A Novel Technique for the Static Alignment of the CLIC Beam Delivery System

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...after **very inspiring discussions** with **Pantaleo Raimondi**, INFN-LNF

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Alignment Procedure

- With the multipole magnets turned **OFF**
  - Orbit Steering, 1-to-1
  - Target Dispersion Steering
- With the multipole magnets turned **ON**
  - Beam-based centering of the multipole magnets
  - Target Dispersion Steering
  - Target Beta-Beating Steering
  - Coupling Correction

Basic Equations

Given a system:

$$\mathbf{y} = \mathbf{f}\left(\mathbf{x}\right) \tag{1}$$

its Taylor expansion around $\mathbf{x_0}, \mathbf{y_0} = \mathbf{f}(\mathbf{x_0})$ is

$$\mathbf{y} = \mathbf{y_0} + \left.\frac{\partial \mathbf{f}}{\partial \mathbf{x}}\right|_{\mathbf{x_0}} \left(\mathbf{x} - \mathbf{x_0}\right) + \ldots \tag{2}$$

$\mathbf{A} = \left.\frac{\partial \mathbf{f}}{\partial \mathbf{x}}\right|_{\mathbf{x_0}}$ is the Jacobian, or response matrix, of the system.

The linear approximation of eq. (1) around $(\mathbf{x_0}, \mathbf{y_0})$ is therefore:

$$\mathbf{y} = \mathbf{y_0} + \mathbf{A}\left(\mathbf{x} - \mathbf{x_0}\right) \tag{3}$$

Linear Approximation and Least Squares Method

This is our "**model**":

$$\mathbf{y} = \mathbf{y_0} + \mathbf{A}\left(\mathbf{x} - \mathbf{x_0}\right)$$

Where, in our case:

$\mathbf{x}$:	is the vector of the correctors
$\mathbf{y}$:	is the vector of the observables
$\mathbf{A}$:	is the response matrix
$\mathbf{x_0}, \mathbf{y_0}$:	is the central point :
	correctors to zero $\longrightarrow$ observables for the reference trajectory

$\Rightarrow$ the observables we will use are: orbit, dispersion, beta-beating and coupling.

Given an arbitrary system configuration, $\mathbf{y} = \mathbf{y}_{\text{Measured}}$, the corresponding **correctors**, $\mathbf{x}$, that match this status, can be found solving the least squares minimization of the function:

$$\chi^{\mathbf{2}} = \; \| \; \mathbf{y}_{\text{Measured}} \; - \; \{\mathbf{y_0} + \mathbf{A}(\mathbf{x}_{\text{Unknown}} - \mathbf{x_0})\} \; \|^{\;\mathbf{2}}$$

Least Squares Method and Singular Value Decomposition

The solution, $\mathbf{x}$, of the previous equation is given by $\frac{\partial \chi^{\mathbf{2}}}{\partial \mathbf{x}} = \mathbf{0}$:

$$\mathbf{y}_{\mathsf{M}} - \mathbf{y_0} \;=\; \mathbf{A}\;(\mathbf{x} - \mathbf{x_0})$$

Being $\mathbf{x_0} = 0$,

$$\mathbf{y}_{\mathsf{M}} - \mathbf{y_0} \;=\; \mathbf{A}\,\mathbf{x}$$

The matrix $\mathbf{A}$ is likely not squared, having usually more observables than correctors $\rightarrow$ the system is overdetermined. One way to solve overdetermined systems is to use the Singular Value Decomposition of this matrix.

The solution is:

$$\mathbf{x} \;=\; \mathbf{A}^{\dagger}(\mathbf{y}_{\mathsf{M}} - \mathbf{y_0})$$

where $\mathbf{A}^{\dagger}$ is the pseudo-inverse of $\mathbf{A}$ in the SVD-sense.

Beam Delivery System

In this context, the correctors, $\mathbf{x}$, are called

θ_x	horizontal correctors
θ_y	vertical correctors

whereas the observables, $\mathbf{y}$, are:

$\mathbf{b}_x$	horizontal bpm readings
$\mathbf{b}_y$	vertical bpm readings
$\boldsymbol{\eta}_x$	horizontal dispersion at each bpm
$\boldsymbol{\eta}_y$	vertical dispersion at each bpm
$\boldsymbol{\beta}_x$	horizontal beta – beating at each bpm
$\boldsymbol{\beta}_y$	vertical beta – beating at each bpm
$\mathbf{C}_x$	horizontal coupling at each bpm
$\mathbf{C}_y$	vertical coupling at each bpm

How to Measure Dispersion, Coupling and Beta-Beating (1/2)

To measure the **dispersion**, it is necessary to use one or more *test-beams* with different energies. We used two test beams with energy difference $\delta = \pm 0.005$:

$$\boldsymbol{\eta} = \frac{b_{+\delta} - b_{-\delta}}{2\delta}$$

To measure the **horizontal beta-beating**, it is necessary to have the first corrector kicking in $x = \pm 1$, then measure the horizontal response of the system:

$$\boldsymbol{\beta}_x = \frac{b_{x|\theta_{1,x=+1}} - b_{x|\theta_{1,x=-1}}}{2\Delta\theta_{1,x}}$$

