

# NNLO QCD Predictions for Triphoton Production

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*based on*

Stefan Kallweit, VS, Marius Wiesemann [[Phys. Lett. B 812 \(2021\) 136013](#)]



**Standard Model at the LHC 2021 (Online)**

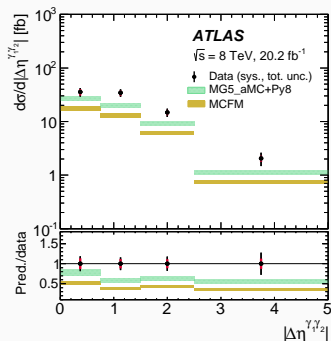
28<sup>th</sup> April 2021



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FÜR PHYSIK

# Motivation for $pp \rightarrow \gamma\gamma + X$

- Tensions between NLO QCD predictions and ATLAS 8 TeV measurement [[arXiv:1712.07291](https://arxiv.org/abs/1712.07291)]
- Main irreducible background for anomalous quartic gauge (e.g.  $Z \rightarrow \gamma\gamma$ ) and anomalous Higgs ( $H\gamma\gamma$ ) couplings
- Fixed order predictions: clean theoretical framework

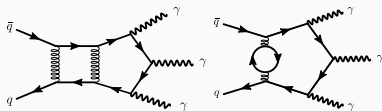


Studied earlier in [[Chawdry, Czakon, Mitov, Poncelet '19](#)] within STRIPPER framework [[Czakon '10,'11](#)]

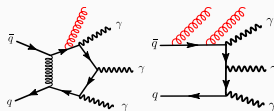
- Very challenging  $\implies$  independent calculation crucial
- We achieve flexible and public implementation

Triphoton is the **first** and **only**  $2 \rightarrow 3$  process at LHC known to NNLO QCD so far

Why?



Two-loop amplitudes



IR divergences

Astoundingly more complex than  $2 \rightarrow 2$ !

Tools and techniques for lower multiplicity **do not scale** well



**new** approaches and frameworks

- Transcendental functions (Feynman integrals)

[Papadopoulos, Tommasini, Wever '15] [Gehrmann, Henn, Lo Presti '18]

[Abreu, Dixon, Herrmann, Page, Zeng '18]

[Chicherin, Gehrmann, Henn, Wasser, Zhang, Zola '18] [Abreu, Page, Zeng '18]

[Chicherin, VS '20] [Abreu, Ita, Moriello, Page, Tschernow, Zeng '20]

[Canko, Papadopoulos, Syrrakos '20]

- Taming algebraic complexity

[Gehrmann, Henn, Lo Presti '15] [Dunbar, Perkins '16] [Peraro '16] [Badger, Brønnum-Hansen, Hartanto, Peraro '18]

[Abreu, Febres Cordero, Ita, Page, VS '18] [Abreu, Dormans, Febres Cordero, Ita, Page '18]

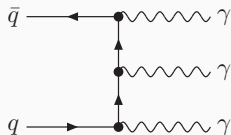
[Abreu, Dormans, Febres Cordero, Ita, Page, VS '19] [Badger, Chicherin, Gehrmann, Heinrich, Henn, Peraro, Wasser, Zhang, Zola '19]

[Abreu, Dixon, Herrmann, Page, Zeng '18] [Chicherin, Gehrmann, Henn, Wasser, Zhang, Zola '18] [Chawdry, Lim, Mitov '18]

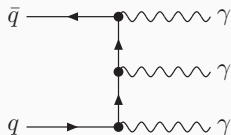
[Boels, Jin, Luo '18] [Böhme, Georgoudis, Larsen, Schönemann, Zhang '18] [Wang, Li, Basat '19] [Guan, Liu, Ma '19]

[Chawdry, Czakon, Mitov, Poncelet '19] [Hartanto, Badger, Brønnum-Hansen '19] [Laurentis, Maltre '20] [Badger, Hartanto, Zola '21],

