Exploring the Universe with *Dark* *Light* *Scalars*

Based on 2010.10880

Bugeon Jo, Hyeontae Kim, Hyung Do Kim, CSS

Chang Sub Shin (IBS-CTPU)

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Outline

Motivation

- Origin of the light scalar DM

- Multi-component DM with very different features

Issues in Axion-Glueball DM Cosmology

- Gluo-thermodynamics with the axion

- Structure of the multi branched axion potential

Implications of Subcomponent Glueball Dark Matter

- Intro (SMBHs at $z\sim7$)

- Dark matter interpretation (gravo-thermal collapse)
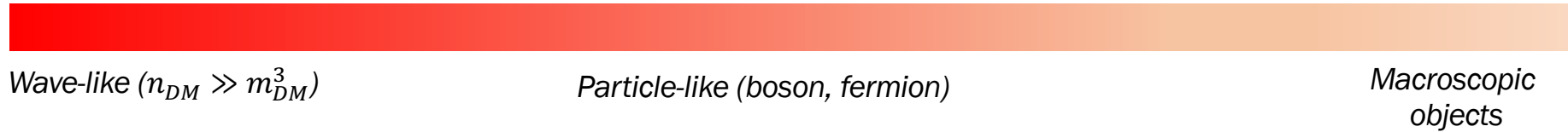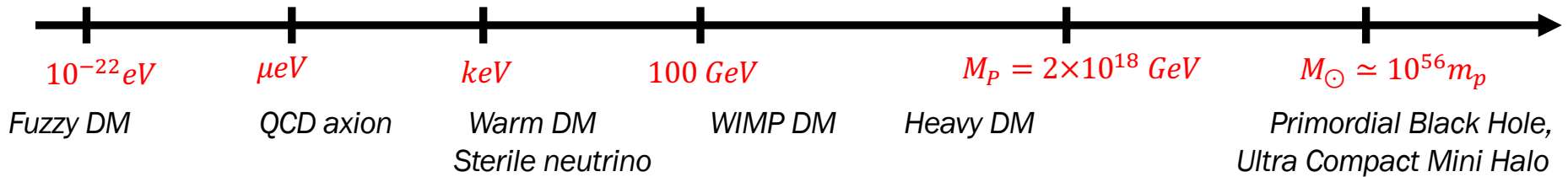
- Our model and Caveats

Summary

Motivation

Motivation

The nature of dark matter is still shrouded in mystery

$$\bar{\rho}_{DM} \simeq 1.2 \times 10^{-6} \text{ GeV/cm}^3 = m_{DM}\bar{n}_{DM}$$

$m_p = 1 \; GeV$
$m_e = 0.5 \; MeV$
$m_\nu = 0.1 \; eV$

$10^{-22}eV$	μeV	keV	$100 \; GeV$	$M_P = 2 \times 10^{18} \; GeV$	$M_\odot \simeq 10^{56} m_p$
Fuzzy DM	*QCD axion*	*Warm DM* *Sterile neutrino*	*WIMP DM*	*Heavy DM*	*Primordial Black Hole,* *Ultra Compact Mini Halo*

Wave-like ($n_{DM} \gg m_{DM}^3$)   *Particle-like (boson, fermion)*   *Macroscopic objects*

Unfortunately, there is no good guiding principle for the mass of dark matter these days

Origin of the scalar mass → Hints for new physics

Ex) Higgs boson (the weak scale is radiatively unstable)
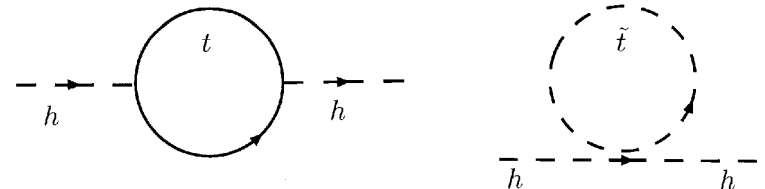
$$\delta_{qc} m_H^2 \sim \Lambda_{UV}^2 \gg m_W^2 \qquad \textit{Supersymmetry, Composite Higgs, Relaxion, Scale invariance etc.}$$

Motivation

Considering scalar dark matter: Need to explain the origin of its mass

$\rightarrow$ *It also determines its interaction structure, too*

Natural scalar dark matter candidates

1) **Axion-like** *particle* (ϕ): *for the compact field* $(\phi \rightarrow \phi + 2\pi f_a)$,

   <u>*Approximate global symmetry*</u>: $\phi \rightarrow \phi + c$ *is broken non-perturbatively*

   (by instanton, confinement, etc.)

   e.g. $\dfrac{\phi}{16\pi^2 f_a}\text{Tr}\left[G_{\mu\nu}\tilde{G}^{\mu\nu}\right] \rightarrow V(\phi) = \sum a_n \Lambda^4 \cos\left(\dfrac{n\phi}{f_a}\right)$

2) **Glueball-like** *particle* (φ_g): *At high scales, there is no scalar degree of freedom*

   <u>*Confining gauge symmetry*</u>: $\text{Tr}\left[G_{\mu\nu}G^{\mu\nu}\right] \rightarrow \varphi_g$: $\quad m_g \sim \sqrt{4\pi}\,\Lambda$ *(confinement scale)*

Both 1) & 2) (and their mixture) can make scalars light. What if we consider both mechanisms simultaneously as a dark sector?

$$L_{dark} = -\frac{1}{4g_h^2}G_{\mu\nu}^a G^{a\mu\nu} + \frac{1}{2}\left(\partial_\mu \phi\right)^2 + \frac{\phi}{32\pi^2 f_a}G_{\mu\nu}^a \tilde{G}^{a\mu\nu}$$

Dark Axion & Dark Glueball DM

$$L_{dark} = -\frac{1}{4g_h^2}GG + \frac{1}{2}(\partial_\mu \phi)^2 + \frac{\phi}{32\pi^2 f_a}G\tilde{G}$$

Taking SU(N) as the hidden gauge group
Decoupled from the SM from the beginning

Confining phase transition
at $T_g = T_{g,c} \simeq \Lambda$

$$L_{eff} = \frac{1}{2}(\partial_\mu \varphi_g)^2 - \frac{1}{2}m_g^2\varphi_g^2 + \frac{a_3}{3!}\frac{4\pi}{N}m_g\varphi_g^3 + \frac{a_4}{4!}\left(\frac{4\pi}{N}\right)^2\varphi_g^4 + \frac{a_5}{5!}\frac{1}{m_g}\left(\frac{4\pi}{N}\right)^3\varphi_g^5 + \cdots$$

$$+ \frac{1}{2}(\partial_\mu \phi)^2 - N^2\Lambda^4\left(\frac{c_2}{2}\left(\frac{\phi}{N f_a}\right)^2 + \frac{c_4}{4!}\left(\frac{\phi}{N f_a}\right)^4 + \cdots\right)$$

$$+ \left(\frac{\phi}{N f_a}\right)^2\left(\frac{g_1}{4\pi N}m_g^3\varphi_g + \frac{g_2}{2!}m_g^2\varphi_g^2 + \cdots\right) + \sum L_{eff}\left(\phi, \varphi_g(J^{PC})\right)$$

$m_g \simeq 6\Lambda$

$m_a \sim 10^{-12}\text{eV}\left(\frac{\Lambda}{\text{MeV}}\right)^2\left(\frac{10^{15}\,\text{GeV}}{f_a}\right)$

$\tau_{\varphi_g} \sim 10^{18}\text{Gyr}\left(\frac{N}{3}\right)^6\left(\frac{f_a}{10^{13}\text{GeV}}\right)^4\left(\frac{\text{GeV}}{m_g}\right)^5$

Stability ensured by the lightness of scalars

Dark Axion & Dark Glueball DM

$$L_{dark} = -\frac{1}{4g_h^2}GG + \frac{1}{2}(\partial_\mu\phi)^2 + \frac{\phi}{32\pi^2 f_a}G\tilde{G}$$

Confining phase transition at $T_g = T_{g,c} \simeq \Lambda$

* *Taking SU(N) as the hidden gauge group*
* *Decoupled from the SM from the beginning*

Axion + Glueball
Multi-component dark matter

$$L_{eff} = \frac{1}{2}(\partial_\mu\varphi_g)^2 - \frac{1}{2}m_g^2\varphi_g^2 + \frac{a_3}{3!}\frac{4\pi}{N}m_g\varphi_g^3 + \frac{a_4}{4!}\left(\frac{4\pi}{N}\right)^2\varphi_g^4 + \frac{a_5}{5!}\frac{1}{m_g}\left(\frac{4\pi}{N}\right)^3\varphi_g^5 + \cdots$$

$$+ \frac{1}{2}(\partial_\mu\phi)^2 - N^2\Lambda^4\left(\frac{c_2}{2}\left(\frac{\phi}{N f_a}\right)^2 + \frac{c_4}{4!}\left(\frac{\phi}{N f_a}\right)^4 + \cdots\right)$$

