On correlated theoretical uncertainties in LHC data analysis

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Foreword I

Statistical comparison of LHC data vs theory

- requires uncertainties+correlations for data and theory
- theory uncertainties+correlations not easy do assess (sometimes neglected), but their relevance increases with precision of data

Various types of theory uncertainties

- PDFs and input parameters (α_S, m_t, ...)
- MC modelling: parton showers, matching, hadronisation
- missing higher-order uncertainties (MHOUs) $\rightarrow$ main foucs of this talk

Factor-2 variations of renormalisation (μ_R) and factorisation scales (μ_F)

- simple, fully general, and most widely used prescription to estimate MHOUs
- often reasonable for cross section normalisation, but not justified for correlations across kinematic regions (shapes) and processes (ratios)

$\Rightarrow$ important to develop more realistic (and possibly general) methods

Foreword II

Aim of this talk: provide examples of what exists about theory uncertainty correlations (not so much) to stimulate discussion on what is missing (a lot)

Two selected examples from the literature

(1) Theory uncertainties in global NNPDF fits [1905.04311, 1906.10698]

$\rightarrow$ need of general models of MHOUs and implications in global analyses

(2) V+jet backgrounds to monojet searches [Lindert et al, 1705.04664]

$\rightarrow$ detailed model of theory correlations as a key to enhance sensitivity

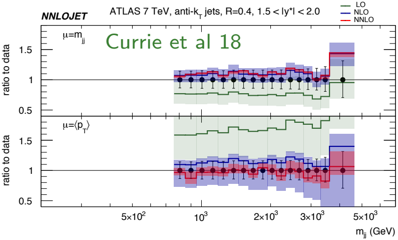
Outline

1. Theory uncertainties in global PDF fits

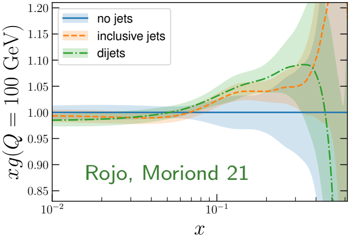
2. V+jet backgrounds to monojet searches

Global PDF fits

$pp \to$ jets



gluon at high-x



PDFs determined by comparing global set of data (D_i) to theory (T_i)

$$\chi^2 = \frac{1}{N_{\rm dat}} \sum_{i,j}^{N_{\rm dat}} (D_i - T_i)\, C_{ij}^{-1}\, (D_j - T_j)$$

- experimental uncertainties + correlations $\Rightarrow$ covariance matrix C_{ij}
- theory uncertainties usually neglected

Theory uncertainties in NNPDF fits [1905.04311, 1906.10698]

Accounted for through

$$\chi^2 = \frac{1}{N_{\rm dat}} \sum_{i,j}^{N_{\rm dat}} (D_i - T_i)\,(C + S)^{-1}_{ij}\,(D_j - T_j)$$

with S_{ij} = extra covariance matrix for theory uncertainties and correlations

$$S_{ij} = \langle \Delta_i \Delta_j \rangle \qquad \Delta_k = \underbrace{\mathcal{T}_k}_{\rm truth} - \underbrace{T_k}_{{\rm N}^k{\rm LO}}$$

Theory uncertainty model based on scale variations

$$\Delta_k = T_k(\xi_R\,\mu_R, \xi_F\,\mu_F) - T_k(\mu_R, \mu_F) \qquad \xi_{R,F} \in [0.5, 2]$$

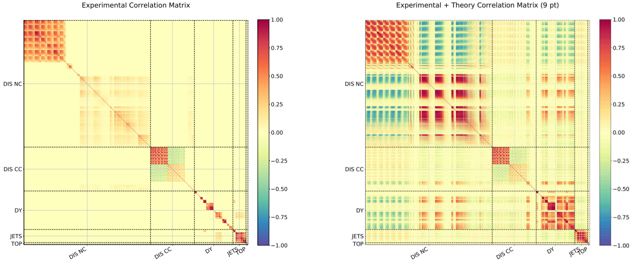
Simple correlation model

- ξ_R correlated for data in the same process class (DIS NC, DIS CC, DY, JET, TOP)
- ξ_F correlated throughout (universality of PDF evolution)

Effect of TH uncertainties on NNPDF fits

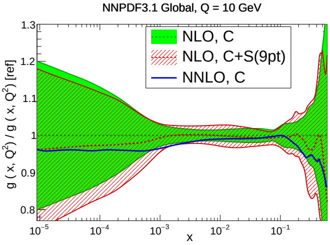
Assessed comparing fits based on $(T + \Delta T)^{\rm NLO}$ vs $T^{\rm NNLO}$

$\Rightarrow$ significant impact since $\Delta T^{\rm NLO} \gtrsim$ exp errors



- new pattern of (anti)correlations across and within process classes
- weight of data with larger TH uncertainty reduced
- shifts in central values towards true results, improved fit quality
- slightly increased PDF uncertainty

Effect of TH uncertainties on NNPDF fits



Gluon PDF

- at high-x: including ΔT^{NLO} lowers gluon PDF and increases uncertainty
- consistent with large shift induced by NNLO theory

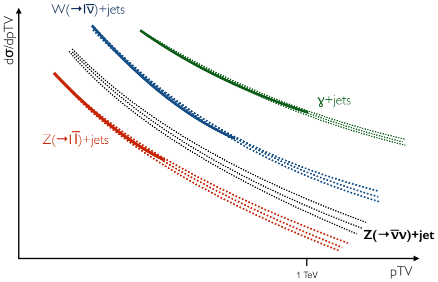
General considerations

- Global fits require general theory uncertainty prescriptions applicable to a wide range of processes+observables
- Scale variations are the only general prescription on the market
- Their main effect is a normalisation shift, and it would be important to develop alternative methods that provide more realistic shape uncertainties and correlations

Outline

Searches of DM in jet+MET signatures

Precision of data calls for percent-level control of irreducible $Z(\nu\nu)$ + jet background



Obtained from visible $Z/W/\gamma+$ jet data + theory extrapolation

- simultaneous profile likelyhood fit of 3 backgrounds + signal
- requires precise $\mathrm{d}\sigma_V/\mathrm{d}p_T$ theory for different processes and p_T regions

prior theory uncertainties and their correlations are crucial!

State of the art $V+$jet backgrounds for monojet searches

[J. M. Lindert, S. Pozzorini, R. Boughezal, A. Denner, S. Dittmaier, A. Huss, A. Gehrmann-De Ridder, T. Gehrmann, N. Glover, S. Kallweit, P. Maierhöfer, M. L. Mangano, T.A. Morgan, A. Mück, M. Schönherr, F. Petriello, G. P. Salam, arXiv:1705.04664v2 (Oct 17)]

Simple processes and observables $\rightarrow$ high precision

- vector-boson p_T distributions for *inclusive* $\gamma/Z/W+\text{jet}$ production
- NNLO QCD + NLO EW + NNLO Sudakov EW

Model of theory uncertainties for global $\gamma/Z/W+\text{jet}$ fit

$$\frac{\mathrm{d}}{\mathrm{d}p_T}\sigma^{(V)}_{\mathrm{TH}}(\vec{\varepsilon}_{\mathrm{TH}}) = \left[\underbrace{K^{(V)}_{\mathrm{QCD}\otimes\mathrm{EW}}(p_T)}_{\text{correction}} + \sum_{i=1} \varepsilon_{\mathrm{TH},i}\, \underbrace{\delta^{(i)}K^{(V)}(p_T)}_{\text{uncertainties}}\right]\frac{\mathrm{d}}{\mathrm{d}p_T}\sigma^{(V)}_{\mathrm{LO}}$$