To measure the **vertical beta-beating**, it is necessary to have the first corrector kicking in $y = \pm 1$, then measure the vertical response of the system:

$$\boldsymbol{\beta}_y = \frac{b_{y|\theta_{1,y=+1}} - b_{y|\theta_{1,y=-1}}}{2\Delta\theta_{1,y}}$$

How to Measure Dispersion, Coupling and Beta-Beating (2/2)

To measure the **horizontal coupling**, it is necessary to have the first corrector kicking in $y = \pm 1$, then measure the horizontal response of the system:

$$\mathbf{C}_x = \frac{b_{x|\theta_{1,y=+1}} - b_{x|\theta_{1,y=-1}}}{2\Delta\theta_{1,y}}$$

To measure the **vertical coupling**, it is necessary to have the first corrector kicking in $x = \pm 1$, then measure the vertical response of the system:

$$\mathbf{C}_y = \frac{b_{y|\theta_{1,x=+1}} - b_{y|\theta_{1,x=-1}}}{2\Delta\theta_{1,x}}$$

⇒ Notice that to obtain these 6 quantities,

$$\underbrace{\boldsymbol{\eta}_x, \boldsymbol{\eta}_y}_{\delta=\pm 0.5\%}, \underbrace{\boldsymbol{\beta}_x, \mathbf{C}_x}_{\theta_{1,x=\pm 1}}, \underbrace{\boldsymbol{\beta}_y, \mathbf{C}_y}_{\theta_{1,y=\pm 1}},$$

a total of **six** measurements is required.

Alignment Algorithm (1/2)

Multipoles OFF

1) **Orbit correction**

$$\begin{pmatrix} \mathbf{b}_x \\ \mathbf{b}_y \end{pmatrix} = \begin{pmatrix} R_{xx} & 0 \\ 0 & R_{yy} \end{pmatrix} \begin{pmatrix} \boldsymbol{\theta}_x \\ \boldsymbol{\theta}_y \end{pmatrix}$$

2) **Target Dispersion Steering**

$$\begin{pmatrix} \mathbf{b}_x \\ \mathbf{b}_y \\ \boldsymbol{\eta}_x - \boldsymbol{\eta}_{0,x} \\ \boldsymbol{\eta}_y - \boldsymbol{\eta}_{0,y} \end{pmatrix} = \begin{pmatrix} R_{xx} & 0 \\ 0 & R_{yy} \\ D_{xx} & 0 \\ 0 & D_{yy} \end{pmatrix} \begin{pmatrix} \boldsymbol{\theta}_x \\ \boldsymbol{\theta}_y \end{pmatrix}$$

$\Rightarrow$ it requires **one** or **two test beams**, with $E = E_0\,(1 \pm 0.005)$, to measure the dispersion.

Alignment Algorithm (2/2)

Multipoles ON

3) **Beam-based centering** of each individual multipolar elements (see later for details)

4) **Coupling** and **Beta-Beating Steering**

$$\begin{pmatrix} \mathbf{b}_x \\ \mathbf{b}_y \\ \boldsymbol{\eta}_x - \boldsymbol{\eta}_{0,x} \\ \boldsymbol{\eta}_y - \boldsymbol{\eta}_{0,y} \\ \boldsymbol{\beta}_x - \boldsymbol{\beta}_{0,x} \\ \boldsymbol{\beta}_y - \boldsymbol{\beta}_{0,y} \\ \mathbf{C}_x \\ \mathbf{C}_y \end{pmatrix} = \begin{pmatrix} R_{xx} & 0 \\ 0 & R_{yy} \\ D_{xx} & 0 \\ 0 & D_{yy} \\ B_{xx} & 0 \\ B_{yx} & 0 \\ 0 & C_{xy} \\ 0 & C_{yy} \end{pmatrix} \begin{pmatrix} \boldsymbol{\theta}_x \\ \boldsymbol{\theta}_y \end{pmatrix}$$

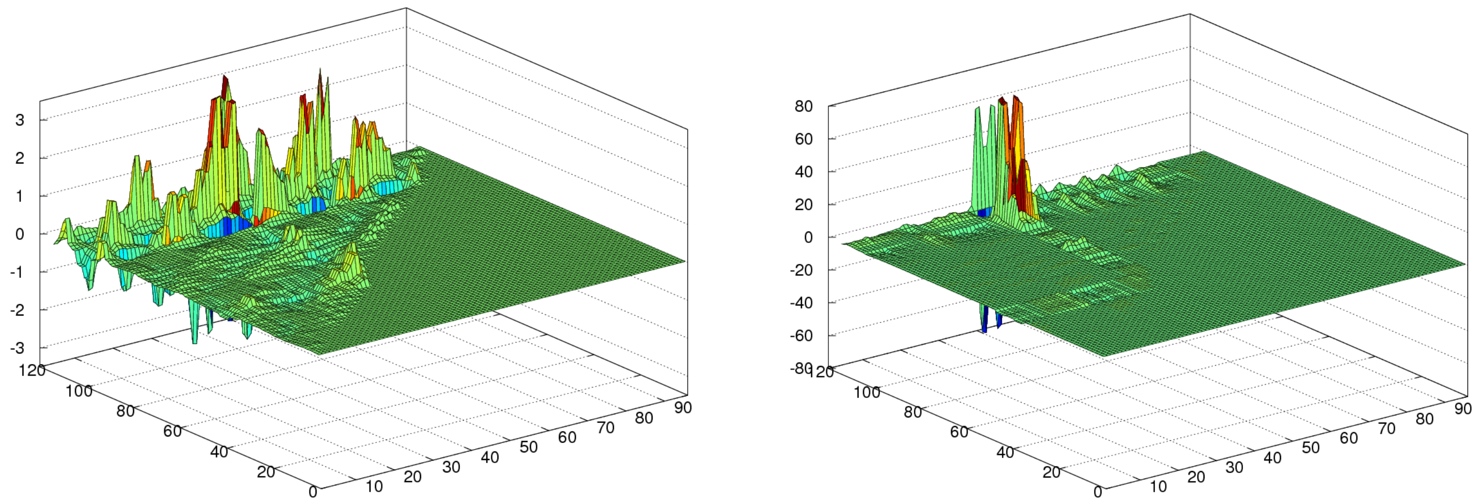
$\Rightarrow$ it requires **four shots** -nominal energy- with the first corrector ON, $\Delta\theta_{1,x|y} = \pm$small kick, to measure beta-beating and coupling.

