Interpretation of $\Lambda$ hyperon spin polarization measurements

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Non-central heavy-ion collisions create fireballs with large global orbital angular momenta

\[ L_{\text{init}} \approx 10^5 \hbar \]

Part of the angular momentum can be transferred from the orbital to the spin part

\[ J_{\text{init}} = L_{\text{init}} = L_{\text{final}} + S_{\text{final}} \]

Emitted particles are expected to be globally polarized along the system’s angular momentum


Figure: M. Lisa, talk at “Strangeness in Quark Matter 2016”

Figure: R. Ryblewski
Measurement of $\Lambda$ and $\bar{\Lambda}$ spin polarization in heavy-ion collisions


... the hottest, least viscous – and now, most vortical – fluid produced in the laboratory...

$$\omega = (P_\Lambda + P_{\bar{\Lambda}})k_B T/\hbar \sim 0.6 - 2.7 \times 10^{22} \text{ s}^{-1}$$

Self-analysing parity-violating hyperon weak decay allows to measure polarization of $\Lambda$

$$\frac{dN}{d\Omega^*} = \frac{1}{4\pi} (1 + \alpha_H \mathbf{P}_H \cdot \mathbf{p}_p^*)$$

figure: T.Niida
Measurement of $\Lambda$ and $\bar{\Lambda}$ spin polarization in heavy-ion collisions

as the outcome of the spin polarization experiments, one cites the magnitude of the polarization along a specific direction in the center-of-mass frame - total angular momentum of the system $L$

to determine the magnitude of the polarization, however, one studies distributions of various three-momentum components of protons emitted in the weak decay of Lambdas which are measured in their rest frame

these two frames are linked by a non-trivial Lorentz transformation interpretation of the results will depend on it!

figure:B. I. Abelev et al. (STAR) PRC 76, 024915 (2007)
Center-of-mass (COM) frame in heavy-ion collision

herein, assume that the reaction plane angle in the laboratory (LAB) frame can be well measured hence $\Psi_{RP} = 0$
Transformation to $\Lambda$ rest frame

**Canonical Boost**

$$\mathcal{L}_\nu^\mu (-v_\Lambda) = \begin{bmatrix}
\frac{E_\Lambda}{m_\Lambda} & -\frac{p_1}{m_\Lambda} & -\frac{p_2}{m_\Lambda} & -\frac{p_3}{m_\Lambda} \\
-\frac{p_1}{m_\Lambda} & 1 + \alpha p_1^1 p_\Lambda^1 & \alpha p_1^1 p_\Lambda^2 & \alpha p_1^1 p_\Lambda^3 \\
-\frac{p_2}{m_\Lambda} & \alpha p_2^1 p_\Lambda^1 & 1 + \alpha p_2^2 p_\Lambda^2 & \alpha p_2^2 p_\Lambda^3 \\
-\frac{p_3}{m_\Lambda} & \alpha p_3^1 p_\Lambda^1 & \alpha p_3^2 p_\Lambda^2 & 1 + \alpha p_3^3 p_\Lambda^3
\end{bmatrix}$$

$$\alpha \equiv 1/(m_\Lambda (E_\Lambda + m_\Lambda))$$

**Rest Frame**

$$p^\mu = (E, p_1, p_2, p_3)$$

$$\mathcal{L}^\mu_\nu (-v_\Lambda)$$

$$S'(p_\Lambda)$$

$$p'_\Lambda = (m_\Lambda, 0, 0, 0)$$
Transformation of system’s angular momentum

\[ J^{\mu \nu} = [L^{\mu \nu} + S^{\mu \nu}] \]

\[ L^k = -\frac{1}{2} \epsilon^{kij} L^{ij} \quad K^i = -L^0 + 0 \]

\[ L'_{\mu \nu} (-v_\Lambda) \]

\[ L' = \gamma_\Lambda L - \frac{\gamma_\Lambda^2}{\gamma_\Lambda + 1} v_\Lambda (v_\Lambda \cdot L) \]

\[ \gamma_\Lambda = \frac{E_\Lambda}{m_\Lambda} \]

Λ sees direction different from y!