- Gaussian nuisance parameters ($\Delta\varepsilon_{\mathrm{TH},i}=\pm 1$) to be fitted in EXP analysis
- Uncertainties $\delta^{(i)}K^{(V)}(p_T)$ embody correlations across processes and p_T regions

Monte Carlo reweighting (for *inclusive* monojet searches)

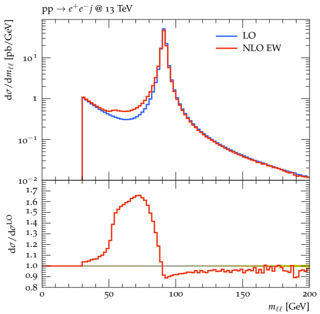
$$\frac{\mathrm{d}}{\mathrm{d}p_T}\frac{\mathrm{d}}{\mathrm{d}\Phi}\sigma^{(V)}(\vec{\varepsilon}_{\mathrm{MC}},\vec{\varepsilon}_{\mathrm{TH}}) := \underbrace{\frac{\mathrm{d}}{\mathrm{d}p_T}\frac{\mathrm{d}}{\mathrm{d}\Phi}\sigma^{(V)}_{\mathrm{MC}}(\vec{\varepsilon}_{\mathrm{MC}})}_{\text{fully differential \& particle level}} \underbrace{\left[\frac{\frac{\mathrm{d}}{\mathrm{d}p_T}\sigma^{(V)}_{\mathrm{TH}}(\vec{\varepsilon}_{\mathrm{TH}})}{\frac{\mathrm{d}}{\mathrm{d}p_T}\sigma^{(V)}_{\mathrm{MC}}(\vec{\varepsilon}_{\mathrm{MC}})}\right]}_{\text{inclusive \& parton level}}$$

Bottom line of reweighting

- distribution in $p_{T,\mathrm{rew}}$ and uncertainties entirely from $\mathrm{d}\sigma_{\mathrm{TH}}$ (fixed-order calculations)
- extrapolation $p_{T,\mathrm{rew}} \to p_{T,\mathrm{exp}}$ and related uncertainties from $\mathrm{d}\sigma_{\mathrm{MC}}$

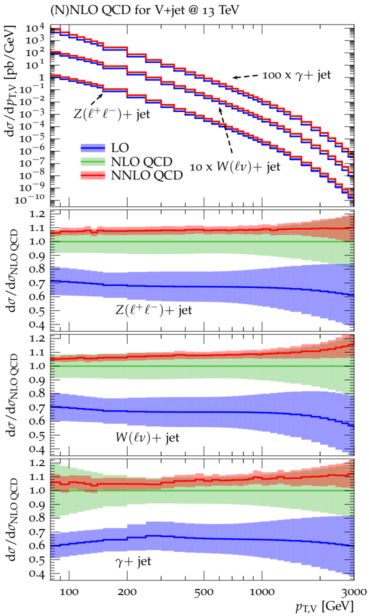
Careful definition of physics objects $(\gamma, \ell^{\pm}, \nu)$ and cuts that enter $p_{T,\mathrm{rew}}$

- optimal TH precision and $\gamma/Z/W$ correlation
  - $\Rightarrow$ inclusiveness wrt QCD+QED radiation
  - $\Rightarrow$ special γ-isolation (see backup)
- exclude aspects better described by $\mathrm{d}\sigma_{\mathrm{MC}}$
  - $\Rightarrow$ no lepton isolation, no hadronisaztion/UE, . . .



$\Rightarrow m_{\ell\ell} > 30\mathrm{GeV}$ cut

NNLO QCD [Gehrmann-De Ridder et al, Boughezal et al, Campbell et al ’15–’19]



Corrections and scale uncertainties

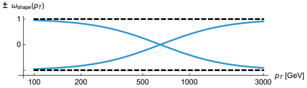
- very similar for $V = W, Z, \gamma$

Uncertainty model

$$\sum_{i=1}^{3} \varepsilon_i^{(V)} \underbrace{\delta^{(i)} K^{(V)}(p_T)}_{\text{uncertainty}} \frac{\mathrm{d}}{\mathrm{d}p_T} \sigma_{\mathrm{LO}}^{(V)}$$

$i = 1$ correlated 7pt $\mu_{R,F}$ variations for $\gamma/W/Z$+jet

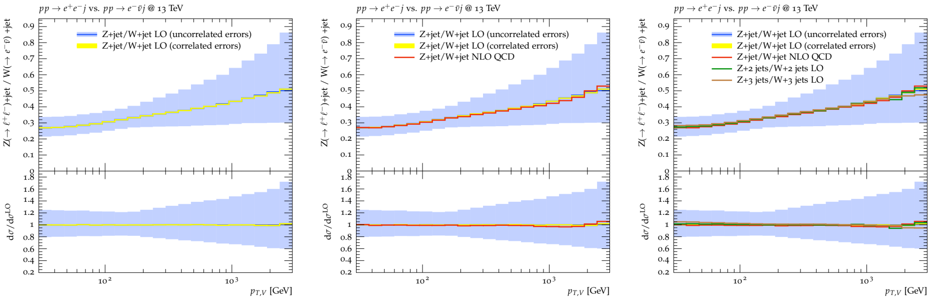
$i = 2$ p_T-shape uncertainty: smooth anticorrelation of $\mu_{R,F}$ variations



$i = 3$ process correlation uncertainty:

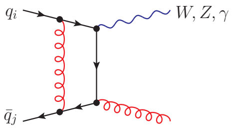
decorrelation of non-universal effects

(un)correlated scale uncertainties for $\sigma^{(Z+j)}/\sigma^{(W+j)}$ ratio



Uncorrelated uncertainty $\gg$ correlated uncertainty and unrealistic

- $V + 1, 2, 3$ jets at NLO confirm that higher-order QCD largely universal (2016)
- expected from structure of QCD corrections and further confirmed at NNLO in 2017 (see below)



Quantitative estimate of non-universal effects?

Process-correlation uncertainties

Idea

- scale variations are largely insensitive to non-universal effects
- for a (conservative) uncertainty estimate one can use the (small) non-universality observed in the highest available perturbative corrections

Quasi-universality for $V = Z, W, \gamma$

$$\mathrm{d}\sigma^{(V)}_{\mathrm{NLO}} = \Big[1 + \underbrace{\kappa^{(Z)}_{\mathrm{NLO}}}_{\mathrm{universal}} + \underbrace{\delta\kappa^{(V)}_{\mathrm{NLO}}}_{\mathrm{non-universal}}\Big]\mathrm{d}\sigma^{(V)}_{\mathrm{LO}}, \qquad \delta\kappa^{(V)}_{\mathrm{NLO}} = \kappa^{(V)}_{\mathrm{NLO}} - \kappa^{(Z)}_{\mathrm{NLO}} \ll \kappa^{(V)}_{\mathrm{NLO}}$$

Prescription for NLO process-correlation uncertainty

- $\delta\kappa^{(V)}_{\mathrm{NLO}}$ downgraded to uncertainty and uncorrelated across processes