Orbit Response Matrix

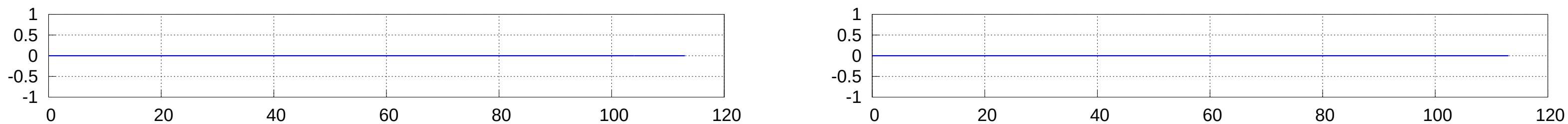
Jacobian of the system:

$$\mathbf{R} = \frac{\partial \mathbf{b}}{\partial \theta}; \qquad \mathbf{R}_{ij} = \frac{b_{i;+\Delta\theta_j} - b_{i;-\Delta\theta_j}}{2\Delta\theta_j}$$

Response matrices: R_{xx}, R_{yy}



Target Responses:

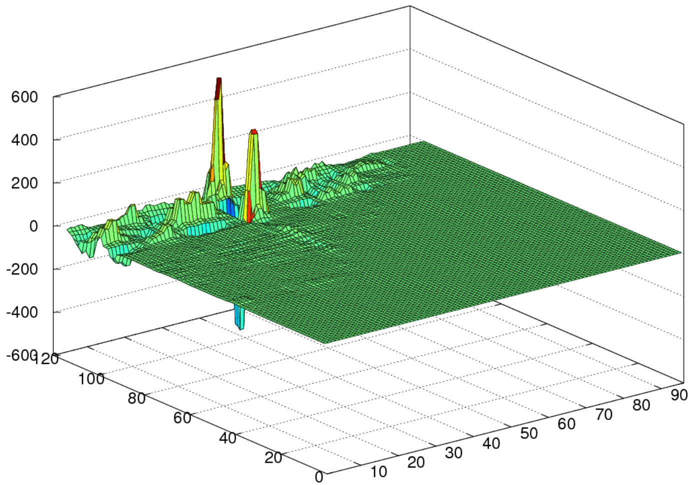


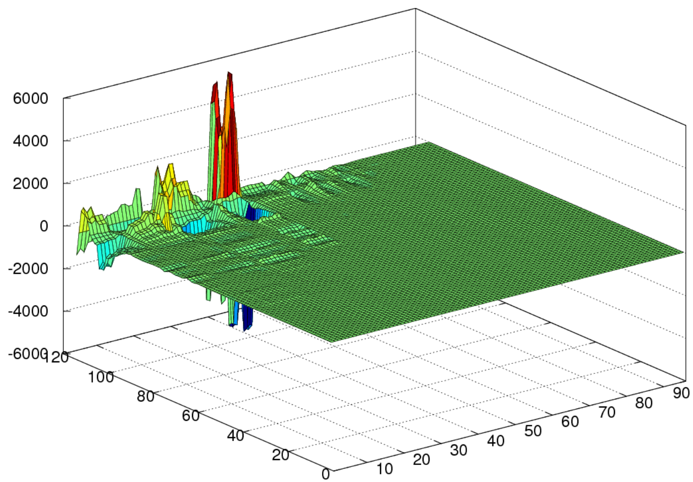
Dispersion Response Matrix

Jacobian of the system:

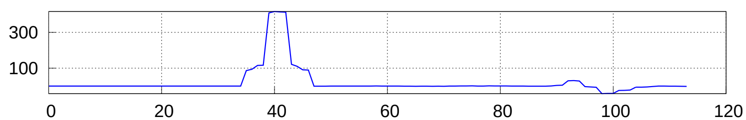
$$\mathbf{D} = \frac{\partial \eta}{\partial \theta} = \frac{\eta_{i;+\Delta\theta_j} - \eta_{i;-\Delta\theta_j}}{2\Delta\theta_j} = \frac{\partial}{\partial \theta}\frac{\partial \mathbf{b}}{\partial \mathbf{E}}$$

