Lattice QCD inputs for the SM: Select Highlights

Aida X. El-Khadra
University of Illinois

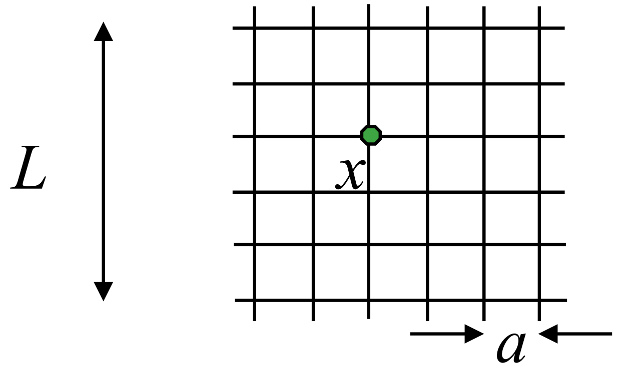
Workshop on the
Standard Model at the LHC 2021
(Online)
26-30 April 2021

Outline

- Lattice QCD Introduction
- Semileptonic B meson decay form factors
  - $|V_{ub}|$ and $|V_{cb}|$
  - LFU
- muon g-2
- Summary and Outlook

Lattice QCD Introduction

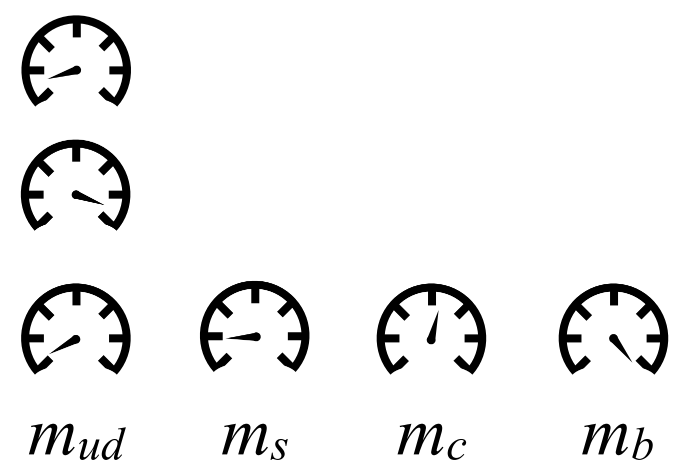
$$\mathcal{L}_{\rm QCD} = \sum_f \bar\psi_f (\not{D} + m_f)\psi_f + \frac{1}{4}{\rm tr}F_{\mu\nu}F^{\mu\nu}$$



- ✦ discrete Euclidean space-time (spacing a)
  derivatives → difference operators, etc…
- ✦ finite spatial volume (L)
- ✦ finite time extent (T)

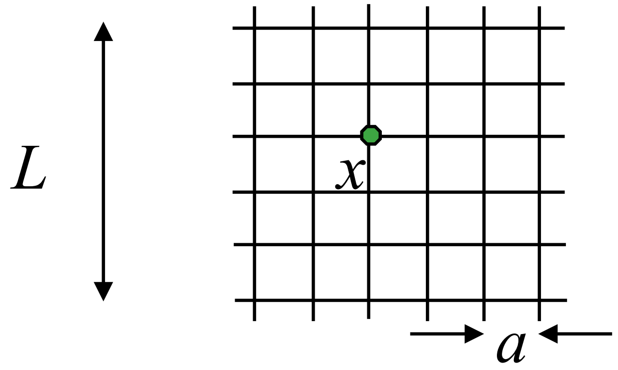
adjustable parameters

- ❖ lattice spacing: $a \to 0$
- ❖ finite volume, time: $L \to \infty,\ T > L$
- ❖ quark masses (m_f): $M_{H,\rm lat} = M_{H,\rm exp}$, $m_f \to m_{f,\rm phys}$
  - tune using hadron masses
  - extrapolations/interpolations



Lattice QCD Introduction

$$\mathcal{L}_{\mathrm{QCD}} = \sum_f \bar{\psi}_f (\not{D} + m_f)\psi_f + \frac{1}{4}\mathrm{tr} F_{\mu\nu}F^{\mu\nu}$$

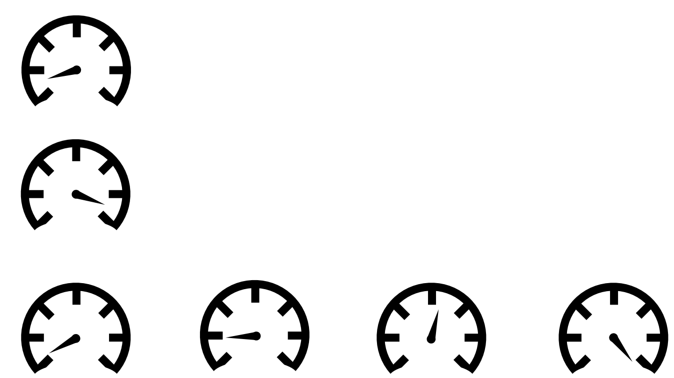


- ✦ discrete Euclidean space-time (spacing a)
  derivatives ➞ difference operators, etc…
- ✦ finite spatial volume (L)
- ✦ finite time extent (T)

Integrals are evaluated numerically using monte carlo methods.

adjustable parameters

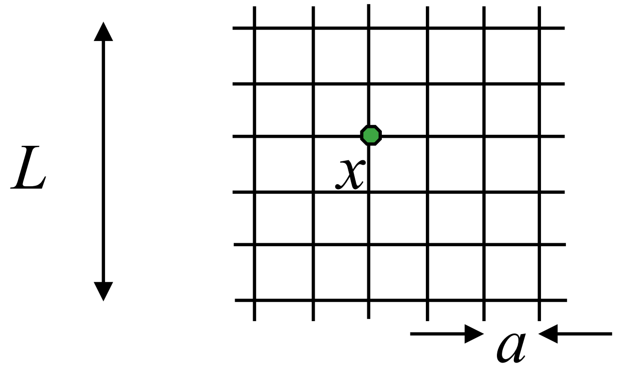
- ❖ lattice spacing: $a \to 0$
- ❖ finite volume, time: $L \to \infty$, $T > L$
- ❖ quark masses (m_f): $M_{H,\mathrm{lat}} = M_{H,\mathrm{exp}}$
  tune using hadron masses $m_f \to m_{f,\mathrm{phys}}$
  extrapolations/interpolations



m_{ud} m_s m_c m_b

Lattice QCD Introduction

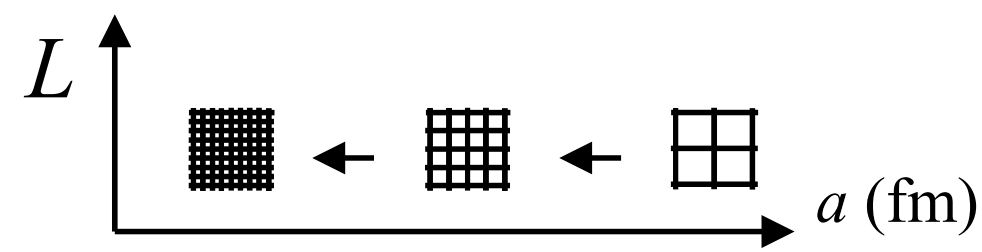
$$\mathcal{L}_{\rm QCD} = \sum_f \bar\psi_f (\not{D} + m_f)\psi_f + \frac{1}{4}{\rm tr} F_{\mu\nu}F^{\mu\nu}$$



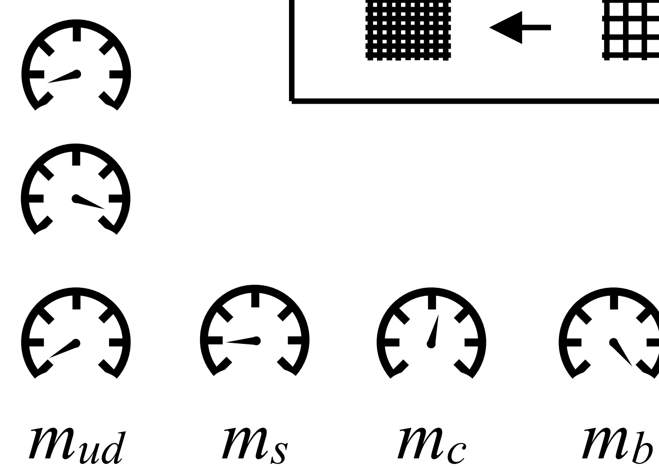
- ✦ discrete Euclidean space-time (spacing a)
  derivatives ➙ difference operators, etc…
- ✦ finite spatial volume (L)
- ✦ finite time extent (T)

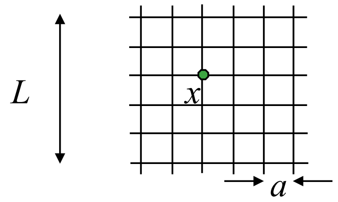
Integrals are evaluated numerically using monte carlo methods.

adjustable parameters



- ❖ lattice spacing: $a \to 0$
- ❖ finite volume, time: $L \to \infty, T > L$
- ❖ quark masses (m_f): $M_{H,\rm lat} = M_{H,\rm exp}$
  - tune using hadron masses $m_f \to m_{f,\rm phys}$
  - extrapolations/interpolations





Lattice QCD Introduction

The State of the Art

Lattice QCD calculations of simple quantities (with at most one stable meson in initial/final state) that **quantitatively account for all systematic effects** (discretization, finite volume, renormalization,…) , in some cases with

- sub percent precision.
- total errors that are commensurate (or smaller) than corresponding experimental uncertainties.

Scope of LQCD calculations is increasing due to continual development of new methods:

- nucleons and other baryons
- nonleptonic decays ($K \to \pi\pi$, …)
- resonances, scattering, long-distance effects, …
- QED effects
- radiative decay rates …

Lattice QCD: Overview

[inspired by A. Kronfeld]

$a_\mu^{\text{HVP LO}}$ a_μ^{HLbL}

g_A, g_T, g_S

$\langle \bar{B}_q^0 | \mathcal{O}_i^{\Delta B=2} | B_q^0 \rangle$

$\langle \bar{D}^0 | \mathcal{O}_i^{\Delta C=2} | D^0 \rangle$

$\hat{B}_K \ \ldots$ $\Lambda_b \to p, \Lambda_c, \Lambda$

nucleon form factors, ..

$f_{+,0}^{B\to D}(q^2), \ldots$

$f_+^{K\to\pi}$ $f_{+,0,T}^{B\to\pi} \ \ldots$

$f_{K^\pm}$ $f_{B_{(s)}} \ \cdots$ $\langle \pi\pi_{(I=2)} | \mathcal{H}^{\Delta S=1} | K^0 \rangle$

$\Delta M_K, \ \epsilon_K$

$\langle \pi\pi_{(I=0)} | \mathcal{H}^{\Delta S=1} | K^0 \rangle$

MEs for light nuclei

$B \to K^* \ell\ell \to K\pi\,\ell\ell \ \ldots$

$K^+ \to \ell^+ \nu\,(\gamma) \ \ldots$

$K^+ \to \pi^+ \ell^+ \ell^-$ $\ldots$

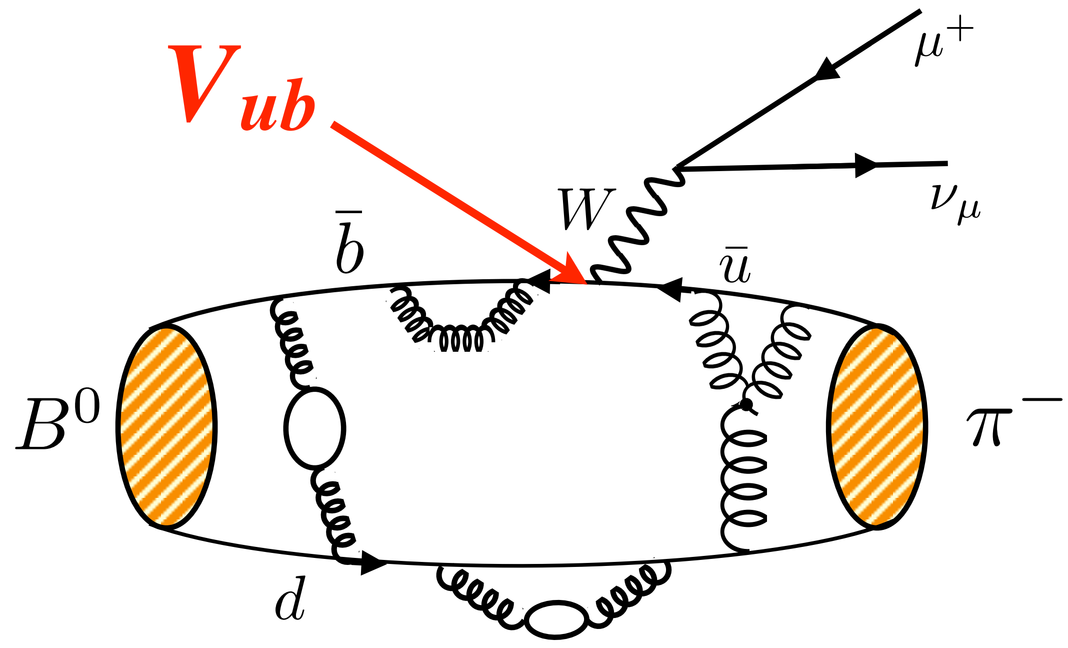
$K^+ \to \pi^+ \nu\bar{\nu}$

$B \to X_c \ell \nu$, other inclusive decay rates, …

Complexity →

LQCD flagship results	Complete LQCD results, large(ish) errors	First results, physical params, incomplete systematics	new methods, pilot projects, unphysical kinematics	new ideas, first studies

