\frac{i}{4}\omega_{\mu ij}\sigma^{ij}\ ,$$
$$\omega_{\mu ij} = g_{\alpha\beta}e_i^\alpha(\partial_\mu e_j^\beta + \Gamma^\beta_{\mu\nu}e_j^\mu)$$
$$\sigma^{ij} = \frac{i}{2}[\gamma^i, \gamma^j]\ .$$

See detail:
Y. Jiang, J. Liao.
PRL. 117 (2016) 19, 192302.
H. Chen, K. Fukushima, X. Huang, and K. Mameda.
PRD 93 (2016) 10, 104052. …

Dirac equation in rotating frame:

$$[i(\gamma^\mu - \gamma^0(\boldsymbol{\omega}\times\mathbf{x})^i\delta_i^\mu)(\partial_\mu + ieA_\mu) + \gamma^0\mathbf{S}_{4\times4}\cdot\boldsymbol{\omega} - m]\psi \;=\; 0$$

- *If the EM field is defined in Lab. frame. (Heavy-ion collisions)*



$$A'_\mu \equiv (A'_0, -\frac{1}{2}By', \frac{1}{2}Bx', 0)$$

From lab. Frame to the rotating frame

$$\boxed{A_\mu = \Big(-\frac{1}{2}B\omega(x^2+y^2), -\frac{1}{2}By, \frac{1}{2}Bx, 0\Big)}$$

Take in, and move to the non-relativistic limit:

$$\left[\frac{(\mathbf{p}-q\boldsymbol{A})^2}{2m} - \frac{q}{m}\mathbf{B}\cdot\mathbf{s} - qA_0 - \boldsymbol{\omega}\cdot(\mathbf{s}+\mathbf{x}\times\mathbf{p})\right]\psi = E\psi$$

No coupling between B and ω !

Two-body Schroedinger equation in both EM and vorticity fields

Two-body Schroedinger equation:

$$\left[\frac{(\mathbf{p}_a - q_a \boldsymbol{A}_a)^2}{2m} + \frac{(\mathbf{p}_b - q_b \boldsymbol{A}_b)^2}{2m} - q_a \mathbf{E}\cdot\mathbf{x}_a - q_b \mathbf{E}\cdot\mathbf{x}_b - \boldsymbol{\omega}\cdot(\mathbf{j}_a + \mathbf{j}_b) - \boldsymbol{\mu}\cdot\mathbf{B} + V\right]\Psi(\mathbf{x}_a, \mathbf{x}_b) = E\Psi(\mathbf{x}_a, \mathbf{x}_b)$$

$$\mathbf{R} \equiv (\mathbf{x}_a + \mathbf{x}_b)/2\,, \qquad \mathbf{P} \equiv (\mathbf{p}_a + \mathbf{p}_b) = -i\hbar\nabla_{\mathbf{R}}\,,$$
$$\mathbf{r} \equiv (\mathbf{x}_a - \mathbf{x}_b)\,, \qquad \mathbf{p} \equiv (\mathbf{p}_a - \mathbf{p}_b)/2 = -i\hbar\nabla_{\mathbf{r}}\,.$$

$$\boldsymbol{\omega}\cdot(\mathbf{R}\times\mathbf{P} + \mathbf{r}\times\mathbf{p} + \mathbf{s})$$

$$\frac{\mathbf{P}^2 + \frac{1}{4}q^2(\mathbf{B}\times\mathbf{r})^2 - q(\mathbf{B}\times\mathbf{r})\cdot\mathbf{P}}{4m_q} + \frac{\mathbf{p}^2 + \frac{1}{4}q^2(\mathbf{B}\times\mathbf{R})^2 - q(\mathbf{B}\times\mathbf{R})\cdot\mathbf{p}}{m_q}$$

Can not find a conserved value and total wavefunction can not be separated into a center-of-mass and a relative part.