Self Interacting Dark Matter: φ_g

Wave Dark Matter: ϕ

$$m_g \simeq 6\Lambda$$
$$m_a \sim 10^{-12}\text{eV}\left(\frac{\Lambda}{\text{MeV}}\right)^2\left(\frac{10^{15}\text{ GeV}}{f_a}\right)$$

$$\tau_{\varphi_g} \sim 10^{18}\text{Gyr}\left(\frac{N}{3}\right)^6\left(\frac{f_a}{10^{13}\text{GeV}}\right)^4\left(\frac{\text{GeV}}{m_g}\right)^5$$

* *Stability ensured by the lightness of scalars*

Issues in Axion-Glueball DM Cosmology

Axion Potential in the Deconfining Phase

Before the confining phase transition, the axion gets a scalar potential via instanton contributions
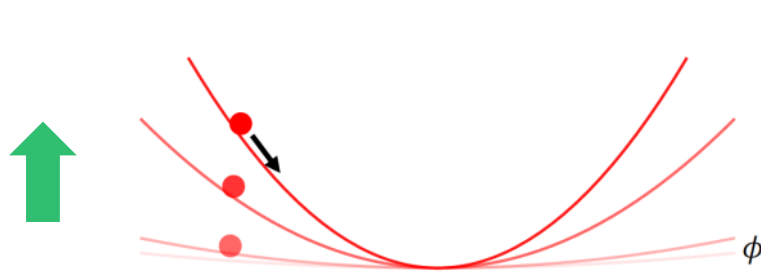
$$V(T_g, \phi) \simeq T_g^4 \, e^{-\frac{8\pi^2}{g_h^2(T_g)}} (1 - \cos\theta) \ \text{ for } \ T_g > T_{g,c}$$

$$= \frac{1}{2} m_a^2 \left(\frac{T_{g,c}}{T_g}\right)^{\frac{11}{3}N-4} \phi^2 + \cdots \qquad \boxed{\theta \equiv \phi/f_a}$$

Energy flow from the gluons to the axions as the gluon temperature decreases ($\rho_g \to \rho_a$).

Naively, this implies $\rho_g = \rho_g(T_g) = \dfrac{\pi^2}{15}(N^2 - 1)\, T_g^4$ *decreases faster than* $1/a^4$

What about the entropy? $\qquad s_g = \dfrac{\rho_g + p_g}{T_g} = \dfrac{4\,\pi^2}{45}(N^2 - 1)\, T_g^3 \propto \dfrac{1}{a^3}$?

→ *Conceptual problems*

Is there also the flow of the entropy?

Is entropy conserved (i.e. $s_g \propto 1/a^3$)?

What is the correct form of the entropy for coupled fluids?

Gluon fluid

Axion fluid
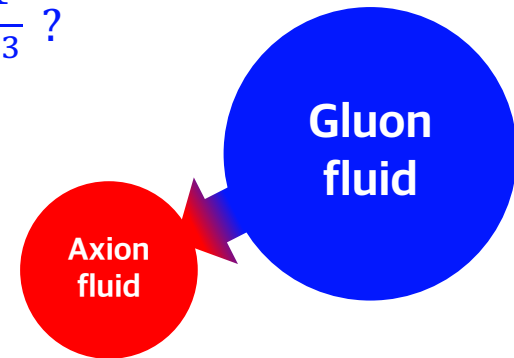
Gluo-thermodynamics with the Theta term

Gluons in thermal equilibrium with T_g; Thermodynamic relations hold during the cosmic expansion.

$$f = -p \qquad\qquad s = -\frac{df}{dT} \qquad\qquad \rho = Ts - p$$

f = free energy density

When $\phi = 0$, the free energy density depends only on T_g:

$$f_g(T_g) = -\frac{T_g}{V}\ln Z_g = -\frac{T_g}{V}\ln\int dA_a \exp\left[-\int_0^{1/T_g} dt\, d^3\vec{x}\left(\frac{1}{4g_h^2}G^a G^a\right)\right]$$

Whereas for $\phi \neq 0$ ($\theta \neq 0$)

$$p_{g+\theta} = -f_{g+\theta} = \frac{T_g}{V}\ln\int dA_a \exp\left[\int_0^{1/T_g} dt\, d^3\vec{x}\left(\frac{1}{4g_h^2}G^a G^a + i\frac{\theta}{32\pi^2}G^a\tilde{G}^a\right)\right]$$

$$= -f_g(T_g) - V(T_g,\phi) \equiv p_g(T_g) - V(T_g,\phi)$$

$$s_{g+\theta} = -df_{g+\theta}/dT_g, \qquad\qquad \rho_{g+\theta} = T_g s_{g+\theta} - p_{g+\theta} \equiv \rho_g(T_g,\phi) + V(T_g,\phi)$$

$\rho_g(T_g,\phi)$ (depends not only on T_g but also on ϕ) and $p_g(T_g)$ are interpreted as the energy density and the pressure of the gluons only. Then, we confirm

$$s_{g+\theta} = \frac{dp_g}{dT_g} - \frac{\partial V(T_g,\phi)}{\partial T_g} = \frac{\rho_g + p_g}{T_g} = s_g \propto \frac{1}{a^3}! \quad \text{(entropy conservation)}$$

Axion Potential in the Confining Phase

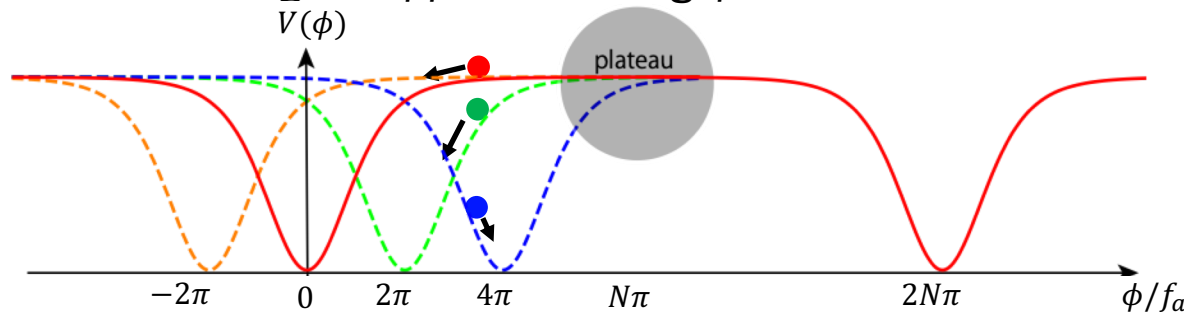*Instanton approximation is no longer valid.* Branch structure (k=1,...,N) emerges

$$L = -\frac{1}{4g_h^2}GG + \frac{\theta}{32\pi^2}G\tilde{G}$$

$$e^{iS[\theta+2\pi]} = e^{iS+2in\pi} = e^{iS[\theta]}$$
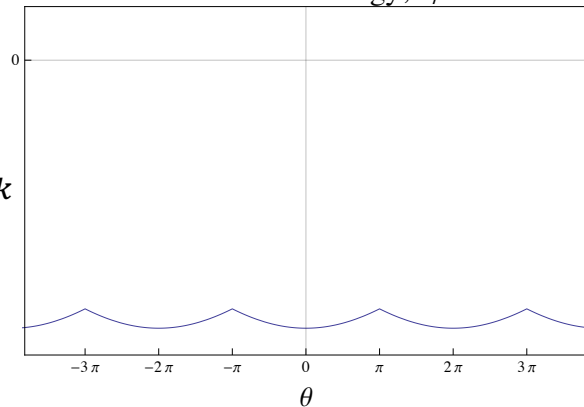
$$E_k(\theta + 2\pi) \neq E_k(\theta)$$

Witten hep-th/9807109

Phenomenological applications: e.g. pure natural inflation 1706.08522, 1711.10490)
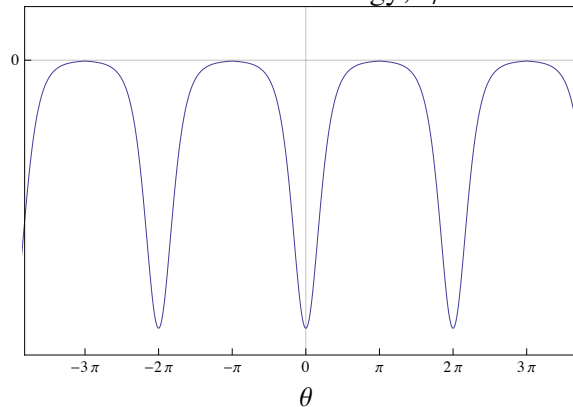

* Is there any nontrivial evolution of the axion DM?

* What is the tunneling rate between different branches?