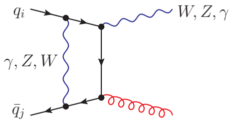
$\Rightarrow$ small uncertainties consistent with NNLO corrections to $\sigma^{(V)}/\sigma^{(Z)}$ (see below)

Prescription for N^{k}LO uncertainty

$$\delta\kappa^{(V)}_{\mathrm{N}^k\mathrm{LO}} = \Big[\kappa^{(V)}_{\mathrm{N}^k\mathrm{LO}} - \kappa^{(Z)}_{\mathrm{N}^k\mathrm{LO}}\Big]_{\mathcal{O}(\alpha_S^k)} \qquad \text{downgraded to uncertainty}$$

nNLO EW corrections

Exact $\mathcal{O}(\alpha)$ correction for $pp \to V + 1, 2$ jets [Denner, Dittmaier, Hofer, Kallweit, Kasprzik, Kühn, Kulesza, Lindert, Maierhöfer, Muck, S.P., Scharf, Schönherr, Schulze, Uccirati]



- -25% at $p_{\mathrm{T}} = 1\,\mathrm{TeV}$
- dominant part well understood: EW Sudakov logs

NLL Sudakov logs for $pp \to V+$ jet [Kühn, Kulesza, Schulze, S.P.; Becher, Garcia i Tormo]

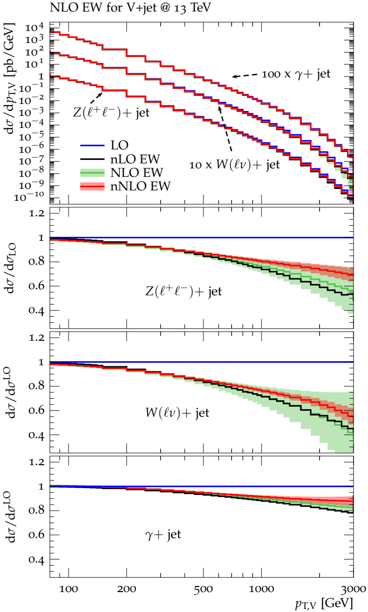
$$\delta^{(1)}_{\mathrm{Sud}} = \sum_{i,j} C^{(1)}_{2,ij} \ln^2\left(\frac{Q^2_{ij}}{M^2}\right) + C^{(1)}_1 \ln^1\left(\frac{Q^2}{M^2}\right) \quad \Rightarrow \quad -25\%$$

$$\delta^{(2)}_{\mathrm{Sud}} = \sum_{i,j} C^{(2)}_{4,ij} \ln^4\left(\frac{Q^2_{ij}}{M^2}\right) + C^{(2)}_3 \ln^3\left(\frac{Q^2}{M^2}\right) + \mathcal{O}\left[\ln^2\left(\frac{Q^2}{M^2}\right)\right] \quad \Rightarrow \quad +5\%$$

Coefficients $C^{(i)} \propto$ SU(2) charges $\Rightarrow$ breaking of universality

$\Rightarrow$ dominant impact on σ_Z/σ_V ratios and $\mathrm{d}\sigma/\mathrm{d}p_{\mathrm{T}}$ shape as well as final uncertainties

nNLO EW corrections to $V+$jet p_T spectra



nNLO EW = NLO EW + NNLO Sudakov

$$\kappa_{\mathrm{nNLO\,EW}} = \left(\frac{\alpha}{\pi}\right)\left[\delta^{(1)}_{\mathrm{hard}} + \delta^{(1)}_{\mathrm{Sud}}\right] + \left(\frac{\alpha}{\pi}\right)^2 \delta^{(2)}_{\mathrm{Sud}}$$

Sudakov logs dominant at NLO ($\delta^{(1)}_{\mathrm{hard}} \ll \delta^{(1)}_{\mathrm{Sud}}$)

$\Rightarrow$ supports NLL Sudakov approx. at NNLO EW

Model of EW uncertainties (see backup)

- predictable N^3LO EW Sudakov logs correlated
- estimated unknown NNLO EW terms of type

  $\delta^{(2)}_{\mathrm{Sud,NNLL}}$, $\delta^{(1)}_{\mathrm{hard}}\delta^{(1)}_{\mathrm{Sud}}$, $\delta^{(2)}_{\mathrm{hard}}$ uncorrelated

QCD⊗EW combination and uncertainties

Additive combination ⇒ large $\mathcal{O}(\alpha\alpha_S)$ uncertainties when $\kappa_{\rm QCD}$ & $\kappa_{\rm EW}$ large

$$\mathrm{d}\sigma_{\rm QCD\oplus EW} = \Big(1 + \kappa_{\rm EW} + \kappa_{\rm QCD}\Big)\,\mathrm{d}\sigma_{\rm LO}$$

Factorisation justified for EW Sudakov logs × soft QCD effects [Manohar, Becher,...]

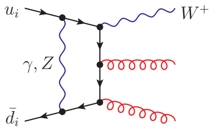
$$\mathrm{d}\sigma_{\rm QCD\otimes EW} = \Big[\big(1+\kappa_{\rm EW}\big)\big(1+\kappa_{\rm QCD}\big) + \underbrace{\delta\kappa_{\rm NNLO\,mix}}_{\text{expected} \ll \kappa_{\rm EW}}\,\kappa_{\rm QCD}\Big]\,\mathrm{d}\sigma_{\rm LO}$$

uncertainty $\delta\kappa_{\rm NNLO\,mix}$ estimated through NLO EW multileg ($V+2j$) calculations

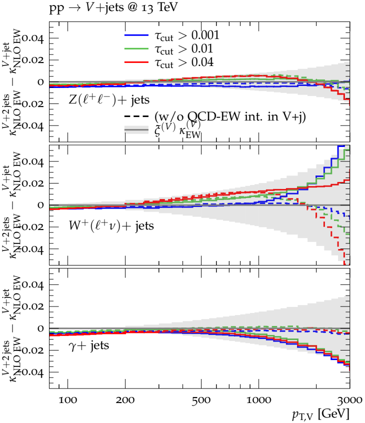
$$\mathrm{d}\sigma^{(V+2j)}_{\rm NLO\,EW}(\tau_{\rm cut}) \quad = \quad \Big[1 + \kappa^{(V+1j)}_{\rm NLO\,EW} + \delta\kappa^{(V)}_{\rm NNLO\,mix}(\tau_{\rm cut})\Big]\mathrm{d}\sigma^{(V+2j)}_{\rm LO}(\tau_{\rm cut})$$

Stable estimator in the IR limit $\tau_{\rm cut} \to 0$

$$\delta\kappa^{(V)}_{\rm NNLO\,mix}(\tau_{\rm cut}) = \kappa^{(V+2j)}_{\rm NLO\,EW}(\tau_{\rm cut}) - \kappa^{(V+1j)}_{\rm NLO\,EW}$$



Estimate for QCD-EW factorisation uncertainty



Difference between EW K-factors

$$\kappa^{(V+2j)}_{\text{NLO EW}} - \kappa^{(V+1j)}_{\text{NLO EW}} \lesssim 1\% \quad \text{up to 1 TeV}$$

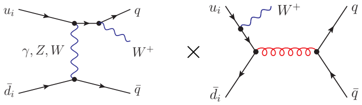
$\Rightarrow$ bulk of $\mathcal{O}(\alpha\alpha_S)$ effects factorises