transform like the spatial components of an antisymmetric tensor

relativistic correction

Λ sees direction different from y!
Transformation of system’s angular momentum

midrapidity Λ’s

change of the system’s angular momentum direction due to relativistic effects

COM

\[ \hat{L} = \frac{L}{L} = (0, -1, 0) \]

\[ \hat{K} = 0 \]

\[ S'(p_\Lambda) \]

\[ \hat{L}^1 = \left(1 - (v_\Lambda^2)^2\right)^{-1/2} \frac{\gamma_\Lambda}{\gamma_\Lambda + 1} v_\Lambda^1 v_\Lambda^2, \]

\[ \hat{L}^2 = \left(1 - (v_\Lambda^2)^2\right)^{-1/2} \left( \frac{\gamma_\Lambda}{\gamma_\Lambda + 1} v_\Lambda^2 v_\Lambda^2 - 1 \right) \]

\[ \hat{L}^3 = \left(1 - (v_\Lambda^2)^2\right)^{-1/2} \frac{\gamma_\Lambda}{\gamma_\Lambda + 1} v_\Lambda^3 v_\Lambda^2. \]

\[ \hat{K'} = \frac{(v_\Lambda^3, 0, -v_\Lambda^1)}{\sqrt{(v_\Lambda^1)^2 + (v_\Lambda^3)^2}} \]
\( \Lambda \) weak decay: \( S'(p_\Lambda) \) vs \( S^*(p_\Lambda) \) rest frames

\[
\begin{align*}
\hat{p}'_p &= (\sin \theta'_p \cos \phi'_p, \sin \theta'_p \sin \phi'_p, \cos \theta'_p) \\
\hat{P}' &= (\sin \Theta' \cos \Phi', \sin \Theta' \sin \Phi', \cos \Theta')
\end{align*}
\]

\[
\hat{P}' = \mathcal{R}_{y'}(\Theta') \mathcal{R}_{z'}(\Phi') \hat{P}' = (0, 0, 1)
\]
$\Lambda$ weak decay: $S'(p_\Lambda)$ vs $S^*(p_\Lambda)$ rest frames

$S'(p_\Lambda)$

$S^*(p_\Lambda)$

\[ \hat{p}_p = (\sin \theta'_p \cos \phi'_p, \sin \theta'_p \sin \phi'_p, \cos \theta'_p) \]
\[ \hat{P}' = (\sin \Theta' \cos \Phi', \sin \Theta' \sin \Phi', \cos \Theta') \]

proton momentum in $S^*(p_\Lambda)$

\[ \hat{p}_{p,x}^* = \cos(\Phi' - \phi'_p) \sin \theta'_p \cos \Theta' - \cos \theta'_p \sin \Theta' \equiv \sin \theta^* \cos \phi^* \]
\[ \hat{p}_{p,y}^* = -\sin(\Phi' - \phi'_p) \sin \theta'_p \equiv \sin \theta^* \sin \phi^* \]
\[ \hat{p}_{p,z}^* = \cos(\Phi' - \phi'_p) \sin \theta'_p \sin \Theta' + \cos \theta'_p \cos \Theta' \equiv \cos \theta^* \]

\[ \hat{P}^* = R_{y'}(\Theta') \ R_{z'}(\Phi') \ \hat{P}' = (0, 0, 1) \]
\[ S^*(p_\Lambda) \]

\[ \frac{dN_{p}^{\text{pol}}}{d\Omega^*} = \frac{1}{4\pi} \left( 1 + \alpha_\Lambda \mathbf{P}^* \cdot \mathbf{p}_p^* \right) \]

\[ S'(p_\Lambda) \]

\[ \frac{dN_{p}^{\text{pol}}}{d\Omega'} = \frac{1}{4\pi} \left[ 1 + \alpha_\Lambda P' \sin \Theta' \cos \Phi' \sin \theta' \sin \Theta' + \cos \theta' \cos \Theta' \right] \]

The magnitude and direction of the polarization can be directly obtained from the averaged values of the three momentum components measured in \( S'(p_\Lambda) \):

\[ \langle \hat{p}'_{p,x} \rangle = \int \frac{dN_{p}^{\text{pol}}}{d\Omega'} (\sin \theta_p')^2 \cos \phi_p' d\theta_p' d\phi_p' = \frac{1}{3} P' \alpha_\Lambda \sin \Theta' \cos \Phi', \]

\[ \langle \hat{p}'_{p,y} \rangle = \int \frac{dN_{p}^{\text{pol}}}{d\Omega'} (\sin \theta_p')^2 \sin \phi_p' d\theta_p' d\phi_p' = \frac{1}{3} P' \alpha_\Lambda \sin \Theta' \sin \Phi', \]

\[ \langle \hat{p}'_{p,z} \rangle = \int \frac{dN_{p}^{\text{pol}}}{d\Omega'} \sin \theta_p' \cos \theta_p' d\theta_p' d\phi_p' = \frac{1}{3} P' \alpha_\Lambda \cos \Theta'. \]

\[ \mathbf{P}' = P' (\sin \Theta' \cos \Phi', \sin \Theta' \sin \Phi', \cos \Theta') = \frac{3}{\alpha_\Lambda} \langle \hat{p}'_{p,x} \rangle (\langle \hat{p}'_{p,y} \rangle, \langle \hat{p}'_{p,z} \rangle) \]
Interpretation of the $\Lambda$ polarisation measurement

