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# Neutrino oscillations in a medium

Based on arXiv:1909.10478 & 2012.09474 with Ki-Young Choi (SKKU) & Jongkuk Kim (KIAS),

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## Outline

• Neutrinos interacting with a hot/dense medium

Coherent forward scattering modifies/drives neutrino oscillations Wolfenstein 78 Adiabatic conversion of solar neutrinos Mikheyev-Smirnov 85

• Study of dispersion relation at a finite temperature

Thermal mass of fermions Weldon 82

Re-derivation of Wolfenstein potential Mannheim 88; Notzold-Raffeld 88; Pal-Pham 89; Nieves 89; ...

• Dispersion of neutrinos in non-standard media

An approximate solution in high-momentum limitGe-Murayama 19; Choi-EJC-Kim 19General solutions in various limitsChoi-EJC-Kim 20

• Implications:

Medium-induced masses and oscillations; cosmological/astrophysical limits



#### Neutrino oscillations in matter

#### L. Wolfenstein

Carnegie-Mellon University, Pittsburgh, Pennsylvania 15213 (Received 6 October 1977; revised manuscript received 5 December 1977)

The effect of coherent forward scattering must be taken into account when considering the oscillations of neutrinos traveling through matter. In particular, for the case of massless neutrinos for which vacuum oscillations cannot occur, oscillations can occur in matter if the neutral current has an off-diagonal piece connecting different neutrino types. Applications discussed are solar neutrinos and a proposed experiment involving transmission of neutrinos through 1000 km of rock.



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general dispersion relations

dark matter

 $\nu$ -DM interaction

by medium-induced mass-squared

#### Neutrino oscillations in vacuum

• Propagation Hamiltonian:

$$i\frac{d}{dt}\psi = H\psi$$

$$\psi = (v_1, v_2, v_3)^T \qquad \psi = (v_e, v_\mu, v_\tau)^T \qquad v_\alpha = U_{\alpha i} v_i$$

$$H \approx p + \frac{m_i^2}{2p} \qquad H \Rightarrow \frac{Um^2 U^+}{2p} = \frac{MM^+}{2p}$$

• Two-flavor evolution:

$$H \approx \frac{\Delta m^2}{4p} \begin{bmatrix} -c_{2\theta} & s_{2\theta} \\ s_{2\theta} & c_{2\theta} \end{bmatrix} \rightarrow P_{e\mu} = |\langle \nu_{\mu} | e^{iHt} | \nu_{e} \rangle|^2 = \sin^2 2\theta \sin^2 \left( \frac{\Delta m^2 L}{4p} \right)$$

#### Wolfenstein potential

 "Oscillations for massless neutrinos can occur as a result of coherent forward scattering provided that this scattering is off-diagonal in ne utrino flavor":

$$\mathcal{H}_{eff} = G_{\alpha\beta} \, \bar{\nu}_{\alpha} \gamma^{\mu} P_L \, \bar{\nu}_{\beta} \, \bar{f} \gamma_{\mu} f \ \Rightarrow H \sim G_{\alpha\beta} \, N_f \quad (\text{momentum-independent})$$

• "In the standard model, vacuum oscillations are modified by the charged current scattering":

$$H_{\nu/\overline{\nu}} \approx \frac{\Delta m^2}{4p} \begin{bmatrix} -c_{2\theta} & s_{2\theta} \\ s_{2\theta} & c_{2\theta} \end{bmatrix} \pm \begin{bmatrix} V_W & 0 \\ 0 & 0 \end{bmatrix} \qquad V_W = \sqrt{2} \ G_F N_e(t) \qquad \text{MSW effect}$$

