SU(2) Anomaly for DE Source

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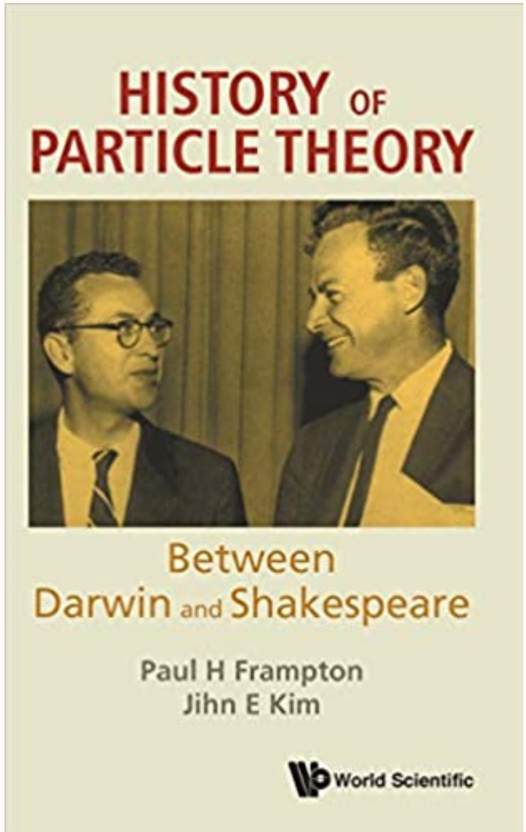
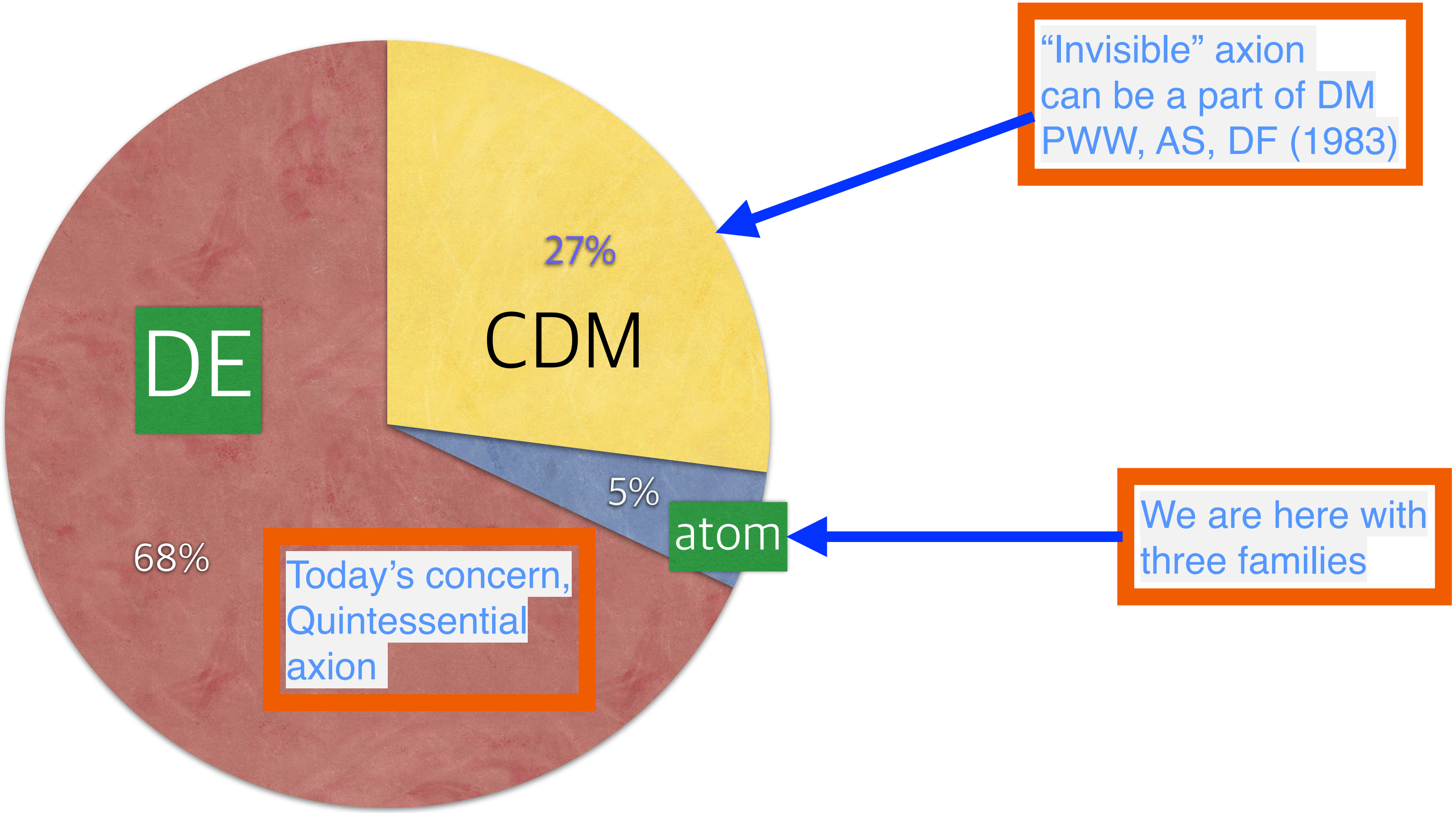
1. Introduction

