Hierarchical axion couplings from axion landscape

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Outline

- *Introduction*
- *Well-motivated axion coupling hierarchies*
- *Hierarchies from axion landscape*
- *Conclusion*

Introduction

Axions or axion-like particles (ALP) are one of the most compelling candidates for BSM physics:

- Axions may solve the naturalness problems

  * QCD axion for strong CP problem   Peccei, Quinn '77; Weinberg '78; Wilczek '78
  * Relaxion for gauge hierarchy problem   Graham, Kaplan, Rajendran '15
  * Inflaton for natural inflation   Freese, Frieman, Olinto '90

- Axions may constitute (part of) the dark side of the Universe

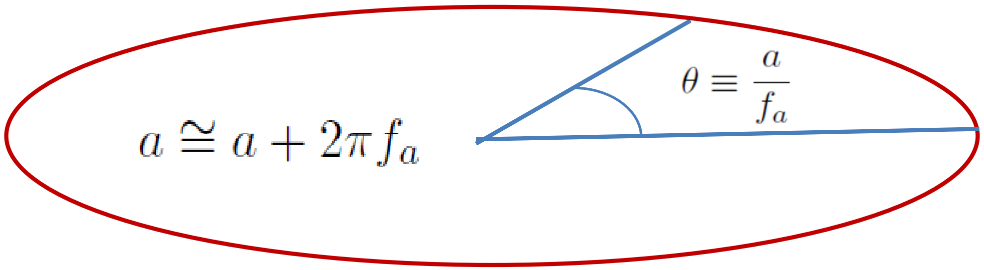
  Dark matter, Dark radiation, Dark energy

- Axions may explain certain astrophysical anomalies

  White dwarf cooling anomaly, γ-ray transparency, γ-ray modulations, ..

- Axions arise naturally in string theory

Axions are periodic scalar field associated with an approximate shift symmetry: $U(1)_{PQ}: \ a \ \rightarrow \ a + constant$



(f_a = axion decay constant)

➔ $\text{axion couplings} \propto \dfrac{1}{f_a}$

Possible origins of axions:

$$\sigma = \rho e^{ia/f_a} \quad \left(f_a = \langle\rho\rangle\right)$$

$$a = \int_{\mathcal{C}_p} A^{(p)}$$

$A^{(p)} = p$-form gauge field in $D(\geq 4+p)$-dim theory

$\mathcal{C}_p = p$-cycle in compact $(D-4)$-dim internal space

$(p-1)$-brane & its magnetic dual $(D-p-3)$-brane which couple to $A^{(p)}$

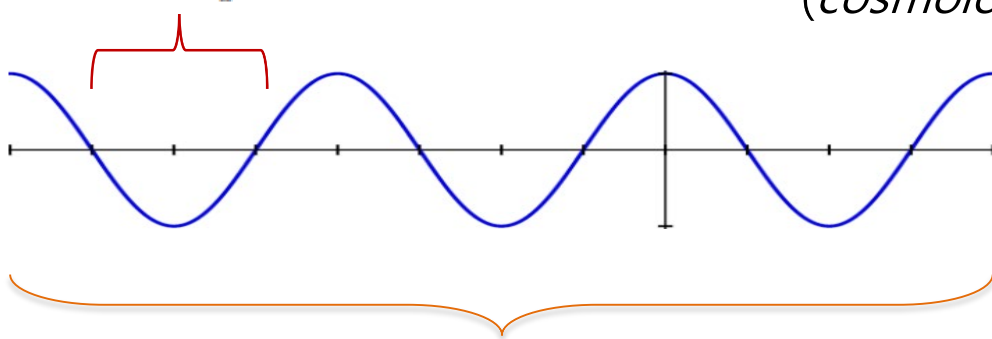
Axion couplings

- Coupling to generate the leading potential:

$$g_{aV} = \frac{N_{\rm DW}}{f_a} \quad \Rightarrow \quad V(a) \simeq -\Lambda^4 \cos(g_{aV}\, a) \equiv -\Lambda^4 \cos\left(N_{\rm DW} \frac{a}{f_a}\right)$$

domain wall number

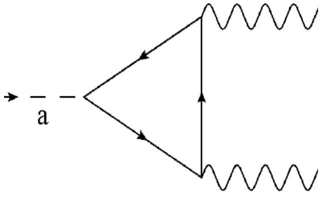
$$\Delta a = \frac{1}{g_{aV}} = \frac{f_a}{N_{\rm DW}}$$ ~ field range for single modulation (*cosmological evolution*)