Fit to $\delta\kappa_{\text{mix}}(p_{\text{T}}) = \xi_{\text{mix}}\kappa_{\text{EW}}(p_{\text{T}})$ yields

$$\delta\, \mathrm{d}\sigma_{\text{NNLO mix}} = \xi_{\text{mix}}\left[\mathrm{d}\sigma_{\text{QCD}\otimes\text{EW}} - \mathrm{d}\sigma_{\text{QCD}\oplus\text{EW}}\right]$$

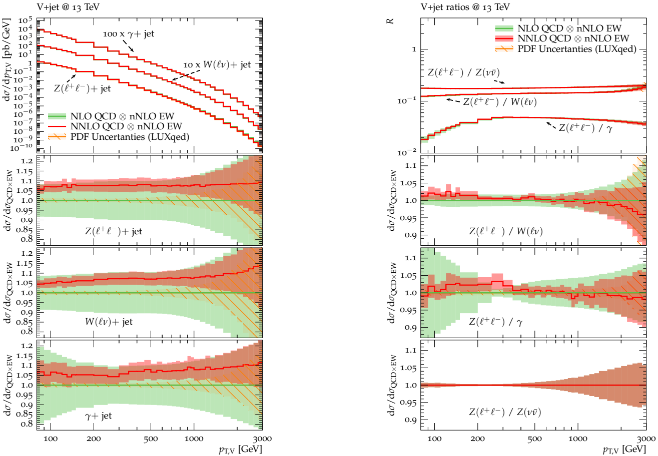
with $\xi_{\text{mix}} = 0.1/0.2/0.4$ for $Z/W/\gamma+$jet

$\Rightarrow$ applied as uncorrelated uncertainty



driven by EW–QCD interferences in the tails

Summary of uncertainties for $\mathrm{d}\sigma/\mathrm{d}p_{\mathrm{T}}$ and ratios

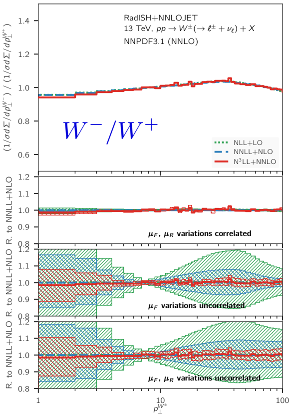


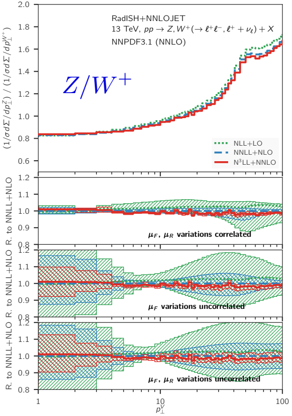
- NNLO ratios confirm strong correlations in QCD uncertainty model
- at 1 TeV uncertainties $\lesssim 5\%$ in distributions reduced to $\lesssim 2\%$ in ratios

Crucial for sensitivity of high-statistics monojet searches!

Recent $\mathrm{N^3LL}$ and $\mathrm{N^3LO}$ results

Z/W p_T-distributions at $\mathrm{N^3LL}$+NNLO [Bizon, Gehrmann-De Ridder, Gehrmann, Glover, Huss, Monni, Re, Rottoli, Walker ’19]



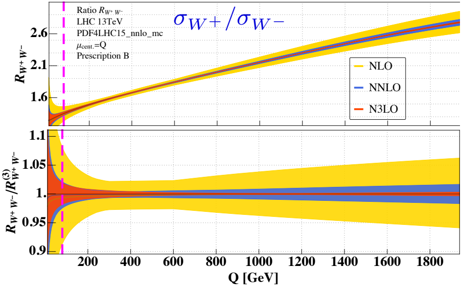


- Very strong correlation of QCD effects confirmed also at small p_T
- uncertainties strongly overestimated by uncorrelated scale variations
- correlated variations and highest available correction suggest $\sim 1\%$ uncertainty

$\mathrm{N^3LO}$ for $pp \to W^\pm \to \nu\ell$ and $pp \to \gamma^* \to \ell^+\ell^-$

[Duhr, Dulat, Mistlberger '20]

New powerful insights into progression of LO, NLO, NNLO, $\mathrm{N^3LO}$ and correlations



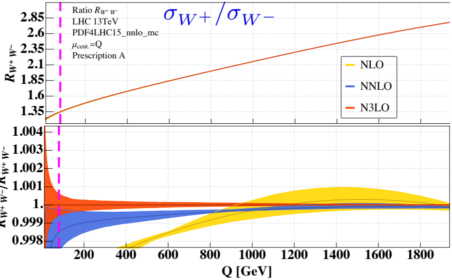
Uncorrelated 7-point $\mu_{R,F}$ variations (numbers for $Q \sim M_W$)

	NNLO	$\mathrm{N^3LO}$	higher	
observed correction	$\sim 0.5\%$	$\sim 0.15\%$	n.a.	very small and convergent
estimated	$\sim 5\%$	$\sim 2\%$	$\sim 0.5\%$	

uncertainty overestimated by factor 10

$\mathrm{N^3LO}$ for $pp \to W^\pm \to \nu\ell$ and $pp \to \gamma^* \to \ell^+\ell^-$

[Duhr, Dulat, Mistlberger '20]



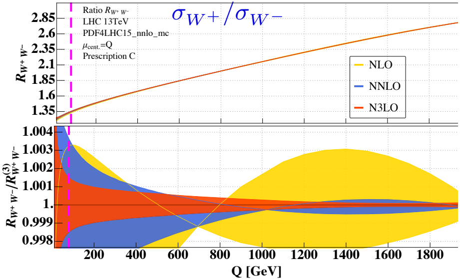
Correlated 7-point $\mu_{R,F}$ variations (numbers for $Q \sim M_W$)

	NNLO	$\mathrm{N^3LO}$	higher	
observed correction	$\sim 0.5\%$	$\sim 0.15\%$	n.a.	very small and convergent
estimated	$\sim 0.1\%$	$\sim 0.1\%$	$\sim 0.05\%$	

uncertainty underestimated by factor 0.5–5

$\mathrm{N}^3\mathrm{LO}$ for $pp \to W^{\pm} \to \nu\ell$ and $pp \to \gamma^* \to \ell^+\ell^-$

[Duhr, Dulat, Mistlberger '20]



Highest available correction (to ratio) as uncertainty (numbers for $Q \sim M_W$)

	NNLO	$\mathrm{N}^3\mathrm{LO}$	higher	
observed correction	$\sim 0.5\%$	$\sim 0.15\%$	n.a.	very small and convergent
estimated	$\sim 1\%$	$\sim 0.5\%$	$\sim 0.15\%$	

overestimates by factor 2-3: best estimator

(but accidental zeros should be avoided)

Take home messages

Global analyses such as PDF (or EFT) fits

- require general model of MHO uncertainties + correlations
- scale variations OK for normalisation but not for kinematic and process correlations

Realistic modelling of MHO uncertainties + correlations

- requires understanding of dominant sources of corrections

$\Rightarrow$ dedicated studies depending on process and observable

Useful guidelines for (un)correlation of MHO uncertainties

- separation of universal contributions (e.g. Sudakov logs) $\Rightarrow$ correlated
- insights from perturbative progression, e.g. using part of highest known correction as uncertainty (see also statistical approaches [Cacciari et al '11–15, Bonvini '20])

EW corrections

- sizeable at high p_T and different correlations (shapes, ratios) wrt QCD corrections

Open question: general and realistic theory uncertainty model possible?

