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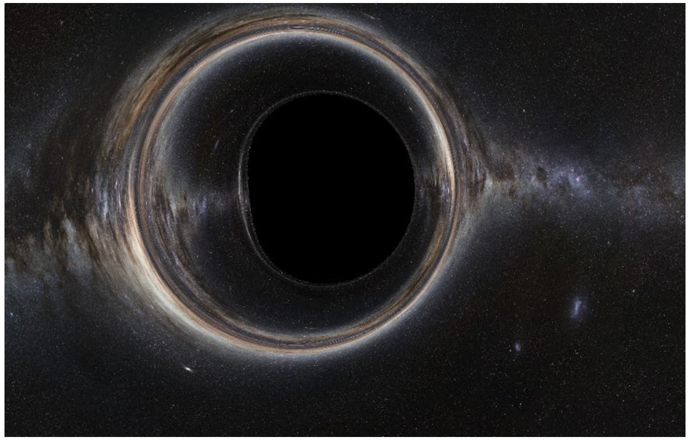
Spinning BPS black holes in AdS and Updates on the search for Multicenter AdS_4 solutions

CERN, Zoom Seminar, March 2, 2021

Based on work in collaboration with K. Hristov, S. Katmadas
and on work to appear with R. Monten

Black holes and quantum gravity

Black holes are often seen as a theoretical laboratory for quantum gravity



Gravity knows about thermodynamics, and it is holographic

$$S_{BH} = \frac{c^3}{G_N \hbar} \frac{A}{4}$$

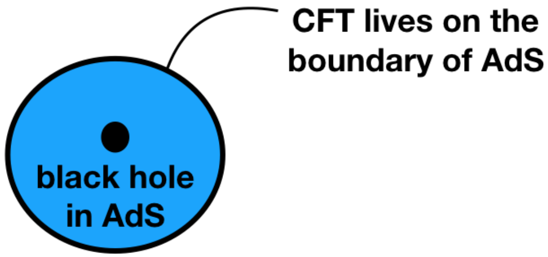
Black holes preserving susy provide a very valuable framework

- one can construct explicit solutions (most often analytical)
- String theory allows to identify the microscopic d.o.f. responsible for their entropy

Supersymmetric (BPS) black holes

Many studies in the context of asymptotically flat black holes have shown a remarkable agreement between macroscopic and microscopic picture [Strominger, Vafa '96]

Quite recently, this was extended to asymptotically AdS black holes: entropy related to the **counting of states** in the dual CFT, living on the boundary.



When bulk and boundary are supersymmetric perform detailed counting of states! Exact quantities (i.e. partition function, indices) computed via *supersymmetric localization* in the dual theory

Entropy matching for static AdS_4 black holes

Static AdS_4 black holes with uplift in M-theory on S^7 known from [Cacciatori, Klemm '09]

- black holes are 1/4 BPS flows from AdS_4 to $AdS_2 \times \Sigma_g$ near horizon geometry
- magnetic gauge field cancels spin connection in the susy equations (topological twist)
- their entropy is function of the charges $S_{BH} = S(Q^I, J)$

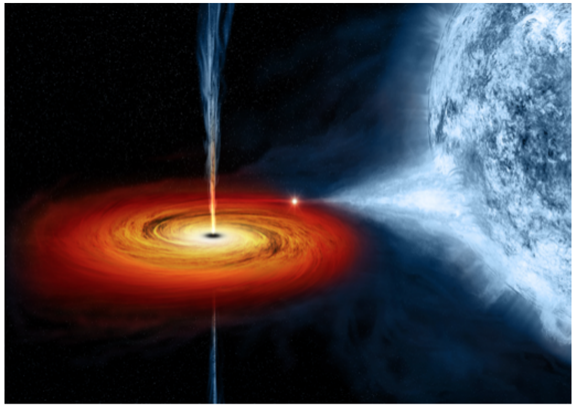
Boundary is $S^1 \times \Sigma_g$: ABJM partition function on $S^1 \times \Sigma_g$ with magnetic fluxes s_i on Σ_g computed via susy localization, in the large N limit [Benini, Hristov, Zaffaroni '15], [Benini, Zaffaroni '16]

$$\log Z_{S^1 \times S^2} \approx -\frac{2\pi N^{3/2}}{3} \sqrt{2 m_1 m_2 m_3 m_4} \sum_{i=1}^{4} \frac{s_i}{m_i} \qquad \sum m_i = 2\pi$$

reproduces Bekenstein-Hawking entropy upon extremization on m_i.

Rotating BPS AdS black holes

AdS_4 extremal rotating black holes can preserve susy! Impossible in 4D Minkowski



BPS bound:

$$M = Q + \frac{J}{l_{AdS}}$$

compatible with extremality bound

We solved first order PDEs from BPS equations analytically. Extremal rotating AdS_4 black holes have Near Horizon geometry in the same class as the Near-Horizon Extremal Kerr (NHEK), present in our universe.