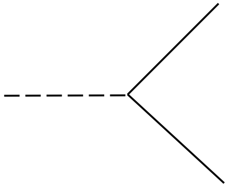


$2\pi f_a$ = full field range

- Couplings to the SM particles

$$\mathcal{L}_{\rm in} = \frac{1}{4}\sum_{A=\gamma,G} g_{aA} F^A \tilde{F}^A + \frac{1}{2}\sum_{\Psi=N,e} g_{a\Psi}\partial_\mu a \bar{\Psi}\gamma^\mu\gamma_5\Psi$$





$$g_{aA} = \frac{g_A^2}{8\pi^2}\frac{c_A}{f_a} \quad (A=\gamma, \text{gluons}) \qquad g_{a\Psi} = \frac{c_\Psi}{f_a} \quad (\Psi = N, e)$$

- Coupling suggested by *the weak gravity conjecture* (WGC):

For axions in theories compatible with quantum gravity, there exist certain instantons whose couplings to axions are stronger than gravity.

Arkani-Hamed et al '07

$$g_{a\mathrm{WGC}} = \frac{N_{\mathrm{WGC}}}{f_a}$$

$$\Rightarrow \quad \mathcal{A}_{\mathrm{ins}} \propto \exp\Big(- S_{\mathrm{ins}} + i N_{\mathrm{WGC}} \frac{a}{f_a} \Big)$$

$$\left(g_{a\mathrm{WGC}} = \frac{N_{\mathrm{WGC}}}{f_a} \gtrsim \frac{S_{\mathrm{ins}}}{M_{\mathrm{Pl}}} \right)$$

- *Relaxion* coupling to generate the barrier potential:

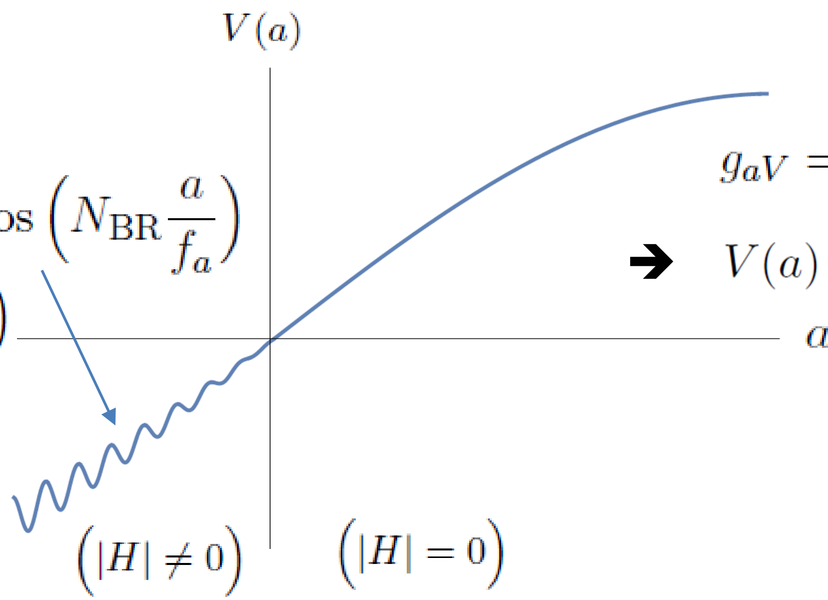
  (= axion-like particle proposed to solve the weak scale hierarchy problem)