[Bendle, Boehm, Heymann, Ma, Rahn, Ritzau, Wittmann, Wu, Zhang '21]



- Five-flavor number scheme (5FNS), no top loops
- Full color, except in double-virtual contributions, missing contributions expected numerically subleading [Chawdry, Czakon, Mitov, Poncelet '19] [Kallweit, VS, Wiesemann '20]



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Calculation within MATRIX framework [Grazzini, Kallweit, Wiesemann '17]

- $q_T$ -subtraction formalism, phase space integration
- NLO  $pp \rightarrow \gamma\gamma\gamma j + X$  through Catani-Seymour [Catani, Seymour '96 '97]
- Tree-level, one-loop from OPENLOOPS [Cascioli, Maierhöfer, Pozzorini '11]
- Two-loop analytic results [Abreu, Page, Pascual, VS '20], employing PentagonFunctions++ [Chicherin, VS '20] for special functions

# Phase space and parameters

$$\sqrt{s} = 7, 8, 13, 14, 27, 100 \text{ TeV}$$

$G_\mu$  scheme for the EW parameters,  $\alpha_s(m_Z) = 0.118$ , NNPDF3.1

$$\text{Dynamical central scale} \quad \mu_R = \mu_F = \frac{1}{4} (p_{T,\gamma_1} + p_{T,\gamma_2} + p_{T,\gamma_3})$$

Fiducial phase space from ATLAS 8 TeV analysis [arXiv:1712.07291]:

$$p_{T,\gamma_1} \geq 27 \text{ GeV}, \quad p_{T,\gamma_2} \geq 22 \text{ GeV}, \quad p_{T,\gamma_3} \geq 15 \text{ GeV}, \quad m_{\gamma\gamma} \geq 50$$

$$0 \leq |\eta_\gamma| \leq 1.37 \text{ or } 1.56 \leq |\eta_\gamma| \leq 2.37$$

$$\Delta R_{\gamma\gamma} \geq 0.45$$

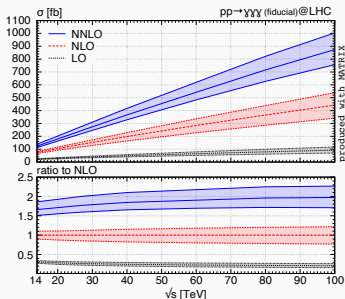
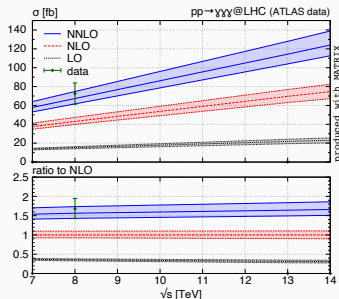
Suppress fragmentation photons: **smooth-cone isolation** [Frixione '98]

$$(n = 1, \delta_0 = 0.4, \text{ fixed } E_T^{\text{ref}} = 10 \text{ GeV})$$

## Uncertainties

- Missing higher orders: 7-point scale variation  $0.5 \leq \mu_R/\mu_F \leq 2$
- Slicing parameter extrapolation ( $r_{\text{cut}} = (\frac{q_T}{m})_{\gamma\gamma} \rightarrow 0$ )  $\implies$  subleading
- PDF uncertainty 2-3% at NLO [arXiv:1712.07291]  $\implies$  negligible at NNLO

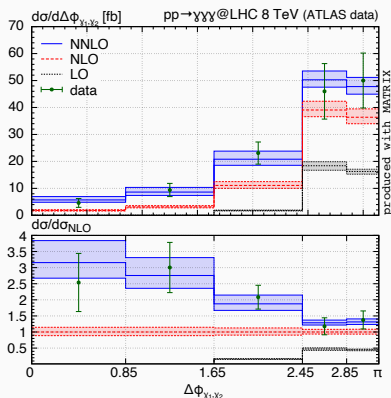
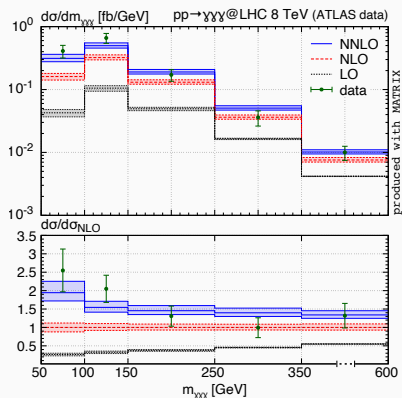
# Fiducial cross sections