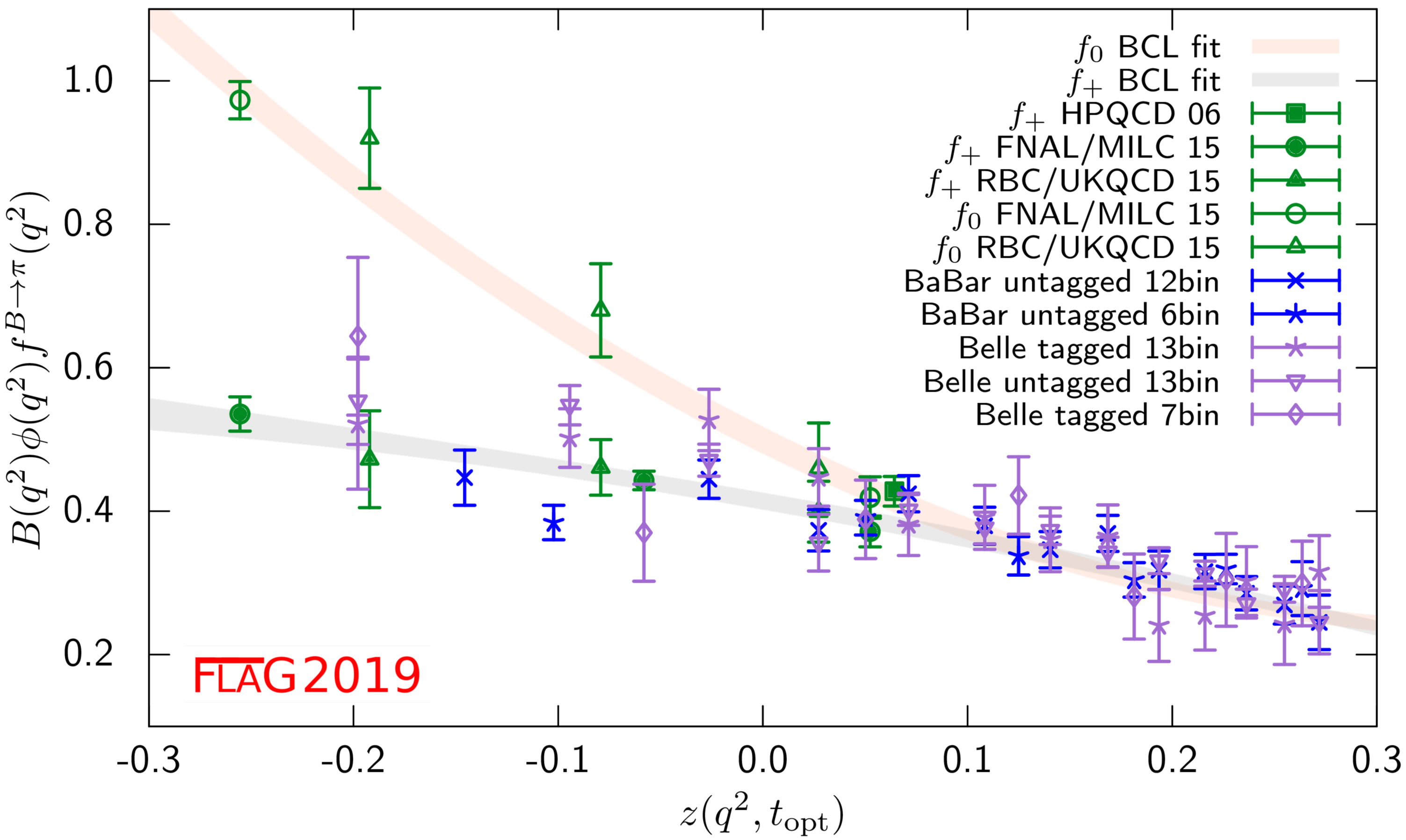
Form factors for $B \to \pi \ell \nu_\ell$ and $|V_{ub}|$



$$\frac{d\Gamma(B\to\pi\ell\nu)}{dq^2} = (\text{known}) \times |V_{ub}|^2 \times |f_+(q^2)|^2 \qquad q^2 = (p_B - p_\pi)^2$$

- ⋆ calculate the form factors in the low recoil (high q^2) range.
- ⋆ use model-independent parameterization of q^2 dependence.
- ⋆ calculate the complete set of form factors, $f_+(q^2)$, $f_0(q^2)$ and $f_T(q^2)$.
- ⋆ for $f_+(q^2)$ compare shape between experiment and lattice.

Form factors for $B \to \pi \ell \nu_\ell$ and $|V_{ub}|$



S. Aoki et al
FLAG 2019 review, 1902.08191
webupdate: flag.unibe.ch/2019/

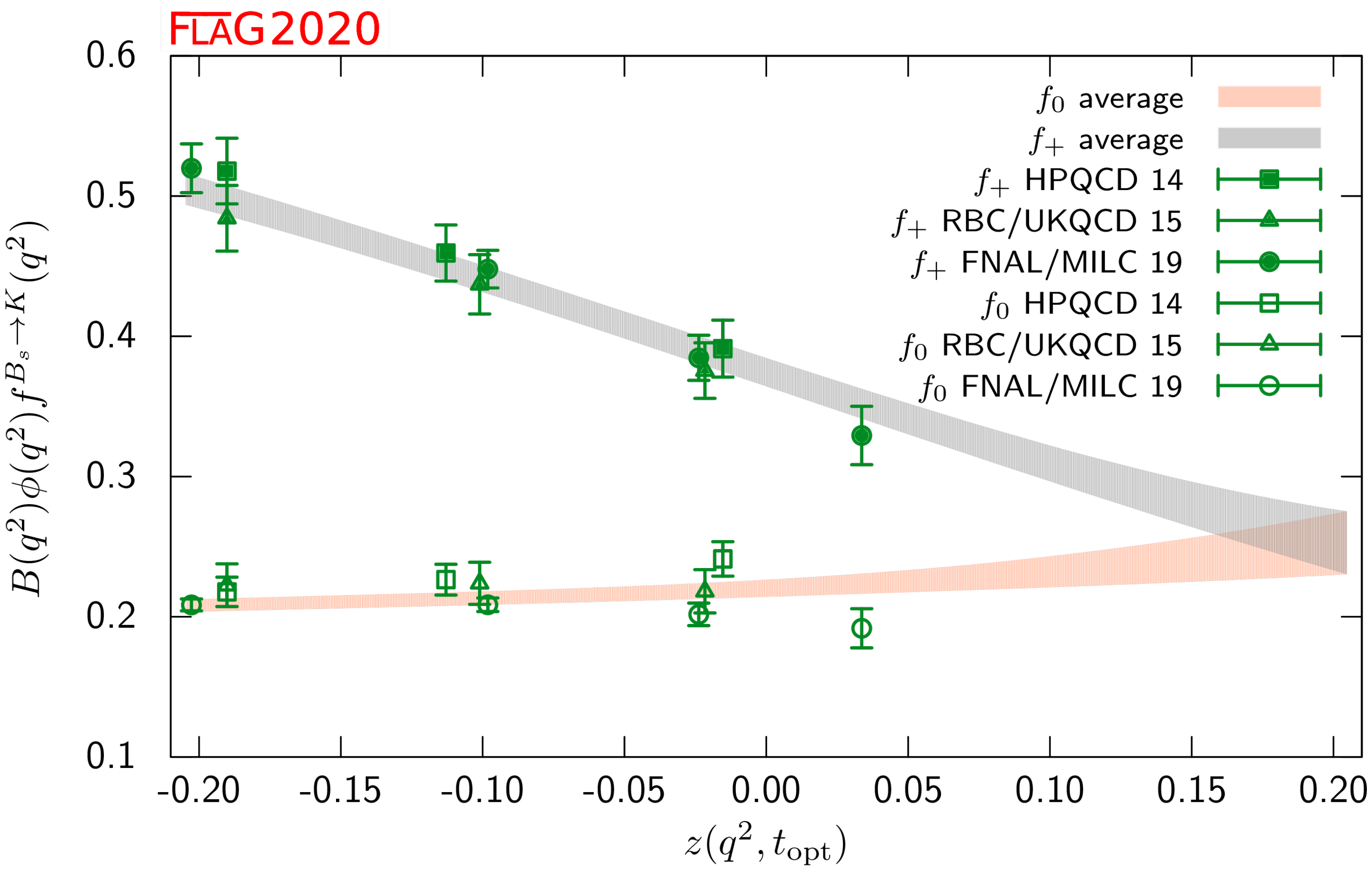
RBC [arXiv:1501.05373, PRD 2015]

FNAL/MILC [arXiv:1503.07839, PRD 2015]

Ongoing work by HPQCD, FNAL/MILC, JLQCD, RBC/UKQCD,…

- shape of f_+ agrees with experiment and uncertainties are commensurate
- fit lattice form factors together with experimental data to determine $|V_{ub}|$ **and** obtain form factors (f_+, f_0) with improved precision…
- similar analysis for $|V_{ub}/V_{cb}|$ from Λ_b decay with LHCb [arXiv:1503.01421, PRD 2015; arXiv:1504.01568, Nature 2015].

Form factors for $B_s \to K \ell \nu_\ell$

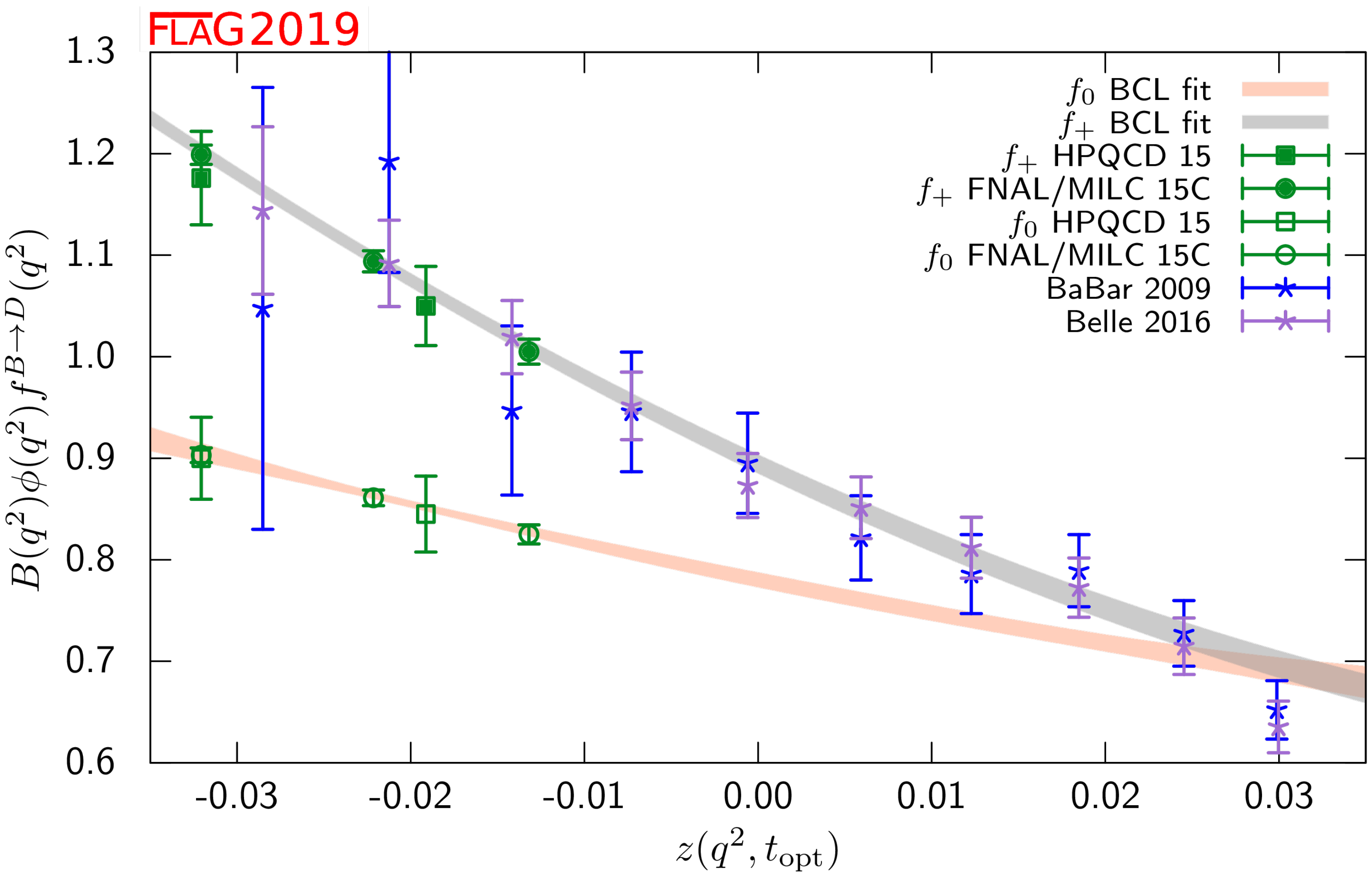


S. Aoki et al
FLAG 2019 review, 1902.08191
webupdate: flag.unibe.ch/2019/

HPQCD [arXiv:1406.2279, PRD 2014]
RBC/UKQCD [arXiv:1501.05373, PRD 2015]
FNAL/MILC [arXiv:1901.02561, PRD 2019]

- Lattice results for $B_s \to K$ and $B_s \to D_s$ form factors can be combined with new LHCb results for B_s decay rates
- Ongoing work by FNAL/MILC, RBC/UKQCD, JLQCD, HPQCD

Form factors for $B \to D \ell \nu_\ell$ and $|V_{cb}|$



S. Aoki et al
FLAG 2019 review, 1902.08191
webupdate: flag.unibe.ch/2019/

HPQCD [arXiv:1406.2279, PRD 2014]
FNAL/MILC [arXiv:1505.03925, PRD 2015]

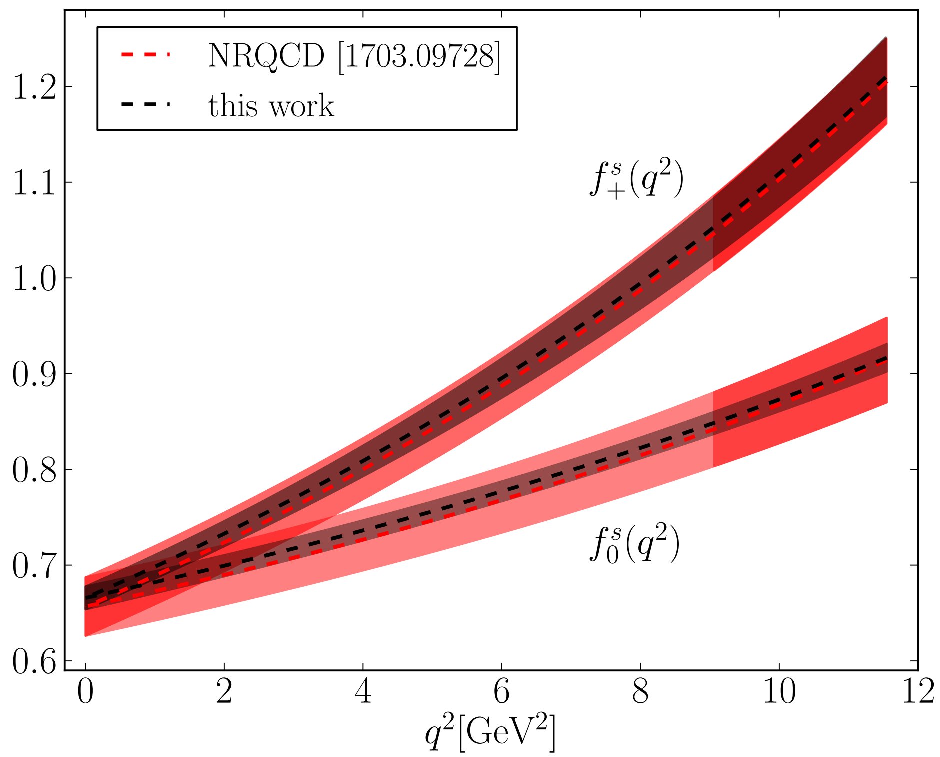
★The form factors obtained from the combined exp/lattice fit are well determined over entire recoil range.