Graham, Kaplan, Rajendran '15

$$g_{a\mathrm{BR}} = \frac{N_{\mathrm{BR}}}{f_a}$$

➔ $$V_{\mathrm{BR}} = \Lambda_{\mathrm{BR}}^4(|H|)\cos\left(N_{\mathrm{BR}}\frac{a}{f_a}\right)$$

$$\left(\Lambda_{\mathrm{BR}}^4 \propto |H|^n\right)$$



$$g_{aV} = \frac{N_{\mathrm{DW}}}{f_a}$$

➔ $$V(a) \simeq \Lambda^4 \cos\left(N_{\mathrm{DW}}\frac{a}{f_a}\right)$$

There can be ***technically natural hierarchies*** among the different couplings of a given propagating axion:

* Derivative couplings invariant under $U(1)_{PQ} : a \rightarrow a + constant$

$$g_{a\Psi} = \frac{c_\Psi}{f_a} \quad (\Psi = N, e)$$

* PQ-breaking non-derivative couplings:

$$g_{aV} = \frac{N_{\rm DW}}{f_a}, \quad g_{a{\rm WGC}} = \frac{N_{\rm WGC}}{f_a}, \quad g_{a{\rm BR}} = \frac{N_{\rm BR}}{f_a}, \quad g_{a\gamma} = \frac{\alpha_{\rm em}}{2\pi}\frac{c_\gamma}{f_a}, \quad g_{aG} = \frac{\alpha_s}{2\pi}\frac{c_G}{f_a}$$

Quantized *(integer-valued)* to be invariant under $a \rightarrow a + 2\pi f_a$.

- No quantum correction to the ratios between the quantized PQ-breaking couplings, so any hierarchy among the PQ-breaking couplings is stable against quantum corrections.

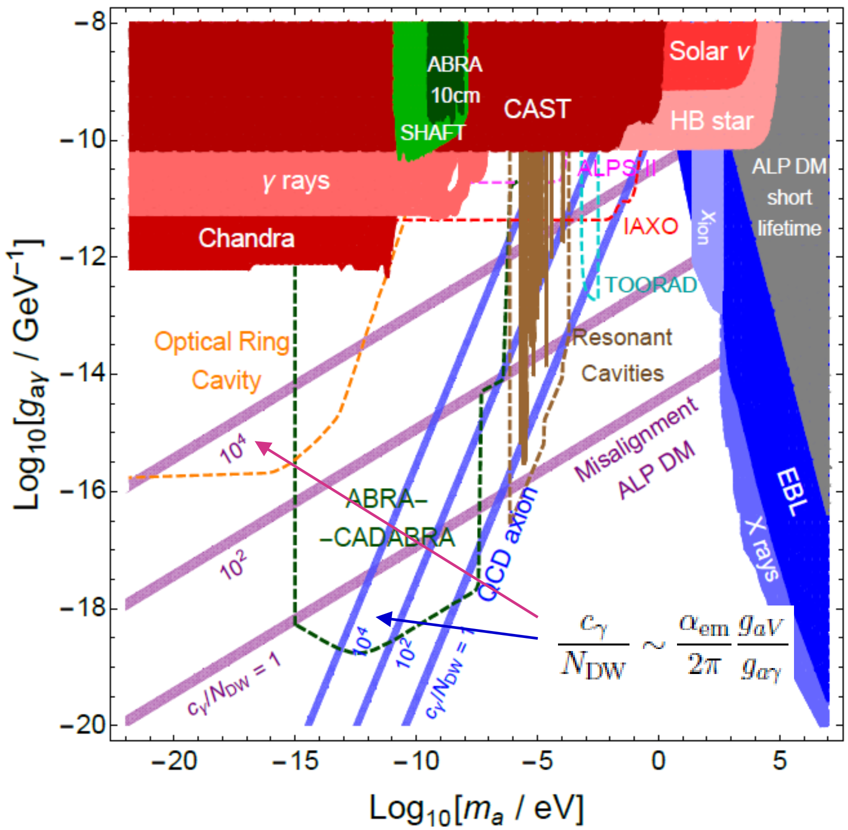
- PQ-invariant couplings >> PQ-breaking couplings are stable also