| $\sqrt{s}$ , TeV | $\sigma_{\text{NNLO}}$ , fb      | $K_{\text{NLO}}$ | $K_{\text{NNLO}}$ |
|------------------|----------------------------------|------------------|-------------------|
| 7                | 57.84(20) $^{+10.7\%}_{-8.3\%}$  | 2.72             | 1.54              |
| 8                | 67.42(20) $^{+11.0\%}_{-8.5\%}$  | 2.82             | 1.56              |
| 13               | 114.60(43) $^{+11.9\%}_{-9.1\%}$ | 3.18             | 1.65              |
| 14               | 123.83(24) $^{+12.0\%}_{-9.2\%}$ | 3.24             | 1.66              |
| 27               | 245.91(48) $^{+13.2\%}_{-9.9\%}$ | 3.76             | 1.77              |
| 100              | 878.9(24) $^{+15.0\%}_{-13.5\%}$ | 4.79             | 1.99              |

- Full agreement with 8 TeV results of [Chawdry, Czakon, Mitov, Poncelet '19]
- Consistent with ATLAS 8 TeV measurement  
72.6  $^{+6.5}_{-6.5}$  (stat)  $^{+9.2}_{-9.2}$  (syst) fb
- Huge  $K_{\text{NLO}}$  and  $K_{\text{NNLO}}$  factors

# Differential cross sections $\sqrt{s} = 8$ TeV



- Major **shape differences** w.r.t. NLO
- Excellent agreement with data (and [Chawdry, Czakon, Mitov, Poncelet '19]) across all differential distributions
- At higher  $\sqrt{s}$  similar pattern, corrections slightly increase



# Perturbative convergence?

## Symptoms

- Giant  $K_{\text{NLO}}$  and  $K_{\text{NNLO}}$  factors
- No overlap in uncertainty bands

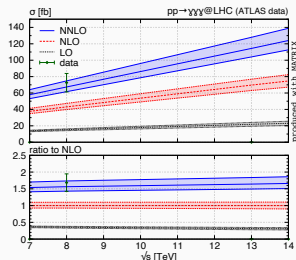
## Lifting fixed-order constraints

- opening of channels  
NLO  $qg \rightarrow \gamma\gamma q$   
NNLO  $qq' \rightarrow \gamma\gamma qq'$ ,  $gg \rightarrow \gamma\gamma q\bar{q}$
- degenerate LO kinematics

Other examples:  $\gamma\gamma$ ,  $Wb\bar{b}$ ,  $H$ , ...

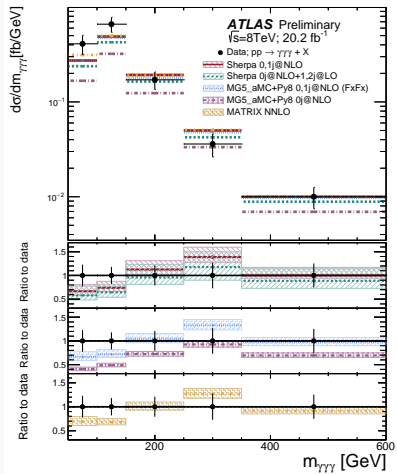
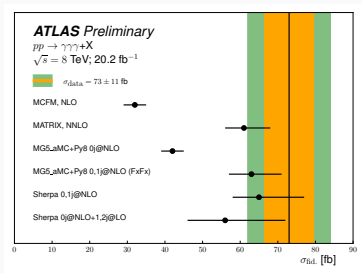