$(\lambda = g_h^2 N)$

Vacuum Casimir Energy, λ/N<<1

Vacuum Casimir Energy, λ/N>>1

$$E_{k+1} = \frac{-\Lambda_*^4}{\left(1 + \left(\frac{\lambda}{4\pi^2}\frac{\theta + 2\pi k}{N}\right)^2\right)^3}$$

Axion potential in holographic description: 1105.3740, Dubovsky, Lawrence, and Roberts

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Matching between Lattice and Dual Descriptions

In lattice calculation from 1702.01049 (for N=3, 4, 6) Bonati, D'Elia, Rossi, Vicari

$$E(\theta) - E(0) = \frac{1}{2}\chi\theta^2(1 + b_2\theta^2 + b_4\theta^4 + \cdots)$$

$$= \frac{1}{2}\chi\theta^2\left(1 + \bar{b}_2\left(\frac{\theta}{N}\right)^2 + \bar{b}_4\left(\frac{\theta}{N}\right)^4 + \cdots\right)$$

$$\boxed{\left(\frac{\chi}{\sigma^2}\right)_{N\to\infty} = 0.02, \qquad \bar{b}_2 = -0.23(3), \qquad |\bar{b}_4| \le 0.1}$$

In holographic dual description from 1105.3740 (β=the radius of extra dimension)

$$E_{k=1}(\theta) = -\frac{2}{3^7\pi^2}\left(\frac{\lambda N^2}{\beta^4}\right)\left(1 + \left(\frac{\lambda}{4\pi^2}\frac{\theta}{N}\right)^2\right)^{-3} = E(0) + \frac{1}{2}\chi_*\theta^2\left(1 + \bar{b}_{2*}\left(\frac{\theta}{N}\right)^2 + \cdots\right)$$

$$\boxed{\frac{\chi_*}{(\pi/\beta)^4} \simeq 0.03\left(\frac{\lambda}{4\pi}\right)^3, \qquad \bar{b}_{2*} = -\frac{2}{\pi^2}\left(\frac{\lambda}{4\pi}\right)^2 \simeq -0.2\left(\frac{\lambda}{4\pi}\right)^2}$$
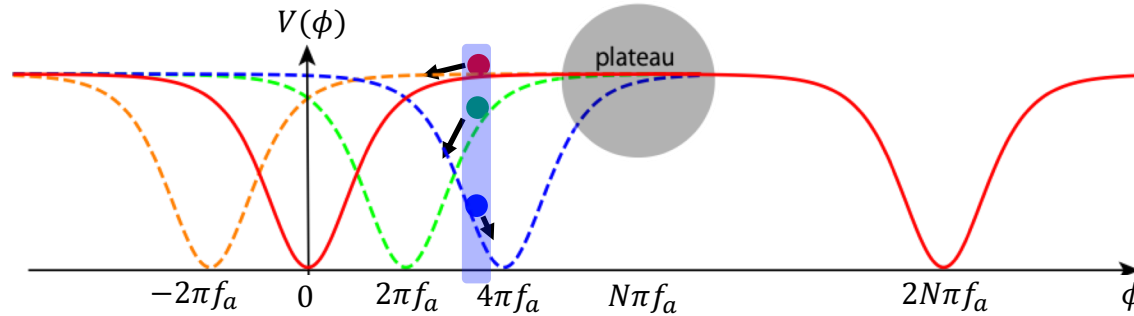
Matching
$$\lambda = g_h^2 N \simeq 4\pi$$

Effective Axion Potential

The axion potential for kth branch (k=1,...,N) with $h(x) = h(x + 2\pi)$
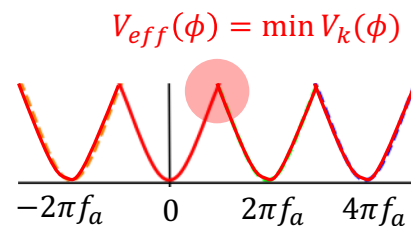
$$V_{k+1}(\phi) = N^2 \Lambda^4 h\left(\frac{\phi}{N f_a} + \frac{2\pi k}{N}\right)$$

Using the tunneling rate estimated in dual picture (1105.3740) with the relation $\lambda \sim 4\pi$, we get

$$\frac{\Gamma(k \to k-1)}{V_3} \sim c\,\Lambda^4\,\exp\left[-O(10^{-11})N\frac{(N/k)^3}{(1+(k/N)^2)^2}\right]$$

$\Gamma_k/H \gg 1$ around the phase transition unless $N > 10^3$ → The transitions between different branches happen <u>instantaneously</u> as the confining phase transition occurs.
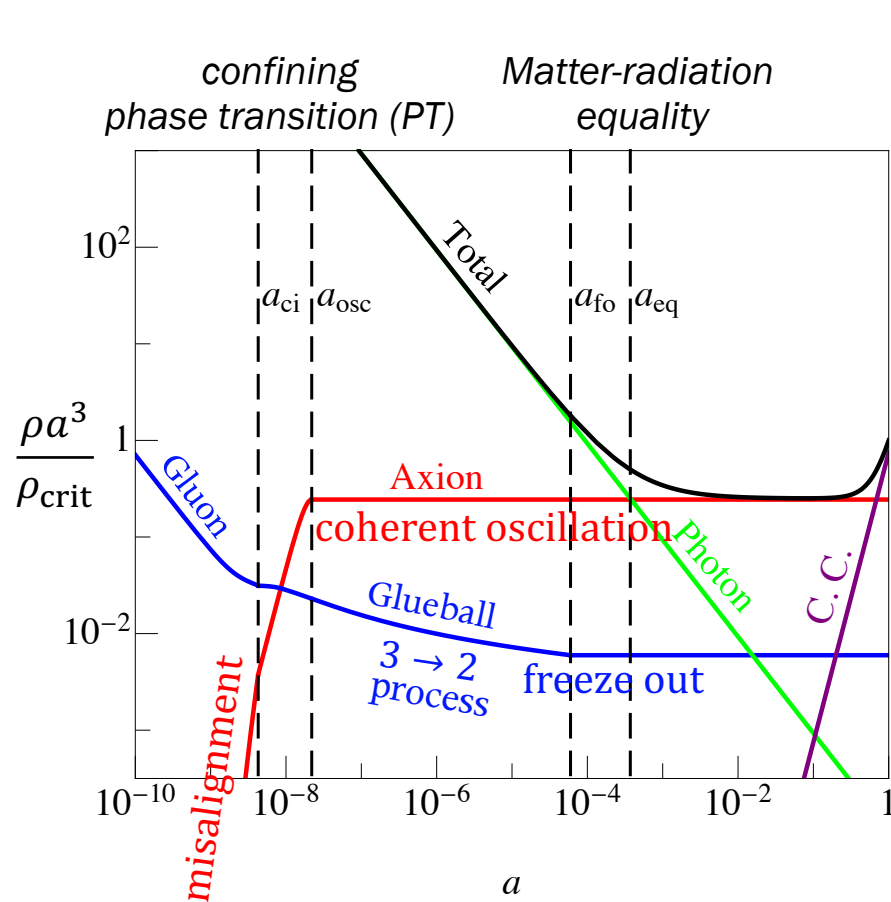
$$V_{eff}(\phi) = \min V_k(\phi)$$

$$V_{eff}(\phi) \sim \frac{\Lambda^4}{f_a^2}\phi^2\left(1 + O\left(\frac{\phi^2}{N^2 f_a^2}\right)\right)$$