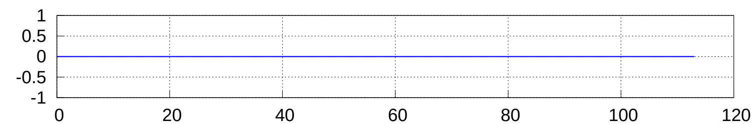
Response matrices: D_{xx}, D_{yy}





Target Responses:



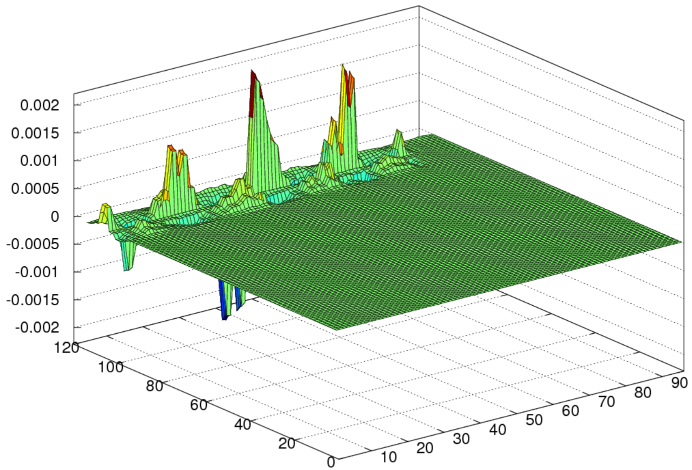


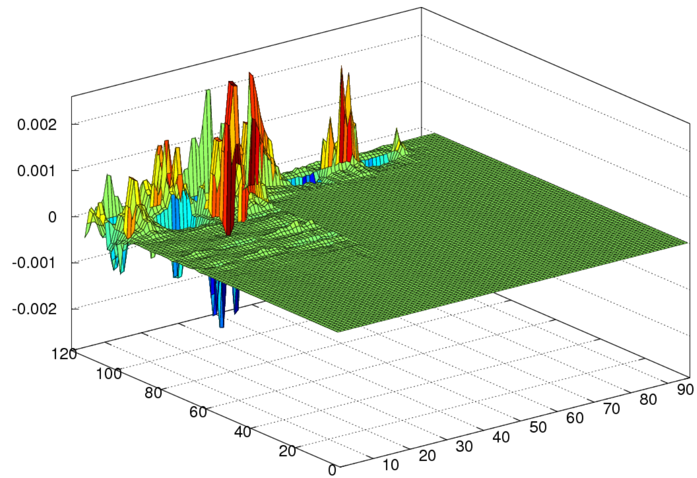
Beta-Beating Response Matrix

Jacobian of the system:

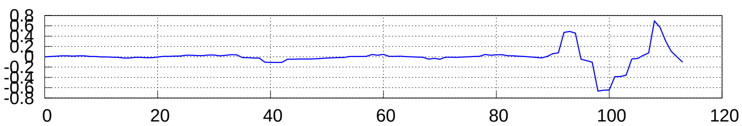
$$\mathbf{B_{x|y}} = \frac{\partial \beta}{\partial \theta} = \frac{\partial}{\partial \theta}\frac{\partial \mathbf{b_{x|y}}}{\partial \theta_{\mathbf{1,x|y}}}$$

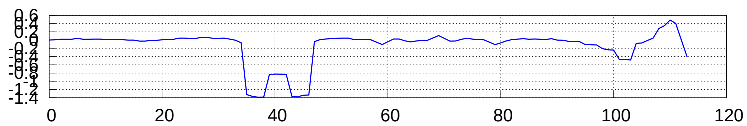
Response matrices: B_{xx}, B_{yx}





Target Responses:



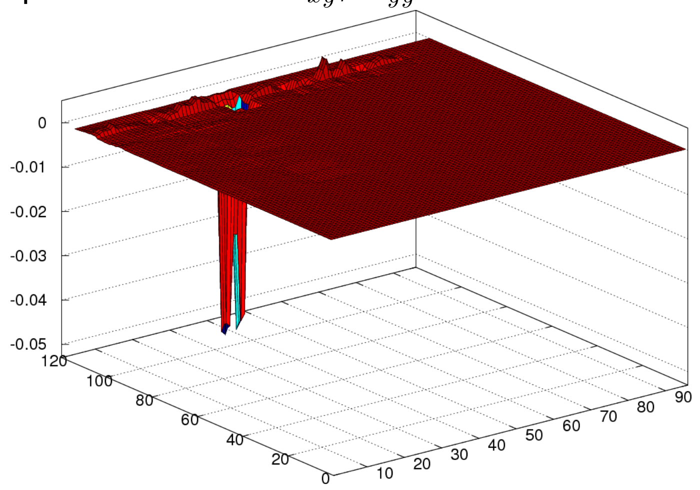


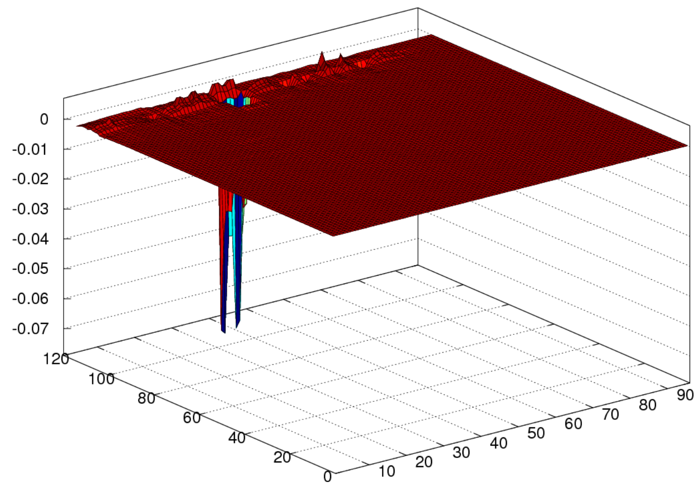
Coupling Response Matrix

Jacobian of the system:

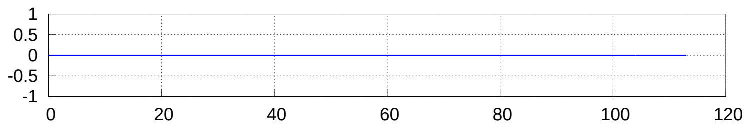
$$\mathbf{C}_{\mathbf{x}|\mathbf{y}} = \frac{\partial \mathbf{c}}{\partial \theta} = \frac{\partial}{\partial \theta} \frac{\partial \mathbf{b}_{\mathbf{x}|\mathbf{y}}}{\partial \theta_{\mathbf{1},\mathbf{y}|\mathbf{x}}}$$

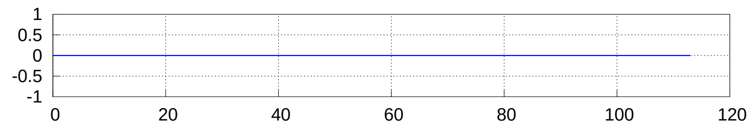
Response matrices: C_{xy}, C_{yy}





Target Responses:





The Actual Systems of Equations

For simplicity I did not mention that we have also:

- the ω-terms, ie. the weights
- the SVD-term β to control and limit the amplitude of the correctors

So the actual systems of equations are the following:

1) Target Dispersion Steering

$$\begin{pmatrix} & & \mathbf{b} \\ \omega_1 & \cdot & (\boldsymbol{\eta} - \boldsymbol{\eta}_0) \\ & & \mathbf{0} \end{pmatrix} = \begin{pmatrix} & & \mathbf{R} \\ \omega_1 & \cdot & \mathbf{D} \\ \beta & \cdot & \mathbf{I} \end{pmatrix} \begin{pmatrix} \boldsymbol{\theta}_x \\ \boldsymbol{\theta}_y \end{pmatrix}$$

2) Coupling and Beta-Beating Steering:

$$\begin{pmatrix} & & \mathbf{b} \\ \omega_2 & \cdot & (\boldsymbol{\eta} - \boldsymbol{\eta}_0) \\ \omega_3 & \cdot & (\boldsymbol{\beta} - \boldsymbol{\beta}_0) \\ \omega_3 & \cdot & \mathbf{C} \\ & & \mathbf{0} \end{pmatrix} = \begin{pmatrix} & & \mathbf{R} \\ \omega_2 & \cdot & \mathbf{D} \\ \omega_3 & \cdot & \mathbf{B} \\ \omega_3 & \cdot & \mathbf{C} \\ \beta & \cdot & \mathbf{I} \end{pmatrix} \begin{pmatrix} \boldsymbol{\theta}_x \\ \boldsymbol{\theta}_y \end{pmatrix}$$

$\Rightarrow$ We have **four degrees of freedom** to tune: $\boxed{\omega_1,\ \omega_2,\ \omega_3 \text{ and } \beta}$.

Beam-Based Centering of the Multipoles

Sextupoles, Octupoles and Decapoles can strongly deflect the beam when they are off-centered.

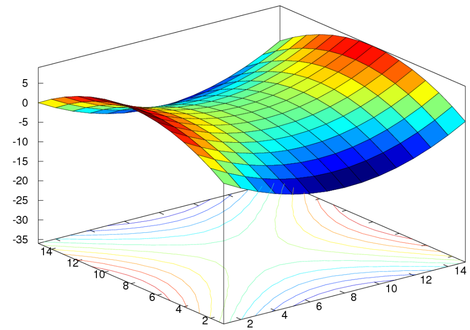
The kick that they induce depends on the difference between the beam position and the magnetic center of the magnet: $\mathrm{d}\mathbf{x}$, $\mathrm{d}\mathbf{y}$.

We scan, horizontally and vertically, the position of each multipole and register the change in beam position at the downstream bpm. We scan in the range $\mathrm{d}\mathbf{x}, \mathrm{d}\mathbf{y} \in [-0.5, 0.5]$ mm.