- Scale uncertainties unreliable until all constraints are lifted
- For triphoton reasonable uncertainties expected at NNLO



# Perturbative convergence?

Comparison to merged 0,1j NLO sheds some light [ATL-PHYS-PUB-2021-001]



## Summary

Triphoton production in NNLO QCD with  $q_T$ -subtraction in MATRIX

- First on-the-fly NNLO calculation of a  $2 \rightarrow 3$  process, publicly available in the next release of MATRIX
- NNLO corrections large and essential for precision phenomenology

## Outlook

- NNLO+PS
- EW corrections, subleading-color contributions
- Better understanding of uncertainties, e.g. interplay of scale choice and isolation criteria [Gehrmann, Glover, Huss, Whitehead '20], etc.

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### Self-promotion corner

Double-virtual contributions for NNLO QCD three-jet are now available [arXiv:2102.13609]!

# Acknowledgments

This project has received funding from the European Research Council (ERC) under the European Union's Horizon 2020 research and innovation programme, *Novel structures in scattering amplitudes* (grant agreement No. 725110).



**Backup**

# $q_T$ subtraction (slicing)

$$h_1 + h_2 \rightarrow F + X$$

[Catani, Grazzini '07]

$q_T$  — transverse momentum of color singlet system  $F$ ,  $r = \frac{q_T}{m_F}$   
 $q_T = 0$  in LO kinematics

$$\begin{aligned} \sigma_{\text{NNLO}}^F &= \int_{\Phi_F} d\sigma_B + \int_{\Phi_F} d\sigma_V + \\ &\quad \left( \int_{\Phi_{F+1}} d\sigma_R + \int_{\Phi_{F+2}} d\sigma_{RR} + \int_{\Phi_{F+1}} d\sigma_{RV} \right) + \int_{\Phi_F} d\sigma_{VV} \\ &\downarrow \quad \downarrow \quad \downarrow \quad \downarrow \\ d\sigma_{\text{NNLO}}^F &= \mathcal{H}_{\text{NNLO}} \otimes d\sigma_B + \left( d\sigma_{\text{NLO}}^{F+\text{jet}} - d\sigma_{\text{NNLO}}^{\text{CT}} \right) \\ &\quad \downarrow \\ &\quad \mathcal{H}_{\text{NNLO}} \otimes d\sigma_B + \left( d\sigma_{\text{NLO}}^{F+\text{jet}} - d\sigma_{\text{NNLO}}^{\text{CT}} \right) \theta(r - r_{\text{cut}}) + \mathcal{O}(r_{\text{cut}}) \end{aligned}$$

$d\sigma_{\text{NLO}}^{F+\text{jet}}$

finite with  $r > 0 \implies$  Catani-Seymour calculation [Catani, Seymour '96 '97]

$d\sigma_{\text{NNLO}}^{\text{CT}}$

universal counterterm, known from small  $q_T$  resummation [Bozzi, Catani, de Florian, Grazzini '05]

$\mathcal{H}_{\text{NNLO}}$

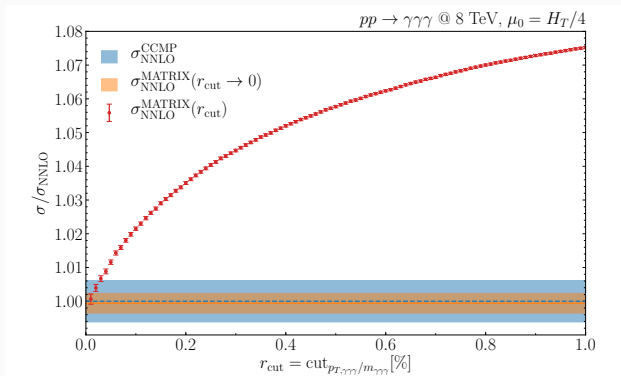
particular IR renormalization of loop corrections [Catani, Cieri, de Florian, Ferrera, Grazzini '13]