#### Generalizing the Wolfenstein effect

• In a medium with arbitrary  $N_e$  and  $N_{\bar{e}}$ 

#### "Free" neutrinos in a medium

• Self-energy correction by the coherent forward scattering:

$$\mathcal{L}_{kin} \Rightarrow \overline{\nu_L} (p^{\mu} - k^{\mu} \Sigma_W) \gamma_{\mu} \nu_L$$

$$S_W = g_{\nu}^2 \frac{N_{e+\bar{e}}}{m_e} \frac{-2m_e E + \epsilon (m_W^2 - p^2 - m_e^2)}{(m_W^2 - p^2 - m_e^2)^2 - 4m_e^2 E^2}$$

$$g_{\nu}^2 \equiv \sqrt{2} G_F m_W^2$$

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$$f_{\mu}^2 \equiv (E, \vec{p}) \qquad p^2 = E^2 - p^2$$

$$\epsilon \equiv \frac{N_e - N_{\bar{e}}}{N_e + N_{\bar{e}}}$$

$$k^{\mu} = (m_e, \vec{0})$$

• Dispersion of Weyl neutrino in the high-momentum limit ( $E \approx p$ ):

$$(p - k\Sigma_W)^2 = (E - m_e \Sigma_W)^2 - p^2 = 0 \xrightarrow{m_W^2 \gg 2m_e p} \begin{cases} E_{\nu/\overline{\nu}} \approx p \pm \epsilon g_{\nu}^2 \frac{N_{e+\overline{e}}}{m_W^2} + \cdots & \text{Potential} \\ E_{\nu/\overline{\nu}} \approx p + g_{\nu}^2 \frac{N_{e+\overline{e}}/m_e}{2p} + \cdots & \text{Oscillation} \end{cases}$$
(For Majorana neutrino, p.11)

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## Dispersion of neutrinos in a medium

## General formulation

• Eqs. of motion in a Lorenz invariant medium:

$$(p - p\Sigma_{1L}^{u} - k\Sigma_{2L}^{u})u_{L}(p) = (M^{+} + \Sigma_{0}^{+})u_{R}(p)$$
$$(p - p\Sigma_{1R}^{u} - k\Sigma_{2R}^{u})u_{R}(p) = (M + \Sigma_{0}^{-})u_{L}(p)$$
$$(3,4) \qquad (1,2,3,4) \qquad (5)$$
$$\Sigma_{1,2}^{+} = \Sigma_{1,2}$$

• For *v*-spinors:  $\Sigma_1^v(p) = \Sigma_1^u(-p)$  $\Sigma_2^v(p) = -\Sigma_2^u(-p)$  • Neutrinos interacting with dark matter and mediator

$$\mathcal{L}' = g_{\alpha i} \, \overline{f_{iL}} \gamma^{\mu} \nu_{\alpha L} \, X_{\mu} + h. \, c. \quad (1)$$

$$g_{\alpha i} \, \overline{f_R} \, \nu_{\alpha L} \, \phi_i + h. \, c. \qquad (2)$$

$$g_{\alpha i} \,\overline{f_{iR}} \nu_{\alpha L} \,\phi + h.c. \qquad (3)$$

$$g_{\alpha\beta} \overline{\nu_{\beta R}^c} \nu_{\alpha L} \phi + h.c.$$
 (4)

$$g_{\alpha\beta} \,\overline{\nu_{\beta R}^c} \nu_{\alpha L} \,\phi + y \,\phi \,\overline{f_R} f_L + h. \,c. \,\,(5)$$

## General eqs. for dispersion

• For  $k = (m_{\phi}, \vec{0}) \& p = (E, \hat{z}p)$ , EoM (for  $\nu_L / \nu_R$ ) is solved by