$$\text{for } \phi < \pi f_a$$

Background Density Evolution

$$L_{dark} = -\frac{1}{4g_h^2}GG + \frac{1}{2}(\partial_\mu \phi)^2 + \frac{\phi}{32\pi^2 f_a}G\tilde{G}$$
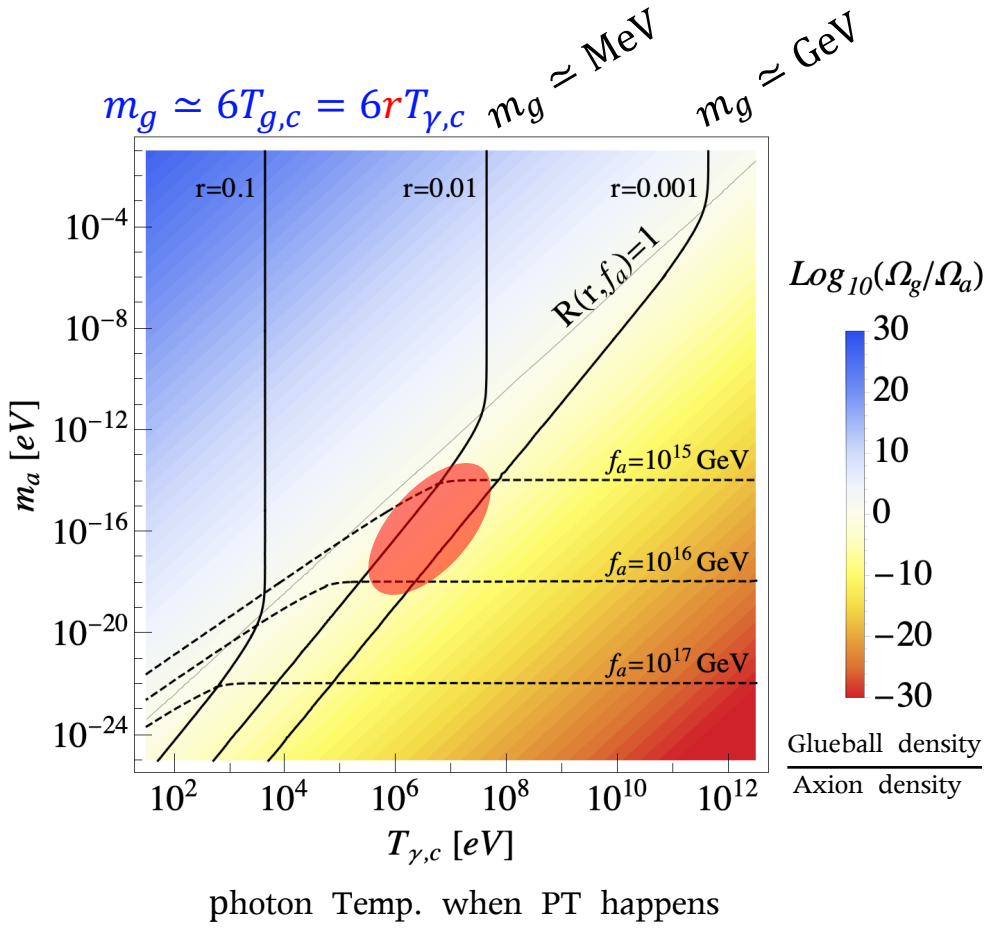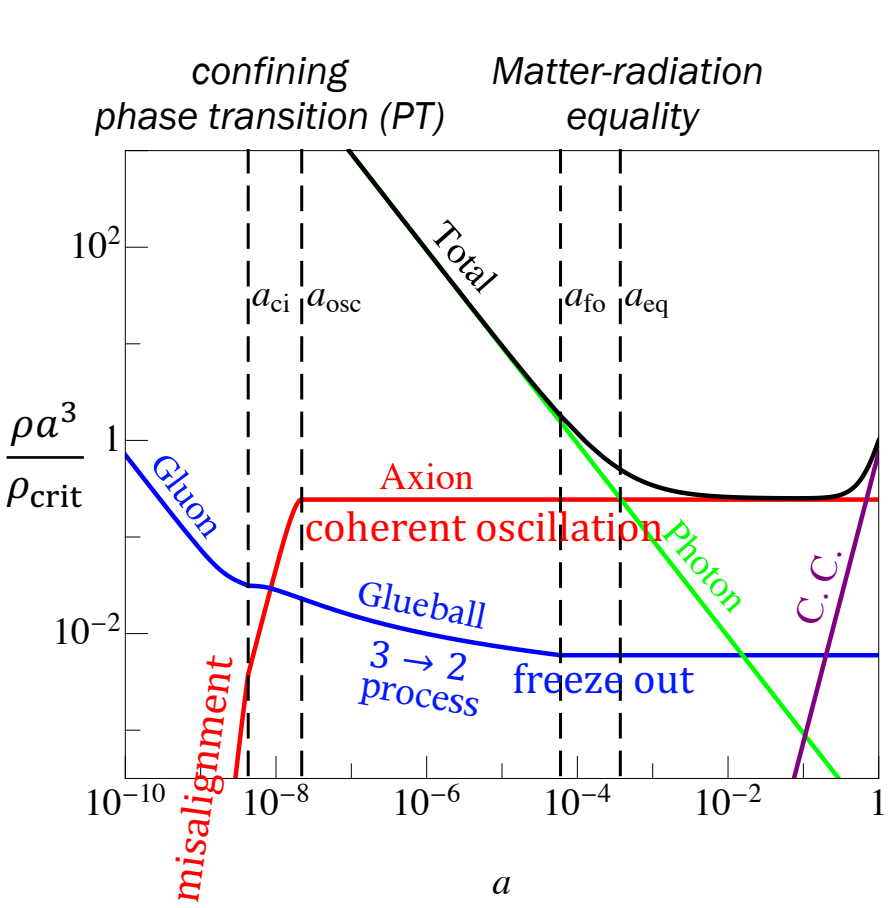
$$r = \left(\frac{T_g}{T_\gamma}\right)_{BBN}$$

photon Temp. when PT happens

Background Density Evolution

$$L_{dark} = -\frac{1}{4g_h^2} GG + \frac{1}{2}(\partial_\mu \phi)^2 + \frac{\phi}{32\pi^2 f_a} G\tilde{G}$$

$$r = \left(\frac{T_g}{T_\gamma}\right)_{BBN}$$

confining phase transition (PT)

Matter-radiation equality

$m_g \simeq 6T_{g,c} = 6rT_{\gamma,c}$ $m_g \simeq$ MeV $m_g \simeq$ GeV

$\frac{\rho a^3}{\rho_{\rm crit}}$

a

$Log_{10}(\Omega_g/\Omega_a)$

$\frac{\text{Glueball density}}{\text{Axion density}}$

photon Temp. when PT happens
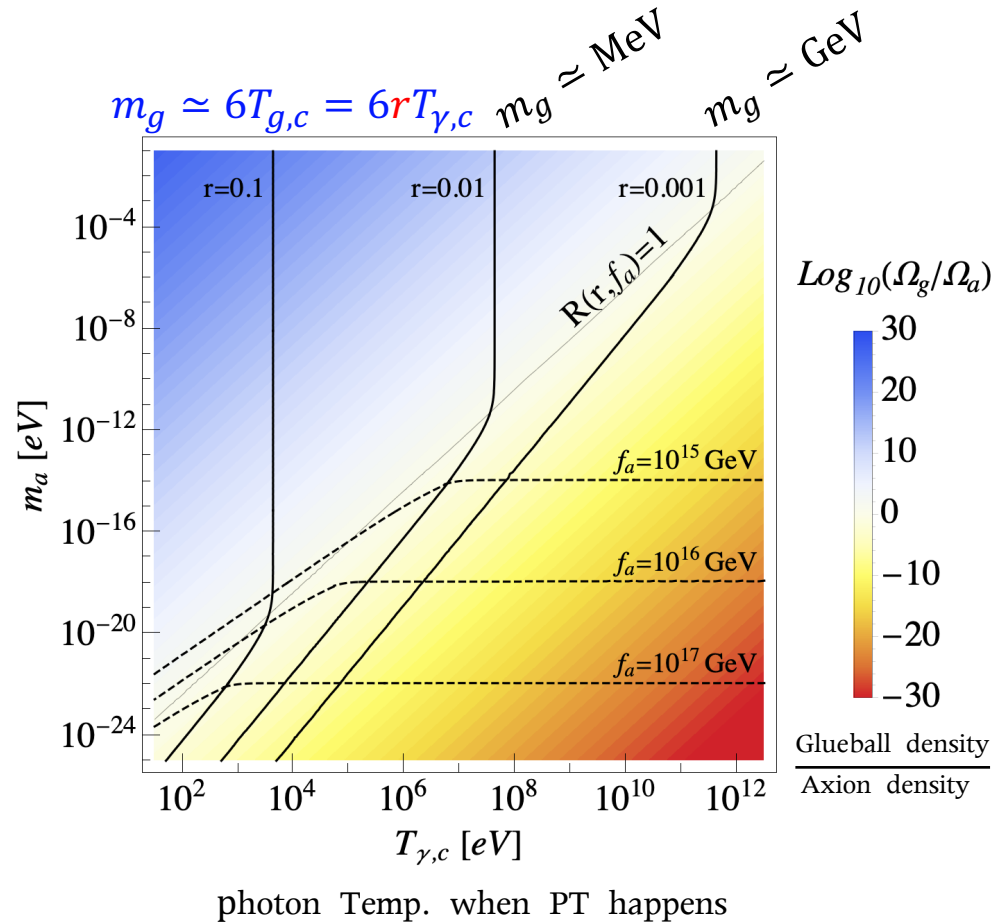
is our interest: dominant axion DM & sub-comp. strongly interacting glueball DM

Implications of
Subcomponent Glueball Dark Matter

Supermassive Black Holes (SMBH) at High z

In the 2010s, new observations of the quasars lead to the discovery of SMBHs around z = 7.