Backup slides

Caveat: fragmentation effects in $\gamma+$jet and $Z/W+$jet

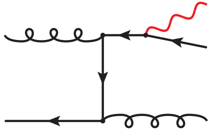
Assumption (to justify estimate of process-correlation uncertainty)

similar QCD dynamics for all $V+$jet processes $\Leftrightarrow$ $\left|\kappa^{(V)}_{\mathrm{NLO}} - \kappa^{(Z)}_{\mathrm{NLO}}\right| \ll \left|\kappa^{(Z)}_{\mathrm{NLO}}\right|$

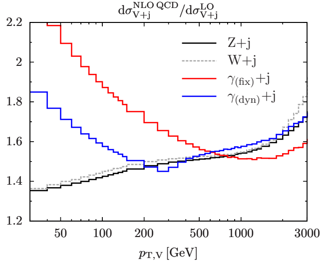
Violated by $q \to qV$ fragmentation effects

$\gamma+$jet: isolation cone of radius R_0 $\Rightarrow$ $\ln(R_0)$

$W/Z+$jet: mass cut-off $M_{Vj} \geq M_V$ $\Rightarrow$ $\ln(M/p_{\mathrm{T}})$



Adopt dynamic γ-isolation with $R_{\mathrm{dyn}}(p_{\mathrm{T},\gamma}) = \min\{M_Z/p_{\mathrm{T},\gamma}, 1.0\}$

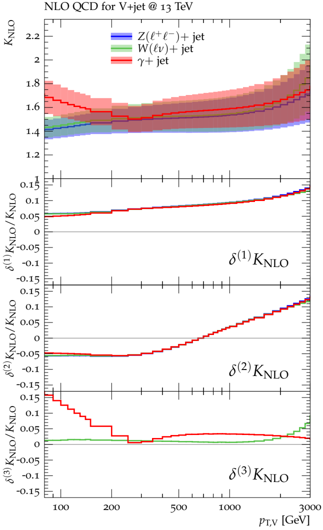


- γ_{dyn} behaves like W/Z at $p_T > M_Z$

$\Rightarrow$ justifies process-correlation estimate $\delta^{(3)}K$

- remnant part $\gamma_{\mathrm{fix}} - \gamma_{\mathrm{dyn}}$ uncorrelated (uncertainty through extra reweighting and MC)

NLO QCD uncertainties: scale variations



Scale uncertainty $\delta^{(1)}K$

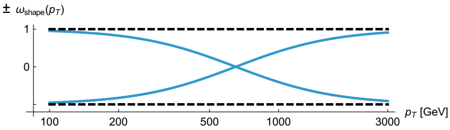
- independent factor-2 variations around

$$\mu_{R,F} = \frac{H_T}{2} = \frac{1}{2}\left(E_{T,V} + \sum_{i\in\{q,g,\gamma\}} p_{T,i} \right)$$

$\Rightarrow$ 5–10% with minor impact on shapes

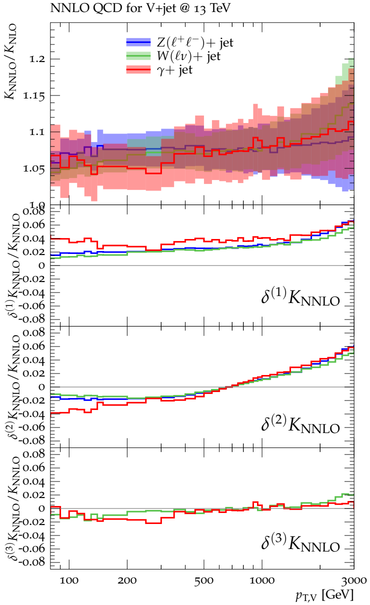
Shape uncertainty $\delta^{(2)}K = \tanh\left[\ln\left(\frac{p_T}{650\ \mathrm{GeV}}\right)\right]\ \delta^{(1)}K$

- max shape distortion within scale variation band



correlation across across $\gamma/W/Z$ pocesses?

Uncertainties from NLO to NNLO QCD



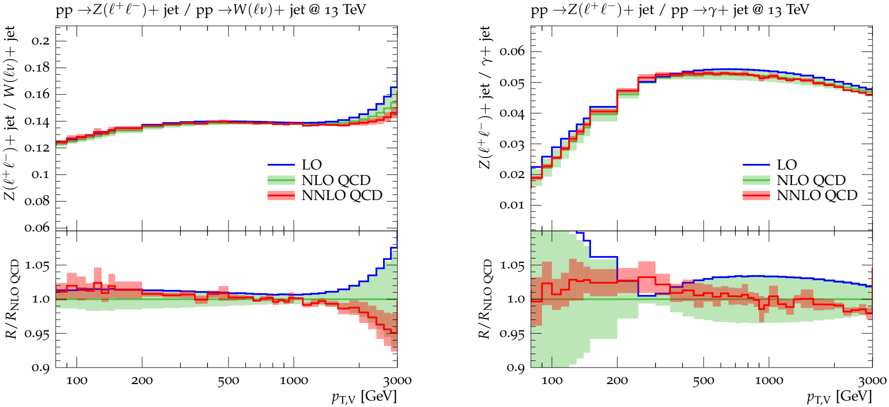
Scale and shape variations $\delta K^{(1,2)}$

- reduced from 5–15% to 2–6%

Process-correlation uncertainties $\delta K^{(3)}$

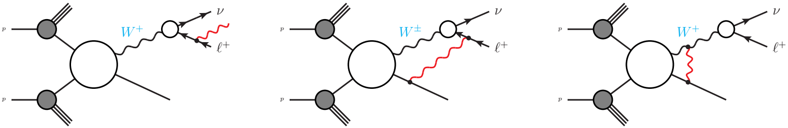
- reduced from 6% to 1–2%
- confirm quasi universality of QCD corrections!

QCD corrections+uncertainties for Z/W and Z/γ ratios



- 1–2% NNLO corrections and residual uncertainty up to 1 TeV
- NNLO confirms NLO uncertainty model! (QCD corrections "quasi universal")

EW corrections: key features



etc.

Corrections to $V \to \ell\ell/\nu\ell$ at scale $Q^2 \sim M_V^2$

- dominated by γ radiation off leptons
- significant impact on lepton kinematics (e.g. distortion of $Z \to \ell\ell$ resonance)

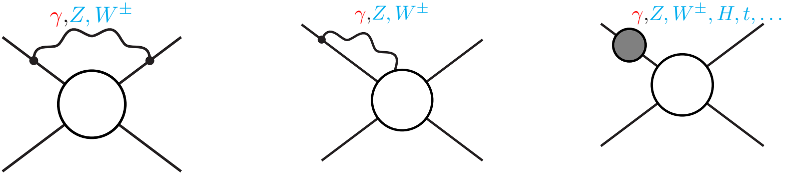
Corrections to $pp \to Vj$ at scales $Q^2 \sim \hat{s} \gg M_W^2$

- dominated by EW Sudakov logarithms

$$\alpha \ln^2\left(\frac{Q^2}{M_W^2}\right) > \alpha_S$$

- significant impact (more than NNLO QCD) on σ_Z/σ_V ratios and $\mathrm{d}\sigma/\mathrm{d}p_\mathrm{T}$ shape