★Can be used for an improved SM prediction of *R*(*D*).

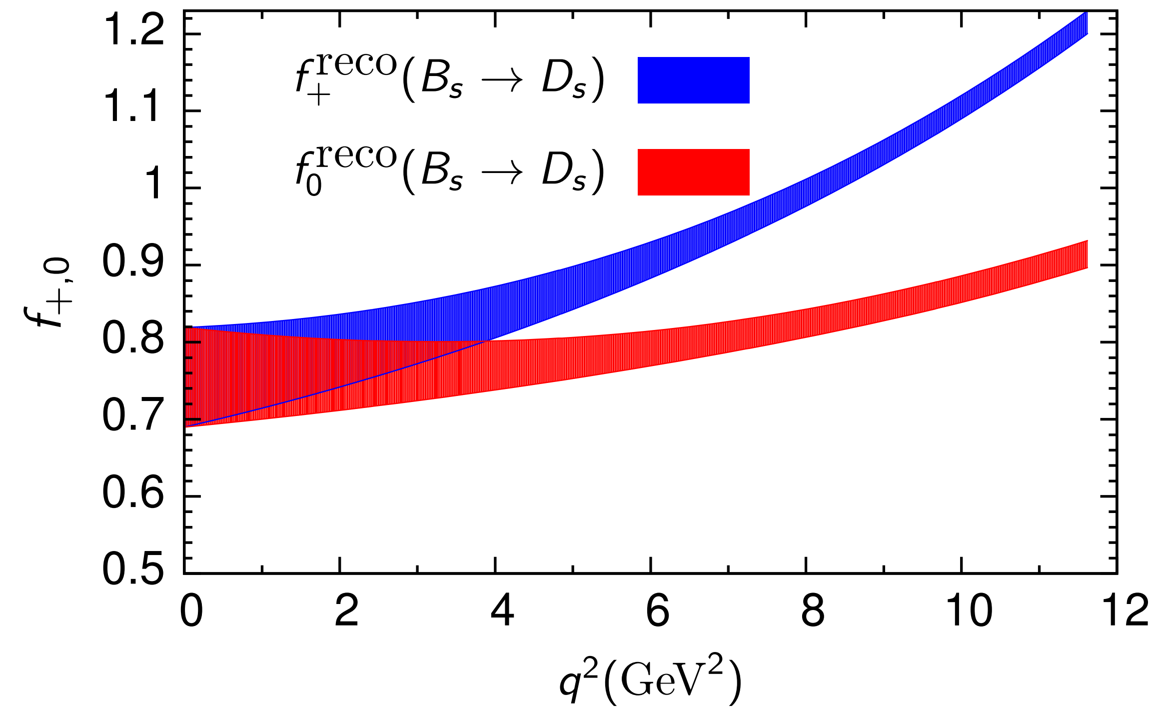
★Ongoing work by FNAL/MILC, JLQCD, RBC/UKQCD, HPQCD

Form factors for $B_s \to D_s \ell \nu_\ell$

HPQCD [arXiv:1906.00701, PRD 2020]



FNAL/MILC [arXiv:1901.02561, PRD 2019]



Reconstructed from $B \to D$ form factors [1505.03925] and B_s/B ratio [1403.0635]

★Can be used to prediction $R(D_s)$.

★New: experimental measurements of differential decay rate by LHCb

★Ongoing work by FNAL/MILC, JLQCD, RBC/UKQCD, HPQCD

Form factors for $B \to D^* \ell \nu_\ell$ and $|V_{cb}|$

$$\frac{d\Gamma}{dw} = (\text{known}) \times |V_{cb}|^2 \times (w^2 - 1)^{1/2} \times \chi(w)|\mathcal{F}(w)|^2$$

$$w = v_B \cdot v_{D^*}$$

★ $\mathcal{F}(w) = f[h_{A_1}(w), h_V(w), h_{A_2}(w), h_{A_3}(w)]$

★ results for form factor at zero recoil:

FNAL/MILC [J. Bailey et al, arXiv:1403.0635, 2014 PRD] $\mathcal{F}(1) = 0.906(4)(12)$

HPQCD [Harrison et al, arXiv:1711.11013, 2018 PRD] $\mathcal{F}(1) = 0.895\,(10)(24)$

★ result for $\mathcal{F}^{B_s \to D_s^*}(1)$: HPQCD [McLean et al, arXiv:1904.02046]

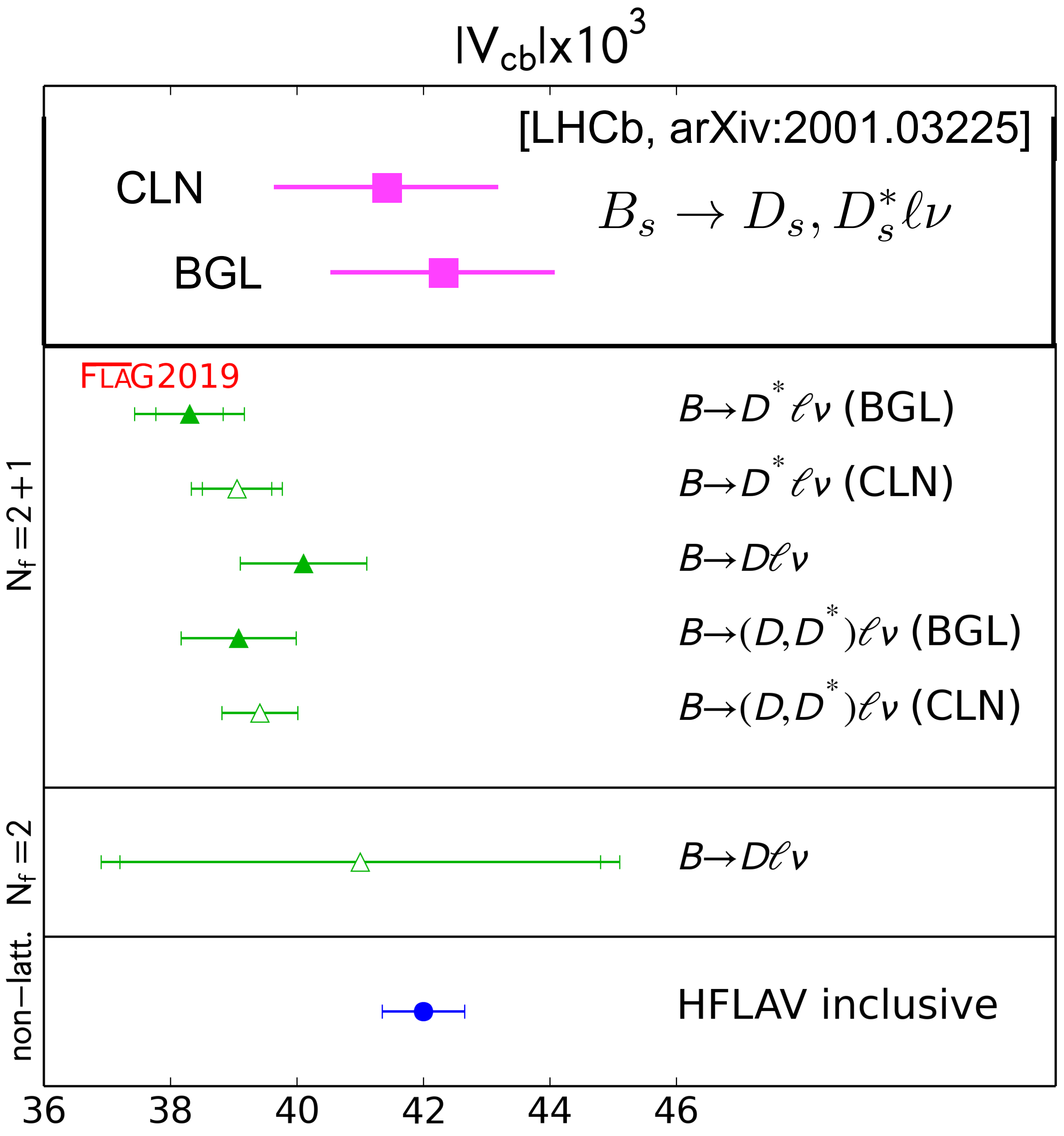
★ Non-zero recoil form factors: ongoing efforts by

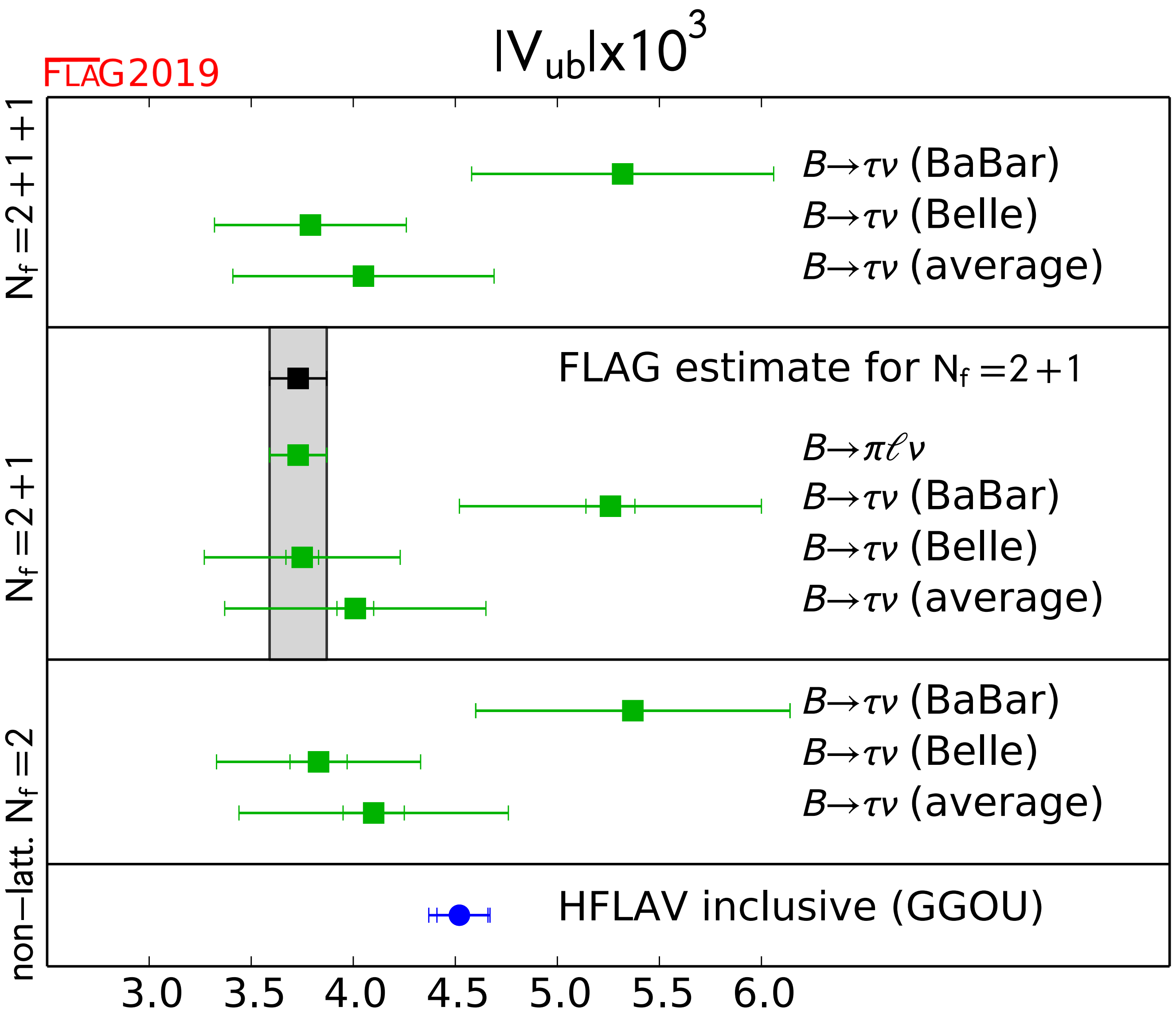
FNAL/MILC [A. Vaquero @ IPPP workshop ``Beyond Flavor Anomalies'']

JLQCD [T. Kaneko @APLAT 2020 conference, arXiv1912.11770]

LANL/SWME [Bhattacharya et al, arXiv:2003.09206]