1) Sextupoles

$$\boldsymbol{\Delta}\mathbf{x}' = -\frac{\mathbf{1}}{\mathbf{2}}\frac{\mathbf{S_N}}{\mathbf{B}\rho}\left(\mathrm{d}\mathbf{x}^{\mathbf{2}} - \mathrm{d}\mathbf{y}^{\mathbf{2}}\right);$$

$$\boldsymbol{\Delta}\mathbf{y}' = +\frac{\mathbf{S_N}}{\mathbf{B}\rho}\,\mathrm{d}\mathbf{x}\,\mathrm{d}\mathbf{y}$$



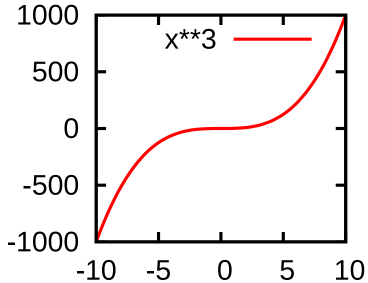
a parabolic fit in x and y gives $\mathrm{d}\mathbf{x}$ and $\mathrm{d}\mathbf{y}$

Beam-Based Centering of the Multipoles

2) Octupoles

$$\mathbf{\Delta x'} = -\frac{\mathbf{1}}{\mathbf{6}}\frac{\mathbf{S_N}}{\mathbf{B}\rho}\left(\mathrm{dx}^{\mathbf{3}} - \mathbf{3}\mathrm{dx}\,\mathrm{dy}^{\mathbf{2}}\right);$$

$$\mathbf{\Delta y'} = -\frac{\mathbf{1}}{\mathbf{6}}\frac{\mathbf{S_N}}{\mathbf{B}\rho}\left(\mathrm{dx}^{\mathbf{2}}\mathrm{dy} - \mathrm{dy}^{\mathbf{3}}\right)$$

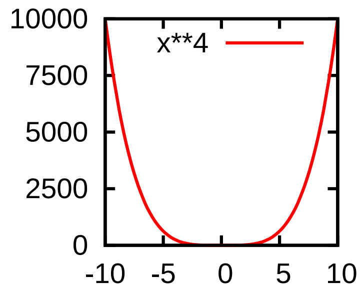


This curve is a cubic, therefore its first derivative is a parabola. A parabolic fit of its derivative, in x and y, gives dx and dy

3) Decapoles

$$\mathbf{\Delta x'} = -\frac{\mathbf{1}}{\mathbf{24}}\frac{\mathbf{S_N}}{\mathbf{B}\rho}\left(\mathrm{dx}^{\mathbf{4}} - \mathbf{6}\mathrm{dx}^{\mathbf{2}}\mathrm{dy}^{\mathbf{2}} + \mathrm{dy}^{\mathbf{4}}\right);$$

$$\mathbf{\Delta y'} = +\frac{\mathbf{1}}{\mathbf{6}}\frac{\mathbf{S_N}}{\mathbf{B}\rho}\left(\mathrm{dx}^{\mathbf{3}}\mathrm{dy} - \mathrm{dxdy}^{\mathbf{3}}\right)$$



This curve is a parabola squared. A parabolic fit of its square root, in x and y, gives dx and dy

Simulation Setup

- CLIC BDS, $L^* = 3.5$ m

- Misalignment 10 μm RMS for:
  - quadrupoles: x and y
  - multipoles: x and y
  - bpms: x and y

- Added two BPMs:
  - one at the IP
  - one 3.5 meters downstream the IP (might this be the same used for the IP-Feedback?)

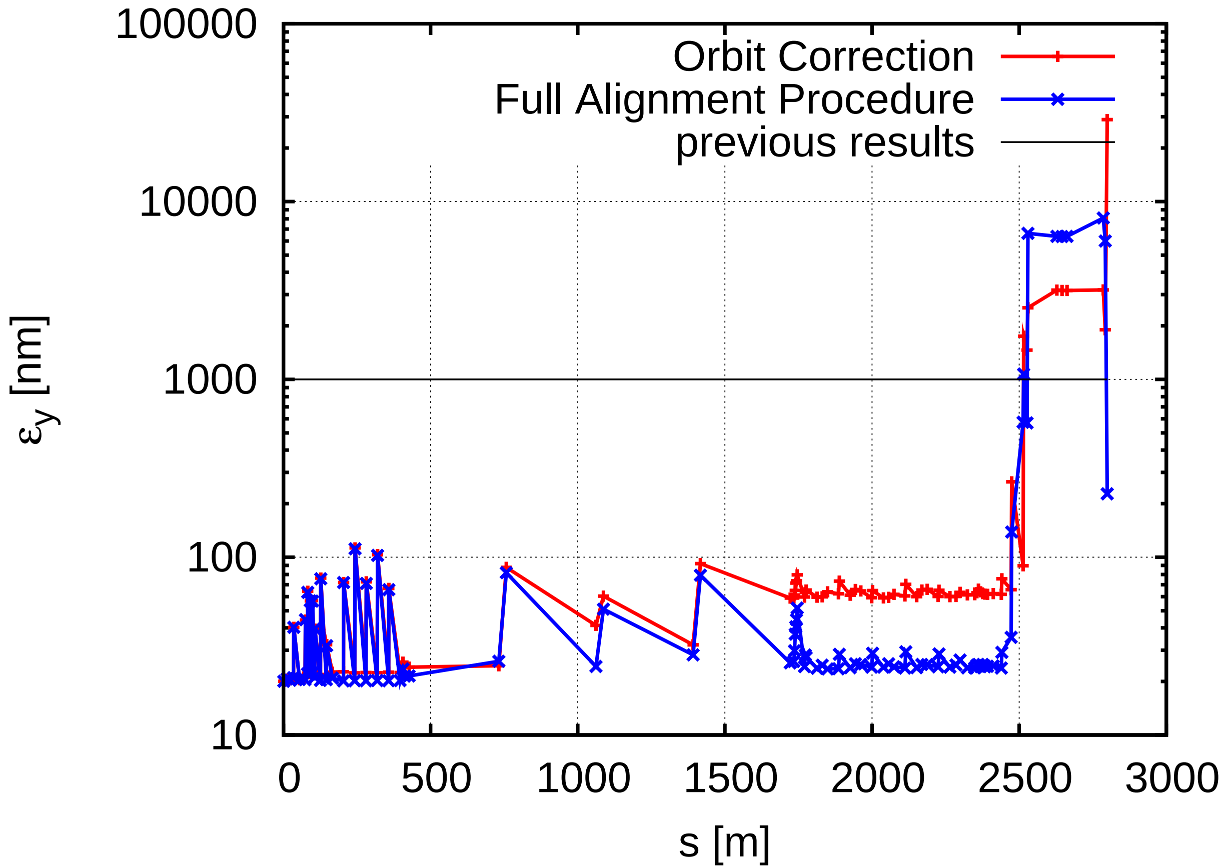
- Studied two different bpm resolutions:
  - 10 nm
  - 100 nm