Around the redshift $z = 7$ ($t \simeq 770$ Myr $\sim 0.05\ t_U$),

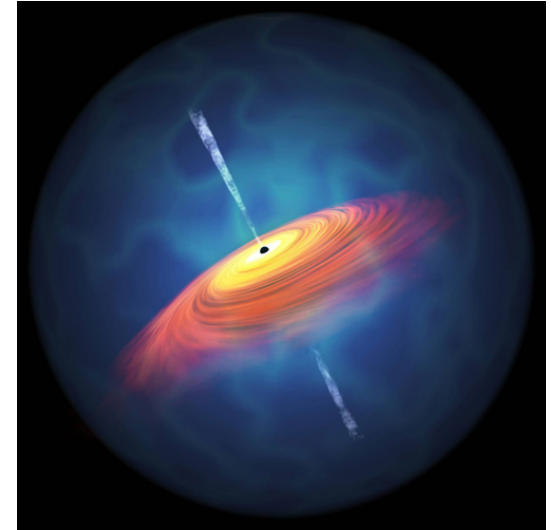*J1342+0928 ($z = 7.54, M_{BH} = 0.8 \times 10^9 M_\odot$, 1712.01860)*

J1120+0641 ($z = 7.09, M_{BH} = 2.0 \times 10^9 M_\odot$, 1106.6088)

J2348-3054 ($z = 6.89, M_{BH} = 2.1 \times 10^9 M_\odot$, 1311.3260)

J0109-3047 ($z = 6.75, M_{BH} = 1.5 \times 10^9 M_\odot$, 1311.3260)

J0305-4150 ($z = 6.61, M_{BH} = 1.0 \times 10^9 M_\odot$, 1311.3260)

J0100+2802 ($z = 6.3, M_{BH} = 1.2 \times 10^{10} M_\odot$, 1502.07418)

The origins of these SMBHs are not clear. It may originate from strongly self-interacting subcomponent DM
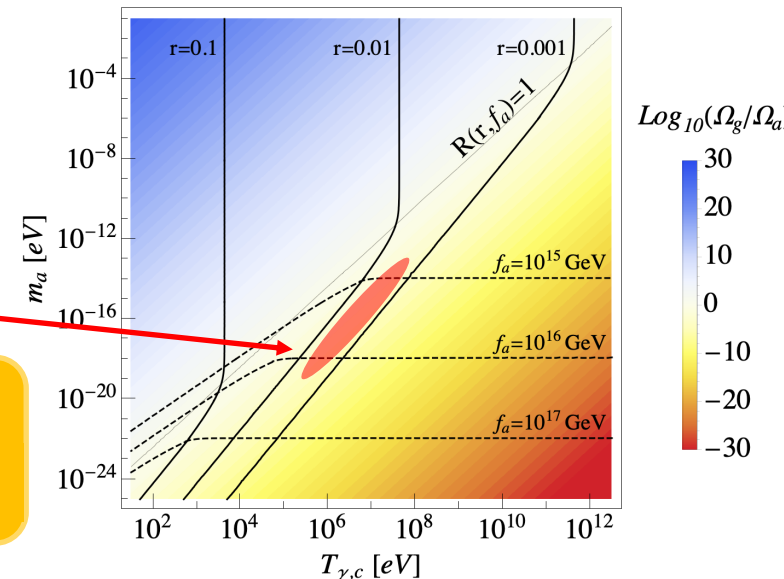
1501.00017 Pollack, Spergel, Steinhardt

(dark glueball subcomponent DM)

$$\sigma_{g2 \to 2} \sim \frac{1}{16\pi}\left(\frac{4\pi}{N}\right)^4 \frac{1}{m_g^2}$$

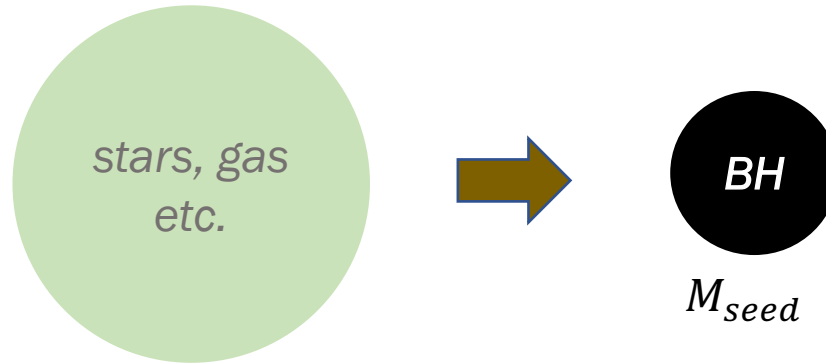$$m_g \sim 0.01 - 1\ \text{MeV}$$
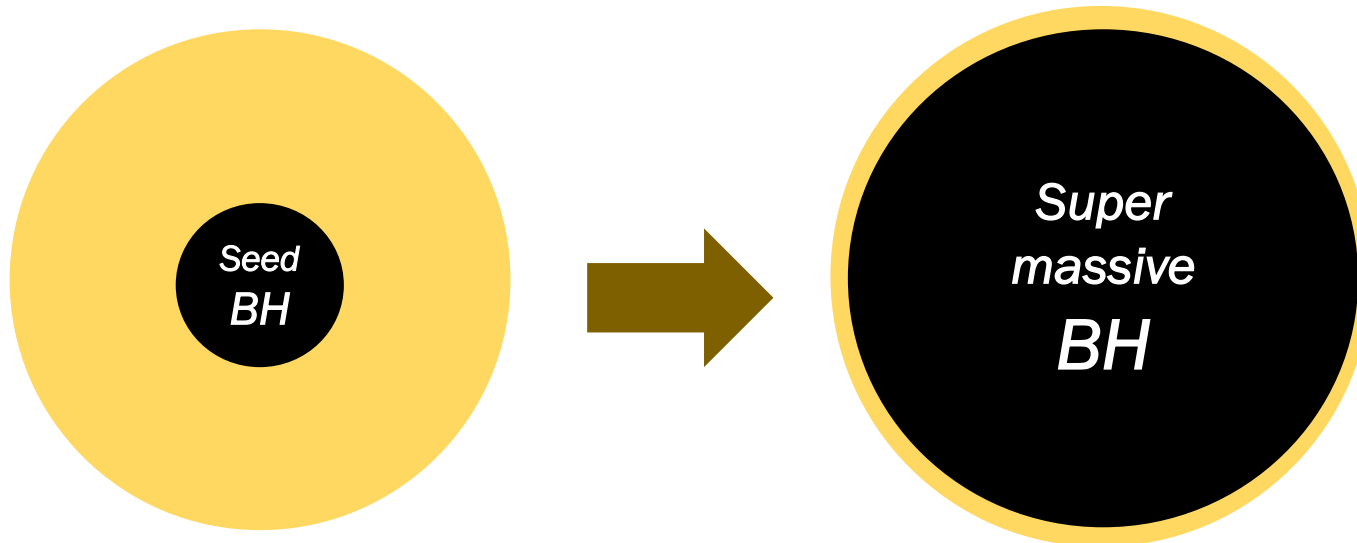$$m_a \sim 10^{-18} - 10^{-14}\ \text{eV}$$

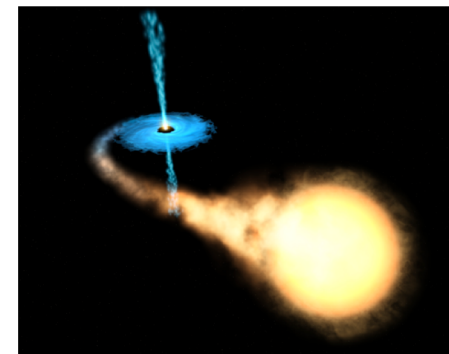# Formation and Growth of BHs

Basic process

1) Formation of the seed black hole (e.g. stellar evolution, direct formation from gas collapse)

M_{seed}

2) Growth by accretion of baryons (and dark matters) or mergers with other black holes

$$M_{BH}(t_{col}) = M_{seed} \rightarrow M_{BH}(t_{obs}) = M_{SMBH} \gg M_{seed}$$

Accretion rate is enough for SMBHs at High z?

If the seed black hole is formed inside the virialized massive halo, it is reasonable that it happens $z < 20 - 30$

From the Eddington limit, $M_{BH}(t_{obs}) \leq M_{seed} \exp\left(\frac{t_{obs} - t_{col}}{t_{Sal}}\right) \equiv M_{seed}\,\mathcal{A}_{acc}$

$(t_{Sal} = 45\text{ Myr})$

During matter domination, the age of the Universe, $t(z) = 550\text{ Myr}\left(\frac{10}{1+z}\right)^{3/2}$

For $z_{obs} = 7$, $z_{col} = 15$, $\mathcal{A}_{acc} \sim (2 - 6)10^4$ → $M_{seed} \sim 10^5\,M_\odot$ at $z_c = 15$

For $z_{obs} = 7$, $z_{col} = 30$, $\mathcal{A}_{acc} \sim (6 - 10)10^5$ → $M_{seed} \sim 10^4\,M_\odot$ at $z_c = 30$

Within the CDM framework

(Madau & Rees 2001, Heger et al. 2003, Wise & Abel 2005)

As the remnants of the Pop III stars ($z \sim 20$), $M_{seed} = O(100)\,M_\odot$

The larger growth rate within a certain period? (super-Eddington accretion)

$$M_{BH}(t) = M_{seed}\,e^{f_{Edd}\,t/\tau_{Sal}}, \qquad (f_{Edd} > 1)$$

Direct collapse of gas into the BH ? $M_{seed} = O(10^{4-6})\,M_\odot$ *but should prevent fragmentation, star formation before the collapse*