$$\begin{split} L \cdot R - m_{\nu}^{2} \pm H &= 0 & L \equiv p - \Sigma_{L} \equiv (L_{0}, \hat{z}L_{z}) \\ R \equiv p - \Sigma_{R} \equiv (R_{0}, \hat{z}R_{z}) \\ H \equiv L_{z}R_{0} - R_{z}L_{0} & \\ 0 &= (E^{2} - p^{2})(1 - \Sigma_{1L})(1 - \Sigma_{1R}) - m_{\nu}^{2} + m_{\phi}^{2}\Sigma_{2L}\Sigma_{2R} \\ -m_{\phi}(E \pm p)\Sigma_{2L}(1 - \Sigma_{1R}) - m_{\phi}(E \mp p)\Sigma_{2R}(1 - \Sigma_{1L}) & \xrightarrow{E \to p} 2p \ m_{\phi} \begin{cases} \Sigma_{2L}(1 - \Sigma_{1R}) \\ \Sigma_{2R}(1 - \Sigma_{1L}) \end{cases} \end{split}$$

• Approximate solutions for  $|\Sigma_1| \ll 1 \& m_{\phi} |\Sigma_2| \ll m_{\nu}$ , p

$$E^{2} \approx p^{2} + m_{\nu}^{2} \left( 1 + \Sigma_{1L}^{(0)} + \Sigma_{1R}^{(0)} \right) \qquad \Sigma^{(0)} = \Sigma(E_{0}, p);$$
  
$$-m_{\phi} \left( E_{0} [\Sigma_{2L}^{(0)} + \Sigma_{2R}^{(0)}] \pm p [\Sigma_{2L}^{(0)} - \Sigma_{2R}^{(0)}] \right) \qquad E_{0} = \sqrt{p^{2} + m_{\nu}^{2}};$$

Wolfenstein:  

$$(\Sigma_{1L,R} = 0; m_{\phi}\Sigma_{2L,R} = \pm V_W)$$

$$E_{\nu,\overline{\nu}}^2 = p^2 + m_{\nu}^2 + V_W^2 \pm 2p V_W$$
(Sun:  $m_{\nu}^2 \sim 2pV_W$ )

## Application to different types of neutrinos

• Weyl 
$$(m_{\nu} = 0)$$
:  $u_L(u_R)$  for  $\nu$   $(\bar{\nu})$ 

 $(E_{\nu,\overline{\nu}} - \mathbf{p})(1 - \Sigma_{1L,R}) - m_{\phi}\Sigma_{2L,R} = 0 \implies E_{\nu,\overline{\nu}} \approx \mathbf{p} + m_{\phi}\Sigma_{2L,R}^{(0)}$ 

• Majorana 
$$(v = v^c)$$
:  $u_L(u_R)$  for  $v(\bar{v})$  for  $E \to p$   
 $(u_R = v_L^c, v_R = u_L^c)$   
 $\Sigma_R^u(p) = [\Sigma_L^v(p)]^* = -[\Sigma_L^u(-p)]^*$   
 $= [\Sigma_L^u(p)]_{\epsilon \to -\epsilon}^*$ 
 $E_{v,\bar{v}} \approx E_0 + \frac{m_v^2}{2E_0} \left( \Sigma_{1L}^{(0)} + \Sigma_{1R}^{(0)} \right)$ 
 $+ \frac{m_\phi}{2} \left( \left( \Sigma_{2L}^{(0)} + \Sigma_{2R}^{(0)} \right) \pm \frac{p}{E_0} \left( \Sigma_{2L}^{(0)} - \Sigma_{2R}^{(0)} \right) \right)$ 
asymmetric

• <u>Dirac</u>  $(\nu \neq \nu^c, m_\nu \neq 0)$  with  $\Sigma_R = 0$ :  $(E^2 - p^2)(1 - \Sigma_{1L}) - m_\nu^2$  $-m_\phi \Sigma_{2L}(E \pm p) = 0$   $E_{\nu_L, \nu_R} \approx E_0 + \frac{m_\nu^2}{2E_0} \Sigma_{1L}^{(0)} + \frac{m_\phi}{2} \Sigma_{2L}^{(0)} \left(1 \pm \frac{p}{E_0}\right) \xrightarrow{\epsilon \to -\epsilon} E_{\overline{\nu}_L, \overline{\nu}_R}$ 