Two-body Schroedinger equation in both EM and vorticity fields

Two-body Schroedinger equation:

$$\left[\frac{(\mathbf{p}_a - q_a \boldsymbol{A}_a)^2}{2m} + \frac{(\mathbf{p}_b - q_b \boldsymbol{A}_b)^2}{2m} - q_a \mathbf{E}\cdot\mathbf{x}_a - q_b \mathbf{E}\cdot\mathbf{x}_b - \boldsymbol{\omega}\cdot(\mathbf{j}_a + \mathbf{j}_b) - \boldsymbol{\mu}\cdot\mathbf{B} + V\right]\Psi(\mathbf{x}_a, \mathbf{x}_b) = E\Psi(\mathbf{x}_a, \mathbf{x}_b)$$

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$$\boldsymbol{\omega}\cdot(\mathbf{R}\times\mathbf{P} + \mathbf{r}\times\mathbf{p} + \mathbf{s})$$

$$\frac{\mathbf{P}^2 + \frac{1}{4}q^2(\mathbf{B}\times\mathbf{r})^2 - q(\mathbf{B}\times\mathbf{r})\cdot\mathbf{P}}{4m_q} + \frac{\mathbf{p}^2 + \frac{1}{4}q^2(\mathbf{B}\times\mathbf{R})^2 - q(\mathbf{B}\times\mathbf{R})\cdot\mathbf{p}}{m_q}$$