Impressive progress in microstate counting via SCIs, also in higher dimensions (e.g. AdS_5)

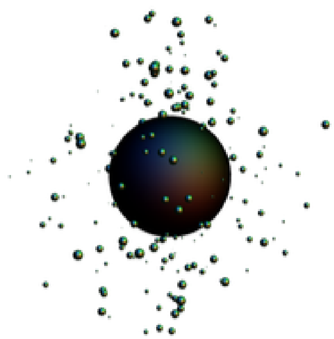
Multicenter black holes

Asymptotically flat multicenter black holes exist and well studied in string theory (e.g. [Denef, '99] + connection to interesting mathematics [Kontsevich, Soibelman '08])

Multicenter in AdS spacetime: long standing challenge. Presence of potential might spoil equilibrium conditions between the centers.

Nevertheless, composite AdS configurations exist: hovering black holes [Iqbal,Horowitz, Santos '14; Horowitz, Santos, CT '18] + dynamical multi black holes in spaces with negative Λ [Chimento,Klemm '13]

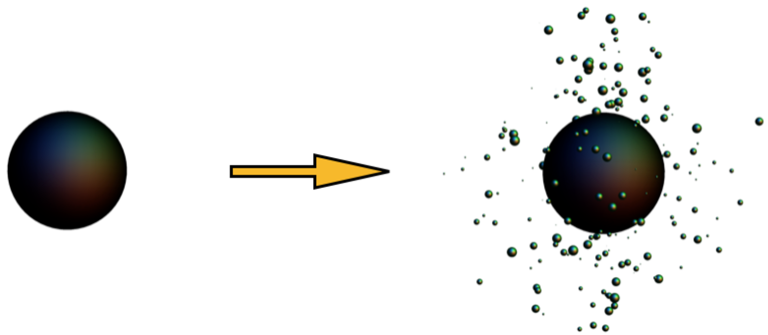
- Work in probe approximation: small probe black hole around a big, massive one [Denef et al, '11–'13]



Multicenter black holes and holographic glass

Black hole bound states model glassy phase of matter in the dual theory [Denef et al.'13]

Fragmented geometries: disordered yet rigid/ exponentially many local free energy minima.
Disorderly frozen matter distribution arising upon cooling



High T: unique, **single** center black hole → liquid phase

Low T: zoo of **multi** center black holes → glass phase

Outline

- Rotating supersymmetric AdS_4 black holes
  - Two classes of BPS solutions
  - Entropy function
  - Status of microstate counting
- Multicenter black holes
  - Background analysis
  - Probe black holes

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- **Rotating supersymmetric AdS_4 black holes**
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$\mathcal{N}=2$ U(1) gauged sugra coupled to vector multiplets

Gravity multiplet coupled to n_V vector multiplets: bosonic fields are the graviton, (n_V+1) vector fields, n_V complex scalars z^i expressed in terms of holomorphic sections $X^I(z)$. Scalar manifold is special Kähler encoded by holomorphic prepotential $F(X^I)$.

Gauging of $U(1)_R$ specified by Fayet-Iliopoulos parameters $G=(g^I, g_I)$
Scalar potential $V(z) \to$ can have supersymmetric AdS black holes

$$S=\int d^4x\,\sqrt{g}\Big[R+g_{ij}\partial_\mu z^i\partial^\mu\bar{z}^j - I_{\Lambda\Sigma}F^{\Lambda}_{\mu\nu}F^{\mu\nu,\Sigma}+R_{\Lambda\Sigma}\epsilon^{\mu\nu\rho\sigma}F^{\Lambda}_{\mu\nu}F^{\Sigma}_{\rho\sigma}+V(z)\Big]$$

BPS equations: susy variations of fermions are zero

$$\delta_\epsilon\psi^I_\mu=0 \qquad \delta_\epsilon\lambda^i_I=0$$

Known rotating supersymmetric black holes

First studies in minimal 4d gauged supergravity. **Two classes** of solutions:

- electric: supersymmetric Kerr–Newman AdS black hole with no static limit [Kostelecky,Perry '92]. Charges satisfy

$$M = \frac{J}{l_{AdS}} + Q$$

- magnetic: minimal spherical case produce naked singularities. Noncompact horizon possible [Caldarelli,Klemm'98]

U(1) FI gauged supergravity + vector multiplets: only isolated examples [Cvetic et al,'05],[Klemm '11]. Lack of systematic! Solutions with both compact horizon and static limit possible.