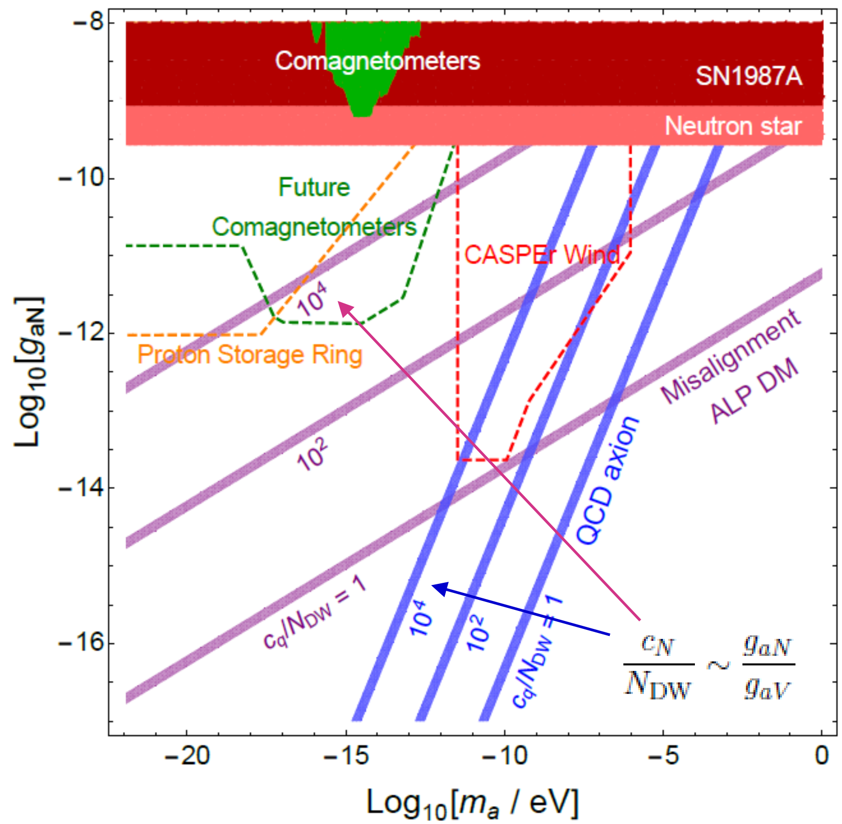
Well-motivated axion coupling hierarchies

- Hierarchy for axion search in laboratory experiments

Farina et al '17; Agrawal et al '18
Marques-Tavares & Teo '18;
Darme et al '20;
Dror & Leedom '20; ...

QCD axion or ALP DM with $g_{a\gamma} \gg \frac{\alpha_{em}}{2\pi} g_{aV}$ $(c_\gamma \gg N_{DW})$ or $g_{aN} \gg g_{aV}$ $(c_N \gg N_{DW})$





More easily accessible region has bigger coupling hierarchy!