Can not find a conserved value and total wavefunction can not be separated into a center-of-mass and a relative part.

Perturbation method (In HIC, the magnetic field eB ≃ 70m_π^2 is much larger than the rotational field mω ≃ m_π^2)

$$\begin{aligned} H &= H_{EM} + H' \\ H' &= -\boldsymbol{\omega}\cdot(\mathbf{R}\times\mathbf{P}_{ps}) - \boldsymbol{\omega}\cdot(\mathbf{l}+\mathbf{s}) + \frac{q}{2}\boldsymbol{\omega}\cdot(\mathbf{R}\times(\mathbf{B}\times\mathbf{r})) \end{aligned}$$

the mixing is due to the perturbation used.

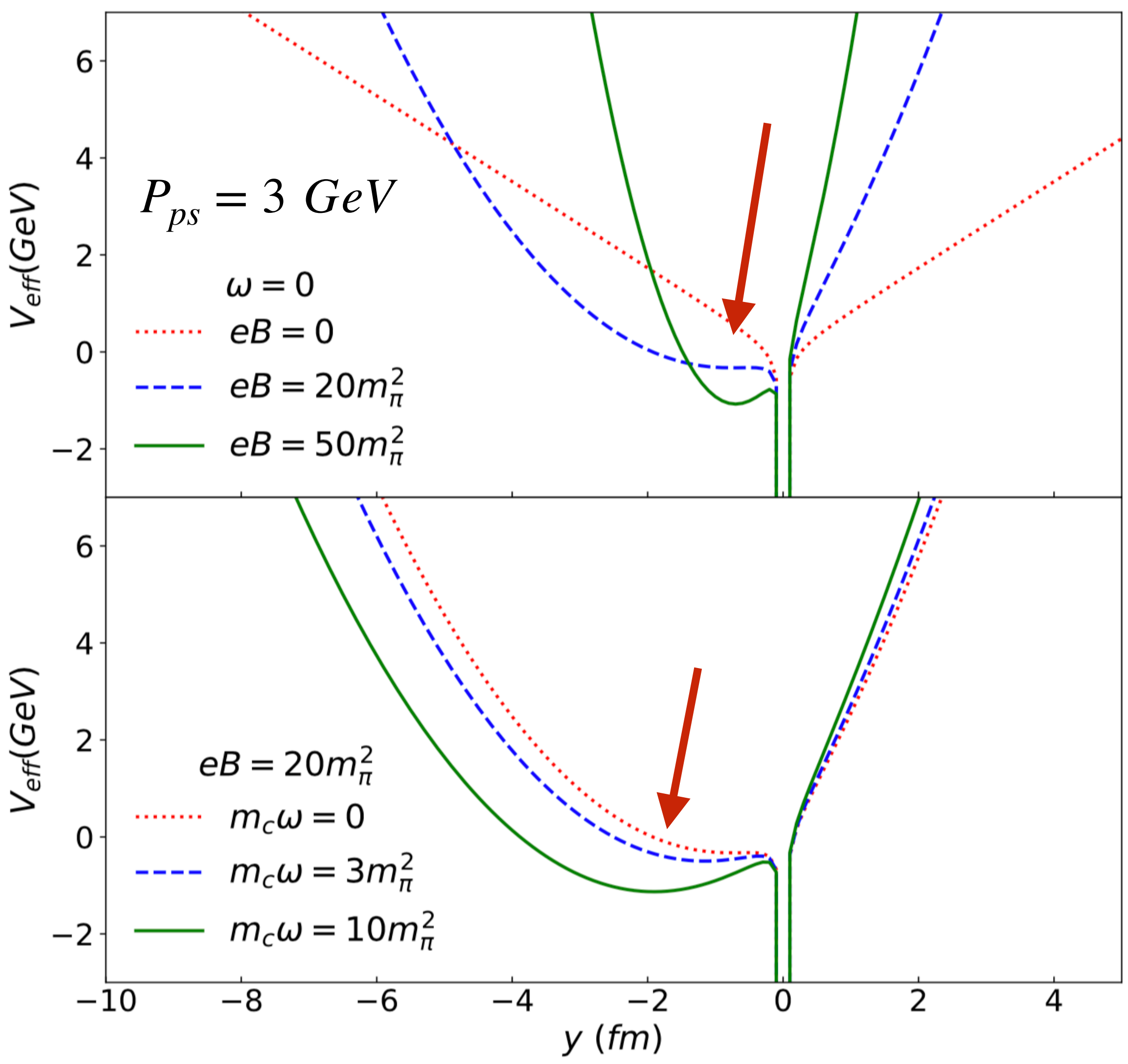
$$\begin{aligned} \epsilon_n &= \epsilon_n^{(0)} + \langle\psi_n^{(0)}|H_r'|\psi_n^{(0)}\rangle, \\ \psi_n &= \psi_n^{(0)} + \sum_{m\neq n}\frac{\langle\psi_m^{(0)}|H_r'|\psi_n^{(0)}\rangle}{\epsilon_m^{(0)} - \epsilon_n^{(0)}}\psi_m^{(0)}, \end{aligned}$$