Chirality is the theme of this talk



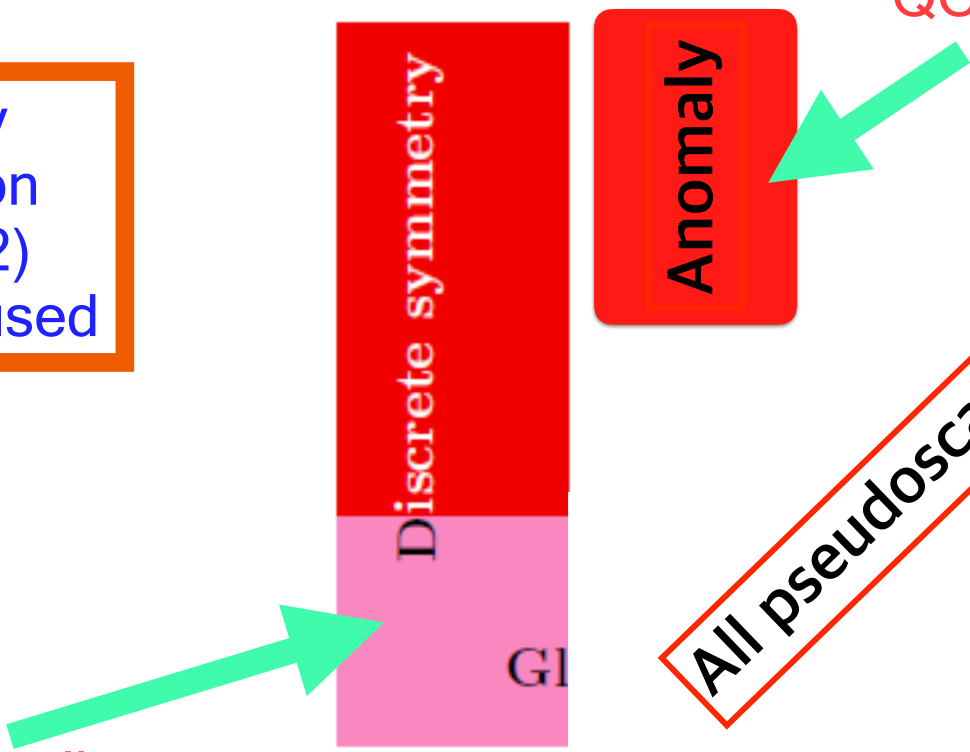
V-A quartet



"Invisible" axion
can be a part of DM
PWW, AS, DF (1983)
27%
CDM
DE
5%
atom
We are here with
three families
68%
Today's concern,
Quintessential
axion