Adding multiplets help in identifying the *entropy function*, to be matched with the CFT index

Matter-coupled rotating black holes

Start from metric with timelike Killing vector

$$ds^2 = -e^{2U}(dt+\omega)^2 + e^{-2U}ds_3^2 \qquad \mathsf{F} = d(\xi(dt+\omega)) + d\mathcal{A}$$

It is convenient to repackage the equations in terms of the symplectic variables

$$\mathcal{I} = e^{-U}\mathrm{Im}\begin{pmatrix} X^I \\ F_I \end{pmatrix} \qquad \mathsf{F} = \begin{pmatrix} F^I_{\mu\nu} \\ G_{I,\mu\nu} \end{pmatrix}$$

Focus on models with

$$F = \frac{X^1X^2X^3}{X^0}$$

Rewrite BPS equations of [Meessen,Ortin '12] in terms of the symplectic section X^I, $F_I = \partial F/\partial X^I$ and other invariants built from derivatives of a symplectically invariant quartic form I_4

$$I_4(\Gamma) = -(q_0p^0 - p^iq_i)^2 + 4q_0q_1q_2q_3 + 4p^0p^1p^2p^3 + 4(p^1p^2q_1q_2 + p^1p^3q_1q_3 + p^2p^3q_2q_3)$$

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Focus on models with

$$F = \frac{X^1X^2X^3}{X^0}$$

Such that

$$\mathrm{Re}\begin{pmatrix} X^I \\ F_I \end{pmatrix} \sim I_4'\left(\mathrm{Im}\begin{pmatrix} X^I \\ F_I \end{pmatrix}\right) \qquad e^{2U} \sim \sqrt{I_4(\mathcal{R})}$$

BPS equations [Meessen,Ortin '12] for solutions with a timelike Killing vector

$$de^x - \langle \tilde{G}, \mathcal{I} \rangle \wedge e^x + \epsilon^{xyz} \langle \mathcal{A}, \tilde{G}^y \rangle \wedge e^z = 0$$

$$\star d\mathcal{I} + \langle \star\tilde{G}, \mathcal{I} \rangle \mathcal{I} - \frac{1}{4} I_4'(\mathcal{I}, \mathcal{I}, \star\tilde{G}) - \rho\, d\omega\, G + \mathcal{F} = 0$$

$$\star d\omega = \langle d\mathcal{I}, \mathcal{I} \rangle - \frac{1}{2} \langle \tilde{G}, I_4'(\mathcal{I}) \rangle$$

Interested in solutions that implement the topological twist. Take as 3d base

$$ds_3^2 = dr^2 + e^{2\psi(r)} ds_\Sigma^2$$

we get $\psi' = \langle G, \mathcal{I} \rangle$ and

$$\tilde{\omega}^{ab} = \epsilon^{ab} \langle G, \mathcal{A} \rangle \qquad \rightarrow \qquad \langle G, \Gamma \rangle = \int_\Sigma R = \kappa$$

Near horizon Solution and sections

Near horizon geometry:

$$ds^2 = -e^{2u}(r dt + \omega_0)^2 + e^{-2u}\frac{dr^2}{r^2} + e^{-2u}\left(\frac{d\theta^2}{\Delta(\theta)} + \nu^2\Delta(\theta)f_\kappa^2(\theta)d\phi^2\right)$$

with Δ, ω_0, u depending only on θ; $f_k(\theta)$ specifies the topology of the horizon

BPS equation for the sections split into two parts which determine

1. angular dependence of the scalars

$$d(e^\psi \mathcal{I}) = -j \sin\theta G \qquad \rightarrow \qquad e^\psi \mathcal{I} = H_0 + j\cos\theta G$$

2. attractor relating H_0 to charges Γ

$$\Gamma = \frac{1}{4}I_4'(H_0, H_0, G) + \frac{1}{2}j^2 I_4'(G)$$

Full Solution and sections

Full geometry:

$$ds^2 = -e^{2U}(dt+\omega)^2 + e^{-2U}dr^2 + e^{-2U+2\psi}\left(\frac{d\theta^2}{\Delta(\theta)} + \Delta(\theta) f_\kappa^2(\theta) d\phi^2\right)$$

with ω, U, ψ depending on r, θ; $f_k(\theta)$ specifies the topology of the horizon

BPS equation for the sections solved with minimal modification

$$e^{\psi}\mathcal{I} = H_0 + H_\infty r + j\cos\theta G$$

Rotation 1-form

$$\omega = \left(\omega_\infty(\theta) - j e^{-\psi}\Delta(\theta)\right) f_\kappa^2(\theta) d\phi$$

Entropy

Conserved charges computed via Komar intergrals allow to express entropy $S_{\rm BH}$ in function of charges, i.e. for T^3 model [Hristov, Katmadas, CT, '18]

$$S_{\rm BH} = \pi \frac{l_{\rm AdS}^2}{\sqrt{2}} \sqrt{\sqrt{(1+12p^1)(1+4p^1)^3 - 4J^2 l_{\rm AdS}^{-4}} - (24(p^1)^2 + 12 g_1 p^1 + 1)}$$

J bounded from above. Reduces to static for $J \to 0$

Near horizon geometry

$$ds_4^2 = e^{-2u}\left(-r^2 d\tau^2 + \frac{dr^2}{r^2} + \frac{v^2}{\Delta(\theta)} d\theta^2\right) + e^{2u}\frac{\mathcal{W}}{v^2}\Delta(\theta)\sin^2\theta\left(d\phi + jv\sqrt{\mathcal{W}} r d\tau\right)^2$$

$$\mathcal{W} = I_4(H_0) - \left(1 + I_4(G) j^2\right) j^2$$

Fits into the general NHEK metric of [Compere, '12] with $SL(2,R) \times U(1)$ symmetry.