The correction depends on the vale, direction of $\mathbf{R}$, ω, $\mathbf{B}$

Two-body Schroedinger equation in both EM and vorticity fields

Effective potential:

$$V_{eff}(x,y,z) \;=\; V_c + V_{ss} + \frac{q^2B^2}{4m}(x^2+y^2) + \frac{qB}{2m}(P_{ps} \pm m\langle R\rangle\omega)y$$
$$-\; qEz - \mu\cdot\mathbf{B} - \boldsymbol{\omega}\cdot(\mathbf{l}+\mathbf{s}).$$

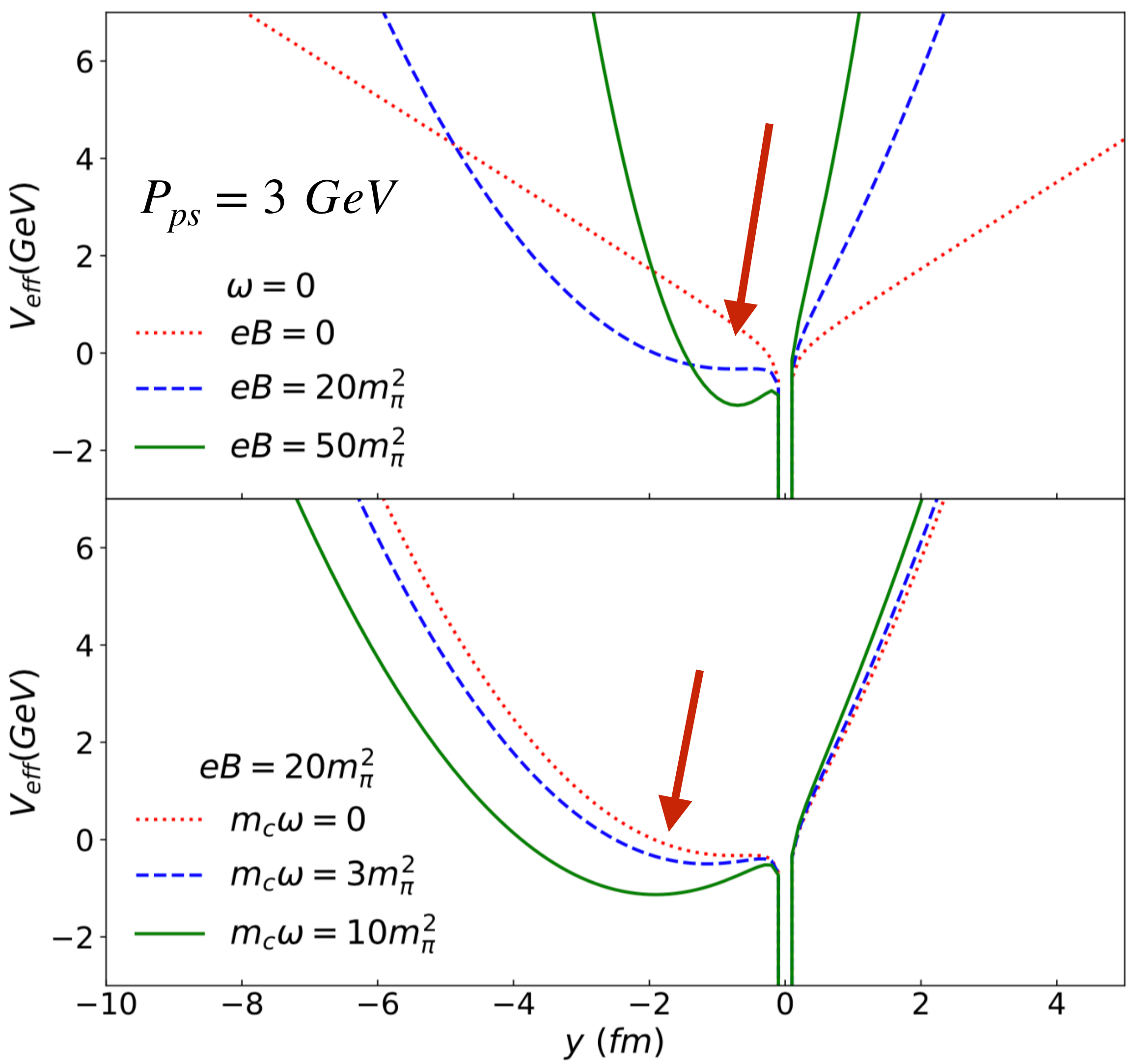


The rotation motion deepens(shallows) the potential well

Two-body Schroedinger equation in both EM and vorticity fields

Effective potential:

$$V_{eff}(x,y,z) = V_c + V_{ss} + \frac{q^2B^2}{4m}(x^2+y^2) + \frac{qB}{2m}(P_{ps} \pm m\langle R\rangle\omega)y - qEz - \mu\cdot\mathbf{B} - \boldsymbol{\omega}\cdot(\mathbf{l}+\mathbf{s}).$$