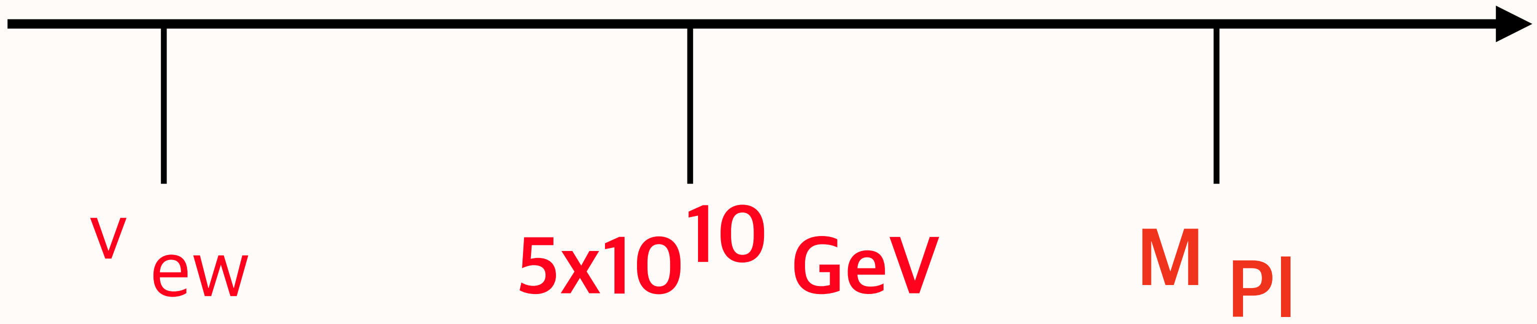
The dominant contribution is QCD anomaly term

Now, tiny contribution from SU(2) anomaly is used



All pseudoscalars are massive

Keep only the leading terms



Common scale for f_a and
source of SUSY breaking

2. SU(2) Anomaly

U(1)_global has anomaly with non-abelian gauge groups: We need global U(1) to have a pseudoscalar particle.

PQ symmetry is an example: Breaking Scale Lambda-QCD.

U(1)-SU(2)-SU(2):

We want to use this anomaly for breaking the DE global symmetry U(1).

What is the scale for DE SU(2)?

SU(2) gauge coupling by running the value at the electroweak scale already given.

With \alpha_2=29.600 +-0.010 at M_Z, SUSY

[1508.04176]

Threshold correction [%]	M_{SUSY} [GeV],	Mg [GeV],	$1/\alpha_{GUT}$	χ^2
+1	$10^{3.96\pm0.10}$	$10^{15.85\pm0.03}$	26.74 ± 0.17	8.2%
±0	$10^{3.45\pm0.09}$	$10^{16.02\pm0.03}$	25.83 ± 0.16	8.2%
−1	$10^{3.02\pm0.08}$	$10^{16.16\pm0.03}$	25.07 ± 0.15	9.5%
−2	$10^{2.78\pm0.07}$	$10^{16.25\pm0.02}$	24.63 ± 0.13	25.1%
−3	$10^{2.60\pm0.06}$	$10^{16.31\pm0.02}$	24.28 ± 0.10	68.1%
−4	$10^{2.42\pm0.05}$	$10^{16.38\pm0.02}$	23.95 ± 0.09	138.3%
−5	$10^{2.26\pm0.05}$	$10^{16.44\pm0.02}$	23.66 ± 0.09	235.7%

An exponential form from instanton interaction

$$\Lambda_{\rm QCD}^4\, e^{-2\pi/\alpha_3}, \quad {\rm At~1~GeV},\ \alpha_3 \sim O(1).$$

$$M_{\rm SU(2)}^4\, e^{-2\pi/\alpha_2}$$

What is appropriate for the scale of the weak-SU(2)? The scale where the pseudoscalar is created.

