Recent results in NNLO and N^{3}LO QCD

Fabrizio Caola

Rudolf Peierls Centre for Theoretical Physics & Wadham College

Standard Model at the LHC 2021, Apr 26 2021

UNIVERSITY OF OXFORD

SOME Recent results in NNLO and N^{3}LO QCD

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Fixed-order calculations

$$\mathrm{d}\sigma = \int dx_1 dx_2 f(x_1) f(x_2)\ \mathrm{d}\sigma_{\mathrm{part}}(x_1,x_2) F_J\ (1 + \mathcal{O}(\Lambda^p_{\mathrm{QCD}}/Q^p))$$

$$\mathrm{d}\sigma_{\mathrm{part}}(x_1,x_2)F_J = \sigma_0 + \left(\frac{\alpha_s}{2\pi}\right)\sigma_1 + \left(\frac{\alpha_s}{2\pi}\right)^2 \sigma_2 + \ldots$$

Fixed order:

- conceptually simple, ``minimally ambiguous" framework
- if input parameters are known well enough: first-principle predictions up to $(\Lambda_{\mathrm{QCD}}/Q)^k$ ~ percent or better → $N^{2/3}LO$
- direct access to fiducial region

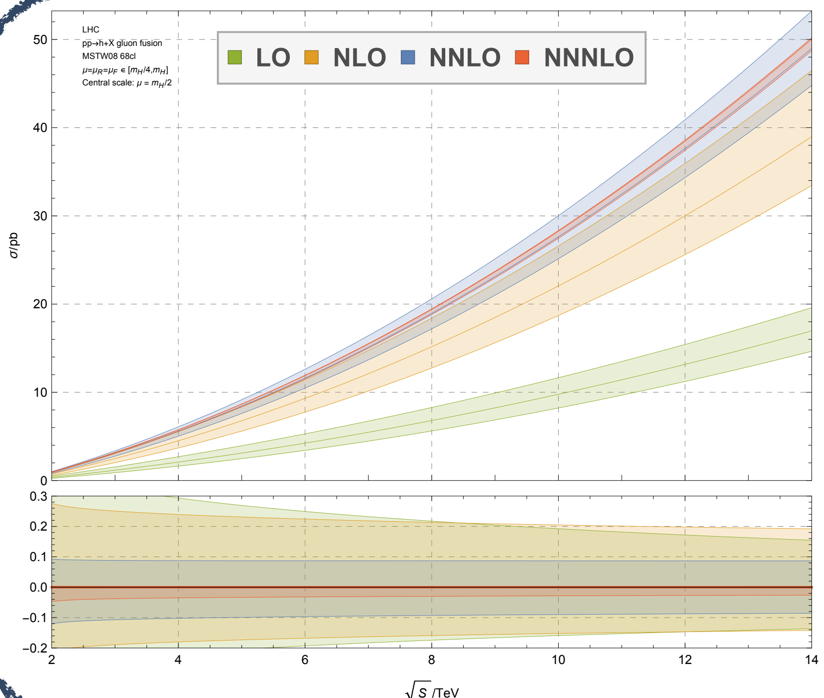
Caveats:

- only available for relatively simple final states
- observable F_J must be ``inclusive enough", i.e. insensitive to IR regions

The N³LO era: inclusive results

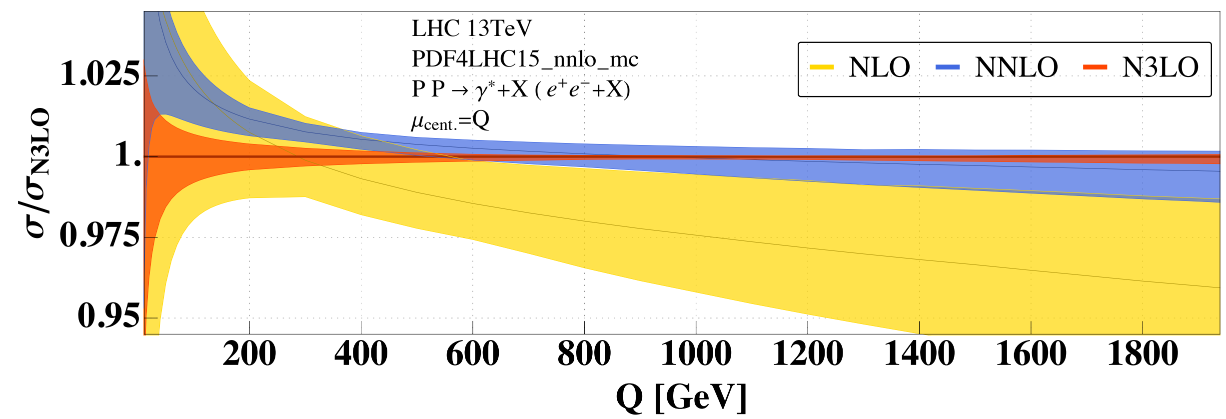
To a large extent, inclusive N³LO for 2 → 1 processes has been solved

[Anastasiou, Duhr, Dulat, Furlan, Gehrmann, Herzog, Lazopoulos, Mistlberger (2016-...);
Duhr, Dulat, Mistlberger (2020-21)]