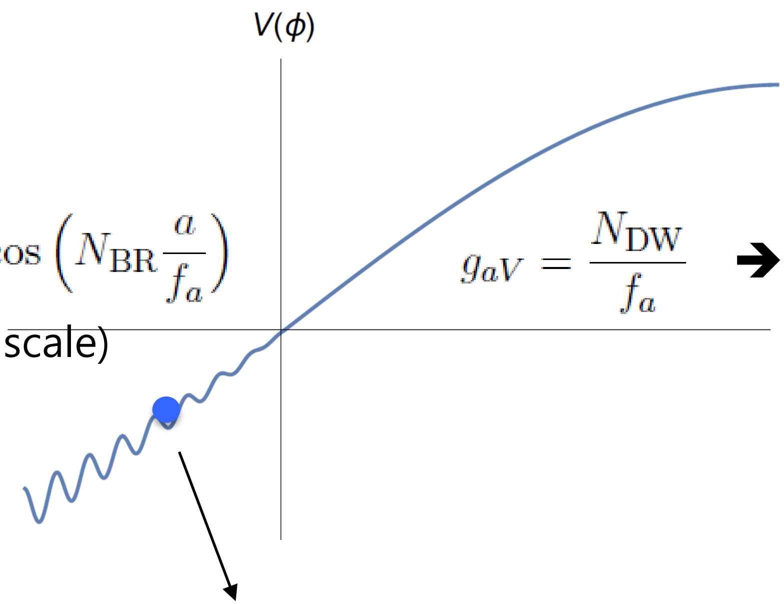
- Hierarchy for relaxion couplings

Graham, Kaplan, Rajendran '15

$$g_{a\mathrm{BR}} = \frac{N_{\mathrm{BR}}}{f_a}$$

➔ $$V_{\mathrm{BR}} = \Lambda^4_{\mathrm{BR}}(|H|)\cos\left(N_{\mathrm{BR}}\frac{a}{f_a}\right)$$

(Λ_{BR} < Weak scale)



$$g_{aV} = \frac{N_{\mathrm{DW}}}{f_a}$$

➔ $$V(a) \simeq \Lambda^4\cos\left(N_{\mathrm{DW}}\frac{a}{f_a}\right)$$

(Λ = Higgs mass cutoff scale >> Weak scale)

$$\text{Slope of } V \sim \frac{N_{\mathrm{DW}}}{f_a}\Lambda^4 \sim \text{Slope of } V_{\mathrm{BR}} \sim \frac{N_{\mathrm{BR}}}{f_a}\Lambda^4_{\mathrm{BR}}$$

$$\Rightarrow \quad \frac{g_{a\mathrm{BR}}}{g_{aV}} = \frac{N_{\mathrm{BR}}}{N_{\mathrm{DW}}} \sim \frac{\Lambda^4}{\Lambda^4_{\mathrm{BR}}} \gtrsim \left(\frac{\text{Higgs mass cutoff scale}}{\text{Weak scale}}\right)^4 \gg 1$$

Relaxion mechanism trades the technically unnatural $\frac{\text{Higgs cutoff scale}}{\text{weak scale}} \gg 1$ for technically natural (but much bigger) $g_{\mathrm{BR}}/g_{aV} \gg 1$.

- Hierarchy for trans-Planckian (or nearly Planckian) axion field excursion

Freese, Frieman, Olinto '90

Frieman et al '95; KC '99

$$\Delta a = \frac{1}{g_{aV}} \sim \begin{cases} \sqrt{N_e} M_P & \text{(large field axion inflation)} \\ M_P & \text{(quintessence (DE) axion)} \\ \frac{1}{20} M_P \left(\frac{\Omega_a h^2}{0.1}\right)^{1/2} \left(\frac{10^{-22}\,\text{eV}}{m_a}\right)^{1/4} & \text{(ultralight ALP dark matter)} \end{cases}$$

Axion weak gravity conjecture (WGC): $g_{a\text{WGC}} = \frac{N_{\text{WGC}}}{f_a} \gtrsim \frac{S_{\text{ins}}}{M_P}$