2-loop beta function :

$$\beta = -(\frac{\alpha_s}{4\pi})^2 \left(\frac{11}{3}C_2(G) - \frac{2}{3}\sum_R T(R) - \frac{\alpha_s}{4\pi}\left(\frac{10}{3}\sum_R C_2(G)T(R) + 2\sum_R \text{Cashimir}_2(\text{SU}(N))T(R) - \frac{34}{3}(C_2(G))^2\right)\right)$$

where

$$C_2(\text{SU}(N)) = N,\ \text{Cashimir}_2(\text{SU}(N)) = \frac{N^2-1}{2N},\ T(R) = \ell(R), \ell(\mathbf{N}) = \frac{1}{2}.$$

Table I was obtained with the following input parameters,

$$\text{At } M_Z = 91.19 \text{ GeV}: \begin{cases} \sin^2\theta_W\Big|_{\overline{\text{MS}}} = 0.23126 \pm 0.00005, \\ \alpha_s = 0.1185 \pm 0.0006, \end{cases}.$$

We obtain

The number of SU(2)-left-handed doublets is [3(colored)+1]x3=12. If counted as left+right, it corresponds to 6.
So, 3Nc-Nf=0. So, this value is not changing.

$$\text{MSSM}: \begin{bmatrix} e^{-2\pi/\alpha_2}\Big|_{M_{\rm GUT}} = 1.69\times 10^{-81}, \\ \begin{cases} M_{\rm SUSY} = 2820+670-540\ \ \text{GeV}, \\ M_{\rm GUT} = (1.065\pm 0.06)\times 10^{16}\,\text{GeV}. \end{cases} \end{bmatrix}$$

$$\text{SM}: \begin{bmatrix} e^{-2\pi/\alpha_2}\Big|_{M_{\rm GUT}} = 1.69\times 10^{-131}, \\ M_{\rm GUT} = (1.096\pm 0.06)\times 10^{15}\,\text{GeV}. \end{bmatrix}$$

If SU(2) gauge force is responsible for DE,

$$\mathrm{MSSM}: \ 1.69\times 10^{-81}\Lambda^4 = (0.003\ \mathrm{eV})^4 \to \Lambda \sim 1.48\times 10^{8}\ \mathrm{GeV},$$
$$\mathrm{SM}: \ 1.065\times 10^{-131}\Lambda^4 = (0.003\ \mathrm{eV})^4 \to \Lambda \sim 5.25\times 10^{20}\ \mathrm{GeV}.$$

So, only the MSSM or SSM has a possibility.

3. QCD axion plus DE quintessence

Let us introduce two complex SM singlet fields which house the QCD axion and quintessential axion,

$$\langle\sigma\rangle = \frac{f_a}{\sqrt{2}}\, e^{ia/f_a},\ \ \langle\sigma_{\rm quint}\rangle = \frac{f_q}{\sqrt{2}}\, e^{ia_q/f_q}$$

	σ	$\sigma_{\rm quint}$
$\mathrm{U}(1)_{\rm PQ}$:	1	Γ_2
$\mathrm{U}(1)_{\rm q}$:	Γ_1	1

Then the cosine potentials become,

$$V = m\Lambda_{\rm QCD}^3 \left(\cos(\frac{a}{f_a} + \Gamma_2 \frac{a_q}{f_q}) + {\rm h.c.}\right) + f_q^4 e^{-2\pi/\alpha_2} \left(\cos(\Gamma_1 \frac{a}{f_a} + \frac{a_q}{f_q}) + {\rm h.c.}\right)$$

The mass matrix becomes,

$$\begin{pmatrix} \frac{m\Lambda_{\rm QCD}^3 + \Gamma_1^2 f_q^4 e^{-2\pi/\alpha_2}}{f_a^2}, & \frac{\Gamma_1 f_q^4 e^{-2\pi/\alpha_2} + \Gamma_2 m\Lambda_{\rm QCD}^3}{f_a f_q} \\ \frac{\Gamma_1 f_q^4 e^{-2\pi/\alpha_2} + \Gamma_2 m\Lambda_{\rm QCD}^3}{f_a f_q}, & \frac{f_q^4 e^{-2\pi/\alpha_2} + \Gamma_2^2 m\Lambda_{\rm QCD}^3}{f_q^2} \end{pmatrix}$$