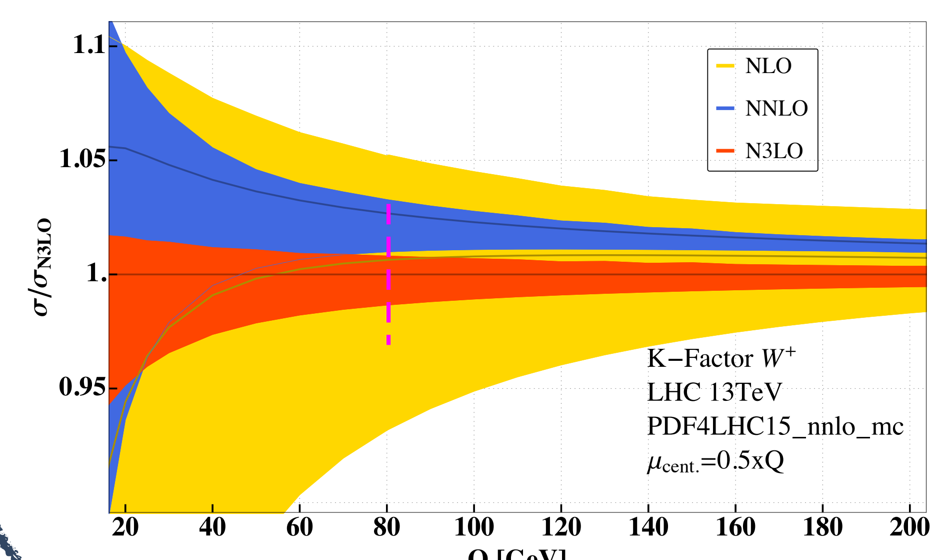


ggH

Drell-Yan

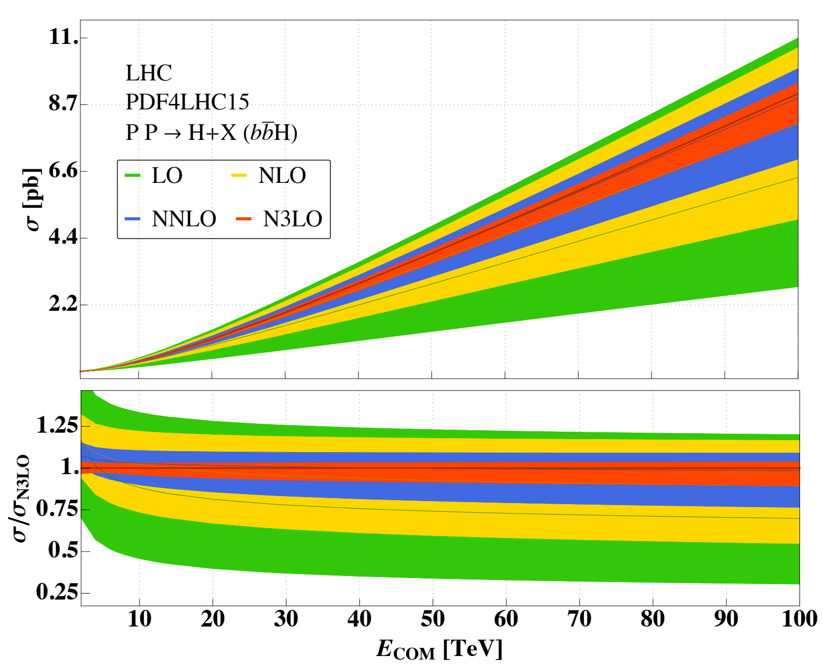


γ^*



W

bbH

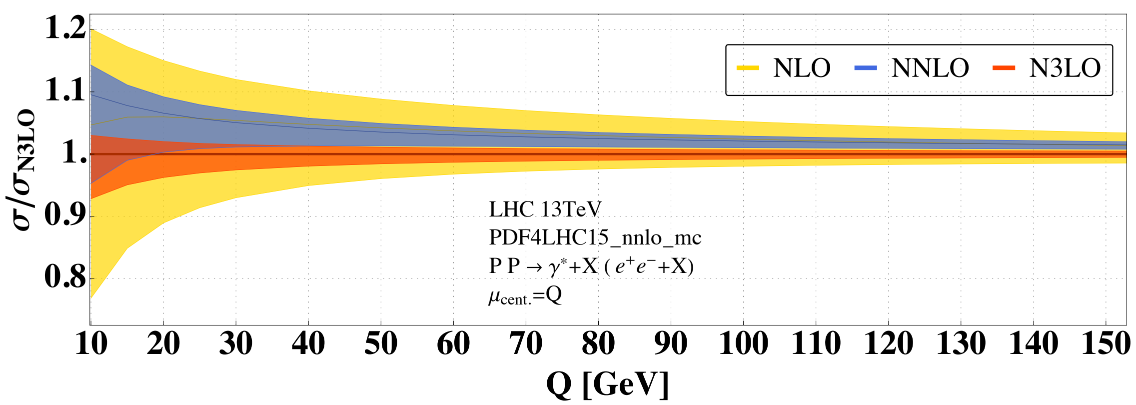


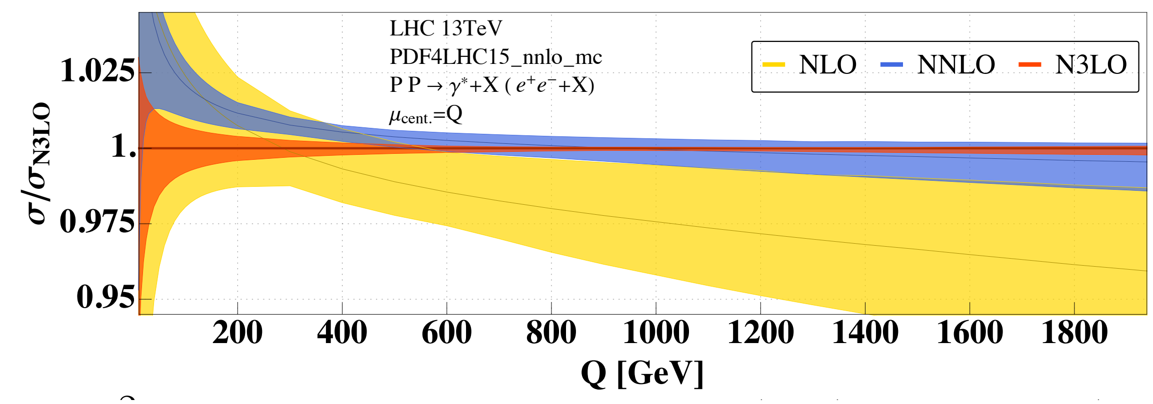
see D. Napoletano's talk

Inclusive Drell-Yan at N³LO

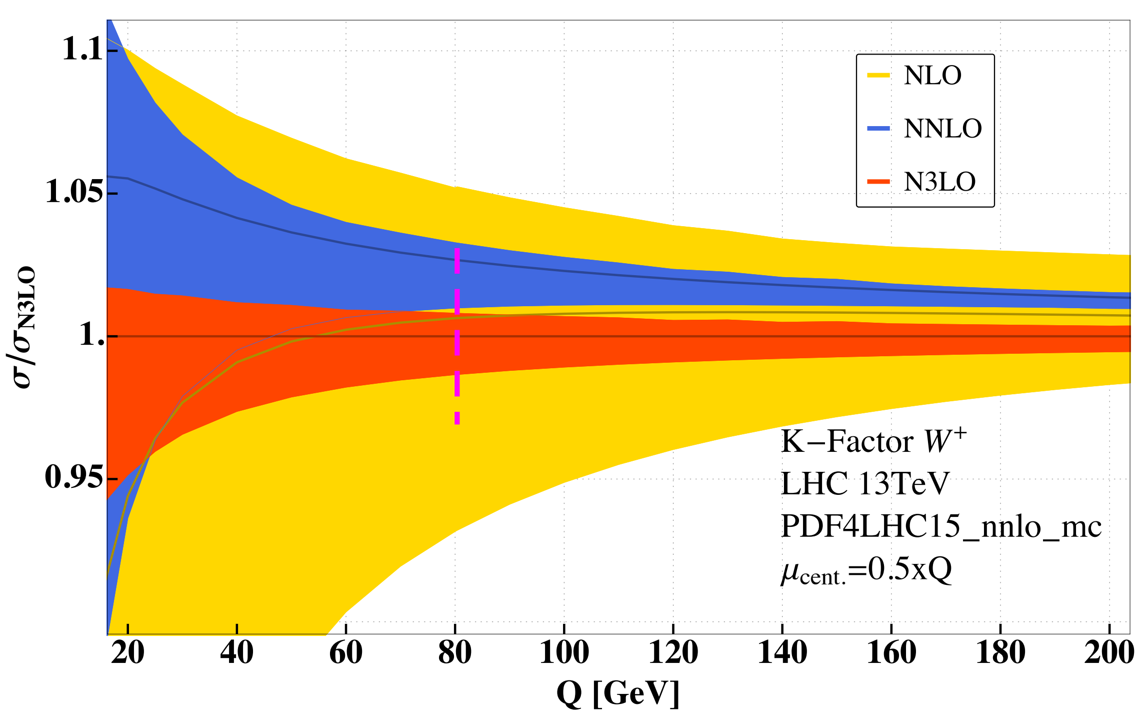
In the EW region Q~100 GeV: ~2-3% N³LO vs per-mill NNLO