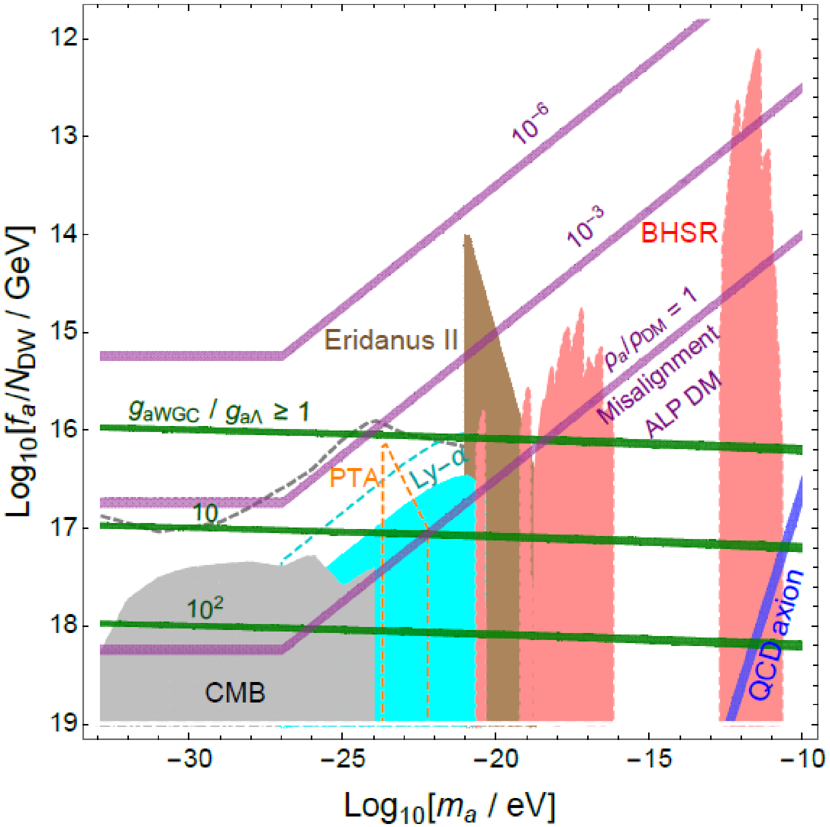
Arkani-Hamed et al '07

$$\Rightarrow \quad \frac{g_{a\text{WGC}}}{g_{aV}} \gtrsim \frac{S_{\text{ins}} \Delta a}{M_P} \sim \begin{cases} \sqrt{N_e} S_{\text{ins}} & \text{(large field axion inflation)} \\ S_{\text{ins}} & \text{(quintessence (DE) axion)} \\ \frac{S_{\text{ins}}}{20} \left(\frac{\Omega_a h^2}{0.1}\right)^{1/2} \left(\frac{10^{-22}\text{eV}}{m_a}\right)^{1/4} & \text{(ultralight ALP dark matter)} \end{cases}$$

(Assuming that the WGC instanton generates nonperturbative correction to the superpotental in SUSY framework, $S_{\text{ins}} \gtrsim \ln\left(m_{3/2} M_P / m_a^2\right)$)

Gravitational probe of ultralight DE-like or DM axions with

$$g_{a\mathrm{WGC}} \gg g_{aV} \;\; (N_{\mathrm{WGC}} \gg N_{\mathrm{DW}})$$



Future CMB and PTA observations will probe the parameter region with

$$\frac{g_{a\mathrm{WGC}}}{g_{aV}} = \frac{N_{\mathrm{WGC}}}{N_{\mathrm{DW}}} \gtrsim 10$$

- Hierarchy with astrophysical or cosmological motivations:

(Dark-)Photophilic axions with

$$g_{a\gamma}\ (\text{or } g_{a\gamma'}) \gg \frac{\alpha_{\rm em}}{2\pi} g_{aV}\ (c_\gamma \text{ or } c_{\gamma'} \gg N_{\rm DW})$$

for magnetogenesis
dissipative inflation
reducing Ω_a
producing $\Omega_{\gamma'}$
avoiding the BH superradiance bound on axions
EDGES 21cm signal
...

Most of the well-motivated hierarchical axion couplings are those of (ultra)light axions and involve large integer-valued parameters.

(Dark-)Photophilic axions with astrophysical or cosmological motivations:

$$\frac{c_\gamma}{N_{\rm DW}} \ \text{or} \ \frac{c_{\gamma'}}{N_{\rm DW}} \gtrsim 10^4$$

Relaxion: $$\frac{N_{\rm BR}}{N_{\rm DW}} \gtrsim \left(\frac{\text{Higgs cutoff scale}}{\text{weak scale}}\right)^4$$

We don't want to introduce such large integer parameters by hand, but generate them as an effective parameter in low energy theory that arises from high scale dynamics which does not involve any large parameter.

Hierarchies from axion landscape

Axion landscape involving

* many axions (e.g. # of p-cycles in string compactification $\gg 1$)

* hierarchical axion potentials (masses):

$$V(a) \propto \begin{cases} \text{quantized flux} \\ \text{non-perturbative effects} \propto e^{-b/g^2} \\ \text{Planck-scale suppressed operators} \propto 1/M_P^n \\ \dots \end{cases}$$

can provide such large integer parameter in low energy effective theory of light axions.