(Loeb & Rasio 1994, Eisenstein & Loeb 1995, …)

Collapsing star cluster? From mergers of Pop II stars, $M_{seed} = O(10^3)\,M_\odot$ *but maybe useful only for explaining observed quasars at z<5*

(Devecchi & Volonteri 2009)

Beyond the CDM framework 1501.00017

Beyond the CDM framework : multi-component with strongly interacting sub-comp. DM

M_h

CDM halo
(main component)

$f_{sub}M_h$

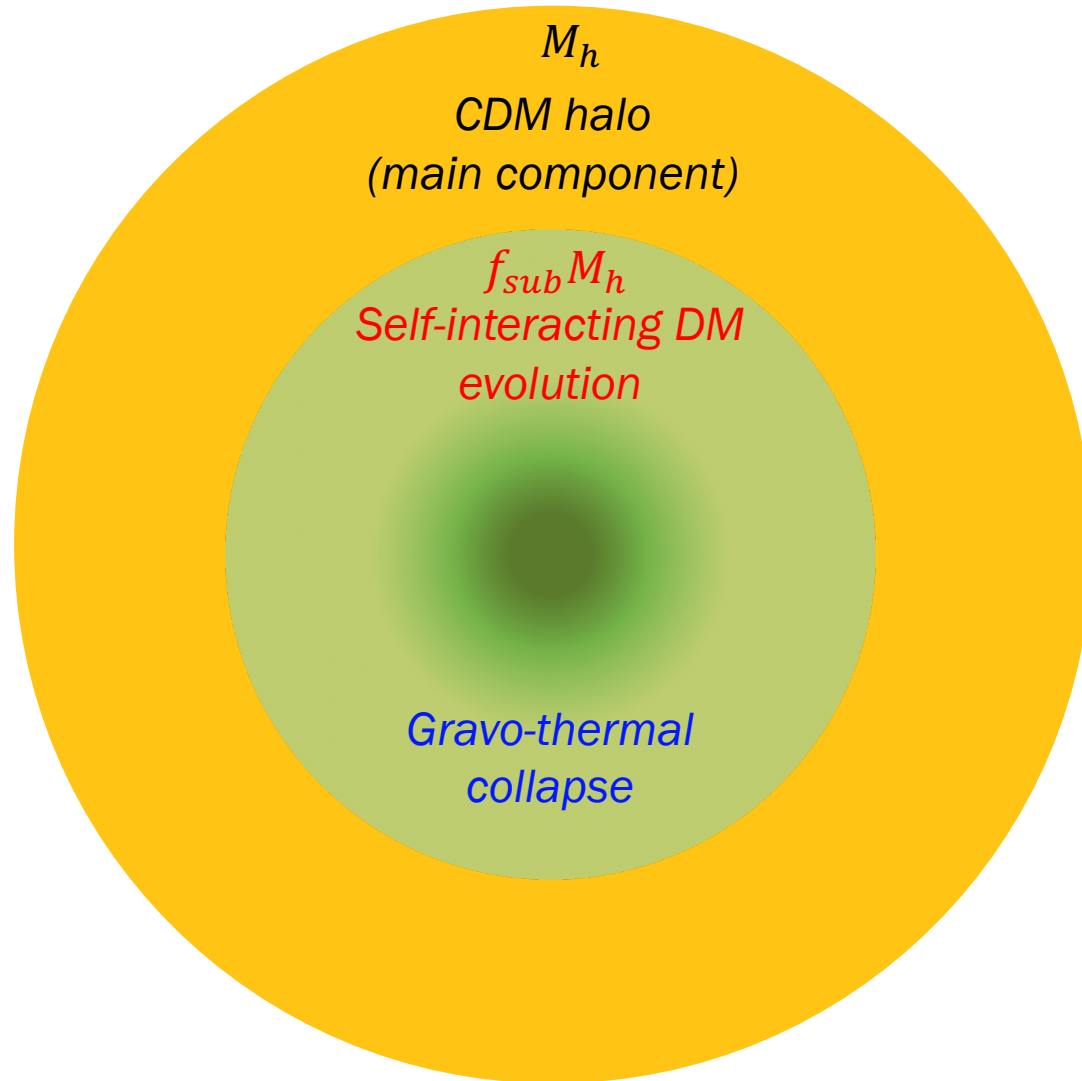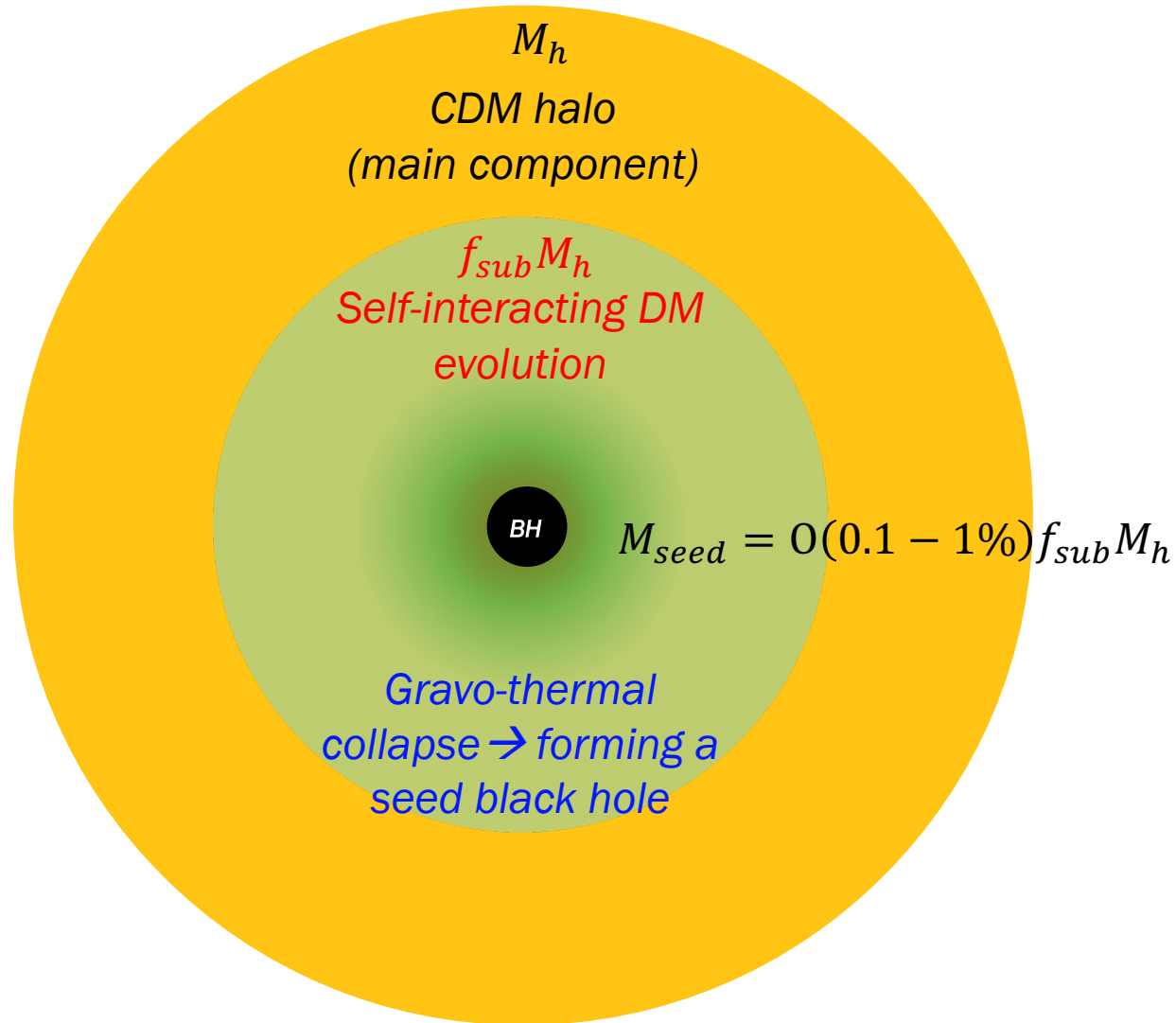Self-interacting DM halo
with a fraction (f_{sub}) < 0.1
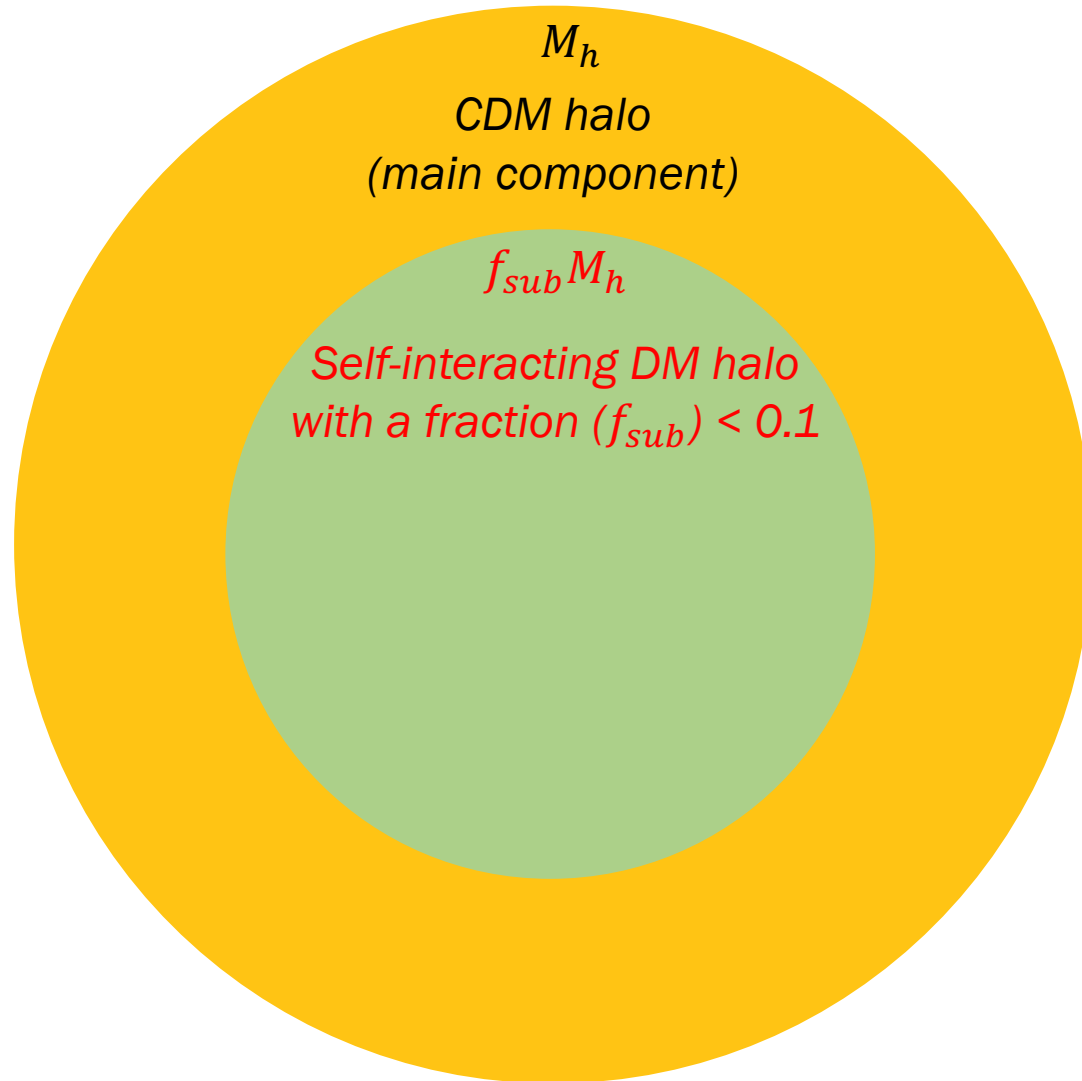
Beyond the CDM framework 1501.00017