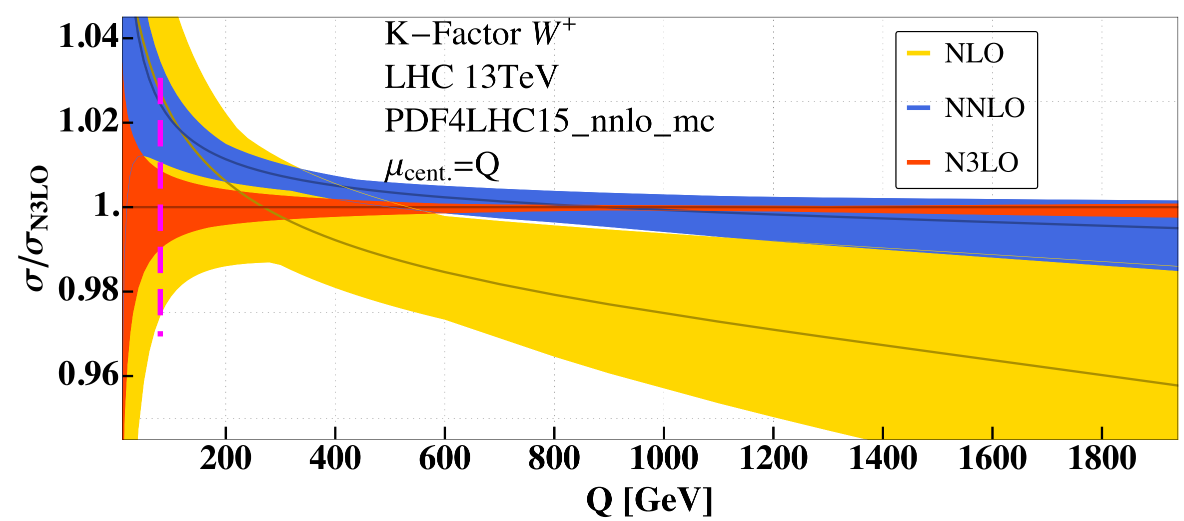
γ^*





W+

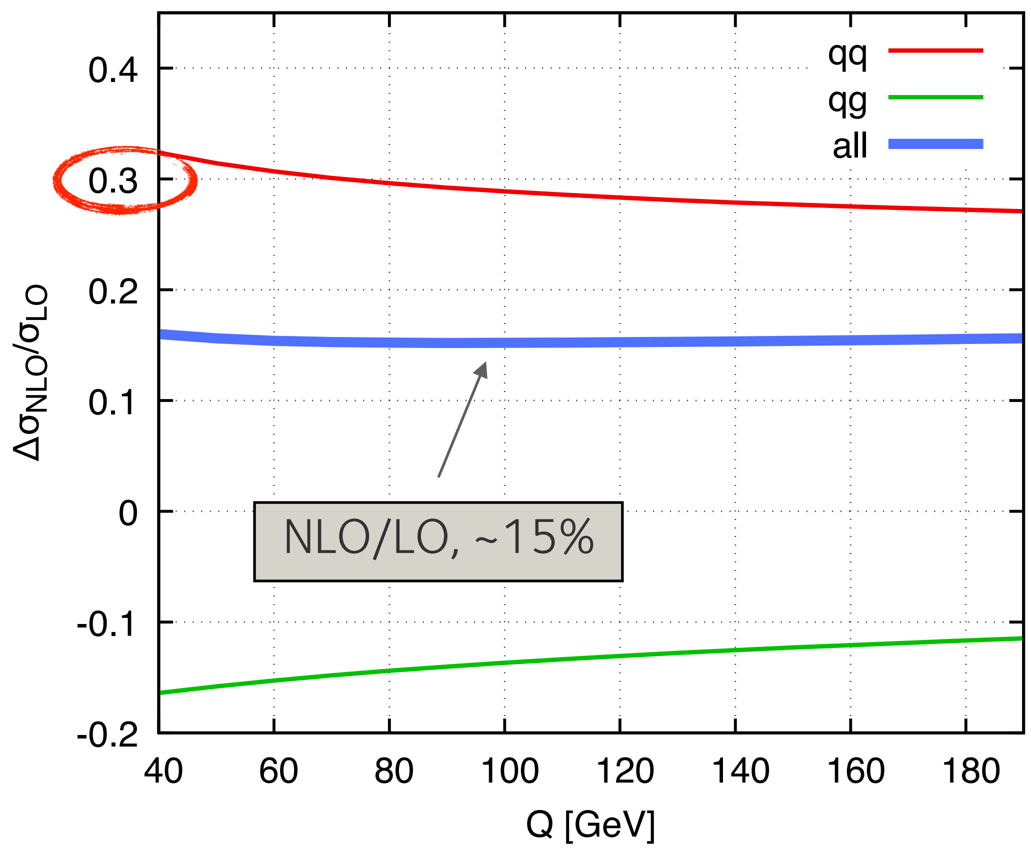


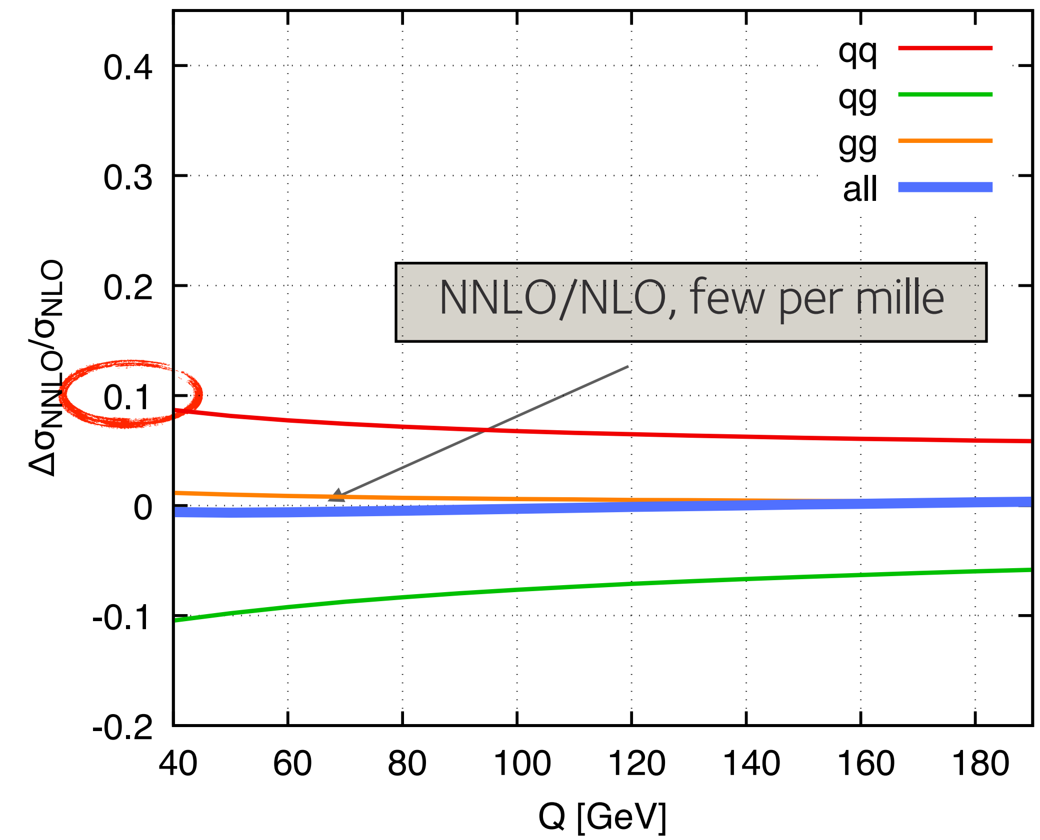


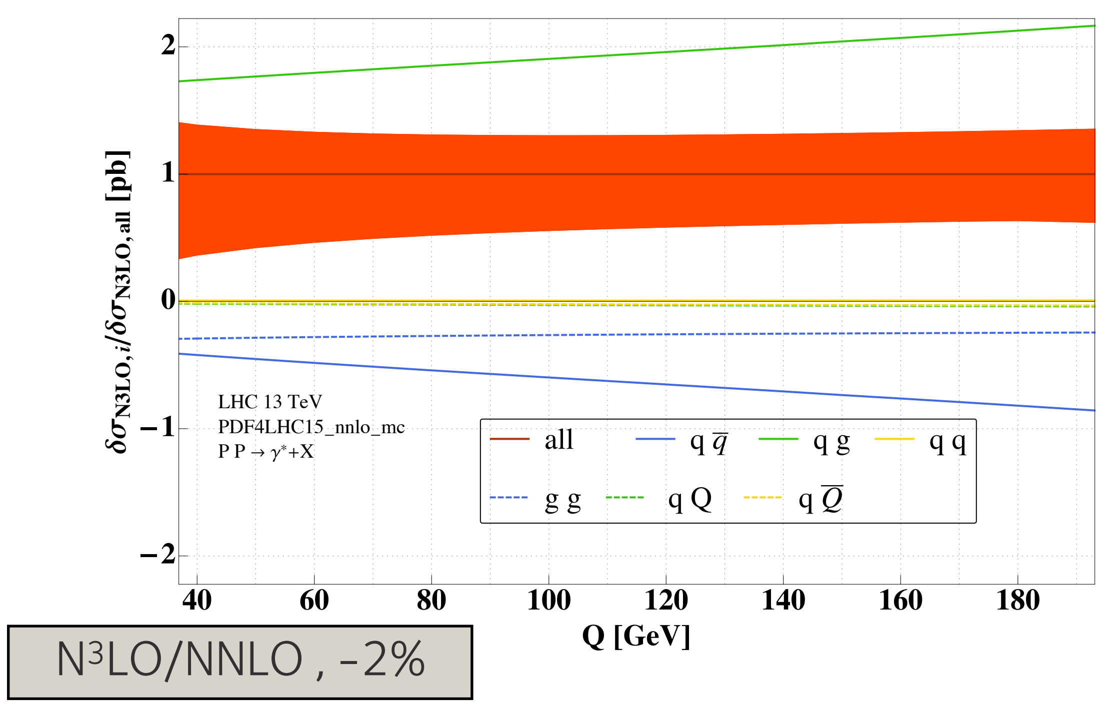
Band only overlap at large Q^2 → trouble in the high-precision region?

Neutral-current DY: flavour decomposition

Per-mille NNLO: unnaturally small. Very large cancellations







- Individual channels (μ=Q) much larger than final result, delicate cancellation pattern
- Individual channels: perturbative convergence
- N3LO ``natural", tiny PDFs changes can significantly affect this picture

N^{3}LO: PDFs

N^{3}LO PDFs not available → order mismatch

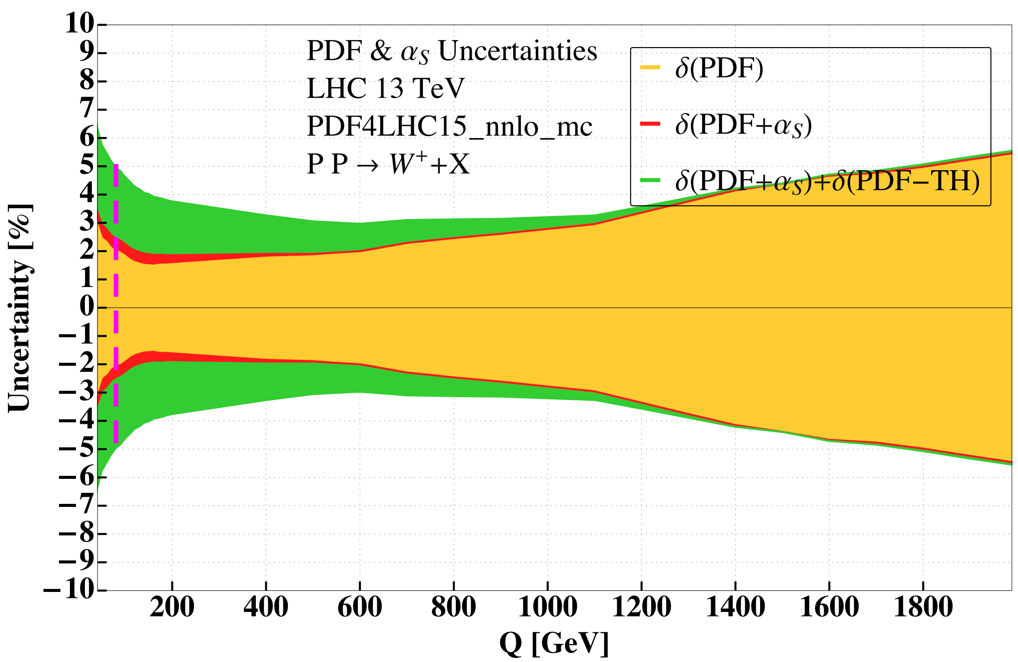
γ^*

Q/GeV	$K_{QCD}^{N^3LO}$	δ(scale)	δ(PDF+α_S)	δ(PDF-TH)
30	0.952	$^{+1.5\%}_{-2.5\%}$	±4.1%	±2.7%
50	0.966	$^{+1.1\%}_{-1.6\%}$	±3.2%	±2.5%
70	0.973	$^{+0.89\%}_{-1.1\%}$	±2.7%	±2.4%
90	0.978	$^{+0.75\%}_{-0.89\%}$	±2.5%	±2.4%
110	0.981	$^{+0.65\%}_{-0.73\%}$	±2.3%	±2.3%
130	0.983	$^{+0.57\%}_{-0.63\%}$	±2.2%	±2.2%
150	0.985	$^{+0.50\%}_{-0.54\%}$	±2.2%	±2.2%

Error: estimate from previous orders

$$\delta(\text{PDF-TH}) = \frac{1}{2}\left|\frac{\sigma_{W^\pm}^{(2),\text{ NNLO-PDFs}} - \sigma_{W^\pm}^{(2),\text{ NLO-PDFs}}}{\sigma_{W^\pm}^{(2),\text{ NNLO-PDFs}}}\right| .$$

W^+

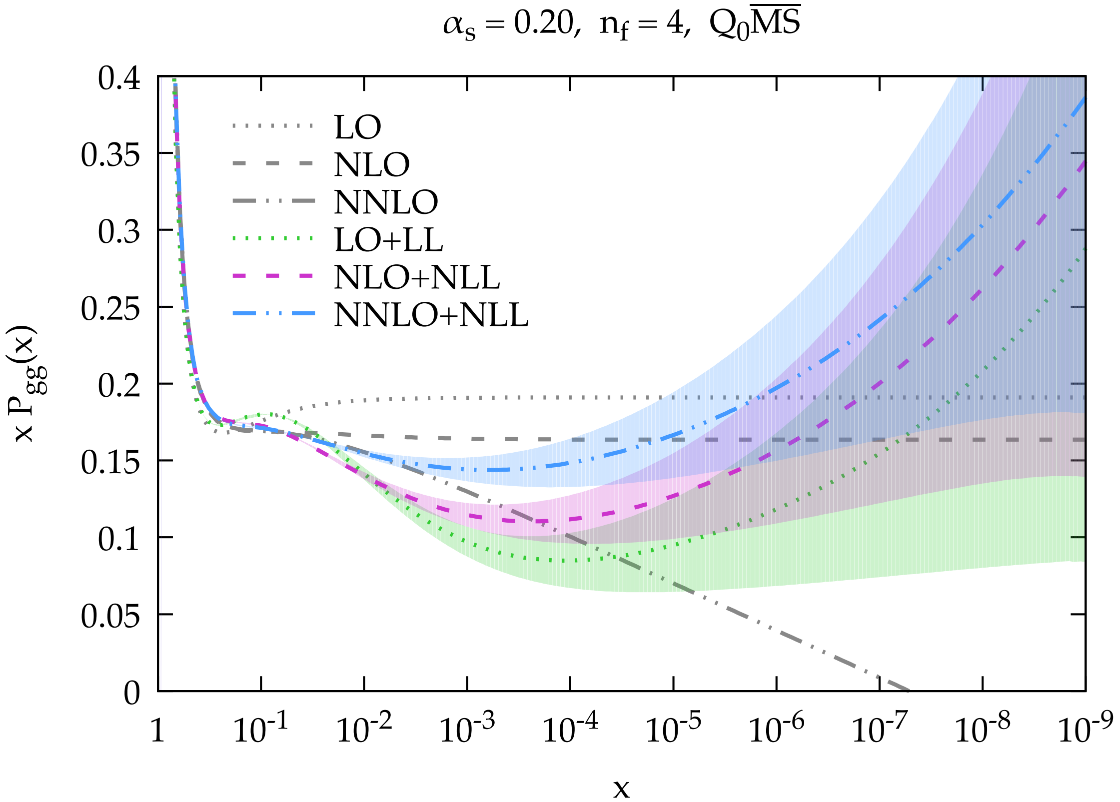


- ~ 2% PDF–TH error in the EW region
- significant fraction of the error budget
- same order of ``standard" PDF+α_s