Holography

Found rotating attractors which extend to 1/4 BPS rotating black holes with boundary

$$ds^2 = r^2\Delta(\theta)\left[-\frac{dt^2}{l_{AdS}^2} + \frac{d\theta^2}{\Delta(\theta)^2} + \frac{\sin\theta^2}{\Delta(\theta)}\left(d\phi + \frac{j}{l_{AdS}^3}dt\right)^2\right]$$

where $l_{AdS}^2 = (I_4(G))^{-1/2}$. Squashing of Σ in parameterized by $\Delta(\theta)$. *

On gravity, provided an entropy function which upon extremization gives S_{BH}. Refined in [Hosseini, Hristov, Zaffaroni '19]

Entropy to be reproduced by Large N ABJM twisted index with angular momentum refinement [Benini, Zaffaroni '16]. Difficult to compute, though some progress in [Closset, Kim, Willett '18]

* Our formalism allows also other kinds of asymptotics

Electric matter-coupled Kerr-Newman black hole

Start from general base

$$ds_3^2 = d\rho + e^{2\phi}(dx^2 + dy^2)$$

Before: $e^{2\phi} = \Phi(x)e^{2\psi(\rho)}$. Radial coordinate $\rho = r$.

Electric matter coupled Kerr-Newman solutions: choose

$$e^{2\phi} = Q(q)P(p) \qquad \rho = qp \qquad x = \alpha(q) + \beta(p)$$

such that

$$ds_3^2 = (q^2P(p) + p^2Q(q))\left(\frac{dp^2}{P(p)} + \frac{dq^2}{Q(q)}\right) + Q(q)P(p)dy^2$$

q radial variable, p, y are coordinates on the sphere. No twist, $g_I P^I = 0$

Electric matter-coupled Kerr-Newman black hole

Entropy obtained by extremizing an *entropy function* with respect to variables m^I conjugate to the electric charges, ω conjugate to J

$$\mathcal{S}(\omega, m^I) = -2\frac{F(m_I)}{\omega} + \sum_I m^I q_I + \omega J \qquad \sum_I 2 g_I m^I - \omega - 2\pi i = 0$$

$F(m^I)$ is the prepotential of the model.

This is the form conjectured by [Choi et al. '18] and tested by them on the only known solutions [Cvetic et al, '05].

Confirmed for full family of new solutions [Hristov, Katmadas, CT '19].

Comparison with the superconformal index

Entropy to be reproduced by the superconformal index.

- Superconformal index found to be $\sim O(1)$ [S. Kim, '09] when fugacities are real
- Should be instead $\sim N^{3/2}$ to match S_{BH}. There is $N^{3/2}$ scaling for complex fugacities [Choi, Hwang, Kim, '19] in Cardy limit. Large black hole entropy matches: **Exciting!**
- Work of [Cabo-Bizet et al '18] [Cassani, Papini '19] showed that the black hole entropy is the Legendre transform of the on-shell gravitational action with respect to chemical potentials entropy in a particular extremal limit.

Further developments regarding minimal ("universal") solutions

- exploiting 3d-3d correspondence [Benini,Gang,Pando Zayas '19]
- i.e. relating the superconformal index to S^3 partition function [Bobev, Crichigno '19]

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Disclaimer

Possible evidence for "new physics" (or new saddles) from 4d superconformal index [Benini, Milan '18], [Cabo-Bizet, Murthy '19]

- [Ardehali, Hong, Liu '20] found micro-canonical entropies $S_C(J_{1,2}, Q_a) = S_{BH}(J_{1,2}, Q_a)/C$ for $C = 2, 3, 4, 5$ presumably corresponding to entropies of new black objects in the bulk.

Very interesting! but I will be agnostic about this.

Multicenter BHs in probe approximation

Exact solutions of Multi-center black holes found [Denef '99] by completing the squares in the sugra Lagrangian and solving the resulting first order equations. Presence of scalar potential obstructs this procedure.

Probe analysis: Stable and metastable probes exist in the background of a $T > 0$ 4d dyonic black hole with scalar profile (neutral scalars) [Anninos, Anous, Denef, Peeters '13]. **True both in Minkowski and AdS.**

In the AdS_4 compactification dual to ABJM theory, one linear combination of the gauge fields is Higgsed, thus massive [Aharony, Bergman, Jafferis, Maldacena '08].