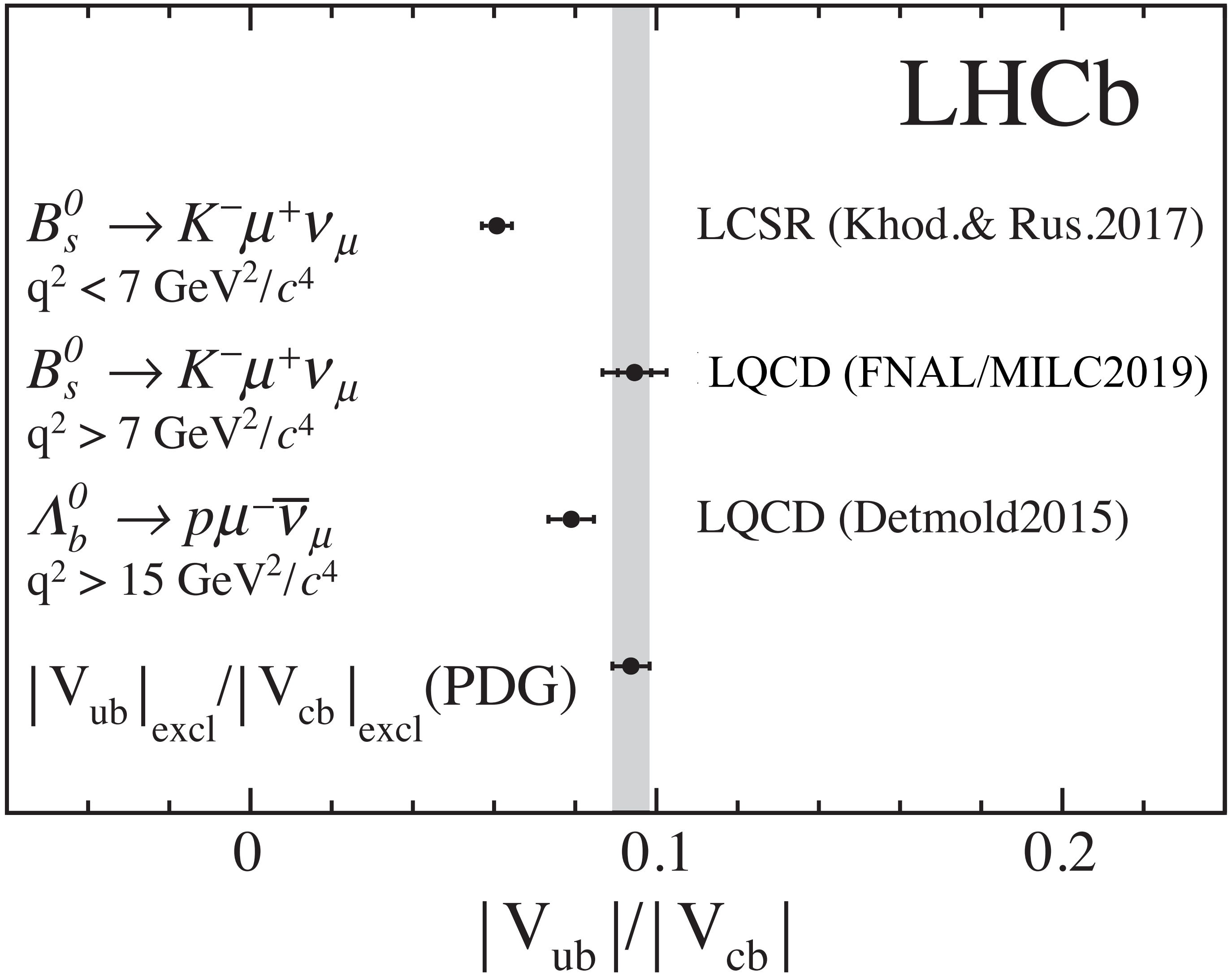
Implications for $|V_{ub}|$





Implications for $|V_{ub}/V_{cb}|$

LHCb [Aaij et al, arXiv:2012.05143, 2021 PRL]

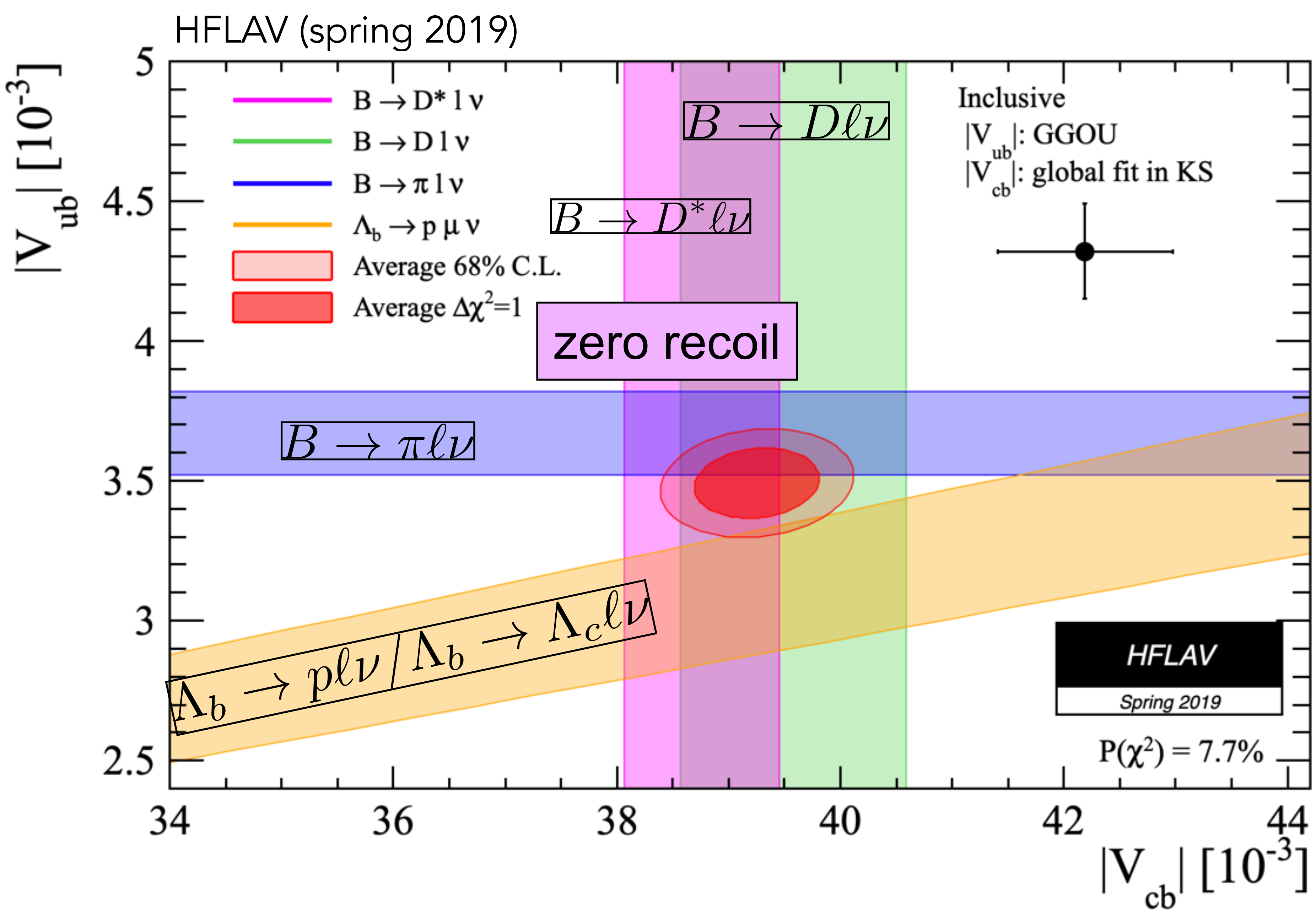


First observation by LHCb!

Measured rates in two large bins
high $q^2 > 7\,\mathrm{GeV}^2$
low $q^2 < 7\,\mathrm{GeV}^2$

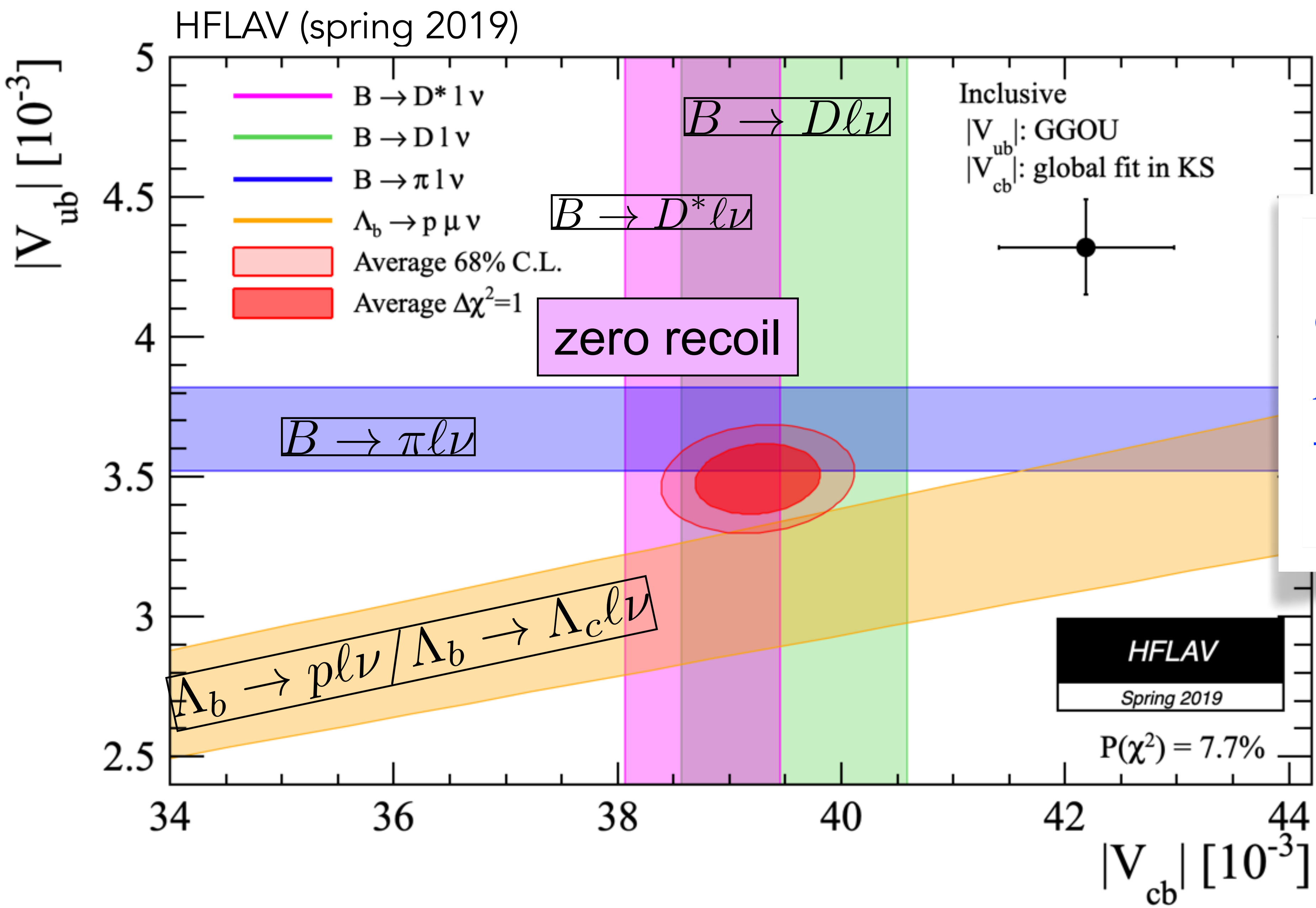
Need smaller bins for shape comparison between experiment and LQCD

Exclusive vs. inclusive $|V_{cb}|$ and $|V_{ub}|$



~3σ tension between inclusive and exclusive $|V_{cb}|$ and $|V_{ub}|$

Exclusive vs. inclusive $|V_{cb}|$ and $|V_{ub}|$

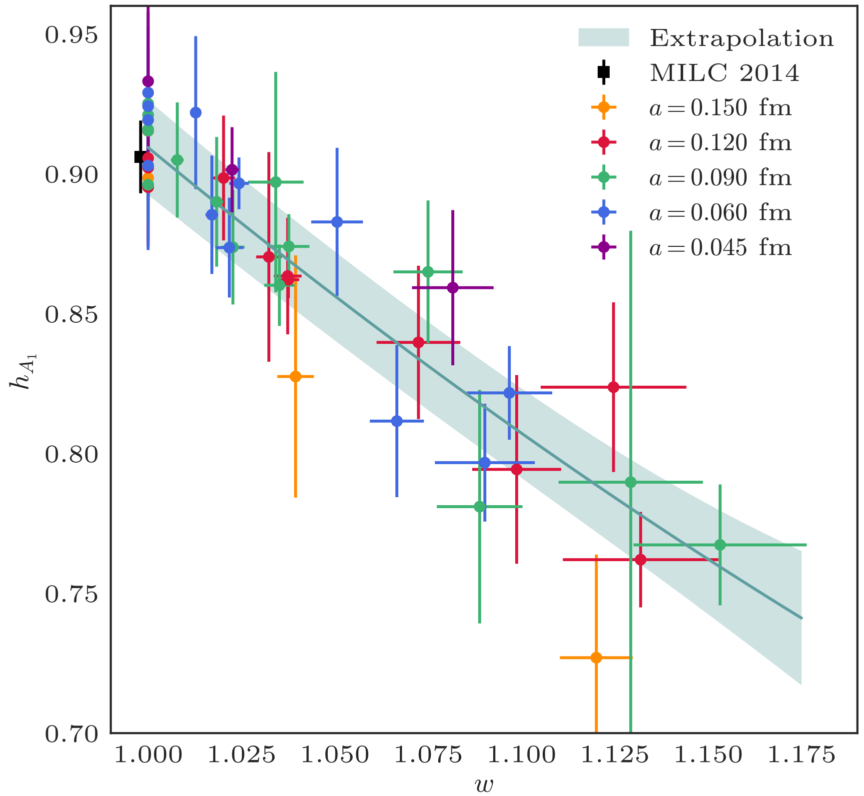


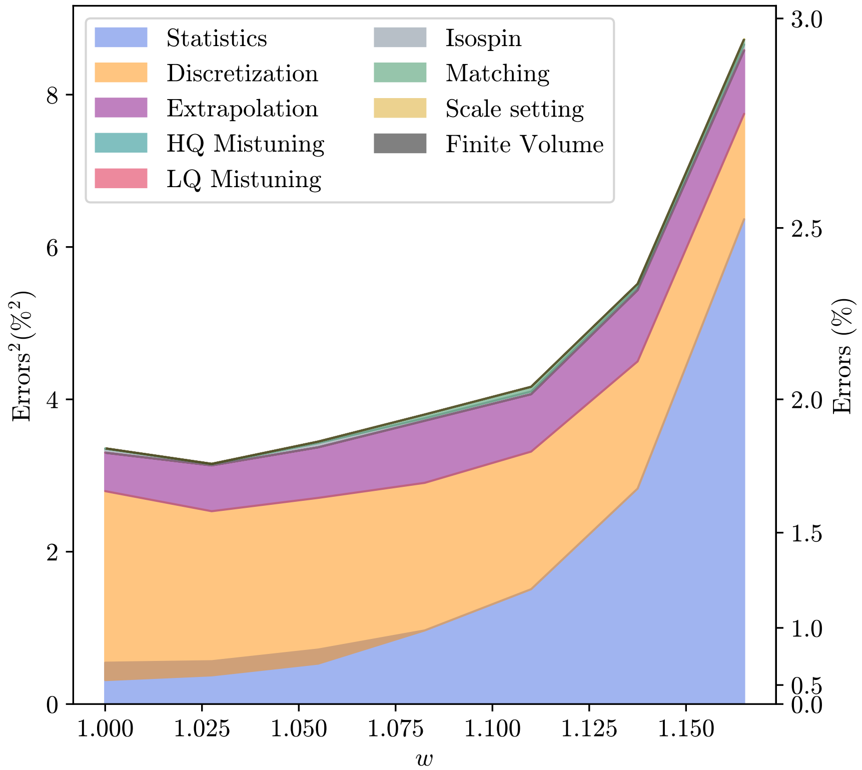
Need LQCD calculations of the $B \to D^*$ form factors at nonzero recoil