N^3LO PDFs: issues

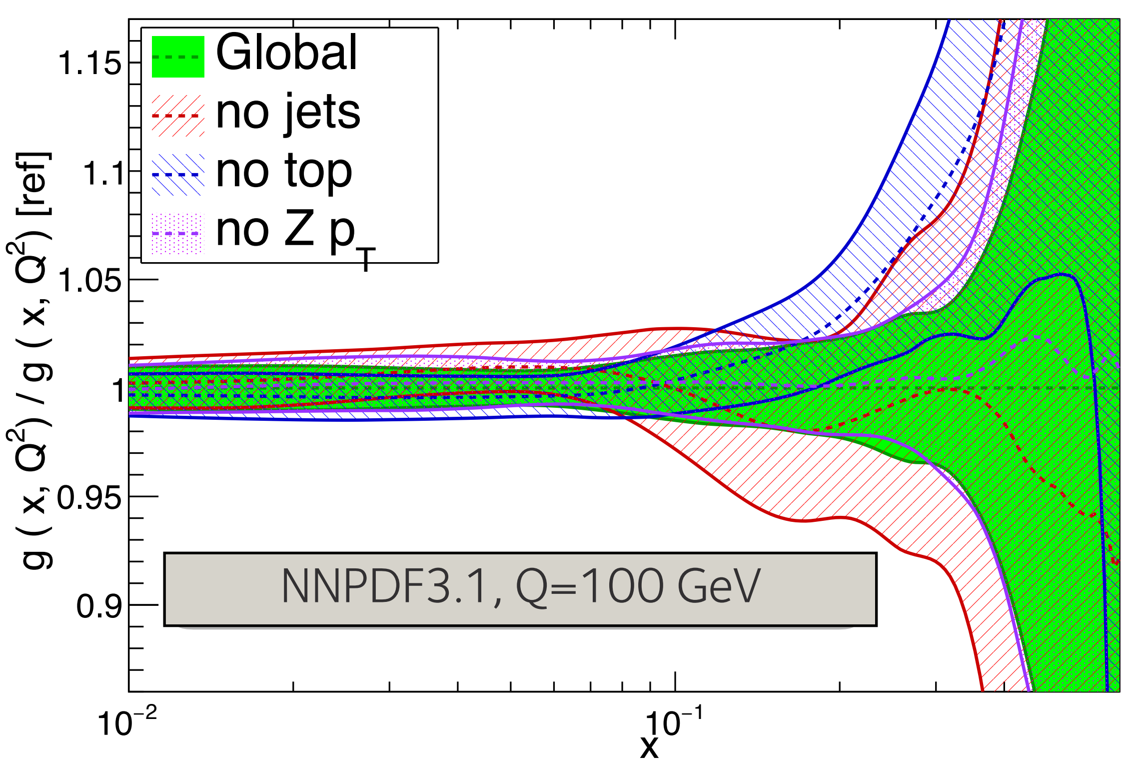
N^3LO: evolution

[Bonvini, Marzani, Muselli (2017-18)]



- N^3LO calculation underway [Herzog, Moch, Ruijl, Ueda, Vermaseren, Vogt, in progrress]
- N^3LO: rapid small-x growth → perturbative instabilities@N^3LO
- NLL resummation known, but large subleading effects [Bonvini, Marzani (2018)]

N^3LO: ``data''



- Collider data crucial to reduce perturbative uncertainty → fully-consistent N^3LO fit would require top, Z p_t, jets @ N^3LO

N^3LO: ratios

To which extent DY QCD corrections are ``universal”?

σ_W/σ_γ

[Duhr, Dulat, Mistlberger (2020)]

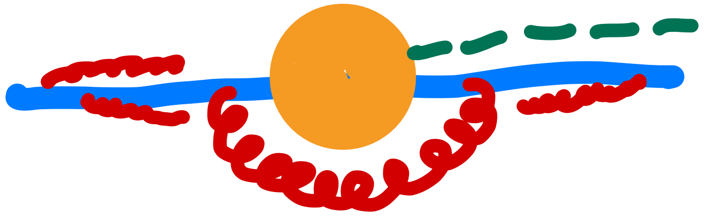
	NLO		NNLO		N^3LO	
μ^{cent}	m_W	$m_W/2$	m_W	$m_W/2$	m_W	$m_W/2$
A	$1.342^{+0.10\%}_{-0.08\%}$	$1.342^{+0.07\%}_{-0.05\%}$	$1.348^{+0.12\%}_{-0.10\%}$	$1.349^{+0.15\%}_{-0.11\%}$	$1.350^{+0.05\%}_{-0.06\%}$	$1.350^{+0.04\%}_{-0.05\%}$
A′	$1.343^{+0.13\%}_{-0.16\%}$	$1.344^{+0.10\%}_{-0.21\%}$	$1.349^{+0.13\%}_{-0.09\%}$	$1.351^{+0.33\%}_{-0.13\%}$	$1.350^{+0.02\%}_{-0.03\%}$	$1.350^{+0.01\%}_{-0.09\%}$
B	$1.342^{+8.82\%}_{-8.08\%}$	$1.342^{+12.9\%}_{-11.4\%}$	$1.348^{+2.26\%}_{-2.31\%}$	$1.349^{+2.24\%}_{-2.27\%}$	$1.350^{+2.21\%}_{-2.14\%}$	$1.350^{+2.21\%}_{-2.14\%}$
B′	$1.343^{+5.28\%}_{-7.40\%}$	$1.344^{+8.09\%}_{-8.97\%}$	$1.349^{+1.85\%}_{-2.63\%}$	$1.351^{+2.21\%}_{-2.24\%}$	$1.350^{+2.60\%}_{-2.25\%}$	$1.350^{+4.65\%}_{-2.70\%}$
C	$1.342^{+0.99\%}_{-0.99\%}$	$1.342^{+0.58\%}_{-0.58\%}$	$1.349^{+0.52\%}_{-0.52\%}$	$1.349^{+0.53\%}_{-0.53\%}$	$1.350^{+0.15\%}_{-0.15\%}$	$1.350^{+0.11\%}_{-0.11\%}$

- A(B): error from correlated (uncorrelated) scale variation
- C: error from perturbative convergence

Reasonable estimate: per-mille. Good enough for e.g. W-mass extraction?

N^3LO: going differential

Colour-singlet production at order α_s^3:



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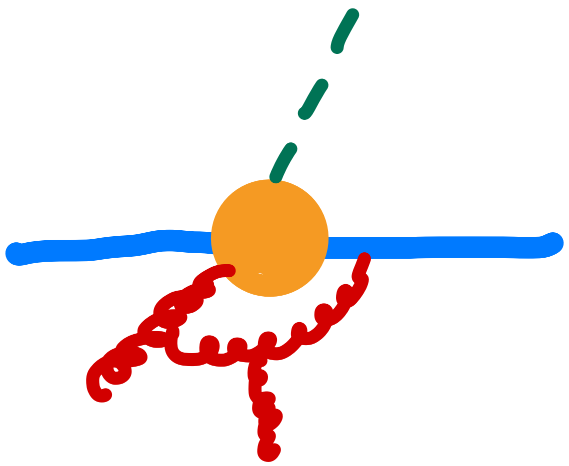


Soft/collinear (+virtual) effects at vanishingly small p_t

↓

Rapidity distribution at vanishingly small p_t

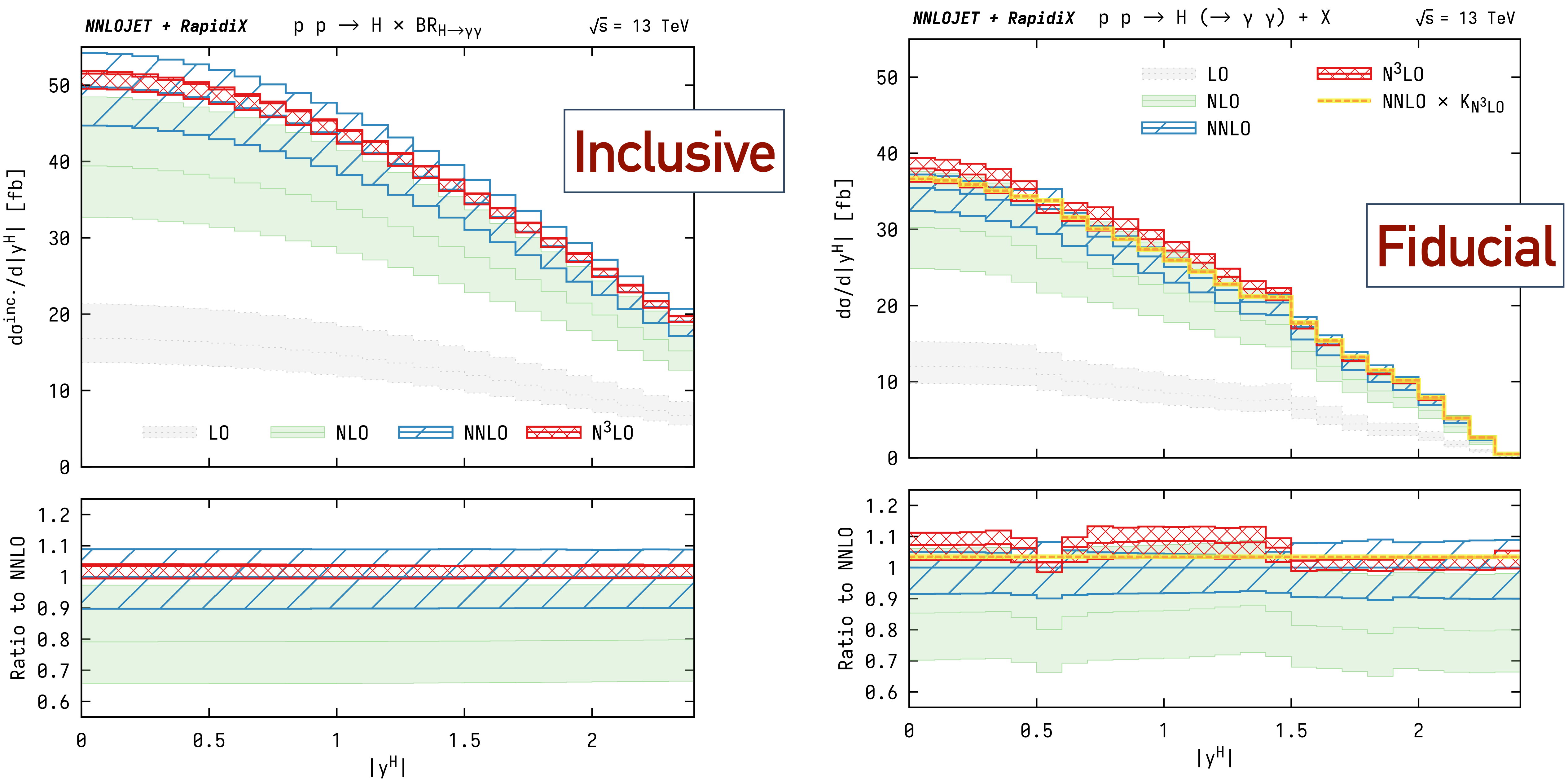
If $p_t \neq 0$: at least one hard emission

↓

V+J@NNLO

Fully-differential Higgs @ N³LO: P2B

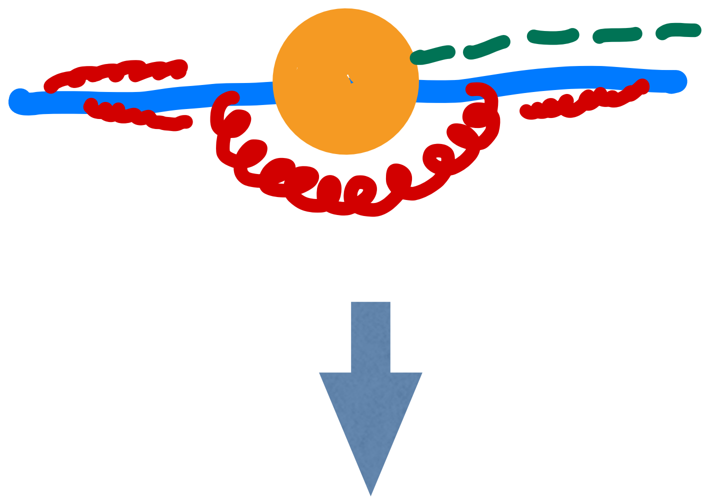
[Chen, Gehrmann, Glover, Huss, Mistlberger, Pelloni (2021)]