Models with multiple axions at UV scale:

Kim, Nilles, Peloso '04;
KC, Kim, Yun '14; KC, Im '15;
Kaplan, Rattazzi '15, ...

$$\mathcal{L} = \frac{1}{2} f_{ij}^2 \partial_\mu \theta^i \partial^\mu \theta^j - V(\theta^i) + \frac{1}{32\pi^2} \vec{k}_A \cdot \vec{\theta} F^A \tilde{F}^A + ... \qquad (i, j = 1, \ldots, N)$$

For hierarchical axion potentials, some axions are heavy enough to be frozen at their VEVs at the energy scale of our interest, while the other axions are light enough to have a dynamical evolution over their full field range.

$$V = V_H + V_L = -\sum_{H=1}^{N_H} \Lambda_H^4 \cos(\vec{q}_H \cdot \vec{\theta}) - \sum_{L=1}^{N_L} \mu_L^4 \cos(\vec{p}_L \cdot \vec{\theta}) + ... \qquad (N_H + N_L = N)$$

$$\left(\theta^i \cong \theta^i + 2\pi, \quad q_{Hi}, p_{Li}, k_{Ai} \in \mathbb{Z}, \quad \mu_L \ll \Lambda_H \right)$$

Light axions correspond to the vacuum manifold of $V_H = -\sum_{H=1}^{N_H} \Lambda_H^4 \cos(\vec{q}_H \cdot \vec{\theta})$

Vacuum solution of $V_H = -\sum_{H=1}^{N_H} \Lambda_H^4 \cos(\vec{q}_H \cdot \vec{\theta})$:

N_L-dim torus parameterized as

$$\theta^i = \sum_{L=1}^{N_L} n_L^i \theta_L \qquad \Big(\theta_L \cong \theta_L + 2\pi, \quad \vec{n}_L \cdot \vec{q}_H = 0, \quad n_L^i \in \mathbb{Z}, \quad \text{g.c.d}(n_L^i) = 1\Big)$$

n_L^i can be obtained by the Smith normal decomposition:

$$q_{Hi} = \sum_{I=1}^{N_H} U_{HI} \lambda_I Q_{Ii} \qquad \Big(\lambda_I \in \mathbb{Z}, \quad U \in GL(N_H, \mathbb{Z}), \quad Q = [\hat{Q}_{Hi}, \hat{Q}_{Li}] \in GL(N, \mathbb{Z})\Big)$$

$$\Rightarrow \qquad [n_H^i, n_L^i]^T = Q^{-1}, \quad \text{Det}(\vec{n}_L \cdot \vec{n}_M) = \text{Det}(\vec{Q}_H \cdot \vec{Q}_I)$$

Taking average over the Gaussian distribution of Q_{Hi} in the limit $N_H \gg 1$,

$$\Big\langle \text{Det}(\vec{n}_L \cdot \vec{n}_M) \Big\rangle = \Big\langle \text{Det}(\vec{Q}_H \cdot \vec{Q}_I) \Big\rangle \sim \Big\langle |\hat{Q}|^2 \Big\rangle^{N_H} \qquad \Big(|\hat{Q}|^2 = \frac{1}{N_H} \sum_{i=1}^{N} \sum_{H=1}^{N_H} Q_{Hi}^2 \Big)$$

Generically $|\hat{Q}|^2 > 1$, so even when all q_{Hi} are integers of order unity, $|\vec{n}_L| \sim |\hat{Q}|^{N_H/N_L}$ is exponentially large in the limit $N_H \gg N_L$.