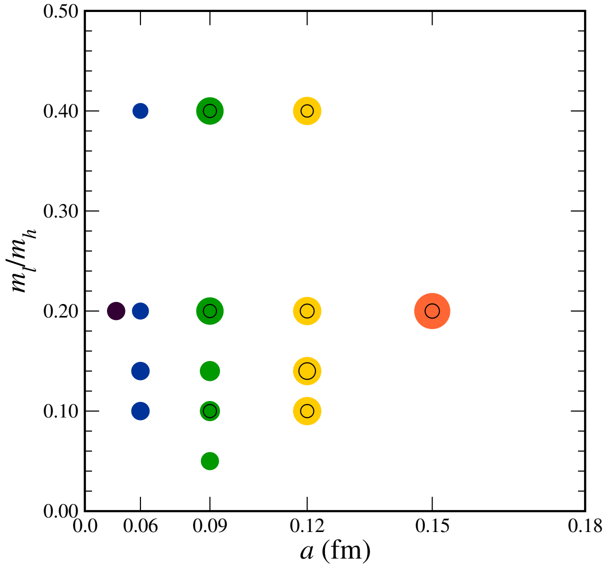
$\sim 3\sigma$ tension between inclusive and exclusive $|V_{cb}|$ and $|V_{ub}|$

Form factors for $B \to D^* \ell \nu_\ell$

A. Vaquero @ IPPP workshop ``Beyond Flavor Anomalies'' [FNAL/MILC, in preparation]







- Preliminary results, combined fit $p - \text{value} = 0.96$
- $h_{A_1}(1) = 0.909(17)$

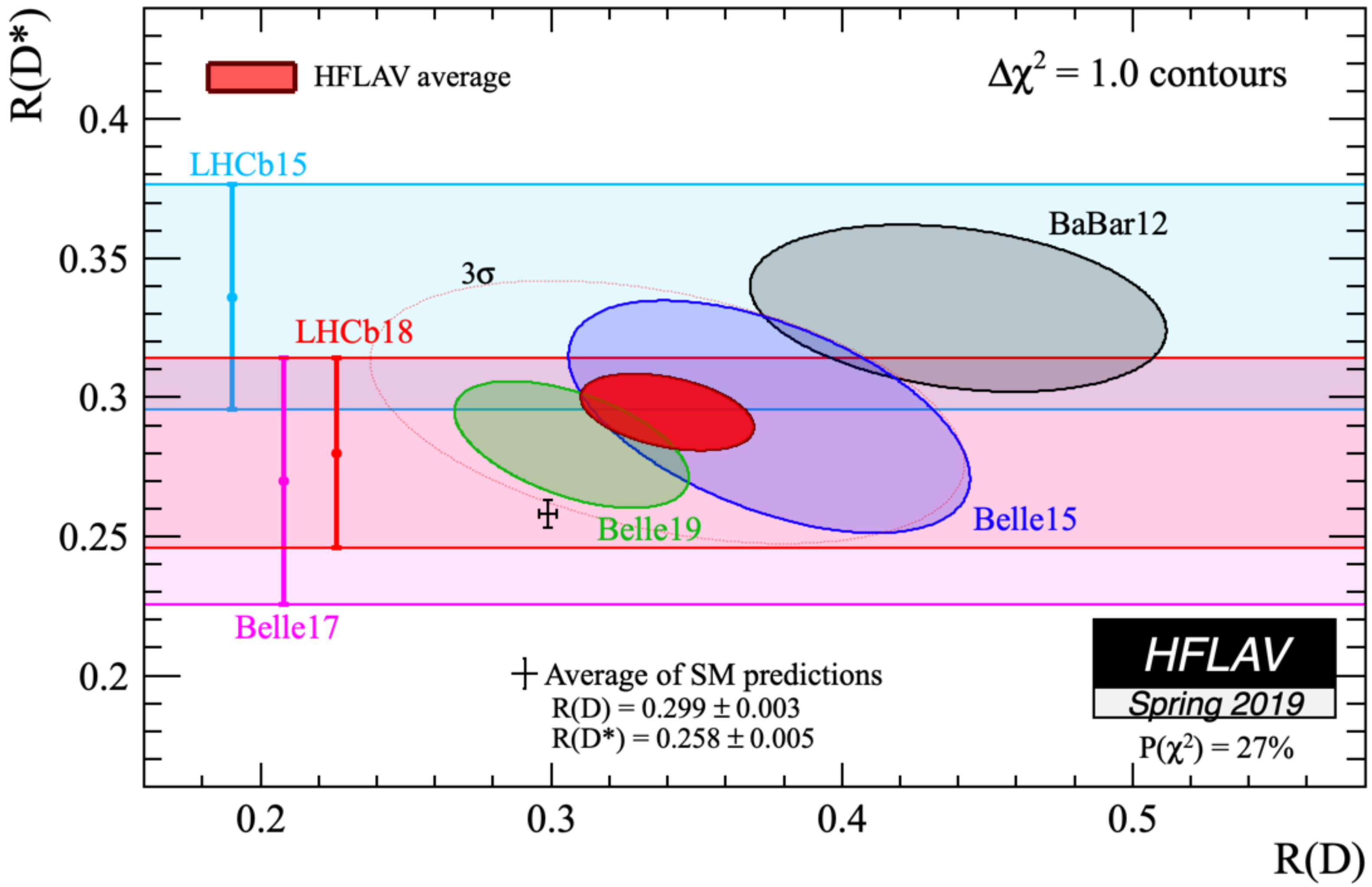
★Results for $h_{A_1}(w), h_{A_2}(w), h_{A_3}(w), h_V(w)$.

★Can be used to calculate $R(D^*)$ (lattice-only)

★Can be used in joint fits with experimental data from BaBar and Belle to determine $|V_{cb}|$ and $R(D^*)$ (lattice + exp)

BSM phenomenology: LFU τ/ℓ

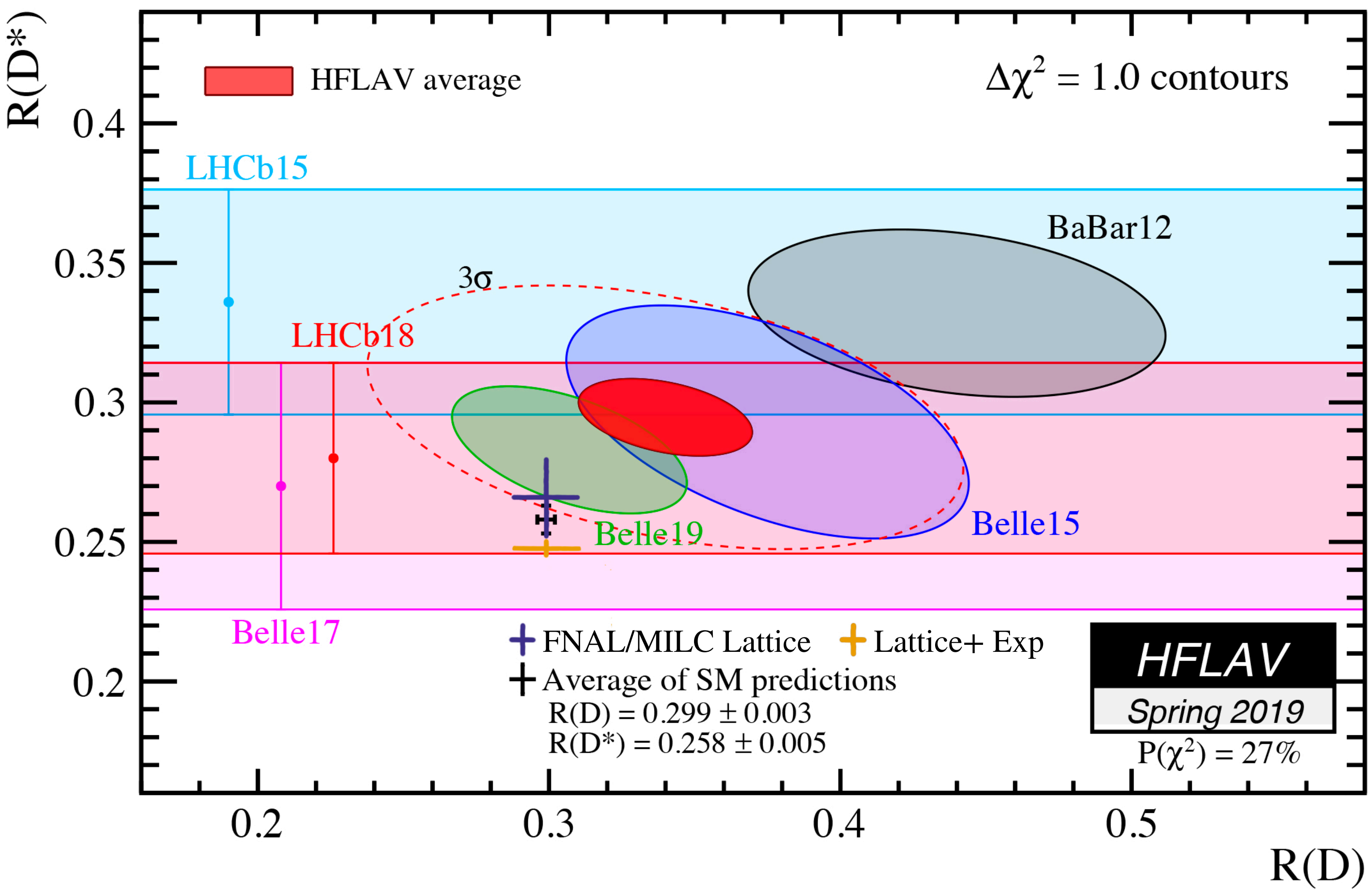
$$R(D^{(*)}) = \frac{\mathcal{B}(B \to D^{(*)}\tau\nu_\tau)}{\mathcal{B}(B \to D^{(*)}\ell\nu)}$$



BSM phenomenology: LFU τ/ℓ

No constraint $w_{\mathbf{Max}}$**:** $R(D^*)_{\mathbf{Lat}} = 0.266(14) \quad R(D^*)_{\mathbf{Lat+Exp}} = 0.2484(13)$
W/ constraint $w_{\mathbf{Max}}$**:** $R(D^*)_{\mathbf{Lat}} = 0.274(10) \quad R(D^*)_{\mathbf{Lat+Exp}} = 0.2492(12)$

Phys.Rev.D 100 (2019), 052007; Phys.Rev.D 103 (2021), 079901; Phys.Rev.Lett. 123 (2019), 091801



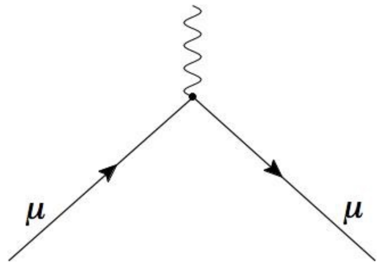
Outline

- Lattice QCD Introduction
- Semileptonic B meson decay form factors
  - $|V_{ub}|$ and $|V_{cb}|$
  - LFU
- muon g-2
- Summary and Outlook

Muon anomalous magnetic moment

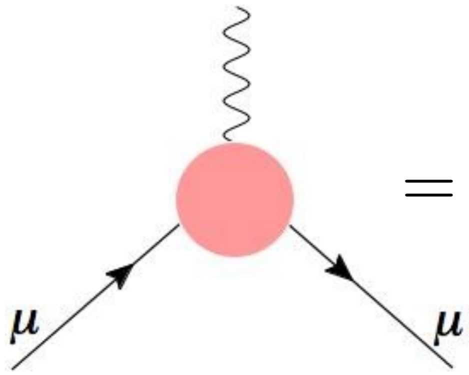
The magnetic moment of charged leptons (e, μ, τ): $\vec{\mu} = g\frac{e}{2m}\vec{S}$

At leading order, $g = 2$:



$$= (-ie)\,\bar{u}(p')\gamma^\mu u(p)$$

Quantum effects (loops):