- Higgs rapidity distribution [Dulat, Mistlberger, Pelloni (2018)]
- Exquisite numerical control of H+j@NNLO [NNLOjet, 2015-2021]
- Combined using P2B [Cacciari, Dreyer, Karlberg, Salam, Zanderighi (2015)]

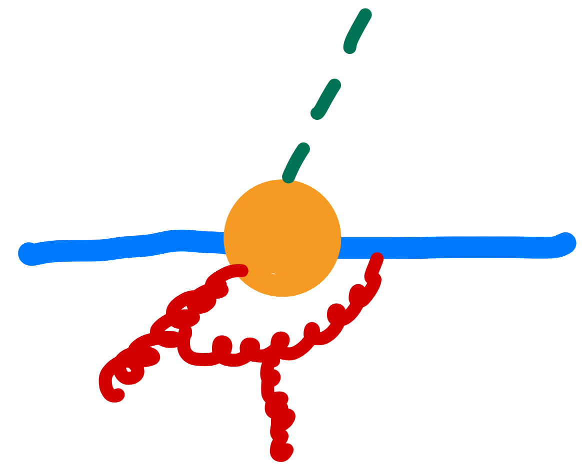
N^3LO without full rapidity distribution

Colour-singlet production at order α_s^3:



Up to power corrections: from resummation

+

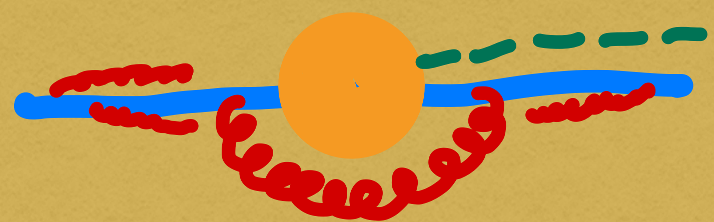


V+J@NNLO

$$\sigma^{\mathrm{N^3LO}} = \int_0^{p_{t,\mathrm{cut}}} \frac{\mathrm{d}\sigma^{\mathrm{N^3LL}}}{\mathrm{d}p_t}\mathrm{d}p_t + \int_{p_{t,\mathrm{cut}}} \frac{\mathrm{d}\sigma^{\mathrm{NNLO}}_{V+J}}{\mathrm{d}p_t}\mathrm{d}p_t + \mathcal{O}\left(p_{t,\mathrm{cut}}^2 \ln^5 p_{t,\mathrm{cut}}\right)$$

N^3LO without full rapidity distribution

Colour-singlet production at order α_s^3:



Last missing ingredient for N^3LL resummation: ``N^3LO beam function”

[Behring, Melnikov, Rietkerk, Tancredi, Wever (2019); Luo, Yang, Zhu, Zhu (2020); Ebert, Mistlberger, Vita (2020)]

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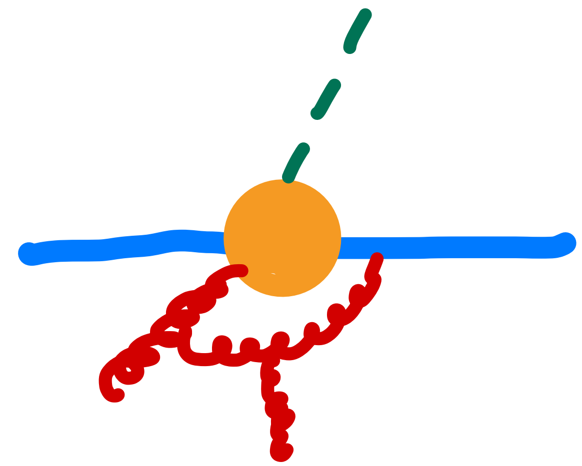


V+J@NNLO

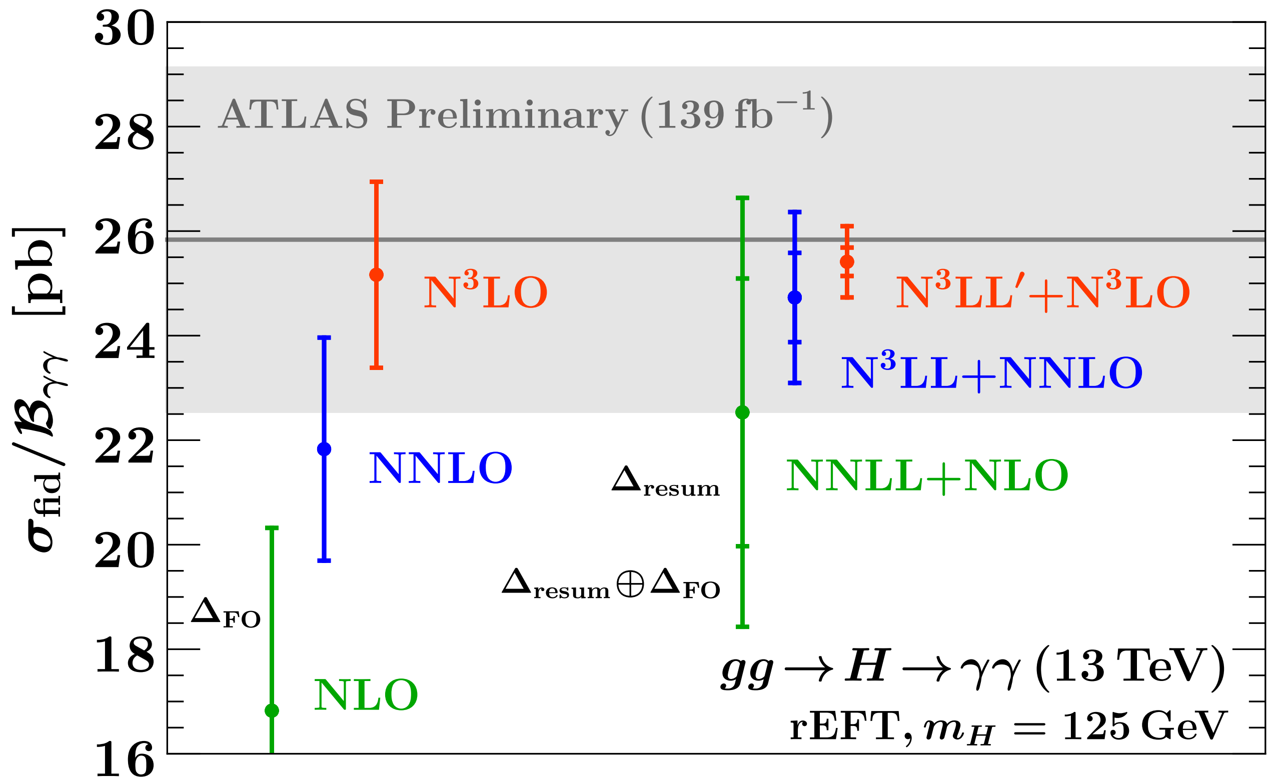
$$\sigma^{\mathrm{N^3LO}} = \int_0^{p_{t,\mathrm{cut}}} \frac{\mathrm{d}\sigma^{\mathrm{N^3LL}}}{\mathrm{d}p_t}\mathrm{d}p_t + \int_{p_{t,\mathrm{cut}}} \frac{\mathrm{d}\sigma^{\mathrm{NNLO}}_{V+J}}{\mathrm{d}p_t}\mathrm{d}p_t + \mathcal{O}\left(p_{t,\mathrm{cut}}^2 \ln^5 p_{t,\mathrm{cut}}\right)$$

Easy to go from N^3LO to N^3LO + N^3LL

N³LO+N³LL: recent results

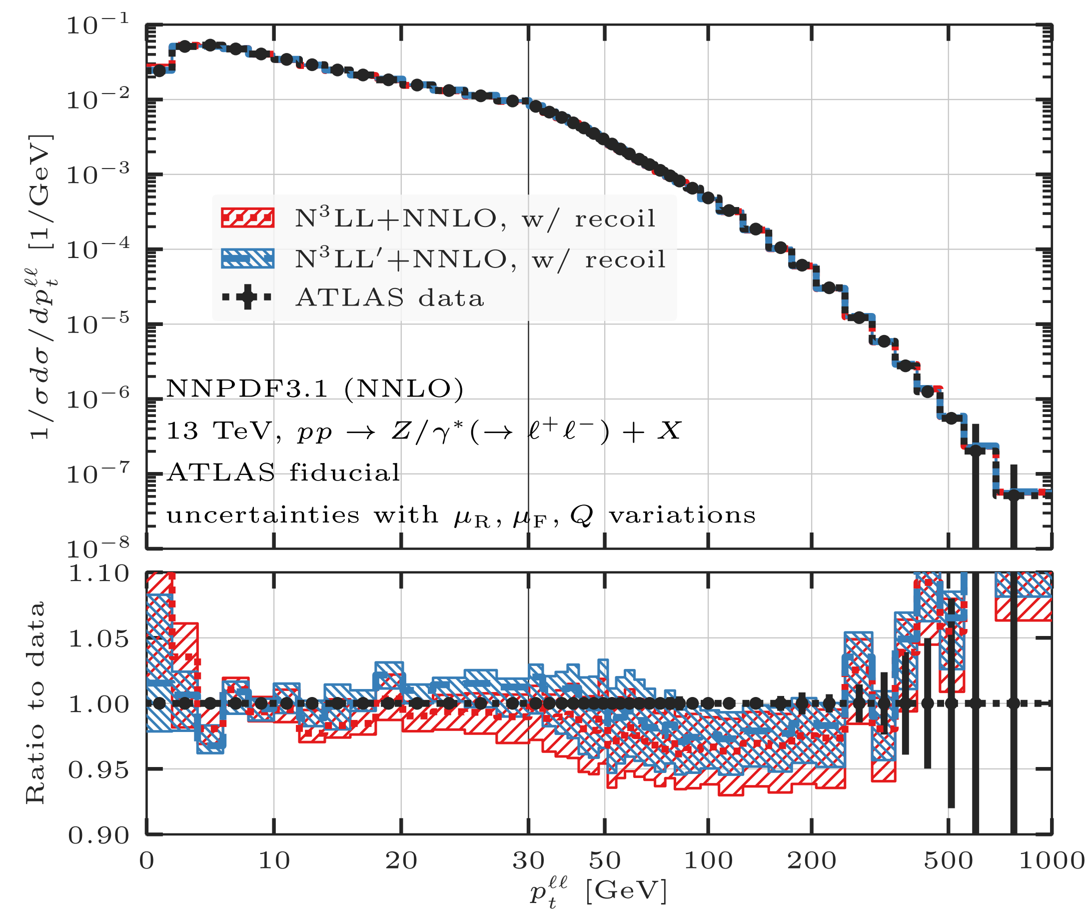
[V+jet@NNLO: NNLOjet, extremely stable down to p_t ~ 0.5 GeV]