Aim: study probe stability in a more general black hole background – charged scalars and massive vector field.

The Model: M theory on Q^{111}

M–theory truncation on homogeneous SE_7 manifold Q^{111}: 4d gauged supergravity with hyper–multiplets with susy AdS_4 vacuum [Cassani, Koerber, Varela '12]

$$S = \int d^4x\sqrt{-g}\left(\frac{R}{2} - g_{i\bar{j}}\partial_\mu z^i \partial^\mu z^{\bar{j}} - h_{uv} D_\mu q^u D^\mu q^v + I_{\Lambda\Sigma} F^{\Lambda}_{\mu\nu} F^{\mu\nu,\Sigma} + \epsilon_{\mu\nu\rho\sigma} R_{\Lambda\Sigma} F^{\mu\nu,\Lambda} F^{\rho\sigma,\Sigma} - V\right)$$

Content: gravity, 3 VM and universal hypermultiplet, prepotential $F = \sqrt{X^0X^1X^2X^3}$

$$k^a_\Lambda = -\{e_0, 2, 2, 2\} \qquad P^3_\Lambda = \{4 - \frac{1}{2}e^{2\phi}e_0, -e^{2\phi}, -e^{2\phi}, -e^{2\phi}\}$$

$$Dq^u = dq^u + k^u_I A^I$$

Dual is a $\mathcal{N} = 2$ Chern–Simons matter theory [Benini, Closset, Cremonesi '09], [Jafferis '09]

The Model: M theory on Q^{111}

One of the vectors becomes massive via Higgs mechanism.

We are interested in black hole (branes) solutions with temperature: background for probe analysis. BPS solutions found in [Gauntlett,Donos, '12] [Halmagyi, Petrini, Zaffaroni '14] Black holes are M2 and M5 branes wrapping noncontractible cycles of the internal manifold

Ansatz: All hypermultiplets scalars except one set to zero. Scalar modes have masses $m^2 = (16, 10, 4, -2, -2, -2, -2)$ corresponding to $\Delta = (6, 5, 4, (2, 1) \times 4)$

Fermions are electrically charged: Dirac-like quantization condition on the black hole magnetic charges

$$P^\Lambda P^3_\Lambda(\overline{u}) \in \mathbb{Z} \qquad P^\Lambda k^u_\Lambda(\overline{u}) \in \mathbb{Z}$$

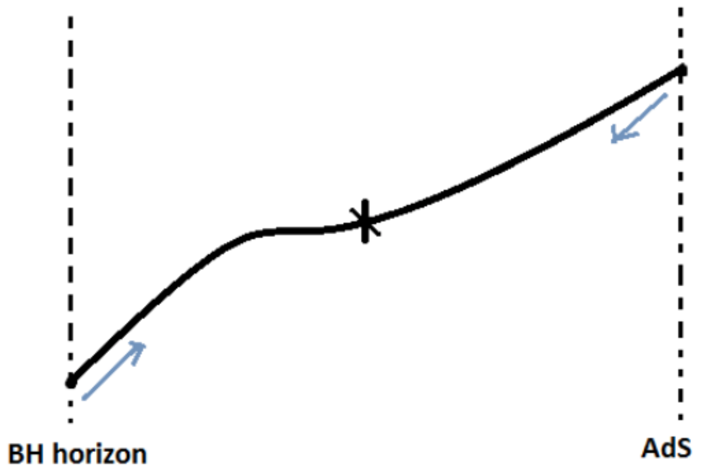
Black hole solutions

Static and spherically symmetric ansatz:

$$ds^2 = -e^{-\beta(r)}h(r)dt^2 + \frac{dr^2}{h(r)} + r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

$$\phi_I = \phi_I(r) \qquad A^\Lambda = \tilde{q}^\Lambda dt - P^\Lambda \cos\theta d\phi$$

Maxwell's equations yield $P^\Lambda k_\Lambda^a = 0$. Massive vector is purely electric.