EW Sudakov logarithms at $Q^2 \gg M_W^2$



Universal effects from soft/collinear virtual gauge bosons [Denner,S.P. '01]

$$\delta\mathcal{M}^{1-\mathrm{loop}}_{\mathrm{LL+NLL}} = \frac{\alpha}{4\pi}\sum_{k=1}^{n}\left\{\frac{1}{2}\sum_{l\neq k}\sum_{a=\gamma,Z,W^\pm} I^a(k)I^{\bar{a}}(l)\ln^2\left(\frac{\hat{s}_{kl}}{M_W^2}\right) + \gamma^{\mathrm{ew}}(k)\ln\left(\frac{\hat{s}}{M_W^2}\right)\right\}\mathcal{M}_0$$

$\Rightarrow$ large negative corrections to any process at high $p_T, E_{T,\mathrm{miss}}, H_T, M_{\mathrm{inv}}, \ldots$

Typical size at 1 TeV

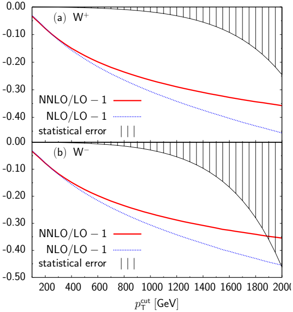
$$\left(\frac{\delta\sigma_1}{\sigma_0}\right)_{\mathrm{LL}} \simeq -\frac{4\alpha}{\pi s_{\mathrm{w}}^2}\ln^2\left(\frac{1\mathrm{TeV}}{M_W}\right) \simeq -26.4\%$$

$\Rightarrow$ actual size strongly dependent on external EW charges, subleading terms, ...

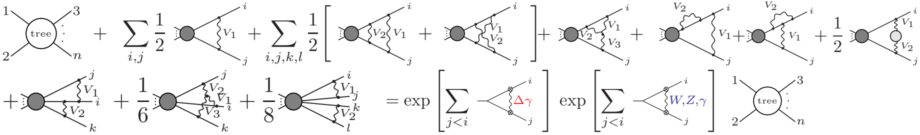
NLO EW and NNLO Sudakov corrections to $V+$jet

EW corrections $\sim -25\%$ for $V+$jet at 1 TeV

- NLO EW + NNLO Sudakov logs [Kühn, Kulesza, S.P.,Schulze '04–'07; Becher, Garcia i Tormo '13]
- NLO QCD+EW with off-shell Z/W decays [Denner,Dittmaier,Kasprzik,Muck '09–'11]
- NLO QCD+EW for $Z/W+1,2$ jets with off-shell decays [Denner, Hofer, Scharf, Uccirati '14; Kallweit, Lindert, Maierhöfer, S.P., Schönherr'15]



Leading effects beyond 1-loop through resummation [Becher, Ciafaloni, Comelli, Denner, Fadin, Jantzen, Kühn, Lipatov, Manohar, Melles, Penin, Pozzorini, Smirnov, . . .]



relevant for percent precision and uncertainty estimates at the TeV scale

nNLO EW corrections and uncertainties: Sudakov logs

General form of EW corrections at $Q^2 \gg M_W^2$ [Kühn et al '05; Manohar et al '07;...]

$$\mathrm{d}\sigma_{\mathrm{EW}} = \exp\left\{\int_{M_W^2}^{Q^2} \frac{\mathrm{d}t}{t}\left[\int_{M_W^2}^{t} \mathrm{d}\tau \frac{\gamma(\alpha(\tau))}{\tau} + \chi(\alpha(t)) + \xi\left(\alpha(M_W^2)\right)\right]\right\} \mathrm{d}\sigma_{\mathrm{hard}},$$

Exponentiation of Sudakov logs (universal anomalous dimensions γ, χ, ξ)

$$\exp\left\{\dots\right\} = 1 + \frac{\alpha}{\pi}\delta_{\mathrm{Sud}}^{(1)} + \left(\frac{\alpha}{\pi}\right)^2 \delta_{\mathrm{Sud}}^{(2)} + \dots.$$

Results for $pp \to V+$jet at NLL accuracy [Kühn, Kulesza, Schulze, S.P. '04-'07; Becher, Garcia i Tormo '13]

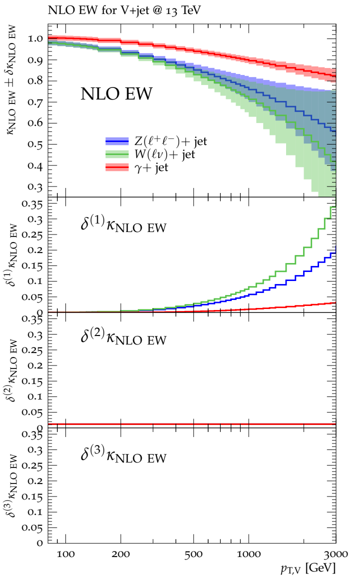
$$\delta_{\mathrm{Sud}}^{(1)} = \sum_{i,j} C_{2,ij}^{(1)} \ln^2\left(\frac{Q_{ij}^2}{M^2}\right) + C_1^{(1)} \ln^1\left(\frac{Q^2}{M^2}\right),$$

$$\delta_{\mathrm{Sud}}^{(2)} = \sum_{i,j} C_{4,ij}^{(2)} \ln^4\left(\frac{Q_{ij}^2}{M^2}\right) + C_3^{(2)} \ln^3\left(\frac{Q^2}{M^2}\right) + \mathcal{O}\left[\ln^2\left(\frac{Q^2}{M^2}\right)\right],$$

Hard contributions known exactly at NLO (free from $\ln(Q^2/M^2)$)

$$\mathrm{d}\sigma_{\mathrm{hard}} = \left[1 + \frac{\alpha}{\pi}\delta_{\mathrm{hard}}^{(1)} + \left(\frac{\alpha}{\pi}\right)^2 \delta_{\mathrm{hard}}^{(2)} + \dots\right] \mathrm{d}\sigma_{\mathrm{Born}},$$

EW uncertainties at NLO



Estimate of missing Sudakov logs

$$\delta^{(1)}\kappa_{\mathrm{NLO\,EW}} \simeq 2 \times \frac{1}{2}\left[\left(\frac{\alpha}{\pi}\right)\delta^{(1)}_{\mathrm{Sud}}\right]^2$$

- $2\times$ inflated naive exponentiation

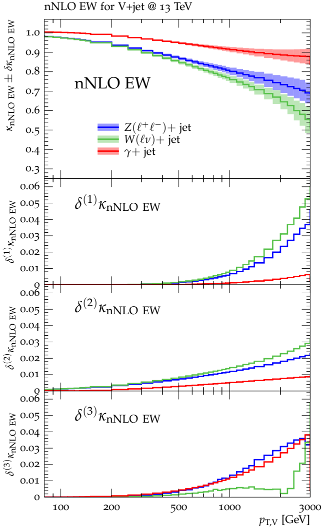
$\Rightarrow$ 5–10% uncertainties around 1 TeV

- consistent with rigorous $\delta^{(2)}_{\mathrm{Sud}}$ calculation ($\sim 5\%$)

Correlation across $V+$jet processes

- $\delta^{(1)}\kappa_{\mathrm{EW}}$ known effect $\Rightarrow$ correlated
- $\delta^{(2,3)}\kappa_{\mathrm{EW}}$ unknown effects $\Rightarrow$ uncorrelated (see later)