Beyond the CDM framework : multi-component with strongly interacting sub-comp. DM

Beyond the CDM framework 1501.00017

Beyond the CDM framework : multi-component with strongly interacting sub-comp. DM

M_h

CDM halo
(main component)

$f_{sub}M_h$
Self-interacting DM
evolution

BH

$M_{seed} = O(0.1 - 1\%)f_{sub}M_h$

Gravo-thermal
collapse → forming a
seed black hole

Beyond the CDM framework 1501.00017

Beyond the CDM framework : multi-component with strongly interacting sub-comp. DM

M_h

CDM halo
(main component)

SMBH
☺
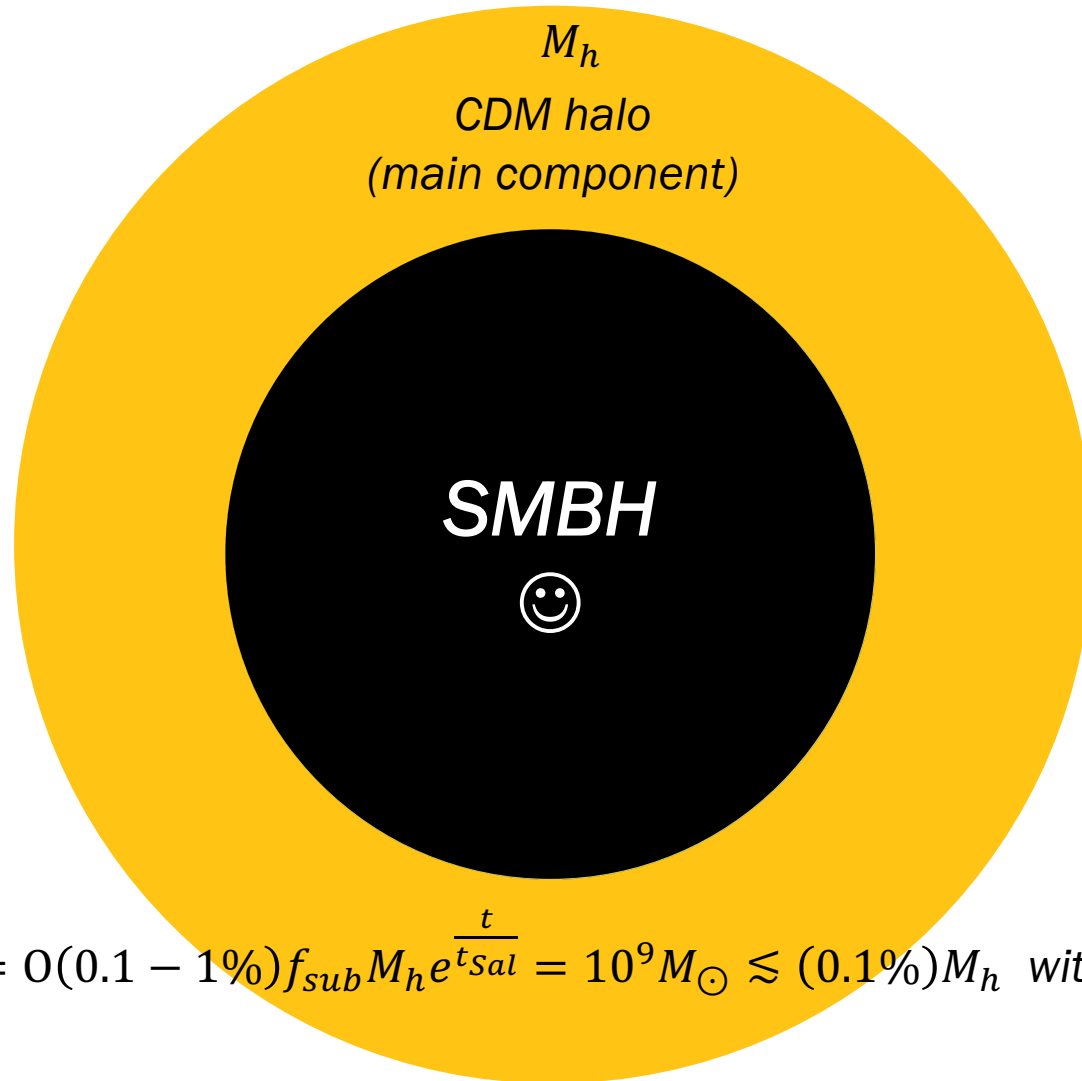
$$M_{BH}(t) = O(0.1 - 1\%) f_{sub} M_h e^{\frac{t}{t_{Sal}}} = 10^9 M_\odot \lesssim (0.1\%) M_h \ \text{with} \ M_h = 10^{12} M_\odot$$

Gravo-Thermal Collapse 1

For the gravo-thermal collapse, maintaining thermal equilibrium is an important condition.