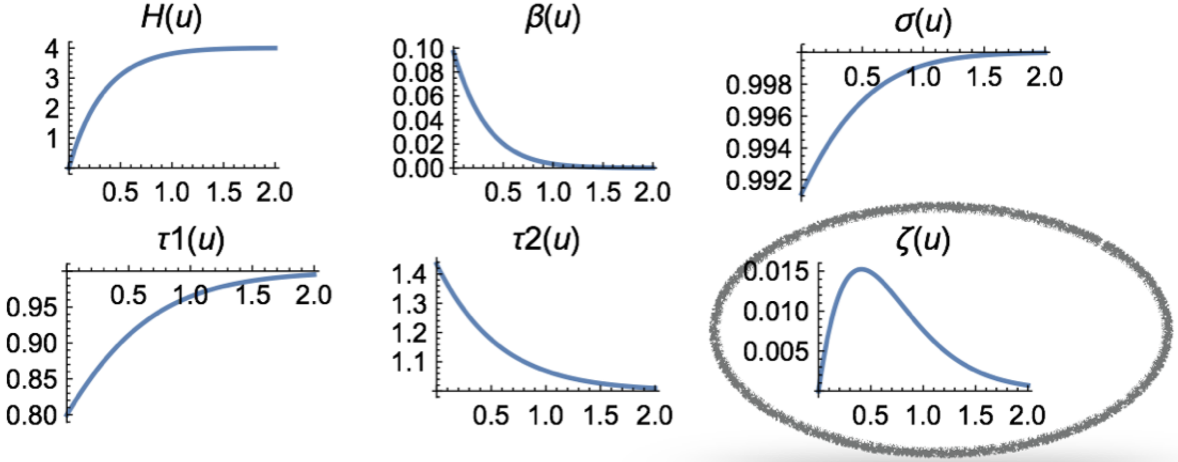


Expansion in series at the black hole horizon and at infinity. Demand to match in between.

In total there are 14 equations. At the end of the day, 7 free parameters = BH charges

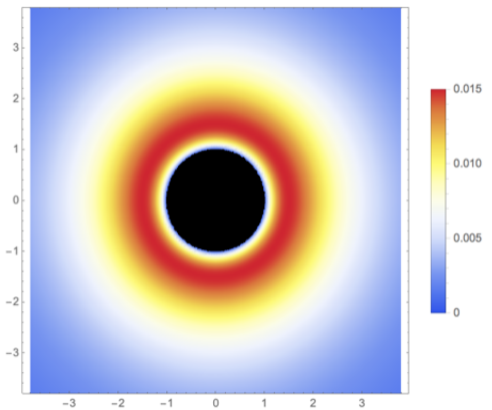
Black hole solutions [Monten, CT, '17]

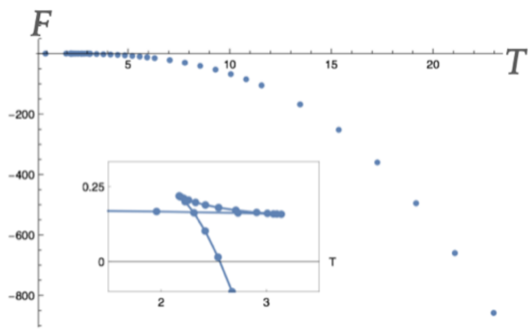
Example of electric BH with massive vector and nontrivial scalar profile:



massive vector field

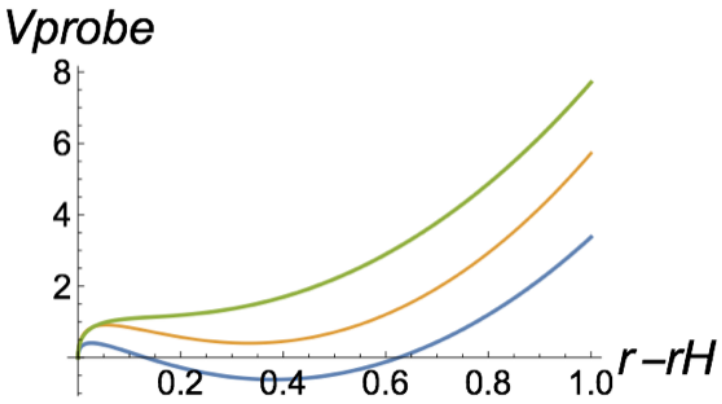
Black hole is surrounded by a massive vector "atmosphere", which hovers outside the black hole horizon





Probes

Expectation: at high temperatures the single-center horizon will be thermodynamically favored (liquid phase), probe will enter the black hole



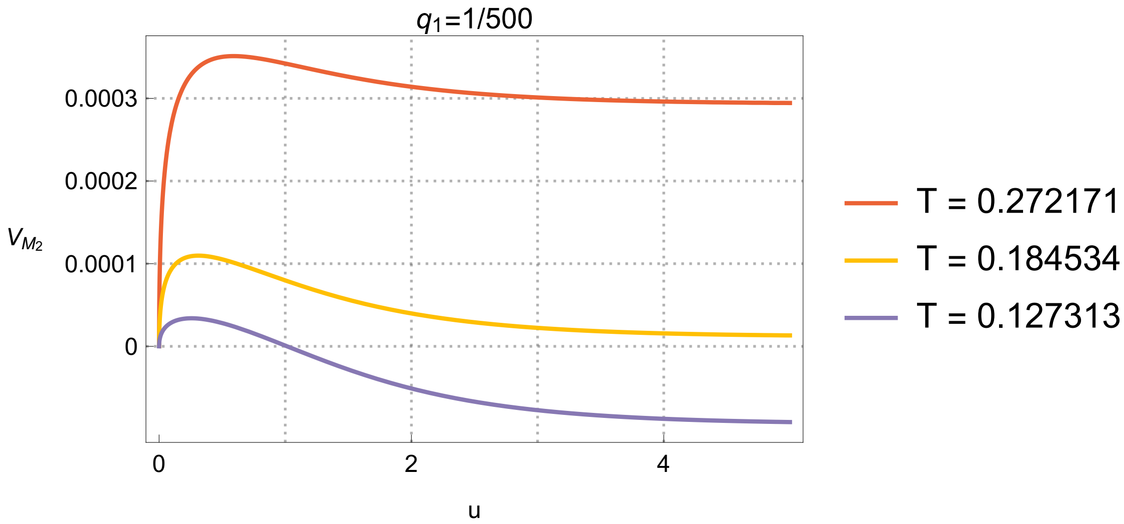
Vprobe is zero at the black hole horizon.