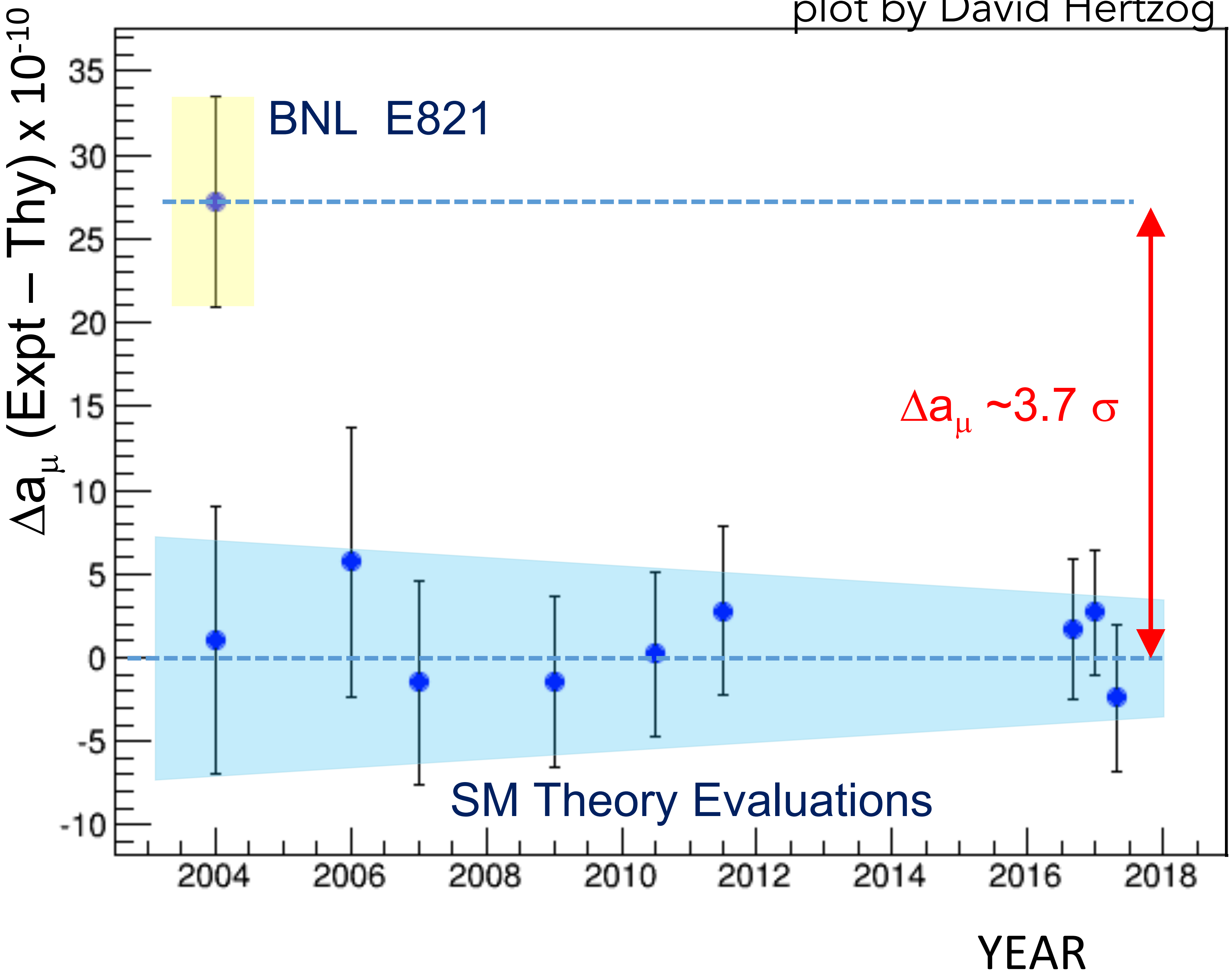
$$= (-i\,e)\,\bar{u}(p')\left[\gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu}q_\nu}{2m}F_2(q^2)\right]u(p)$$

Note: $F_1(0) = 1$ and $g = 2 + 2\,F_2(0)$

Anomalous magnetic moment: $a \equiv \frac{g-2}{2} = F_2(0)$

Muon g-2: history of experiment vs theory

plot by David Hertzog



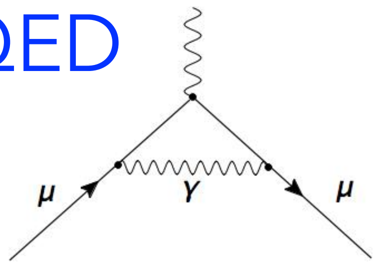
Muon g-2: SM contributions

$$a_\mu = a_\mu(\text{QED}) + a_\mu(\text{Weak}) + a_\mu(\text{Hadronic})$$

Muon g-2: SM contributions

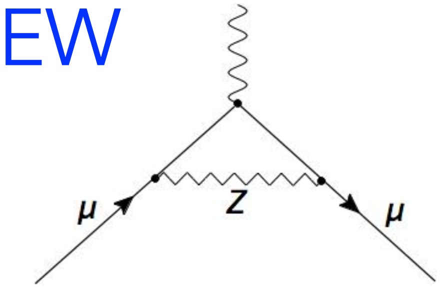
$$\boxed{a_\mu = a_\mu(\text{QED}) + a_\mu(\text{Weak}) + a_\mu(\text{Hadronic})}$$

QED



$+\ldots$ $116\,584\,718.9\,(1) \times 10^{-11}$ 0.001 ppm

EW

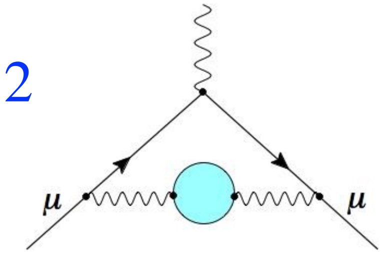


$+\ldots$ $153.6\,(1.0) \times 10^{-11}$ 0.01 ppm

Hadronic…

…Vacuum Polarization (HVP)

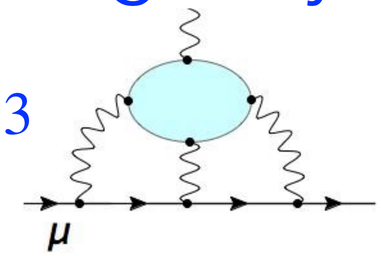
α^2



$+\ldots$ $6845\,(40) \times 10^{-11}$ [0.6%] 0.34 ppm

…Light-by-Light (HLbL)

α^3

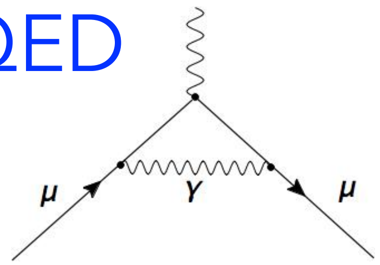


$+\ldots$ $92\,(18) \times 10^{-11}$ [20%] 0.15 ppm

Muon g-2: SM contributions

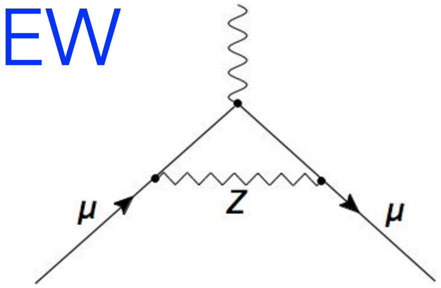
$$\boxed{a_\mu = a_\mu(\text{QED}) + a_\mu(\text{Weak}) + a_\mu(\text{Hadronic})}$$

QED



$+\ldots$ $\quad 116\,584\,718.9\,(1) \times 10^{-11}$ $\quad$ 0.001 ppm

EW

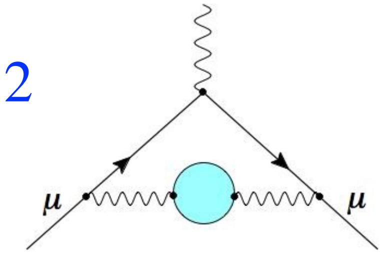


$+\ldots$ $\quad 153.6\,(1.0) \times 10^{-11}$ $\quad$ 0.01 ppm

Hadronic…

…Vacuum Polarization (HVP)

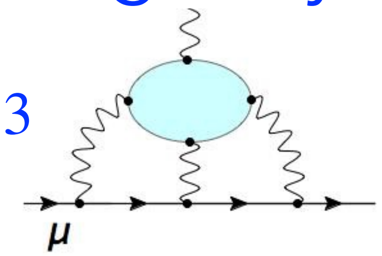
α^2



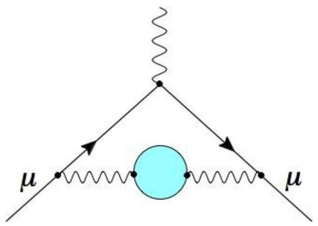
$+\ldots$ $\quad 6845\,(40) \times 10^{-11}$ [0.6%] $\quad$ 0.34 ppm

…Light-by-Light (HLbL)

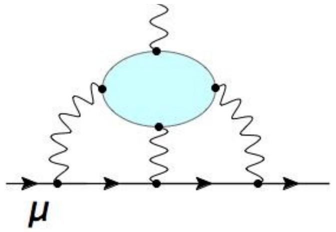
α^3



$+\ldots$ $\quad 92\,(18) \times 10^{-11}$ [20%] $\quad$ 0.15 ppm

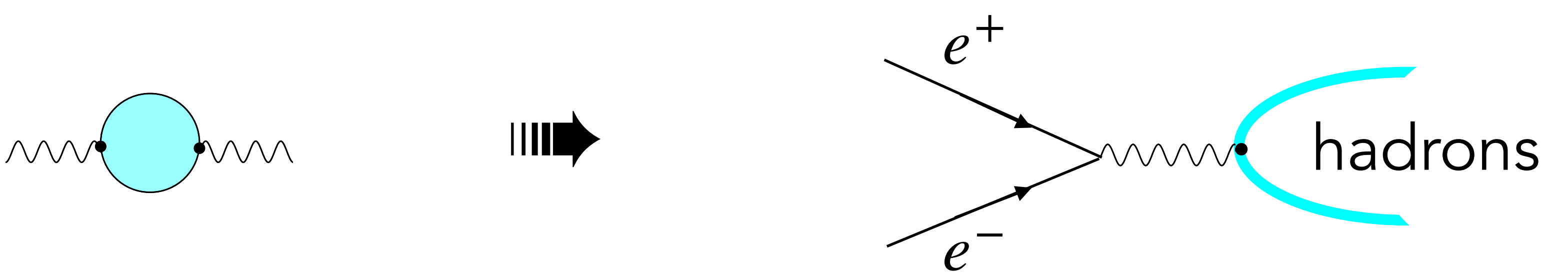


Muon g-2: Hadronic Corrections



1. Dispersive data-driven approach:

Use experimental data together with dispersion theory. For example:



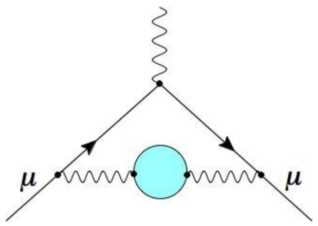
HVP:

Many experiments (over 20+ years) have measured the e^+e^-cross sections for the different channels over the needed energy range with increasing precision. The combined data + dispersion theory yield HVP with a current error ~ 0.6%.

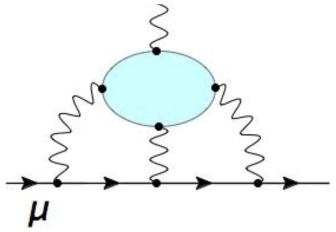
HLbL:

New dispersive approach now allows for data-driven evaluations of HLbL, currently ~20% error ➠ theory error is (almost) completely quantified.

Replaces previous results obtained using simplified models of QCD.



Muon g-2: Hadronic Corrections



2. Euclidean Lattice QCD:

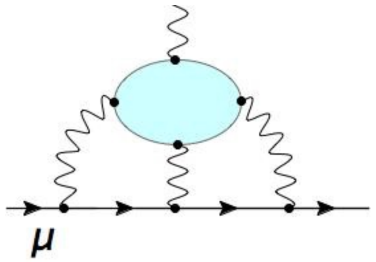
- *ab-initio* method to quantify QCD effects
- already used for simple hadronic quantities with high precision
- requires large-scale computational resources
- allows for entirely SM theory based evaluations

Lattice HVP: ~2% error

- Complete calculations by ~6 different lattice collaborations
- Uncertainties are still larger than data-driven approach, but first lattice result with 0.8% uncertainty [Borsanyi et al, arXiv:2002.12347, 2021 Nature]
- Improved calculations a high priority for the lattice community

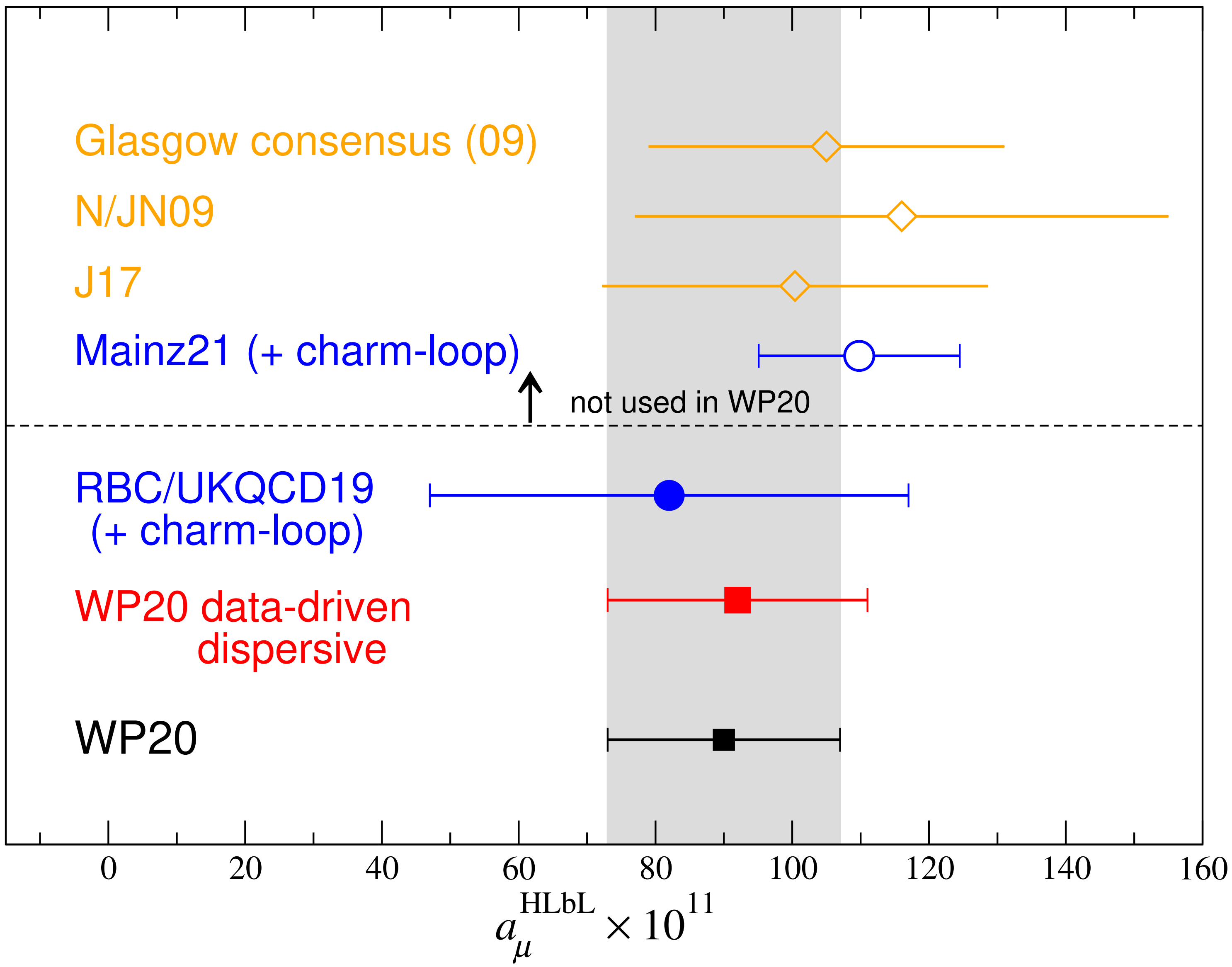
Lattice HLbL: ~45% error

- first complete calculation by RBC/UKQCD [T. Blum et al, arXiv:1911.08123, PRL2020]
- New: complete calculation by Mainz [E.H. Chao et al, arXiv:2104:02632]
- expect improvements from continued computational effort