EW uncertainties at nNLO



Sudakov logs beyond NNLO

$$\delta^{(1)}\kappa_{\mathrm{nNLO\,EW}} \simeq 2 \times \frac{1}{3!}\left[\left(\frac{\alpha}{\pi}\right)\delta^{(1)}_{\mathrm{Sud}}\right]^3$$

NNLO subleading logs and hard effects

$$\delta^{(2)}\kappa_{\mathrm{EW}} \simeq 0.05 \times \frac{\alpha}{\pi}\left(\delta^{(1)}_{\mathrm{Sud}} + \delta^{(1)}_{\mathrm{hard}}\right)$$

$\Rightarrow$ $\delta^{(1)}_{\mathrm{hard}} \times \delta^{(1)}_{\mathrm{Sud}}$ terms with 5% hard NLO effects

$\Rightarrow$ $\delta^{(2)}_{\mathrm{hard}}$ terms with $\delta^{(2)}_{\mathrm{hard}}/\delta^{(1)}_{\mathrm{hard}} \sim 0.05(\pi/\alpha) \sim 20$

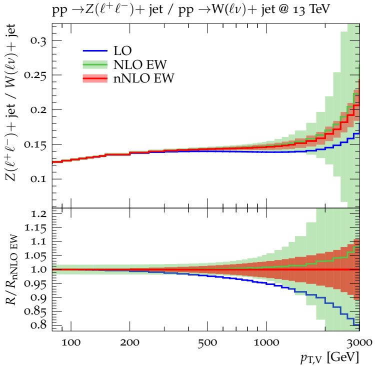
Difference $\delta^{(2)}_{\mathrm{Sud}}$ vs naive expotentiation

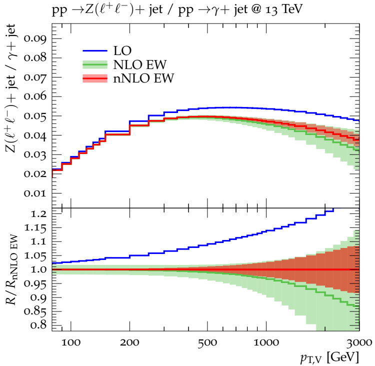
$$\delta^{(3)}\kappa_{\mathrm{EW}} = \left(\frac{\alpha}{\pi}\right)^2\left[\delta^{(2)}_{\mathrm{Sud}} - \frac{1}{2}\left(\delta^{(1)}_{\mathrm{Sud}} + \delta^{(1)}_{\mathrm{hard}}\right)^2\right]$$

$\Rightarrow$ further $\delta^{(1)}_{\mathrm{hard}} \times \delta^{(1)}_{\mathrm{Sud}}$ and $\left(\delta^{(1)}_{\mathrm{hard}}\right)^2$ terms

$\Rightarrow$ nNLO EW uncertainties $\lesssim$ 1-2% up to 1 TeV

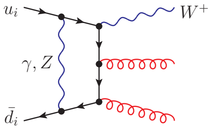
EW corrections and uncertainties for Z/W and Z/γ ratios





- up to 20% NLO EW and 5% nNLO EW effects at TeV scale ($\gg$ QCD corrections)
- nNLO EW results support uncertainty correlation scheme
- NNLO Sudakov logs needed for percent precision at TeV scale

Estimator of non-factorising NNLO mixing effects



Parametrisation of non-fact QCD–EW effects

$$\mathrm{d}\sigma_{\mathrm{NNLO\,mix}} \quad = \quad \left(\kappa_{\mathrm{EW}} + \delta\kappa_{\mathrm{NNLO\,mix}}\right)\kappa_{\mathrm{QCD}}\,\mathrm{d}\sigma_{\mathrm{LO}}$$

Restriction to real–virtual contribution with 2nd jet

$$\mathrm{d}\sigma^{(V+2j)}_{\mathrm{NLO\,EW}}(\tau_{\mathrm{cut}}) \quad = \quad \left[\kappa^{(V+1j)}_{\mathrm{NLO\,EW}} + \delta\kappa^{(V)}_{\mathrm{NNLO\,mix}}(\tau_{\mathrm{cut}})\right]\mathrm{d}\sigma^{(V+2j)}_{\mathrm{LO}}(\tau_{\mathrm{cut}})$$

with N-jettiness resolution cut on 2nd jet

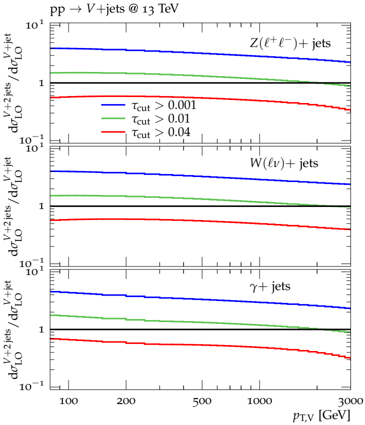
$$\tau_1 = \sum_k \min_i\left(\frac{p_i \cdot q_k}{E_i\sqrt{\hat{s}}}\right) > \tau_{\mathrm{cut}}$$

yields $\delta\kappa_{\mathrm{NNLO\,mix}}$ estimator given by NLO EW K-factor difference

$$\delta\kappa^{(V)}_{\mathrm{NNLO\,mix}}(\tau_{\mathrm{cut}}) = \kappa^{(V+2j)}_{\mathrm{NLO\,EW}}(\tau_{\mathrm{cut}}) - \kappa^{(V+1j)}_{\mathrm{NLO\,EW}}$$

Finite in IR limit $\tau_{\mathrm{cut}} \to 0$, i.e. reasonably stable wrt τ_{cut} variations

Variations of IR cut



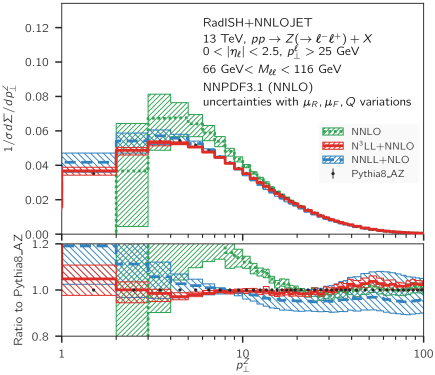
Variations of τ_{cut} by 1.5 orders of magnitude avoiding

- avoiding $\sigma^{V+2j}/\sigma^{V+1j} \gg 1$ = deep IR regime $\Rightarrow$ "excessive" factorisation
- avoiding $\sigma^{V+2j}/\sigma^{V+1j} \ll 1$ = hard regime $\Rightarrow$ non representative of $\sigma_{\text{incl}}^{(V+1j)}$

p_T distributions for Z/W at $\mathrm{N^3LL}$+NNLO [Bizon, Gehrmann-De Ridder, Gehrmann, Glover, Huss, Monni, Re, Rottoli, Walker '19]

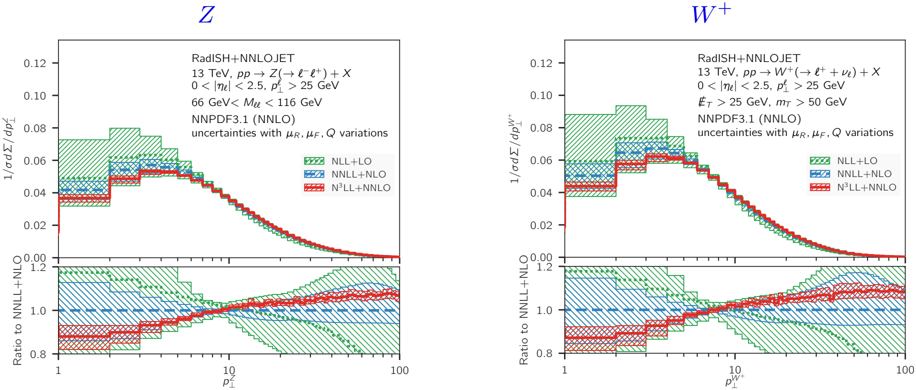
Matching to $\mathrm{N^3LO}$ resummation with RADISH [Bizon, Monni, Re, Rottoli, Torielli]