- Apertures are not taken into account / synchrotron radiation emission is not taken into account

⇒ All simulations have been carried out using placet-octave

Simulation Results

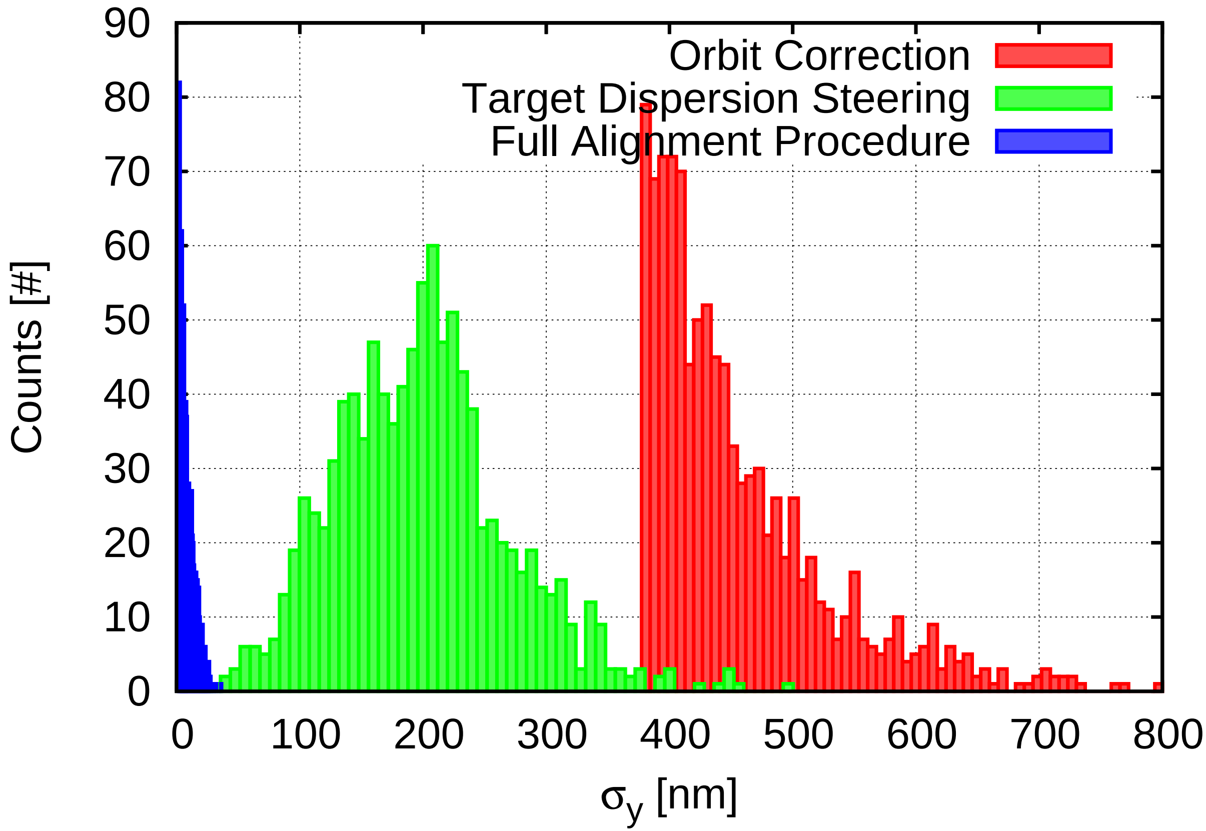
Emittance growth along the CLIC BDS for **100 seeds**, bpm resolution 10 nm:



⇒ Final vertical emittance is 223 nm (previous results were at 1000-10000 nm).