H



[Billis, Dehnadi, Ebert, Michel, Tackmann (2021)]

H/Z



[Re, Rottoli, Torrielli (2021)]

Z

Order	NLO	NNLO	N^{3}LO
$\sigma(pp \to Z/\gamma^* \to l^+l^-)$ [pb]	766.3 ± 0.5	757.4 ± 1.9	746.1 ± 1.9
Order	NLL+NLO	NNLL+NNLO	N^{3}LL+N^{3}LO
$\sigma(pp \to Z/\gamma^* \to l^+l^-)$ [pb]	773.7 ± 0.5	759.8 ± 1.9	749.6 ± 2.0

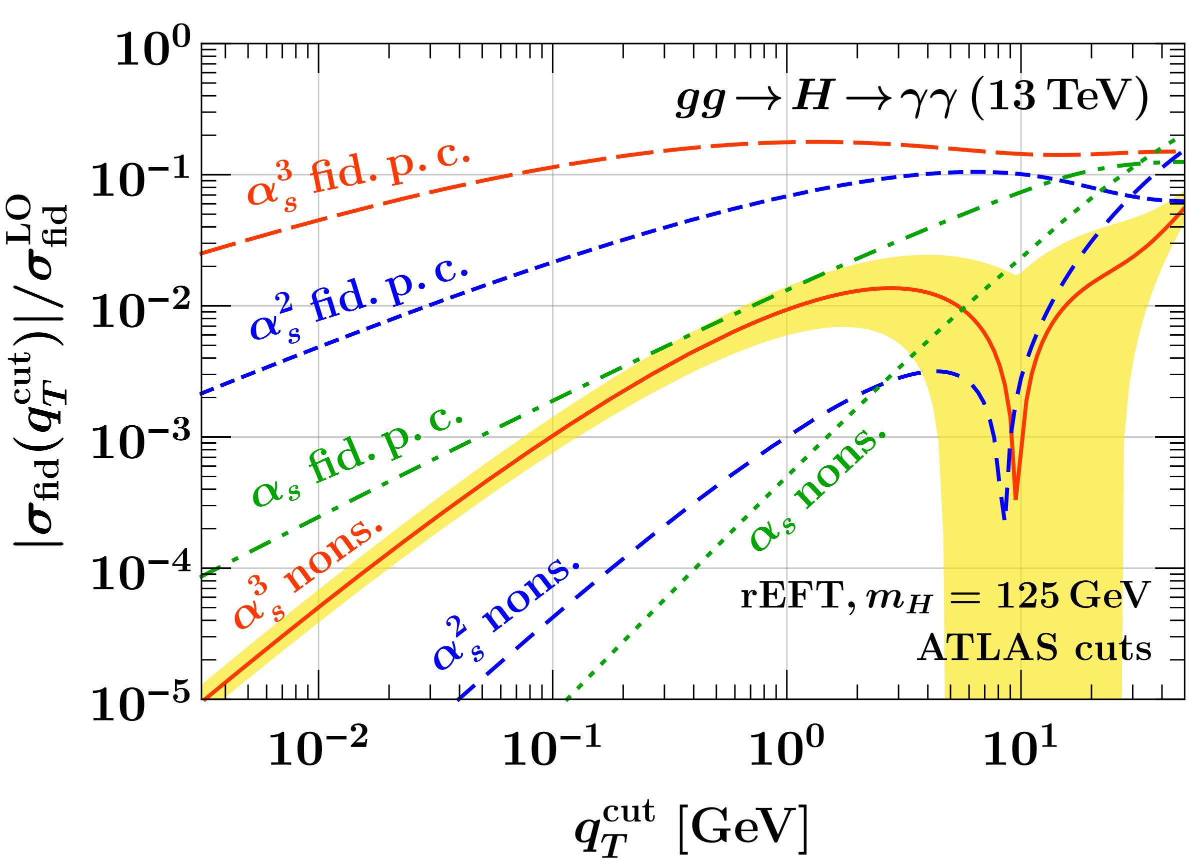
[Camarda, Cieri, Ferrera (2021)]

N^{3}LO from resummation: a word of caution

To extract N^{3}LO: subleading power must be under control

$$\sigma^{\mathrm{N^3LO}} = \int_0^{p_{t,\mathrm{cut}}} \frac{\mathrm{d}\sigma^{\mathrm{N^3LL}}}{\mathrm{d}p_t} \mathrm{d}p_t + \int_{p_{t,\mathrm{cut}}} \frac{\mathrm{d}\sigma^{\mathrm{NNLO}}_{V+J}}{\mathrm{d}p_t} \mathrm{d}p_t + \mathcal{O}\left(p_{t,\mathrm{cut}}^2 \ln^5 p_{t,\mathrm{cut}}\right)$$

- Subleading power ~ $\alpha_s^n (p_t/Q)^2 \ln^{2n-1}(p_t/Q)$ → much lower cutoff w.r.t. NNLO
- Naive estimate: NNLO V+j down to ~1-0.5 GeV → error up to order 1%



- For Higgs, confirmed by (and included in) [Billis, Dehnadi, Ebert, Michel, Tackmann (2021)]
- Good news: first subleading is enough
- N^{3}LO+N^{3}LL: less severe, but more ambiguities (see first slide…)

Fiducial N^{3}LO: a much more serious problem

$$\mathrm{d}\sigma = \int dx_1 dx_2 f(x_1) f(x_2)\, \mathrm{d}\sigma_{\mathrm{part}}(x_1, x_2) F_J\, (1 + \mathcal{O}(\Lambda^p_{\mathrm{QCD}}/Q^p))$$

``observable F_J must be insensitive to IR regions''

violated by ATLAS/CMS experimental cuts

- Drell-Yan: $p_{t,l} > 25$ GeV, $|y_l| < 2.5$ → the infamous ``symmetric cuts''. Well-known source of troubles [Frixione, Ridolfi (1997)]
- Higgs: asymmetric cuts. $p_{t,\gamma 1(2)} < 0.35(0.25)\, m_H$, $|y_\gamma| < 2.37$, with gap

Unfortunately, both symmetric and asymmetric cuts share the same feature: introduce linear p_t dependence on the acceptance at small p_t

[Catani, Cieri, de Florian, Ferrera, Grazzini (2018); Ebert, Michel, Tackmann + Billis, Dehnadi (2017-2021); Salam + Slade (2015, 2021)]

Fiducial N³LO: a much more serious problem

$$\mathrm{d}\sigma = \int dx_1 dx_2 f(x_1) f(x_2)\, \mathrm{d}\sigma_{\mathrm{part}}(x_1, x_2) F_J\ (1 + \mathcal{O}(\Lambda^p_{\mathrm{QCD}}/Q^p))$$

``observable F_J must be insensitive to IR regions"

violated by ATLAS/CMS experimental cuts

Linear p_t dependence → spurious growth of the perturbative series

[Billis, Dehnadi, Ebert, Michel, Tackmann (2021)]

$$\begin{aligned}
\sigma^{\mathrm{FO}}_{\mathrm{incl}} &= 13.80\,[1 + 1.291 + 0.783 + 0.299]\,\mathrm{pb}\,, \\
\sigma^{\mathrm{FO}}_{\mathrm{fid}}/\mathcal{B}_{\gamma\gamma} &= 6.928\,[1 + (1.300 + 0.129_{\mathrm{fpc}}) \\
&\quad + (0.784 - 0.061_{\mathrm{fpc}}) \\
&\quad + (0.331 + 0.150_{\mathrm{fpc}})]\,\mathrm{pb}\,.
\end{aligned}$$

Starting from N³LO: spurious effect can be as large as correction itself

Fiducial N^{3}LO: a much more serious problem

$$\mathrm{d}\sigma = \int dx_1 dx_2 f(x_1) f(x_2)\, \mathrm{d}\sigma_{\mathrm{fin}}(x_1, x_2) F_J \left(1 + \mathcal{O}(\Lambda^p_{\mathrm{IR}}/Q^p)\right)$$

Two options:

- Consider N^{3}LO + N^{3}LL. Straightforward, but introduces new degree of arbitrariness (matching/profile functions, resummation scales…)

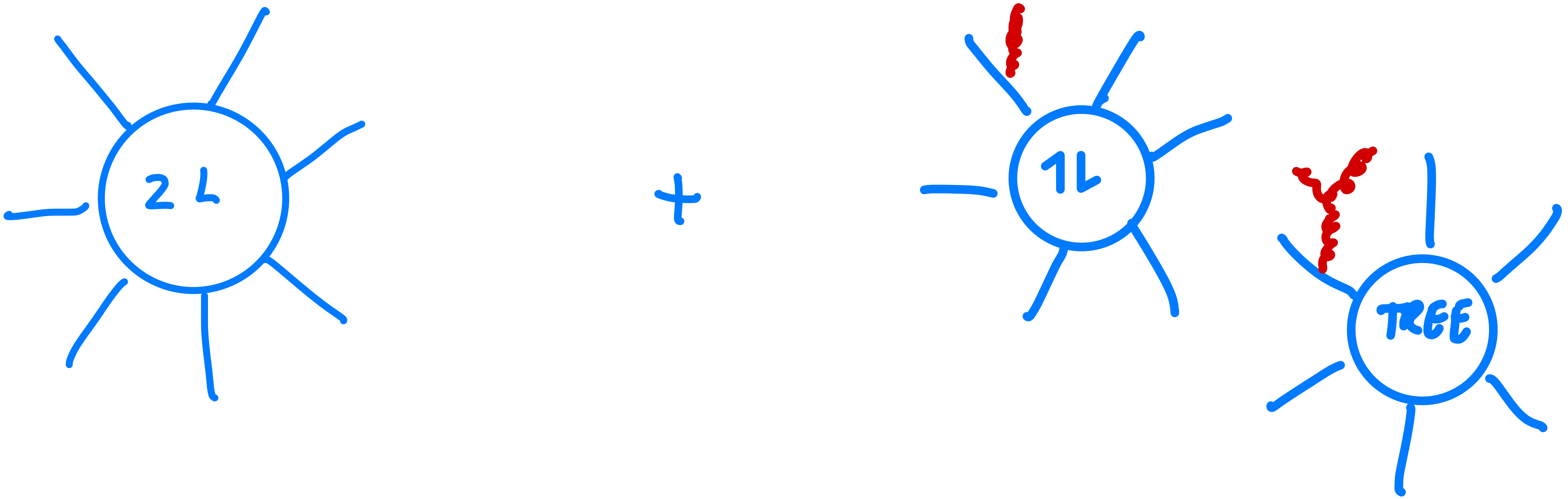
- Devise a set of cuts that are less sensitive to IR physics
  Interesting theoretical and experimental problem