#### Neutrinos in a medium of complex scalar

$$\mathcal{L}' = g \overline{f_R} \nu_L \phi + g^* \, \overline{\nu}_L f_R \phi^*$$

#### Modified neutrino propagator

• Finite temperature/density calculation

 $S_{\nu}^{-1}(p) = (\not p - \not Z) = (\not p - \not p \Sigma_1 - \not k \Sigma_2)$  $\not Z = i g g^+ \int \frac{d^4 k}{(2\pi)^4} \Delta_{\phi}(k) S_f(p+k)$ 

$$\Delta_{\phi}(k) = \frac{1}{k^2} - 2\pi i \,\delta(k^2 - m_{\phi}^2) f_{\phi}(k)$$
  
$$S_f(q) = (q + m_f) \left(\frac{1}{q^2 - m_f^2} + 2\pi i \,\delta(q^2 - m_f^2) f_f(q)\right)$$



Medium 4-momentum:  $k = (k^0, \vec{k}) \simeq (m_{\phi}, 0)$ Distribution functions:

$$\begin{split} f_f(q) &= 0 \\ f_\phi(k) &= \left(\theta(k_0)n_\phi + \theta(-k^0) n_{\overline{\phi}}\right)(2\pi)^3 \delta^3(\vec{k}) \end{split}$$

## Self-energy corrections

$$\Sigma_{1L}^{u,v}(p), \Sigma_{1L,R} = S(p) \pm \epsilon A(p)$$
  
$$\Sigma_{2L}^{u,v}(p), \Sigma_{2L,R} = A(p) \pm \epsilon S(p)$$

$$\epsilon \equiv \frac{N_{\phi} - N_{\overline{\phi}}}{N_{\phi} + N_{\overline{\phi}}} \qquad \delta m^2 \equiv |g|^2 \frac{N_{\phi} + N_{\overline{\phi}}}{2 m_{\phi}}$$

$$S(p) = \delta m^2 \frac{p^2 + m_{\phi}^2 - m_f^2}{\left(p^2 + m_{\phi}^2 - m_f^2\right)^2 - 4m_{\phi}^2 E^2}$$
$$A(p) = \delta m^2 \frac{-2m_{\phi} E}{\left(p^2 + m_{\phi}^2 - m_f^2\right)^2 - 4m_{\phi}^2 E^2}$$

$$\mathbf{S}_{\nu}^{-1}(p) = (\not p - \not p \Sigma_1 - \not k \Sigma_2)$$

Modified dispersion relations  $E = E(p; m_{\nu}^2, \delta m^2, m_f^2, m_{\phi}^2)$ depending on the parameter hierarchies: 4. High density limit:  $\delta m^2 \gg m_f^2, m_{\nu}^2, 2m_{\phi}p$ 

- 1. Decoupling limit:  $m_f^2 \gg \delta m^2$ ,  $m_v^2$ ,  $2m_\phi p \gg m_\phi^2$
- 2. Heavy neutrino limit:  $m_{\nu}^2 \gg \delta m^2$ ,  $m_f^2$ ,  $2m_{\phi}p$
- 3. High momentum limit:  $2m_{\phi}p \gg m_f^2, \delta m^2, m_{\nu}^2$

 $E^2 = p^2 - p^2$ 

## Dispersion of Weyl/Majroana neutrinos

\*) Weyl:  $m_{\nu} = 0$ 

1. Decoupling limit

3. <u>High momentum limit</u>



#### Dispersion of Dirac neutrinos



# Implications

#### Effective mass in a medium

• L-conserving mass-squared without chirality-flip:

$$\delta m^{2} = \frac{gg^{+}}{2} \frac{\rho_{\phi + \overline{\phi}}}{2m_{\phi}} \qquad m_{M} = \frac{1}{2} \left( m_{\nu} + \sqrt{m_{\nu}^{2} + 4\delta m^{2}} \right)$$
$$E \xrightarrow{p \to 0} m_{M,D} \qquad m_{D} = \sqrt{m_{\nu}^{2} + \delta m^{2}}$$