HLbL: Comparison

a_μ^{HLbL}

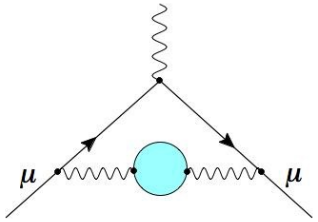


models of QCD +EFT, large Nc

Lattice QCD + QED

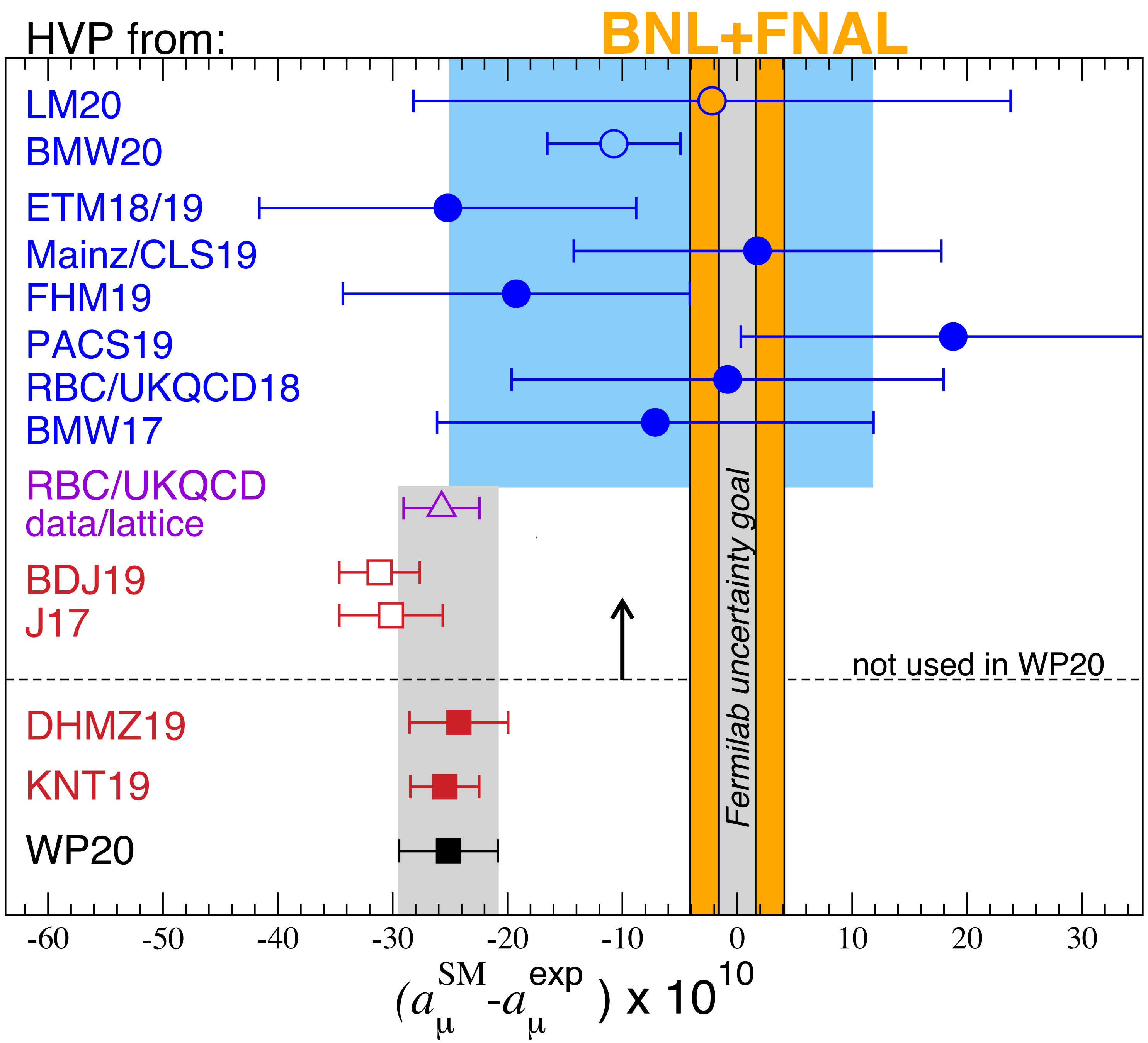
data + dispersive approach

Now well-determined in two approaches, systematically improvable



HVP: Comparison

$$\left[a_\mu^{\rm HVP} + \left[a_\mu^{\rm QED} + a_\mu^{\rm Weak} + a_\mu^{\rm HLbL} \right] \right\rangle \; a_\mu^{\rm SM}$$



Lattice QCD + QED

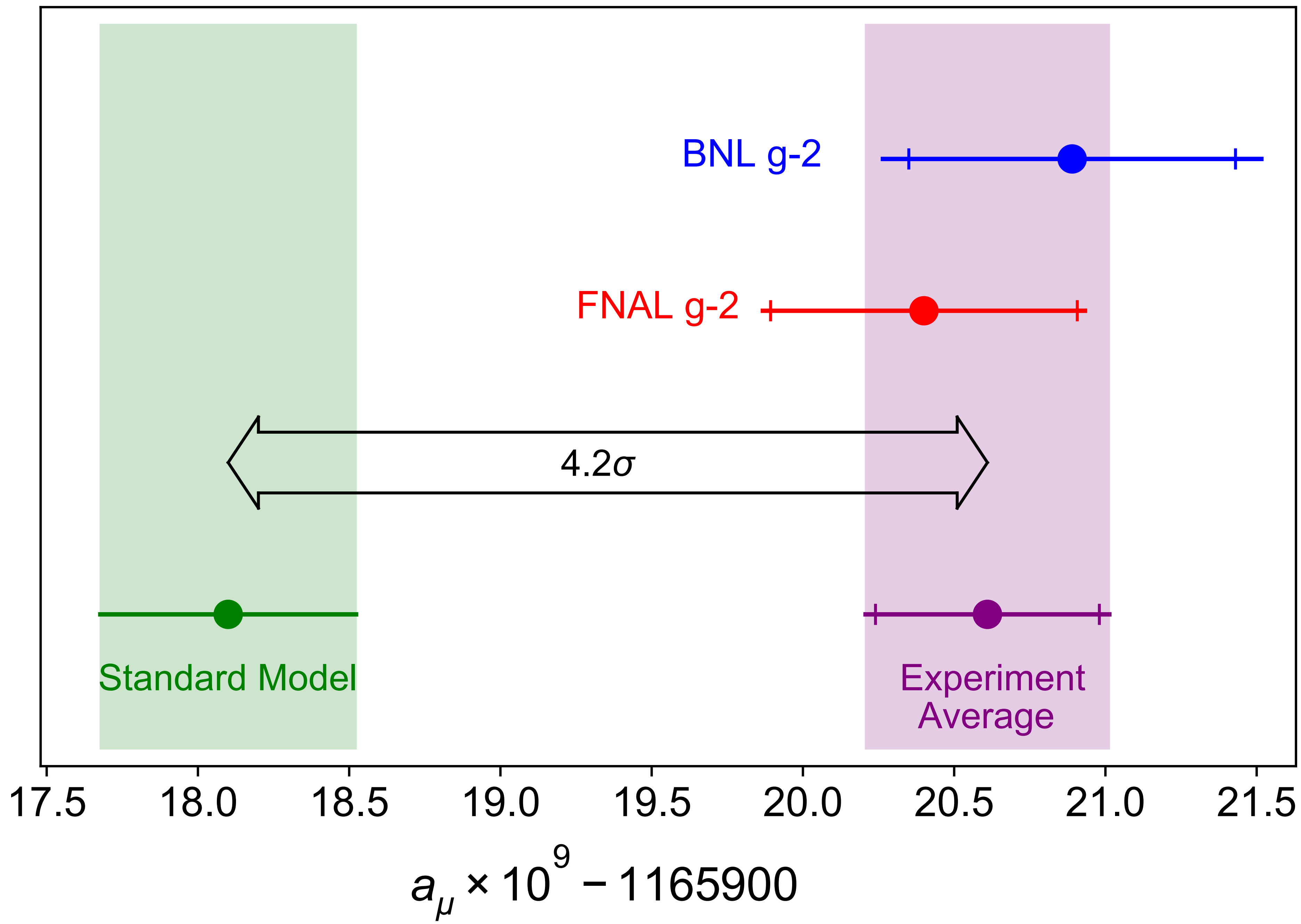
hybrid: combine data & lattice

data driven

+ unitarity/analyticity constraints

Muon g-2: experiment vs theory

[B. Abi et al (Muon g-2 Collaboration), Phys. Rev. Lett. 124, 141801 (2021)]



Summary Table

Contribution	Value $\times 10^{11}$	References
Experimental average (E989+E821)	116592061(41)	Phys.Rev.Lett. 124, 141801
HVP LO (e^+e^-)	6931(40)	Refs. [2–7]
HVP NLO (e^+e^-)	−98.3(7)	Ref. [7]
HVP NNLO (e^+e^-)	12.4(1)	Ref. [8]
HVP LO (lattice, *udsc*)	7116(184)	Refs. [9–17]
HLbL (phenomenology)	92(19)	Refs. [18–30]
HLbL NLO (phenomenology)	2(1)	Ref. [31]
HLbL (lattice, *uds*)	79(35)	Ref. [32]
HLbL (phenomenology + lattice)	90(17)	Refs. [18–30, 32]
QED	116 584 718.931(104)	Refs. [33, 34]
Electroweak	153.6(1.0)	Refs. [35, 36]
HVP (e^+e^-, LO + NLO + NNLO)	6845(40)	Refs. [2–8]
HLbL (phenomenology + lattice + NLO)	92(18)	Refs. [18–32]
Total SM Value	116 591 810(43)	Refs. [2–8, 18–24, 31–36]
Difference: $\Delta a_\mu := a_\mu^{\text{exp}} - a_\mu^{\text{SM}}$	251(59)	

website: https://muon-gm2-theory.illinois.edu

Summary and Outlook

- Lattice QCD calculations of semi-leptonic $B_{(s)}$ meson form factors are very mature, including (almost) complete sets for π, K, D, D_s final states
  - -also true for rare decay form factors (e.g. $B \to K\ell\ell$)
  - -4 groups working on $B_{(s)} \to D^*_{(s)}$ form factors
  - ➠ meeting the growing precision needs of the experimental program
  - ➠ more information on $|V_{ub}|, |V_{cb}|$ incl. vs excl. puzzle
- $\Delta a_\mu = 251\,(59)$ difference between exp and SM at 4.2σ precision will improve in experiment and theory
- scope of LQCD calculations continues to increase (new methods, new formulations, new quantities)

The next few years will be very exciting!

Thank you!

Farah Willenbrock

Appendix

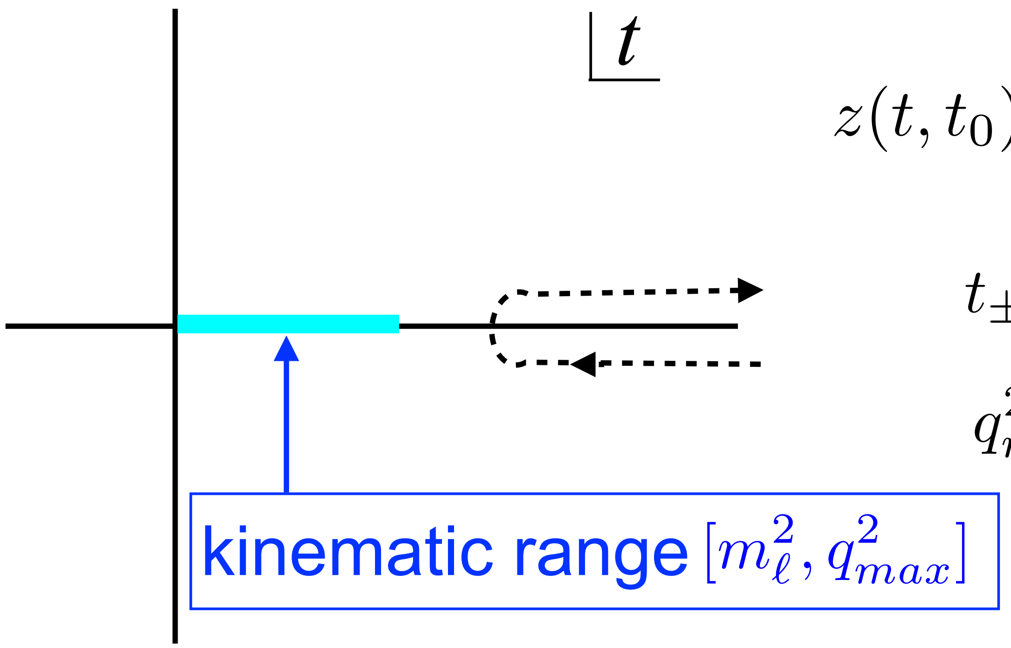
Heavy Quarks

- For light quark ($m_q \ll \Lambda_{\text{QCD}}$) quantities, the leading discretization errors $\sim (a\Lambda)^2$ — if the fermion action is $O(a)$ improved.
- Using the same action for heavy quarks ($m_Q > \Lambda_{\text{QCD}}$) results in leading discretization errors $\sim (am_Q)^2$. The effects are large, if $am_q \not< 1$, which is true for b quarks on most available ensembles.