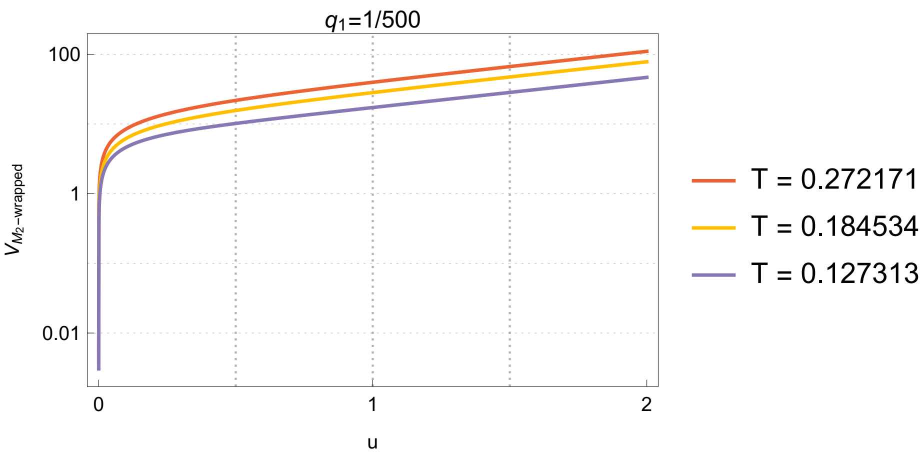
Unstable, stable and metastable probes

- Expectation: at high temperatures the single-center horizon will be thermodynamically favored (liquid phase), probe will enter the black hole
- Compare with the previous case of uncharged scalar. Effect of the interaction probes - condensate

Probes: preliminary results [Monten, CT, to appear]

Electric black branes manifest instability towards nucleation of **spacetime filling** M2 branes





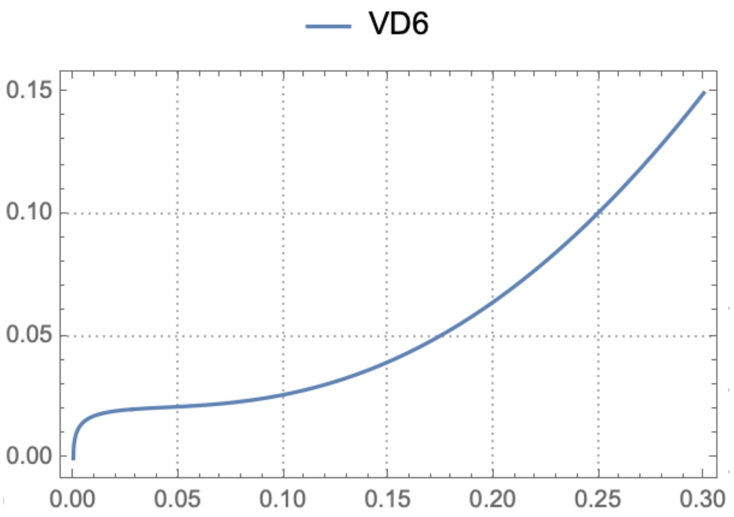
Purely electric spherical black holes: **Wrapped M2 branes** are not stable (similarly to [Klebanov, Pufu, Tesileanu '12]). Need to have mutually nonlocal charges. Need to consider other probes

Probes: preliminary results

DBI action for Fluxed D6 brane (combination of M2, M5 KK monopole)

$$V_{D6} = -\int m_\gamma(z) - \int (q_I A^I - p^I B_I) \qquad m_\gamma = |Z(\gamma, z, \bar{z})|$$

as in [Asplund, Denef, Dzienkowski '15]



Need to go to lower temperatures and appropriately tune charges of the background.

Intuition from the FI case on the parameter space where we can find stable probes [Anninos, Anous, Denef, Peeters '13]

Notice that we set to zero the magnetic component of the Higgsed U(1)

Multicenter black holes in the probe approximation

- Expectation: equilibrium distance is close to the horizon. Should not ”feel” the presence of cosmological constant
- small black hole / asymptotically flat space limit should reproduce the well known BPS equilibrium separation formula. ”Caged wall crossing” due to AdS asymptotics.