• Affecting neutrino oscillations:  $p^2 \gg 2m_{\phi}p \gg \delta$ 

$$p^{2} \gg 2m_{\phi}p \gg \delta m^{2}, m_{\nu}^{2} \implies E \sim \frac{m_{\nu}^{2} + \delta m^{2}}{2p}$$
$$p^{2} \gg \delta m^{2}, m_{\nu}^{2} \gg 2m_{\phi}p \implies E \sim \frac{m_{M,D}^{2}}{2p}$$

## Oscillation as a dark medium effect

• Solar and Atmospheric neutrino oscillations:  $E_{\nu} \approx p$ 

 $p_{sol} = (0.1 - 10) \text{MeV}, \qquad p_{atm} = (0.1 - 100) \text{ GeV} \qquad \frac{\Delta m^2}{2p} = \begin{cases} 3.8 \times 10^{-10:-12} \text{ eV} \\ 1.2 \times 10^{-11:-14} \text{ eV} \end{cases}$ 

• Weyl neutrinos can oscillates due to  $\rho_{\rm DM}^{loc} \approx 0.3 \, {\rm GeV/cm^3} \approx 2.3 \times 10^{-6} \, {\rm eV^4}$ 

$$\delta m_{loc}^2 = g^2 \frac{\rho_{DM}^{loc}}{2m_{DM}^2} \sim \Delta m_{sol, atm}^2 \implies m_{\phi} \approx 0.03g \text{ eV} \left(\frac{2.5 \times 10^{-3} \text{ eV}^2}{\delta m_{loc}^2}\right)^{\frac{1}{2}}$$

• CPV may also appear: 
$$E_{\nu} \ni \pm \epsilon \frac{m_f^2}{2m_{\rm DM}p} \frac{\delta m^2}{2p}$$
;  $\pm \epsilon m_{\rm DM}$ 

## Cosmological limitation

• Neutrinos were heavier at earlier time.

$$\delta m^{2}(z) = \delta m_{loc}^{2} \frac{\rho_{DM}^{0}}{\rho_{DM}^{loc}} (1+z)^{3} \approx 6650 \left(\frac{z}{1100}\right)^{3} \delta m_{loc}^{2}$$

• Around the CMB decoupling, neutrinos cannot be too heavy:

 $\delta m^2(z) \lesssim (0.1 \text{eV})^2 \Rightarrow \delta m_{loc}^2 < 10^{-6} \text{eV}^2$ 

• To explain the neutrino oscillations as an local DM effect, the current DM component needs to be generated well after the decoupling.

## Astrophysical bounds

- The present neutrino-DM scattering cross-section is constrained by SN1987A ( $E_{\nu} \sim 10 \text{ MeV}$ ) and IceCube-170922A ( $E_{\nu} \sim 300 \text{TeV}$ ) neutrino observations:  $\frac{\sigma_{\nu-\phi}}{m_{\phi}} \approx \frac{g^4}{32\pi m_{\phi}^2 E_{\nu}} \lesssim 10^{-22} \text{ cm}^2/\text{GeV} \implies m_{\phi} \gtrsim 2 \times 10^6 g^2 \text{ eV}$
- Combined with the condition for the "massless" oscillation



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## Conclusion

- General dispersion relations of neutrinos propagating in a medium are f ound up to first order in perturbation considering the four limiting situa tions.
- Effective mass is generated and the neutrino oscillations may be due to the medium effect (partly or totally even for Weyl neutrinos).
- The scenario is limited by the cosmological mass generation and the ne utrino-DM interaction is constrained by the astrophysical neutrino obser vations.
- Sensible UV-completion for the origin of scalar DM?

Dark sector coupling only to neutrinos; late generation of ultra-light cold DM