➠Two classes of solutions:

1. avoid $\sim (am_Q)^2$ effects using EFT (HQET, NRQCD)
   but: nontrivial matching and renormalization
   - rel. heavy quarks (Fermilab, Columbia,..): matching rel. lattice action via HQET to continuum
   - lattice NRQCD, HQET: use EFT to construct lattice action
2. brute force: use the same lattice action for heavy quarks as for light quarks
   - generate gauge ensembles with a small enough so that $(am_b) < 1$
   - supplement with HQET inspired extrapolation and/or static limit

The z-expansion

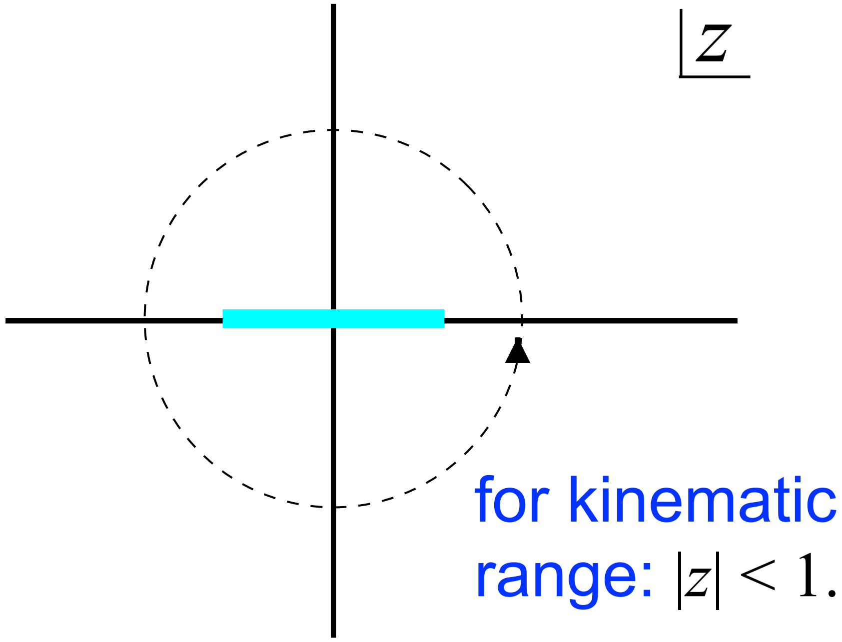


$$t = q^2$$

$$z(t,t_0) = \frac{\sqrt{t_+ - t} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - t} + \sqrt{t_+ - t_0}}$$

$$t_\pm = (m_B \pm m_\pi)^2$$

$$q^2_{max} = t_-$$



The form factor can be expanded as:

$$f(t) = \frac{1}{P(t)\phi(t,t_0)} \sum_{k=0} a_k(t_0) z(t,t_0)^k$$

Bourrely at al (Nucl.Phys. B189 (1981) 157)
Boyd, Grinstein, Lebed (hep-ph/9412324, PRL 95; hep-ph/9504235, PLB 95; hep-ph/9508211, NPB 96; hep-ph/9705252, PRD 97)
Lellouch (arXiv:hep- ph/9509358, NPB 96)
Boyd & Savage (hep-ph/9702300, PRD 97)
Bourrely at al (arXiv:0807.2722, PRD 09)

- $P(t)$ removes poles in $[t_-, t_+]$
- The choice of outer function ϕ affects the unitarity bound on the a_k.
- In practice, only first few terms in expansion are needed.

Muon g-2 Theory Initiative

- Maximize the impact of the Fermilab and J-PARC experiments
  ➠ quantify and reduce the theoretical uncertainties on the hadronic corrections
- summarize the theory status and assess reliability of uncertainty estimates
- organize workshops to bring the different communities together:
  First plenary workshop @ Fermilab: 3-6 June 2017
  HVP workshop @ KEK: 12-14 February 2018
  HLbL workshop @ U Connecticut: 12-14 March 2018
  Second plenary workshop @ HIM (Mainz): 18-22 June 2018
  Third plenary workshop @ INT (Seattle): 9-13 September 2019
  Lattice HVP at high precision workshop (virtual): 16-20 November 2020
  Fourth plenary workshop @ KEK (virtual): 28 June - 02 July 2021

- White Paper posted 10 June 2020:
  [T. Aoyama et al, arXiv:2006.04822, Phys. Repts. 887 (2020) 1-166.]
  132 authors, 82 institutions, 21 countries

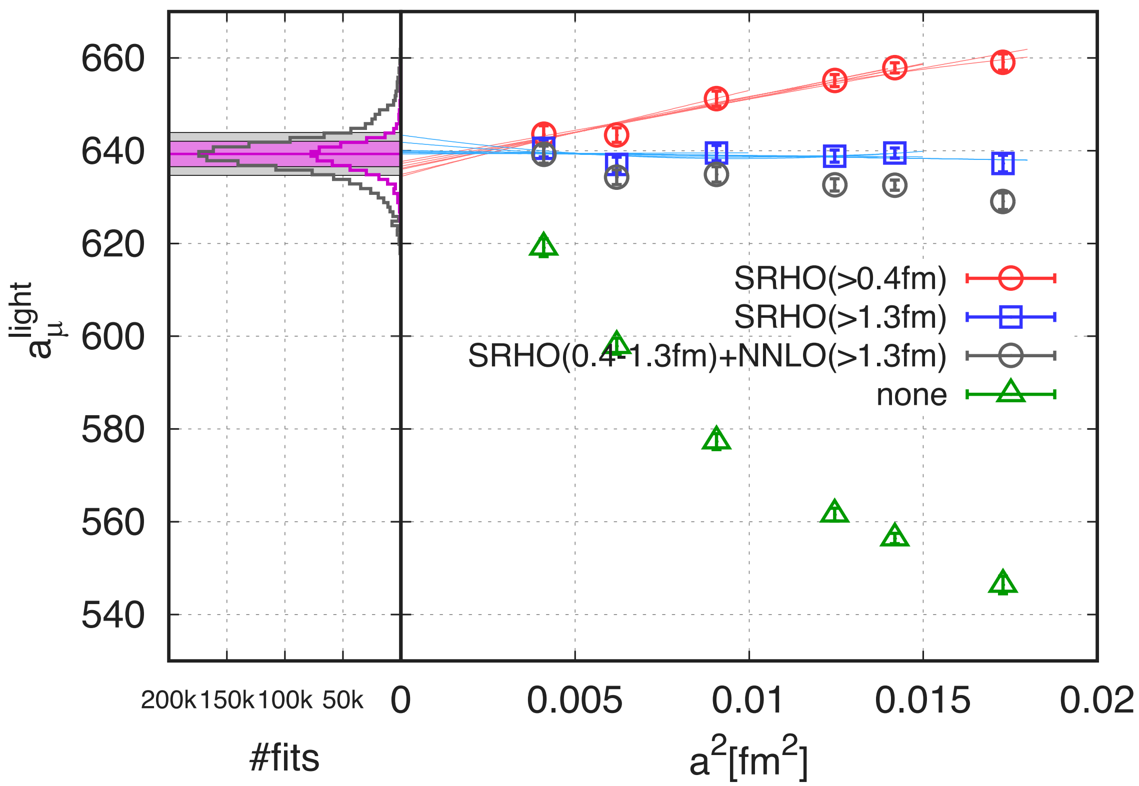
Muon g-2 Theory Initiative

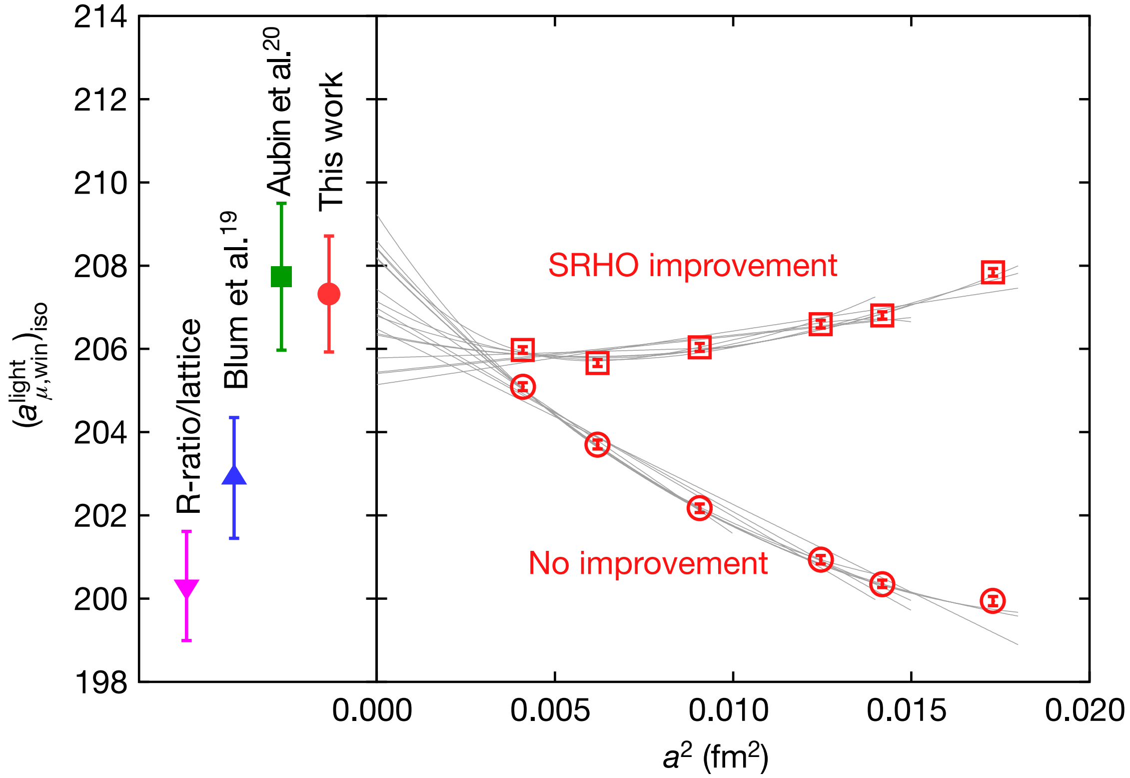
Steering Committee

- Gilberto Colangelo (Bern)
- Michel Davier (Orsay)
- Simon Eidelman (Novosibirsk)
- Aida El-Khadra (UIUC & Fermilab)
- Martin Hoferichter (Bern)
- Christoph Lehner (Regensburg University & BNL)
- Tsutomu Mibe (KEK) J-PARC Muon g-2/EDM experiment
- Lee Roberts (Boston) Fermilab Muon g-2 experiment
- Thomas Teubner (Liverpool)
- Hartmut Wittig (Mainz)

Lattice HVP: results from BMW

[Borsanyi et al, arXiv:2002.12347, 2021 Nature]

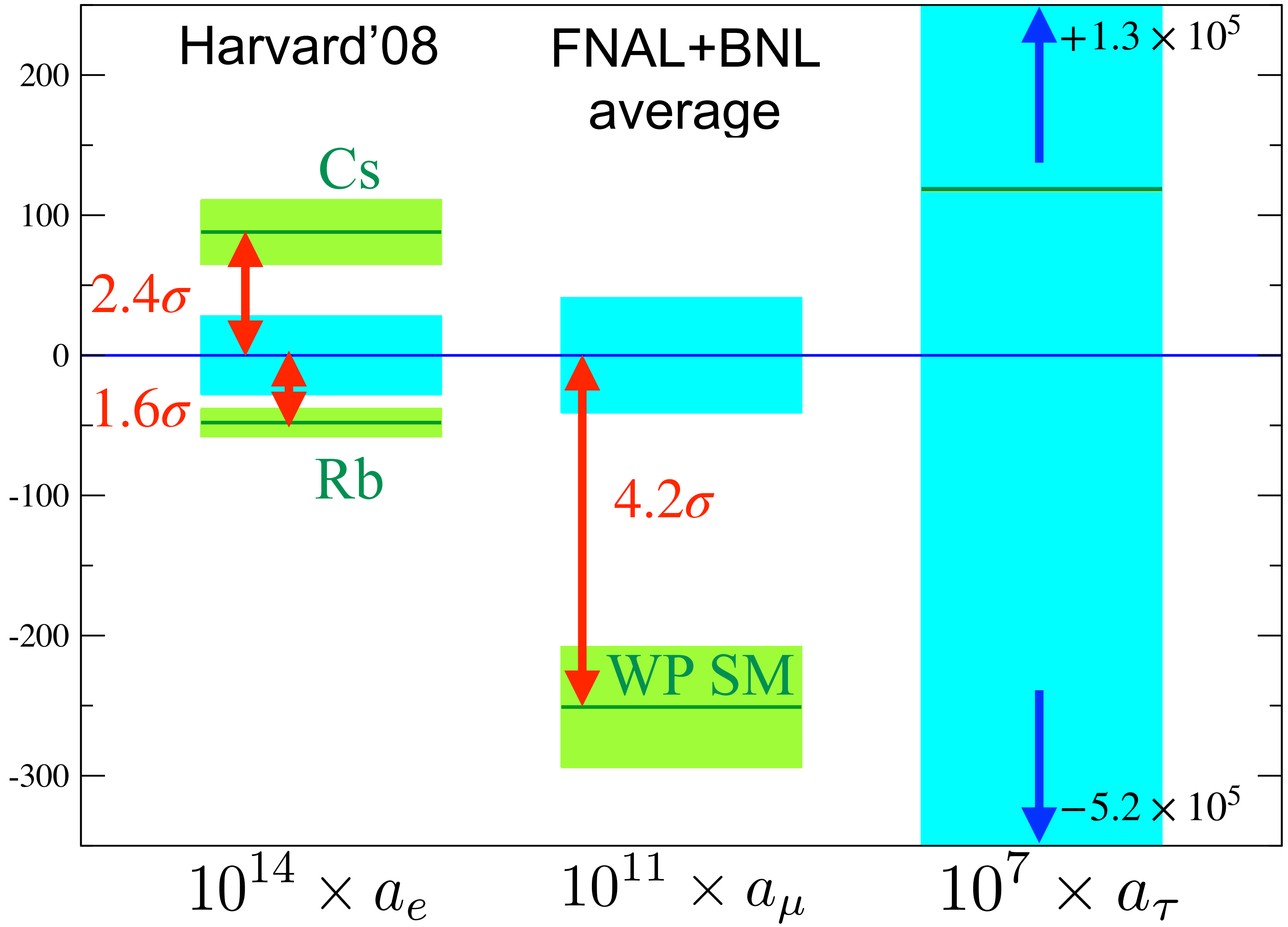




- Small statistical errors and large discretization effects (before corrections)
- Intermediate window a_μ^W:
  -3.7 σ tension with data-driven evaluation (KNT)
  -2.2 σ tension with RBC/UKQCD18
- Need to quantify the differences between data-driven evaluations and the BMW results for the various energy/distance scales

Lepton moments summary

$$a_\ell^{\rm SM} - a_\ell^{\rm Exp}$$



Sensitivity to heavy new physics:

$$a_\ell^{\rm NP} \sim \frac{m_\ell^2}{\Lambda^2}$$

$$(m_\mu/m_e)^2 \sim 4 \times 10^4$$

Cs: α from Berkeley group [Parker et al, Science 360, 6385 (2018)]

Rb: α from Paris group [Morel et al, Nature 588, 61–65(2020)]