Notice that

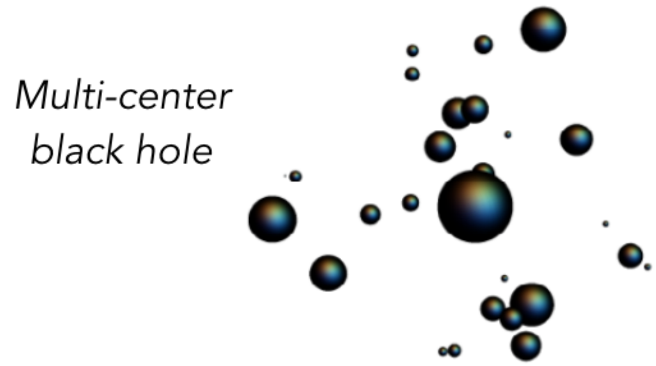
- not SUSY
- subtleties regarding boundary conditions and baryon operators in ABJM-like theories [Bergman, Tachikawa, Zafrir ’20]

Conclusions and perspectives/1

Chart the parameter space of probe charges looking for stable and metastable probes

- Interpretation of the instability for spacetime filling branes nucleation (similar to [Henriksson, Hoyos, Jokela '19] for branes in T^{11}).

- Multiple black holes induce electric and magnetic dipole charges, corresponding to inhomogeneities in charge densities and magnetic fields in the dual field theory
  $\rightarrow$ High viscosity? to be verified



Conclusions and perspectives/2

More specific goals...

- Understand the thermodynamic origin of the entropy function for rotating BHs i.e. from family of finite nonzero T solutions
- Considering more general Sasaki-Einstein truncations, i.e. with hypermultiplets: constraints due to Higgsing

... Towards general lessons for rotating black holes

- NH geometry in the same class as the Near-Horizon Extremal Kerr. Concrete realization of the Kerr/CFT correspondence [Hartman, Guica, Song, Strominger, '08]
- Beyond extremality: JT gravity description of near-extremal AdS black holes [Castro, Pedraza, CT, Verheijden, to appear]

the end. Thank you!

Quartic invariant form:

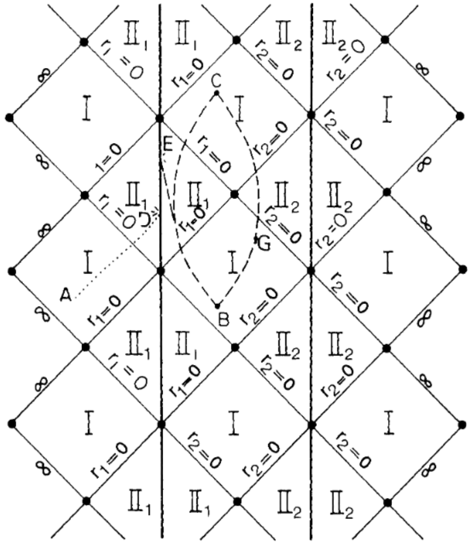
$$I_4(\Gamma) = \frac{1}{4!} t^{MNPQ} \Gamma_M \Gamma_N \Gamma_P \Gamma_Q$$

satisfies

$$I'_4(\Gamma)_M = \Omega_{MN} \frac{\partial I_4(\Gamma)}{\Gamma_N} = \frac{1}{3!} \Omega_{MN} t^{NPQR} \Gamma_P \Gamma_Q \Gamma_R$$

Penrose diagram for Majumdar-Papapetrou solutions

[Hartle-Hawking, Comm.Math.Phys 1972]



BPS squaring for stationary configurations, gauged sugra:

$$S_{4D} = -\frac{1}{16\pi}\int dt \int_{\mathbb{R}^3} \Big[(\mathcal{E},\mathcal{E}) - 4\,(Q + d\alpha + 2e^{-U}\,\mathrm{Re}\,Z(\star\mathcal{G}) + \frac{1}{2}e^{2U} \star d\omega) \wedge \mathrm{Im}\langle \mathcal{E}, e^{U}e^{-i\alpha}\mathcal{V}\rangle$$

$$- 2\left[\langle \mathcal{F} + 2\,\mathrm{Re}\,d(e^{U}e^{-i\alpha}\mathcal{V}\,\omega), \star\mathcal{G}\rangle - \star 1\right]$$

$$- 2\,\left(\star d\psi - 2\,e^{-U}\,\mathrm{Im}(e^{-i\alpha}Z(\mathcal{G}))\right) \wedge \left(d\psi - 2\,e^{-U}\,\mathrm{Im}(e^{-i\alpha}Z(\star\mathcal{G}))\right)$$

$$+ 4e^{-U}e^{2\psi}\,\mathrm{Im}(e^{-i\alpha}Z(G))\,d\left(e^{-2\psi}\star\eta\right)\Big].$$