Masses of the QCD axion and the quintessential axion are

$$m_a^2 = \frac{m\Lambda_{\rm QCD}^3}{f_a^2}\left(1+\frac{\Gamma_2^2 f_a^2}{f_q^2}\right) + O(e^{\frac{-2\pi}{\alpha_2}}),$$

$$m_{a_q}^2 = (\Gamma_1\Gamma_2 - 1)^2 f_q^2 e^{\frac{-2\pi}{\alpha_2}} \frac{1}{1+\Gamma_2^2(f_a^2/f_q^2)} + O(e^{\frac{-4\pi}{\alpha_2}})$$

The QCD axion mass is as expected. The coefficient m\Lammbda^3 is the familiar form in terms of Z=mu/md: Z/(1+Z)^2 times (f_\pi m_\pi)^2. the quintessential axion mass has the extremely small exponential factor.

4. Breaking scale by V

Note that $e^{-2\pi/\alpha_2}$ is almost 0.169×10^{-40}. With $f_q \simeq 10^8\,\mathrm{GeV}$, we obtain $m_q \simeq 2 \times 10^{-13}\,\mathrm{GeV} \approx 0.0002\,\mathrm{eV}$ and the vacuum energy density $f_q^4 e^{-2\pi/\alpha_2} \approx (0.64 \times 10^{-3}\,\mathrm{eV})^4$.

The SU(2) anomaly breaking is OK, but we have to check whether V has more strongly breaking terms.

In particular, any global symmetry is broken by gravity.
Therefore, the following terms can be present.

$$\frac{\Lambda^{n+4}}{M_{\rm P}^n}$$

Allow power (n+4),
but forbid terms up to power (n+3).

Since only SUSY is compatible with SU(2) anomaly for DE, we work with SUSY. Namely, we work with super potential terms

$$W \sim \sigma_q^{n+3}/M_{\rm P}^n \longrightarrow V \sim (n+3)|\langle\sigma_q\rangle|^{n+3}/M_{\rm P}^{n-1}$$

For the VEV of \sigma_q = 1.48x10^8 GeV, we forbid up to

$$\frac{\sigma_q^{11.82}}{M_{\rm P}^{8.82}}$$

Allow n from 9 in the superpotential.

Attempt: Z(4R)

Obviously, we cannot forbid all terms up to n=8.

Z(N) discrete symmetry must be with a large N. Or we need a product of discrete symmetries. Details from string compactification will be presented at CUBES-02.

Attempt: Z(4R)xZ(2)

	NMSSM					anti-SU(5)		PQ	Quintessential
	$\mathbf{10}_g$	$\overline{\mathbf{5}}_g$	$\mathbf{1}_g$	$\mathbf{5}_H$	$\overline{\mathbf{5}}_H$	$\Sigma_{\rm GUT}$,	$\overline{\Sigma}_{\rm GUT}$	σ	σ_q
$\mathbf{Z}_{4R}$	$+\frac{1}{2}$	$+\frac{1}{2}$	$+\frac{1}{2}$	$+1$	$+1$	$+4$,	$+4$	$r_1 = +2$	$r_2 = +2$
$\mathbf{Z}_{4R} \times \mathbf{Z}_2$	0	0	0	$+1$	$+1$	$+1$,	$+1$	$+1$	$+1$

TABLE IV: Working quantum numbers of SUSY chiral fields.

Superpotential with Z(4R)xZ(2) discrete symmetry (e.g. for mu) is allowed, but dangerous terms are forbidden.

$$\sigma^2\sigma_q,\ \sigma\sigma_q^2,\ \Sigma\overline{\Sigma}\sigma,\ \Sigma\overline{\Sigma}\sigma_q,\ \boxed{\mathbf{5}_H\overline{\mathbf{5}}_H\sigma^2}.\ \boxed{\mathbf{5}_H\overline{\mathbf{5}}_H\sigma_q^2}$$

4. Conclusion

1. SU(2) anomaly.

2. QCD axion for DM and quintessential axion as DE.

3. SUSY theory. And the breaking is discussed.

4. Need a care for discrete symmetries to preserve the SU(2) anomaly as the source of DE.