$$\langle \cos \phi'_p \rangle = \int \left( \frac{dN_{p}^{\text{pol}}}{d\Omega'} \right) \sin \theta'_p \cos \phi'_p \, d\theta'_p \, d\phi'_p = \frac{\pi \alpha_{\Lambda}}{8} P' \sin \Theta' \cos \Phi',$$

$$\langle \sin \phi'_p \rangle = \int \left( \frac{dN_{p}^{\text{pol}}}{d\Omega'} \right) \sin \theta'_p \sin \phi'_p \, d\theta'_p \, d\phi'_p = \frac{\pi \alpha_{\Lambda}}{8} P' \sin \Theta' \sin \Phi'. $$

$$P_H = \frac{8}{\pi \alpha_{\Lambda}} \langle \sin \phi'_p \rangle$$

**“y” component of the polarization three-vector measured in the Lambda rest frame**

not the component of the polarization along the total angular momentum vector, as the $y$ directions is COM and $\Lambda$RF differ

- it is tempting to measure also mean $\langle \cos \phi'_p \rangle$
- ratio would give information about the angle $\Phi'$
Correlation with global angular momentum

\[ P' = P' \left( \sin \Theta' \cos \Phi', \sin \Theta' \sin \Phi', \cos \Theta' \right) = \frac{3}{\alpha_{\Lambda}} \left( \langle \hat{p}_{p,x}' \rangle, \langle \hat{p}_{p,y}' \rangle, \langle \hat{p}_{p,z}' \rangle \right) \]

direction of the total angular momentum that is “seen” by the spin of the decaying \( \Lambda \) that has three-momentum in COM

\[ \hat{L}' = \left( 1 - (v_{\Lambda} \cdot \hat{L})^2 \right)^{-1/2} \left( \hat{L} - \frac{\gamma_{\Lambda}}{\gamma_{\Lambda} + 1} v_{\Lambda} (v_{\Lambda} \cdot \hat{L}) \right) \]

projection of the polarization along the direction of the total angular momentum

\[ \hat{L}' \cdot P' = \left( 1 - (v_{\Lambda} \cdot \hat{L})^2 \right)^{-1/2} \left( \hat{L} \cdot P' - \frac{\gamma_{\Lambda}}{\gamma_{\Lambda} + 1} v_{\Lambda} \cdot P' \cdot v_{\Lambda} \cdot \hat{L} \right) \]

In the case of spin polarization of Lambdas, a reference frame where all particles are at rest does not exist, since the analyzed Lambdas have usually different momenta in COM

The advantage of \( \hat{L}' \cdot P' \) compared to \( \hat{L} \cdot P' \) is that the spin polarization of each \( \Lambda \) irrespectively of its three-momentum in COM, is projected on the same physical axis corresponding to \( L \) in COM
Numerical estimates

Assuming

\[ P' = P' \hat{L} \]

\[ \hat{L} \cdot P' = P' \left(1 - \frac{v^2}{2}\right)^{-1/2} \left(1 - \frac{v^2}{1 + \sqrt{1 - v^2}}\right) \equiv P' F_P(v) \]

Average value for Lambdas in momentum range \((m,n)\) GeV

\[
\langle \hat{L}' \cdot P' \rangle_{m-n} = P' \frac{\int_{v(m)}^{v(n)} dv \int d\Omega F_P(v) F_T(v)}{\int_{v(m)}^{v(n)} dv \int d\Omega F_T(v)}
\]

\[
\langle \hat{L}' \cdot P' \rangle_{2-3} = 0.97 \ P'
\]

\[
\langle \hat{L}' \cdot P' \rangle_{3-4} = 0.94 \ P'
\]

\[
\langle \hat{L}' \cdot P' \rangle_{4-5} = 0.92 \ P'
\]

\[
\langle \hat{L}' \cdot P' \rangle_{5-6} = 0.90 \ P'
\]

Relativistic effects studied in this work may reach 10% for the most energetic Lambdas studied at STAR.

\[
v = \sqrt{v_1^2 + v_2^2 + v_3^2}
\]

\[
F_T(v) = N \left[ \exp \left( \frac{m_\lambda}{T_{\text{eff}} \sqrt{1 - v^2}} \right) + 1 \right]^{-1}
\]

\[
v(n) = \tanh \left[ \sinh^{-1} \left( \frac{n \text{ GeV}}{m_\lambda} \right) \right]
\]

\[
T_{\text{eff}} = 150 \ \text{MeV}
\]
Summary

we have discussed the interpretation of the recent measurements of the spin polarization in relativistic heavy-ion collisions

we have shown that the appropriate interpretation of the relation between the spin direction (measured in the RF) and the total angular momentum of the system (measured in the COM frame) requires that the direction of the angular momentum is boosted to the RF

we have given the necessary formula that may be used to average the measured polarization of Lambdas with different momenta in the COM frame
Thank you for your attention!