[Billis, Dehnadi, Ebert

One order below, but complex processes

Fully-differential NNLO bottlenecks:

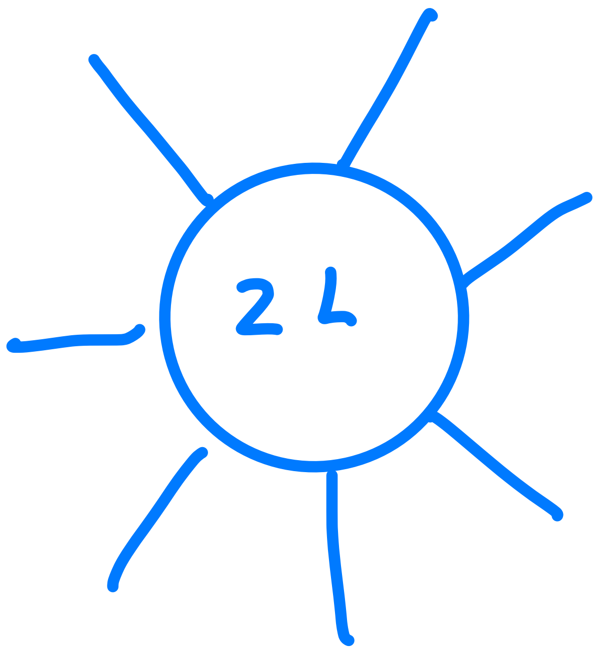
2L amplitudes + dealing with non-trivial IR emission (``subtractions”)



+ stable 1L amplitudes [OpenLoops…]

One order below, but complex processes

[caveat: only include results from last year. Impressive progress]



- new ideas/better implementation of IBP reductions (=from full amplitude to a bunch of independent integrals) [Klapper, Lange, Maierhöfer, Usovitsch, KIRA 2.0; Bendle, Böhm, Ma, Rahn, Ristau, Wittmann, Zu, Zhang; Heller, Manteuffel]
- all Feynman integrals for massless 2→3 scattering available in fast/robust implementation [Chicherin, Sotnikov]
- Steps towards Feynman integrals for V+2j [Abreu, Ita, Moriello, Page, Tschernow]

- Leading-colour 3j amplitude [Abreu, Dormans, Febres-Cordero, Ita, Kraus, Page, Pascual, Ruf, Sotnikov]
- Leading-colour 3γ amplitude [Abreu, Page, Pascual, Sotnikov; Chawdhry, Czakon, Mitov, Poncelet]
- Leading-colour γγj amplitude [Agarwal, Buccioni, Manteuffel, Tancredi; Chawdhry, Czakon, Mitov, Poncelet]
- Towards LC Vjj [Badger, Hartanto, Zoia]
- Towards analytic ttb [Badger, Chaubey, Hartanto, Marzucca]

\+ recent earlier work from the above and Peraro, Gehrmann, Henn, lo Presti, Papadopoulos, Tommasini, Wever, Schabinger, Gluza, Kajda, Kosower, Georgoudis, Larsen, Schönemann, Mitev, Wasser…

One order below, but complex processes

[caveat: <u>only include results from last year</u>. Impressive progress]

Punchline:

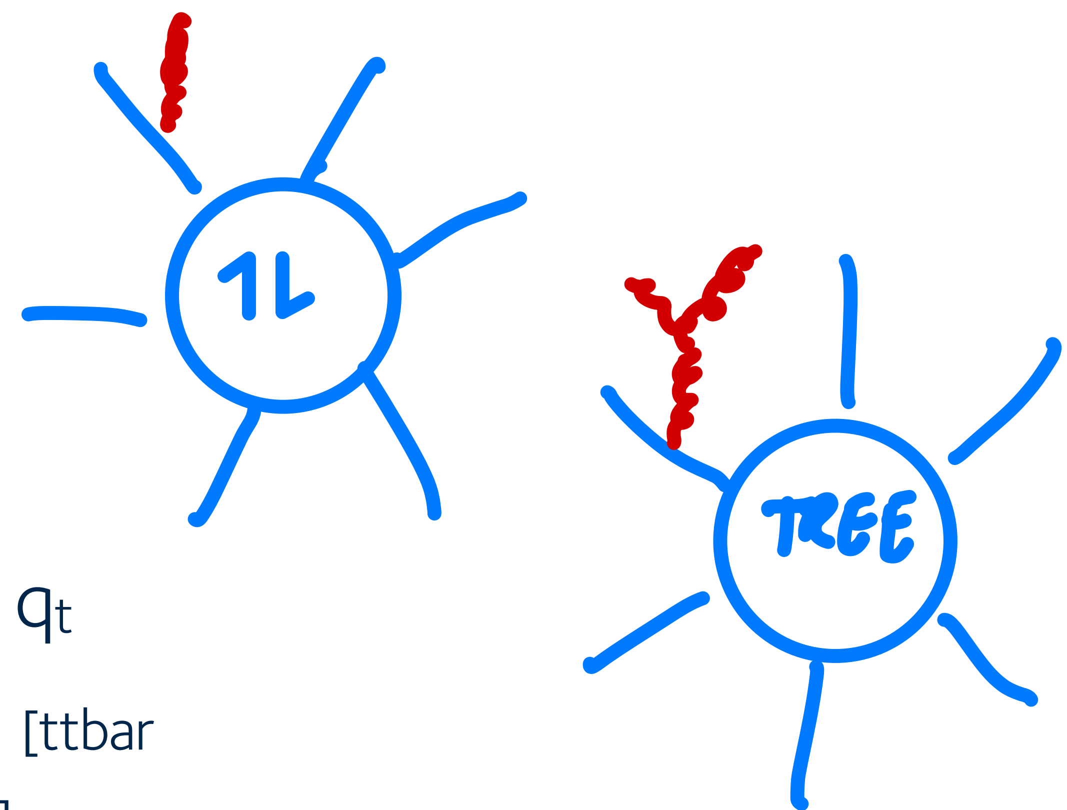
- Leading color for massless 2 → 3 either available or within reach. <u>Efficient, robust implementation that can be used in MC codes</u>
- Progress towards full color
- First steps towards Vjj
- Still progress towards massive scattering
- Mixture of new technical development and systematic approach

let]

+ earlier work from the above and Peraro, Gehrmann, Henn, lo Presti, Papadopoulos, Tommasini, Wever, Schabinger, Gluza, Kajda, Kosower, Georgoudis, Larsen, Schönemann, Mitev, Wasser…

One order below, but complex processes

<u>Infrared subtraction</u>

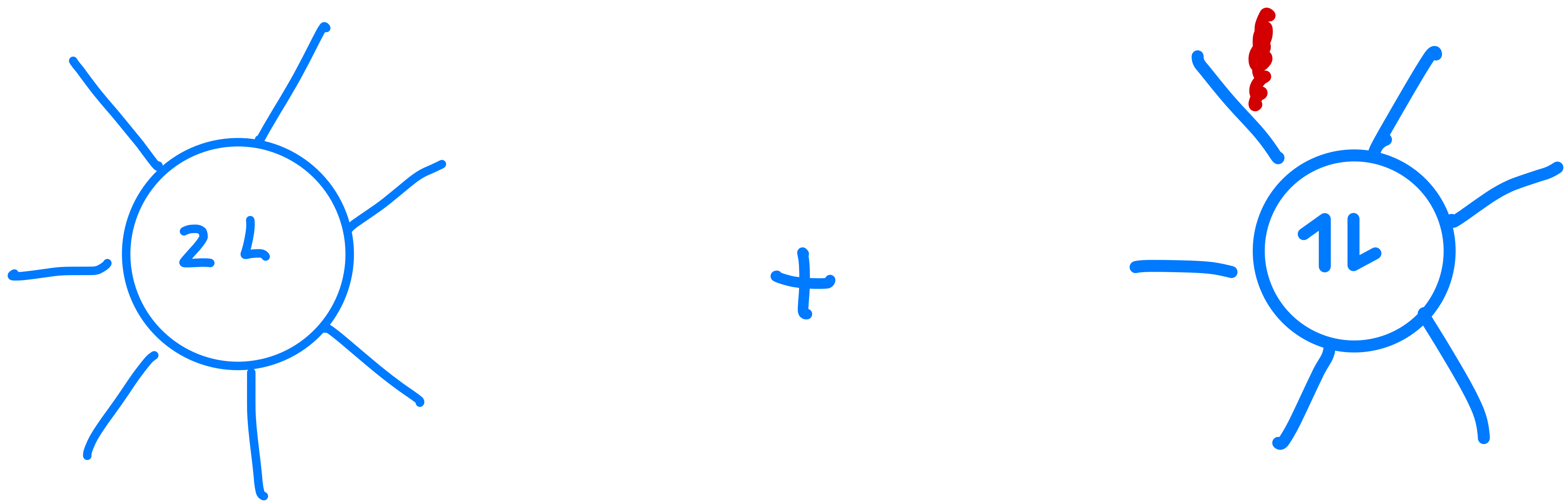


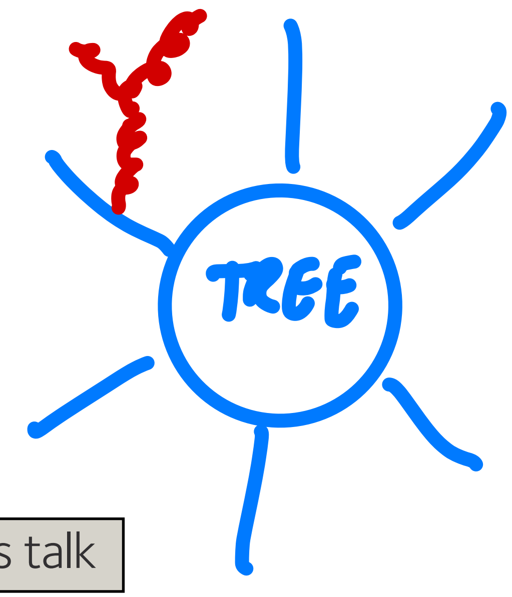
- Several frameworks on the market, some of them ready to deal with generic processes [antennas, stripper, nested soft-collinear]

- Robust, efficient and public implementation of q_t subtraction: Matrix: → color singlet, tt (+H,V) [ttbar extension: Catani, Devoto, Grazzini, Kallweit, Mazzitelli (2019)]

- Advancement made it possible to easily perform mixed QCD-EW corrections [de Florian, Der, Fabre (2018); Cieri, de Florian, Der, Mazzitelli (2020); Delto, Jaquier, Melnikov, Röntsch+Behring, Buccioni, FC (2018-2021), Buonocore, Grazzini, Tramontano + Kallweit, Savoini (2020-21); see also inclusive results by Bonciani, Buccioni, Rana, Vicini (2020) and 2L amplitude by Heller, von Manteuffel, Schabinger, Spiesberger (202)]