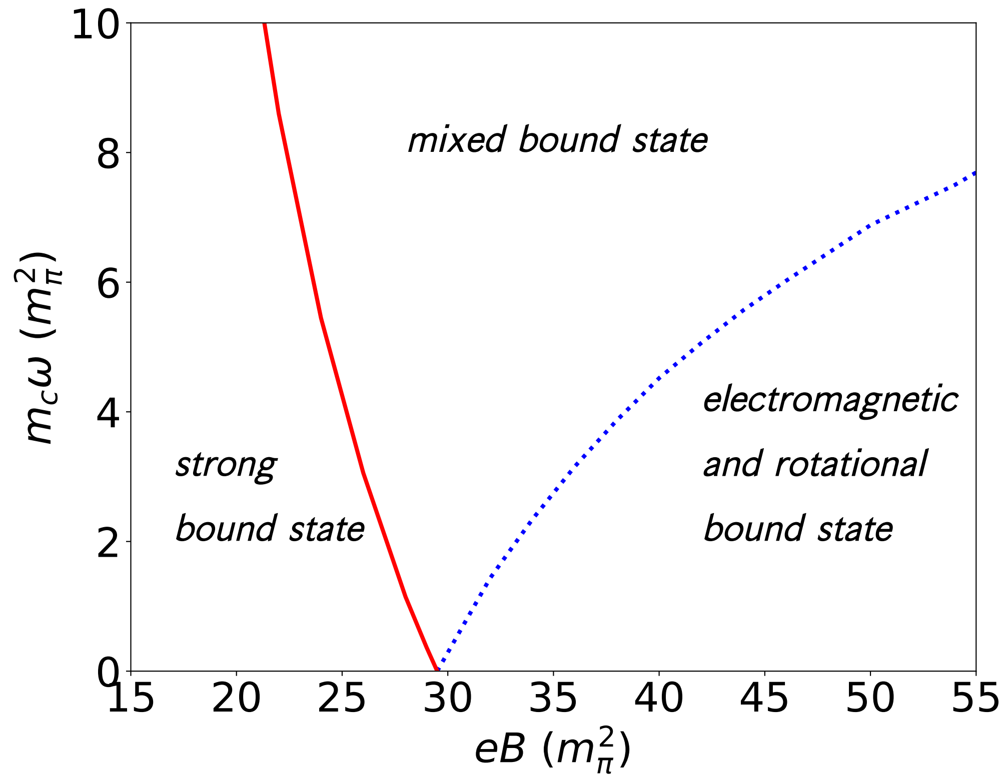
The rotation motion deepens(shallows) the potential well

Dissociation or not in vacuum? ***See detail:***
K. Marasinghe and K. Tuchin,
PRC 84 (2011) 044908.

motional Stark effect

The ionization probability of quarkonium equals its tunneling probability through the potential barrier

Transit from an isotropic bound state of strong interaction to an anisotropic bound state dominated by electromagnetic and rotational interaction. (transition line $\epsilon = 0$)



Possible to be realized in high energy nuclear collisions !

Summary

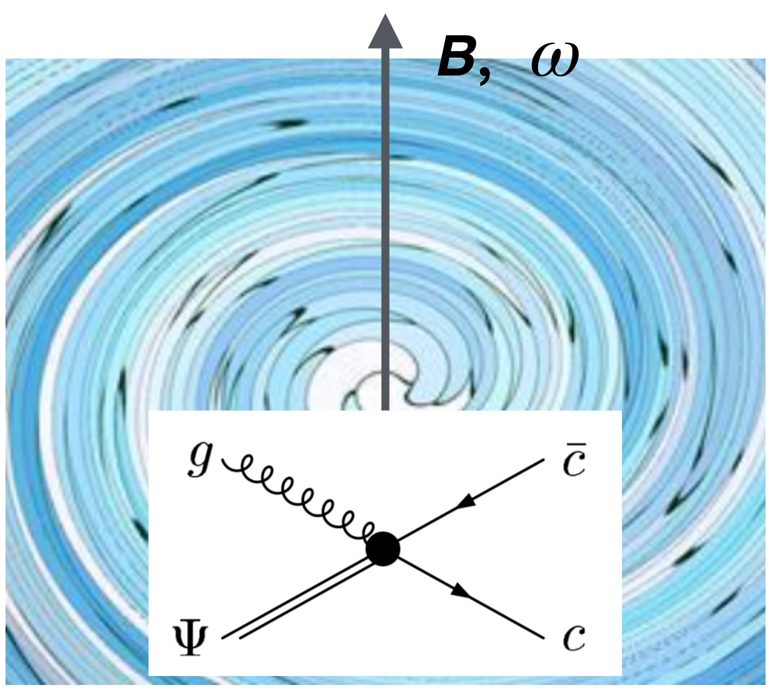
We study the static properties of charmonium states in electromagnetic field, vorticity field, and both two fields via two-body Schroedinger equation. And find the mass and shape of charmonium changed largely.

This may supply a tool to probe and study EM and vorticity field in the experiment.

Outlook

Dissociation cross-section and decay width of charmonium in EM and vorticity field which is crucial to transport framework.

J. Hu, S. Shi, J. Zhao, P. Zhuang. In progress…



Thanks for your attention!