$r_{\text{cut}} \rightarrow 0$

extrapolation automated in MATRIX [Grazzini, Kallweit, Wiesemann '17]

## $r_{\text{cut}} \rightarrow 0$ extrapolation

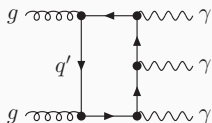
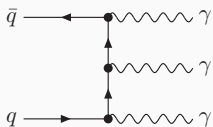
Three smooth-cone **isolated** photons, intricate interplay with  $r_{\text{cut}}$   
 $\Rightarrow$  potentially large uncertainties



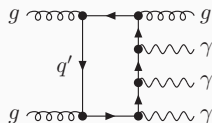
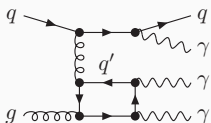
- $q_T$  subtraction fully capable dealing with  $2 \rightarrow 3$  color singlet!
- extrapolation uncertainties controlled, few %, mild phase-space dependence



# Loop-induced contributions

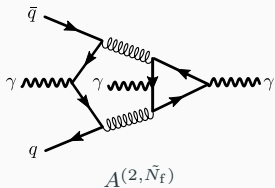
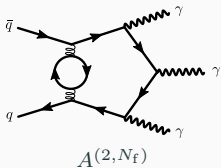
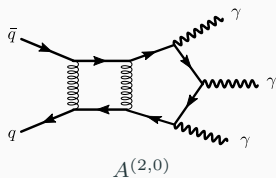


$= 0$



finite (formally  $N^3LO$ ), small  $\lesssim 1\%$

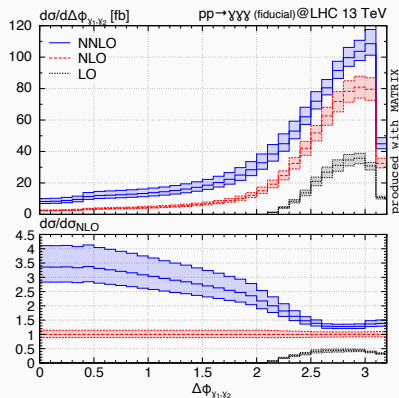
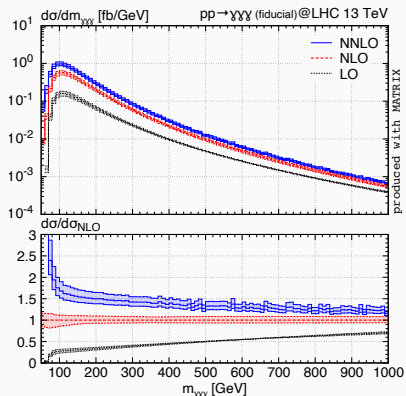
# Contributions to two-loop hard function $\mathcal{H}^{(2)}$



$$\mathcal{H}^{(2)} = \frac{N_c^2}{4} \left( \mathcal{H}^{(2,0)} - \frac{1}{N_c^2} (\mathcal{H}^{(2,0)} + \mathcal{H}^{(2,1)}) + \frac{1}{N_c^4} \mathcal{H}^{(2,1)} \right) + C_F T_F N_f \mathcal{H}^{(2, N_f)} + C_F T_F \left( \sum_{f=1}^{N_f} Q_f^2 \right) \mathcal{H}^{(2, \tilde{N}_f)} \longrightarrow \frac{N_c^2}{4} \mathcal{H}^{(2,0)}$$

- $\mathcal{H}^{(2,1)}$ ,  $\mathcal{H}^{(2, \tilde{N}_f)}$  include non-planar two-loop contributions (difficult, WIP)
- $\mathcal{H}^{(2, N_f)}$ ,  $\mathcal{H}^{(2, \tilde{N}_f)}$  contributions expected numerically subleading

# Differential cross sections $\sqrt{s} = 13$ TeV



- Similar pattern as in 8 TeV, corrections generally increase
- Efficient calculation allows fine binning, predictions for high-energy tails