NNLO: recent results





- First results using 2→3@2L: 3γ production [Chawdhry, Czakon, Mitov, Poncelet (2019); Kallweit, Sotnikov, Wiesemann (2020)] see V. Sotnikov's talk

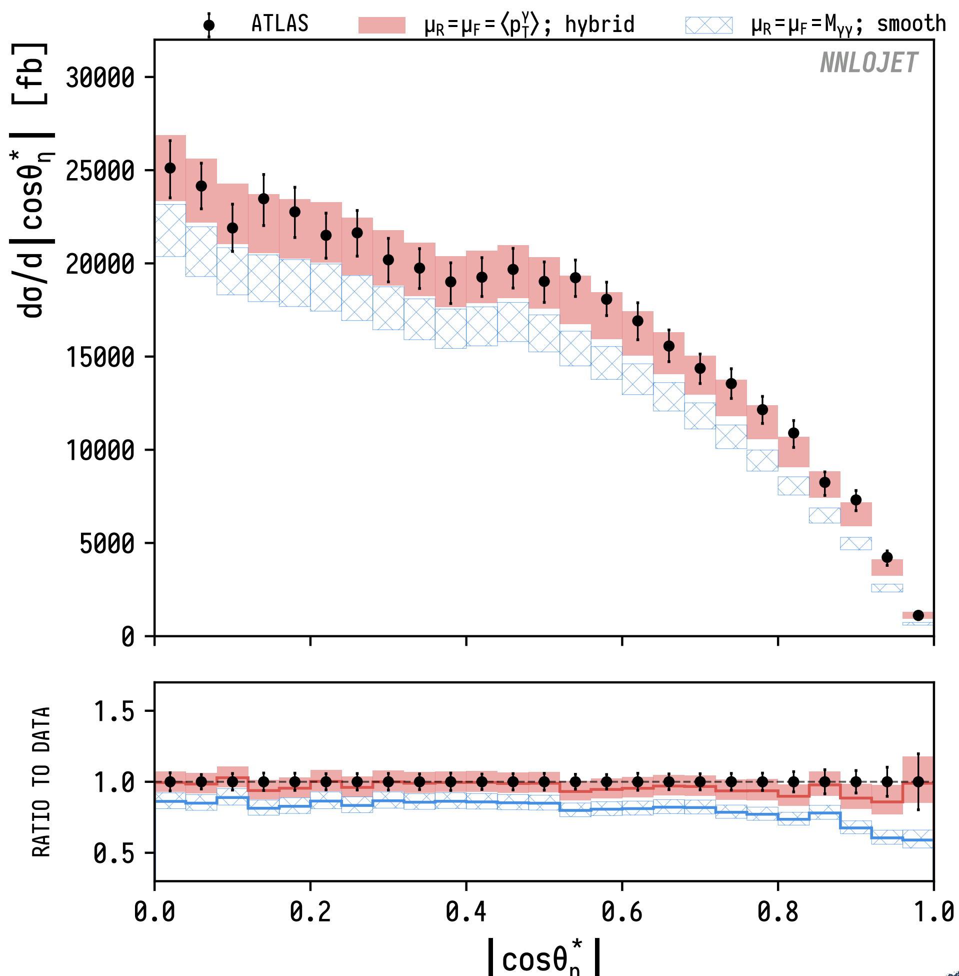
- From ``proof of concept'' to interesting pheno studies: γγ isolation [Gehrmann, Glover, Huss, Whitehead (2020)], jet-binning in VH [Gauld, Gehrmann-de Ridder, Glover, Huss, Maier (202)]

- Mixed QCD-EW: impact on the W mass [Behring, Buccioni, FC, Delto Jaquier, Melnikov, Röntsch (2021)], impact of finite-width corrections [Buonocore, Grazzini, Kallweit, Savoini, Tramontano (2021)]

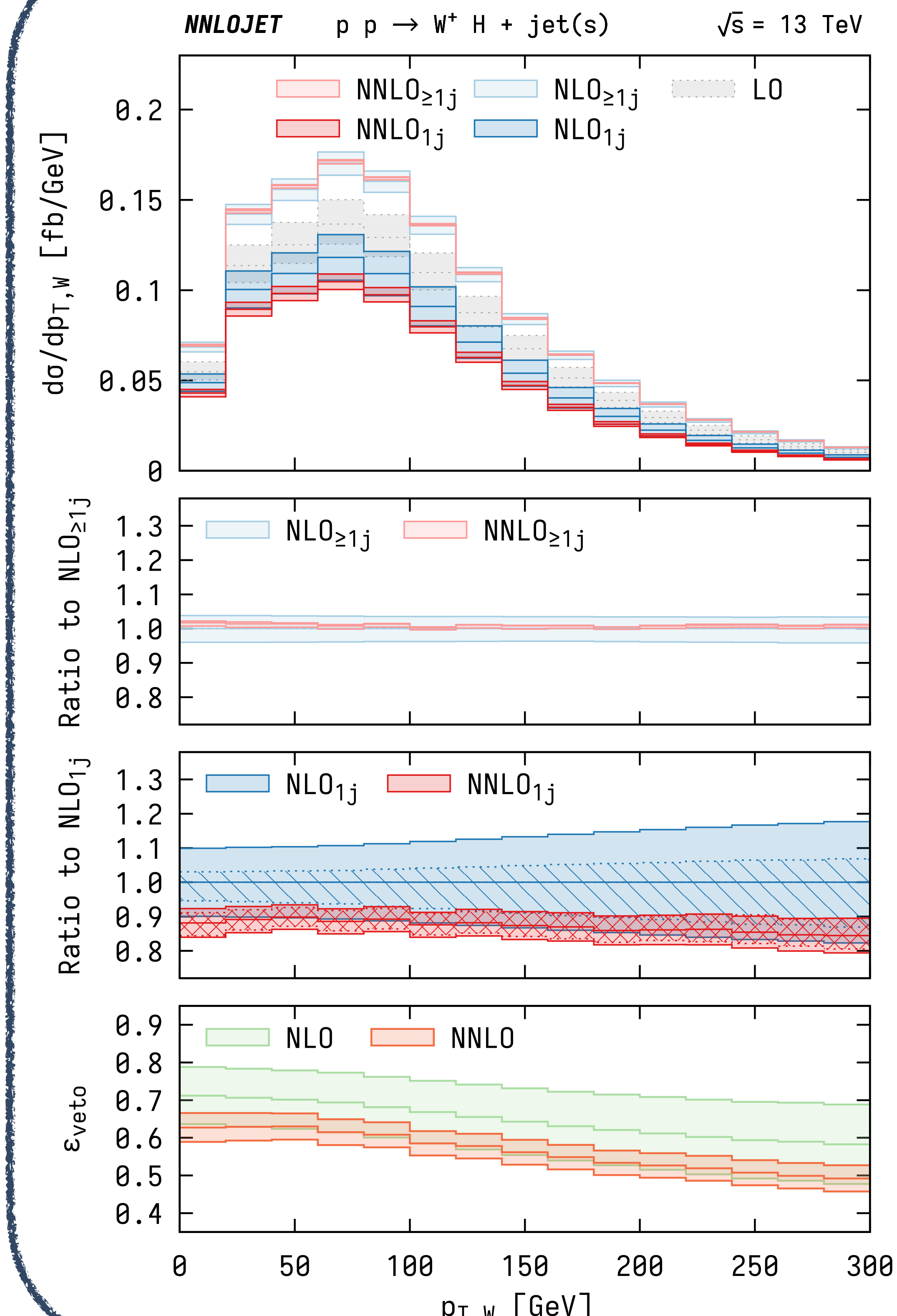
- More exploration in HF: bb [Catani, Devoto, Grazzini, Kallweit, Mazzitelli (2020)], first steps towards ttH [Catani, Fabre, Grazzini, Kallweit (2021)]

NNLO: recent results

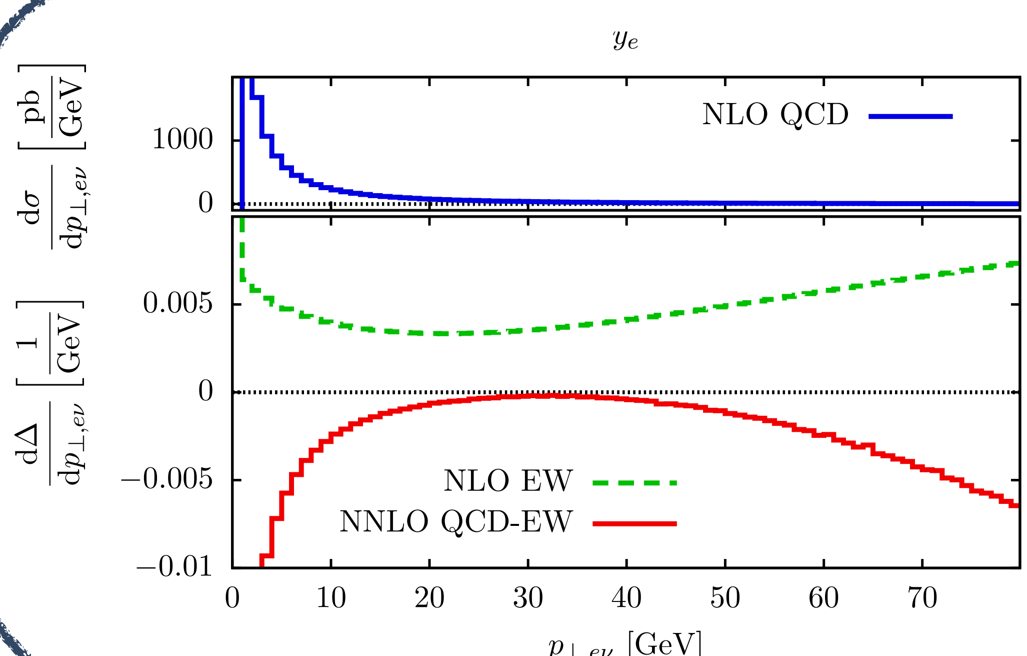
The impact of photon isolation criteria



Precise jet-veto efficiencies for WH



W-mass studies



Conclusions and outlook

- In the recent past: a lot of improvement in fixed-order calculations
  - 2-loop amplitudes
  - subtraction schemes
  - Computational tools (→ ingredients for N^3LL resummation)
- We now see the phenomenological fruits:
  - good subtraction schemes + good computational tools for resummation: fully differential N^3LO
  - good subtraction schemes, new tools for scattering amplitudes: (massless) 2→3@NNLO within reach
- Did not mention many other developments (NNLOPS...)
- As we explore further, and become more and more precise, new interesting issues come about
  - fiducial region and infrared sensitivity

Thank you very much for your attention!