Exponentially large $|\vec{n}_L|$ results in exponentially enlarged decay constants (field range) of the light axions:

$$(f_{\rm eff}^2)_{LM} = n_L^i f_{ij}^2 n_M^j$$

$$\left(n_L^i = \frac{\partial \theta^i}{\partial \theta_L} = \#\text{ of }2\pi\text{-shifts of }\theta^i\text{ for a single }2\pi\text{-shift of }\theta_L\right)$$

and also hierarchical effective couplings of light axions determined by

$$\vec{n}_L \cdot \vec{k}_A, \quad \vec{n}_L \cdot \vec{p}_M, \ldots$$

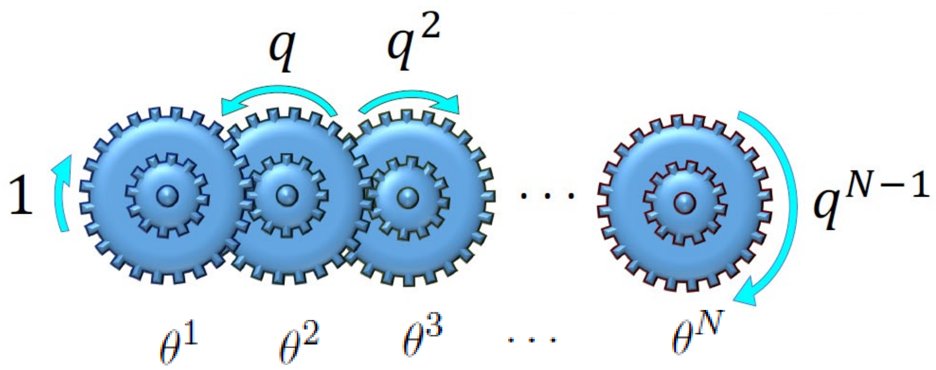
Clockwork axion model:

KC, Kim, Yun '14; KC, Im '15;
Kaplan, Rattazzi '15, ...

Focus on the lightest axion in the axion landscape: $N_H \gg N_L = 1$

Heavy axion potential induced by

$$\vec{q}_1 = (q, -1, \ldots, 0), \quad \vec{q}_2 = (0, q, -1, \ldots, 0), \quad \ldots, \quad \vec{q}_{N-1} = (0, \ldots, 0, q, -1)$$



Gluons couple to the first gear, while the photon couples to the last gear:

$$\vec{k}_G = (1, 0, \ldots, 0) \quad \vec{k}_\gamma = (0, \ldots, 0, 1)$$

Dominant light axion potential from the coupling to the last gear and the subleading barrier potential from the coupling to the first gear:

$$\vec{p}_{\rm BR} = (1, 0, \ldots, 0), \quad \vec{p}_V = (0, \ldots, 0, 1)$$

Exponentially enlarged field range and exponential coupling hierarchy of the light axion:

$$\vec{n}_L = (1, q, \ldots, q^{N-2}, q^{N-1})$$

$$f_L = |\vec{n}_L| f = \sqrt{\frac{q^{2N} - 1}{q^2 - 1}} f$$

$$\frac{g_{a\mathrm{BR}}}{g_{aV}} = \frac{\vec{n}_L \cdot \vec{p}_{\mathrm{BR}}}{\vec{n}_L \cdot \vec{p}_V} = q^{N-1}$$

$$\frac{g_{a\gamma}}{g_{aV}} = \frac{\alpha_{\mathrm{em}}}{2\pi} \frac{\vec{n}_L \cdot \vec{k}_\gamma}{\vec{n}_L \cdot \vec{p}_V} = \frac{\alpha_{\mathrm{em}}}{2\pi} q^{N-1}$$

$$\frac{g_{a\gamma}}{g_{aG}} = \frac{e^2}{g_c^2} \frac{\vec{n}_L \cdot \vec{k}_\gamma}{\vec{n}_L \cdot \vec{k}_G} = \frac{e^2}{g_c^2} q^{N-1}$$

Conclusion

- There are a variety of well-motivated & technically natural axion coupling hierarchies.

- Many of the on-going or planned experiments searching for axions are probing the parameter regions of large coupling hierarchies. There are also many well-motivated astrophysical or cosmological phenomena relying on hierarchical axion couplings.

- Such hierarchical axion couplings can naturally arise in the low energy limit of axion landscape without introducing any large or small model parameter in the UV theory.