- high precision extended to small p_T region
- fully realistic experimental cuts with MC technique



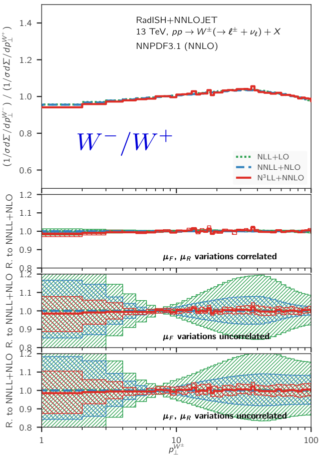
- resummation crucial below 20 GeV
- $\mathrm{N^3LL}$ significantly improves agreement with PY8_AZ (and data) below 10 GeV
- important input for M_W determination (especially W/Z ratio)

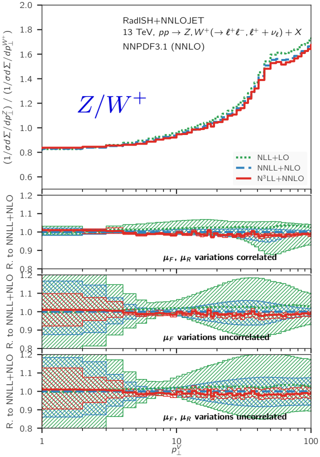
Corrections and scale variations for p_T^Z and $p_T^{W^+}$



- 9-point factor-2 (μ_R, μ_F, μ_Q) variation consistent with observed corrections and decreasing with log+perturbative order
- few percent $\mathrm{N^3LL + NNLO}$ uncertainty down to small p_T
- also at small p_T very similar QCD effects for Z and W (universality of IR effects)

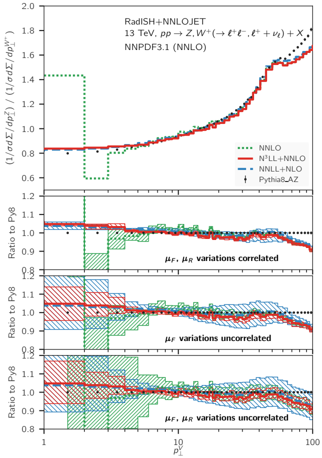
Corrections and uncertainties for ratios





- Very strong correlation of QCD effects : ratios remarkably stable
- uncertainties strongly overestimated by uncorrelated scale variations
- correlated variations and highest available correction suggest $\sim 1\%$ uncertainty

N3LL+NNLO vs PY8 for Z/W^+ ratio [Bizon, Gehrmann-De Ridder, Gehrmann, Glover, Huss, Monni, Re, Rottoli, Walker '19]



At moderate p_T

- very good agreement with PY8

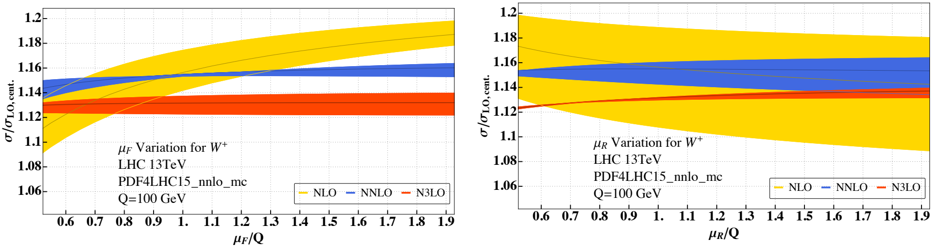
At $p_T < 4$ GeV

- few-percent difference
- consistent with uncorrelated scale variations (very conservative)
- but well below difference bewteen NLL+NLO and $\mathrm{N^3LL}$+NNLO

Drell-Yan at N^{3}LO [Duhr, Dulat, Mistlberger '20]

$\Rightarrow$ Off-shell $\mathrm{d}\sigma/\mathrm{d}Q^2$ for $pp \to W^\pm \to \nu\ell$ and $pp \to \gamma^* \to \ell^+\ell^-$

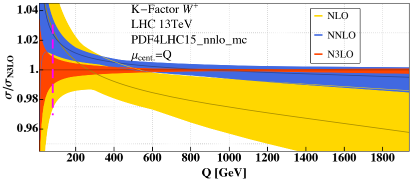
$\Rightarrow$ new powerful insights into progression of LO, NLO, NNLO, N^{3}LO



K-factors and μ_F, μ_R variations for W^+ at 100 GeV

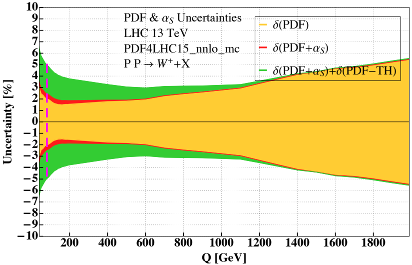
- scale dependence reduced to $\sim 1\%$ at N^{3}LO
- $\sigma_{N^3LO}/\sigma_{NNLO} \sim 0.98$ well ouside NNLO scale-variation band
- similar tension using central scales Q or $Q/2$

$\mathrm{d}\sigma/\mathrm{d}Q$ distribution for $pp \to \ell^+\nu$



7-point factor-2 variations of μ_R, μ_F

- $Q > 500$ GeV : overlapping bands
- $Q \sim M_W$: N3LO outside NNLO band
- related to sizeable cancellation of NNLO corrections between different channels

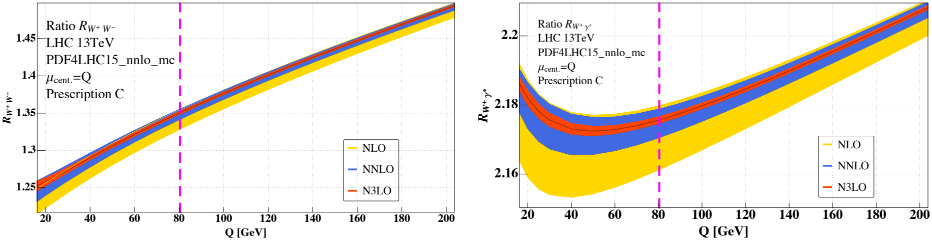


Uncertainty from missing N3LO PDFs

- tension may disappear through small shift of PDFs when upgraded from NNLO to N3LO (presently unavailable)
- uncertainty estimated by downgrading NNLO PDFs to NLO PDFs in σ_{NNLO} is sizeable at $Q < 500$ GeV

Importance of full assessment of PDF uncertainties and their further reduction

$R_{W^+W^-}$ and $W_{W^+\gamma^*}$ ratios for $Q < 200$ GeV



Highest available correction (to ratio) as uncertainty

- Reliable higher-order estimate for both ratios
- Small corrections and residual uncertainties at permille level

further strong indication of universality of QCD corrections to W, Z, γ^*