Simulation Results

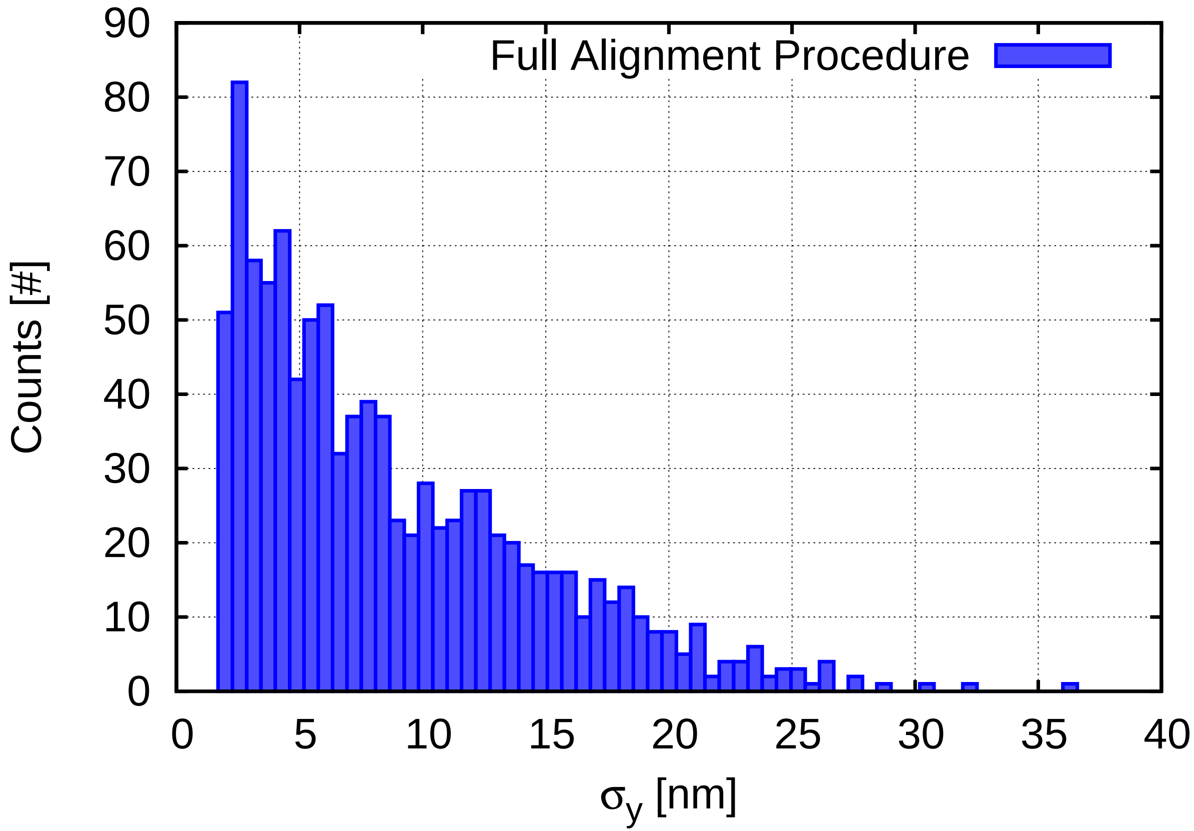
Average **RMS vertical beamsize** at the IP for **1000 seeds**, bpm resolution 10 nm:



⇒ Final average vertical beamsize is 8.9 nm. Success rate is 100%.

Simulation Results

Average **RMS vertical beamsize** at the IP for **1000 seeds**, bpm resolution 10 nm:



⇒ Final average vertical beamsize is 8.9 nm. Success rate is 100%.

Simulation Summary Table

After a scan of the weights ω_1, ω_2 and ω_3, having fixed β:

β	bpm res. [nm]	ω_1	ω_2	ω_3	vertical beam size @ IP [nm]
0	10	0.04	1.64	10.0	9.9
1	10	0.15	1.97	4.88	7.8
0	100	0.002	0.62	24.2	20.3
1	100	0.018	1.80	15.7	16.3

After a scan of ω_1, ω_1, ω_1 and β:

β	bpm res. [nm]	ω_1	ω_2	ω_3	vertical beam size @ IP [nm]
0.85	10	0.14	1.95	1.85	7.6
5.25	100	3.95	0.65	140.0	10.0

$\Rightarrow$ All results are the average of 100 seeds

Conclusions and Next Steps

A novel technique for the BBA of the CLIC BDS has been presented.

It takes into account additional observables, such as beta-beating and coupling measurements, to further improve the beam quality.

An additional step of Beam-Based Centering of the Magnetic Multipoles has been also performed, to reduce the impact of the strong multipolar fields.

⇒ The first results of this technique show an excellent performance of the algorithm, with a 100% success rate, reaching a final vertical beam size of 7.6 nm for 10 nm bpm resolution, and 10 nm for 100 nm bpm resolution.

Next steps,

- Study in further detail this approach and its fine tuning
- Apply Tuning Knobs at the IP
- Apply it and test it on ATF2