Therefore the *"relaxation time"* *should be shorter than the age of the Universe for a given z.*

How short? *Numerical calculation is necessary*
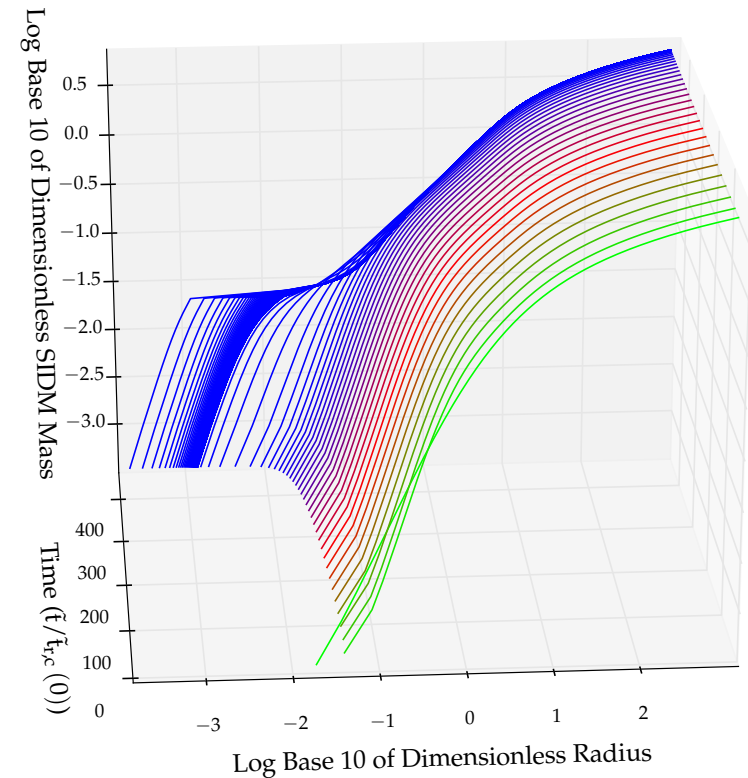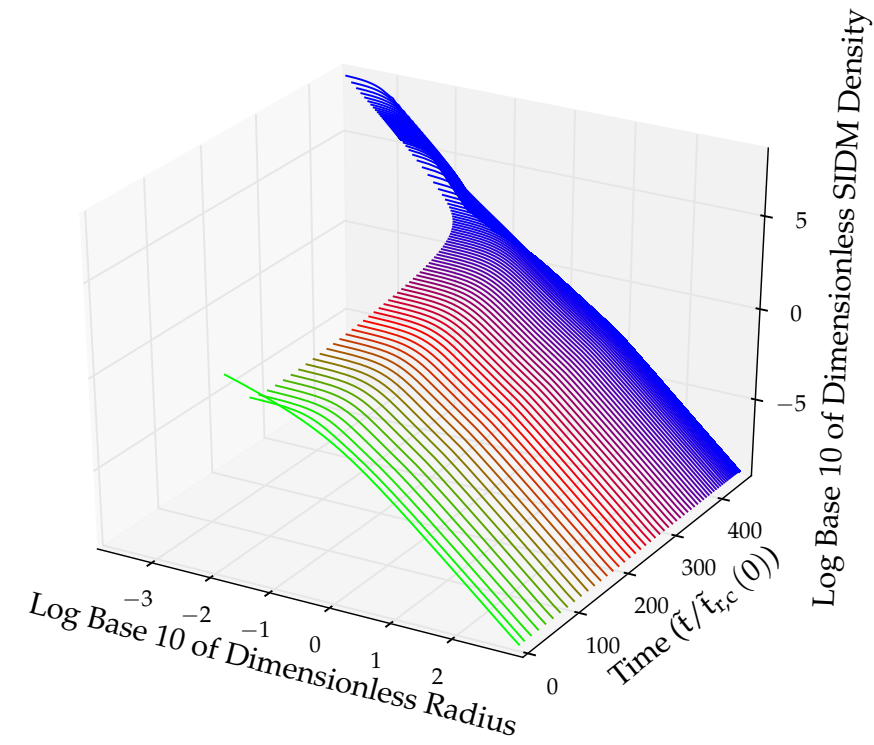
1501.00017 for the isolated halo with $f_{sub}=1$

Gravo-Thermal Collapse 2

For the gravo-thermal collapse, maintaining thermal equilibrium is an important condition.

Therefore the "relaxation time" should be shorter than the age of the Universe for a given z.

How short? Numerical calculation is necessary
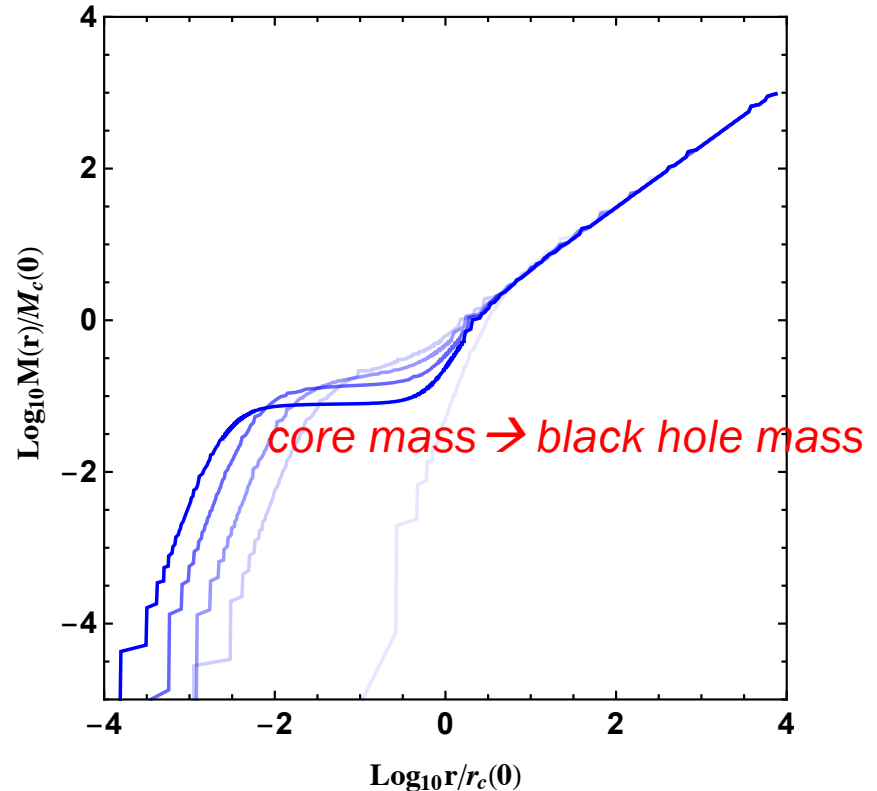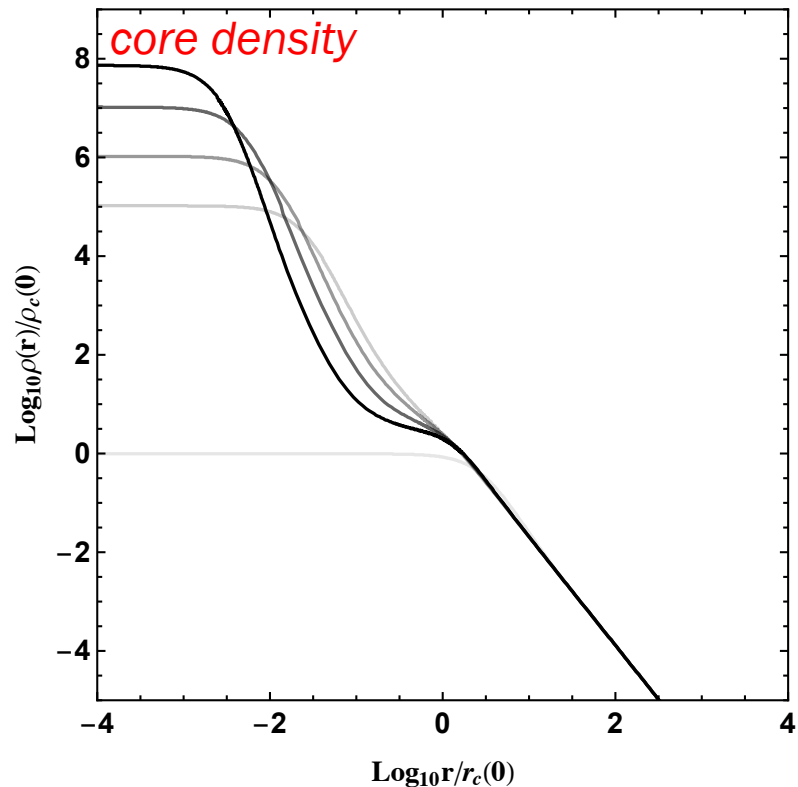
Balberg et.al. 0110561 for the isolated halo with $f_{sub}=1$

Gravo-Thermal Collapse 3

Numerical calculations estimate the collision time as

$$\Delta t_{col} \simeq 480 \, t_{relax}(t_i)$$

$$t_{relax}(t_i) = \frac{m_{DM}}{\sigma_{el} \rho_s(t_i) v_s(t_i)}$$

It is non-trivial to estimate the collision time for the collapse of sub-component dark matter.

There are two papers to estimate the collision time and the seed black hole mass for the isolated halo with a small fraction (f_{sub}) of self-interacting DM

$$\Delta t_{col} \simeq 480 \, t_{relax}(t_i), \qquad M_{seed} \simeq \frac{0.02}{\ln c} f_{sub} M_h$$

1501.00017, fluid approximation

$$t_{relax}(t_i) = \frac{m_{subDM}}{\sigma_{el} \, f_{sub} \rho_s(t_i) v_s(t_i)}$$

$$\Delta t_{col} \simeq \frac{480}{f_{sub}^2} \, t_{relax}(t_i), \qquad M_{seed} \simeq 0.006 f_{sub} M_h$$

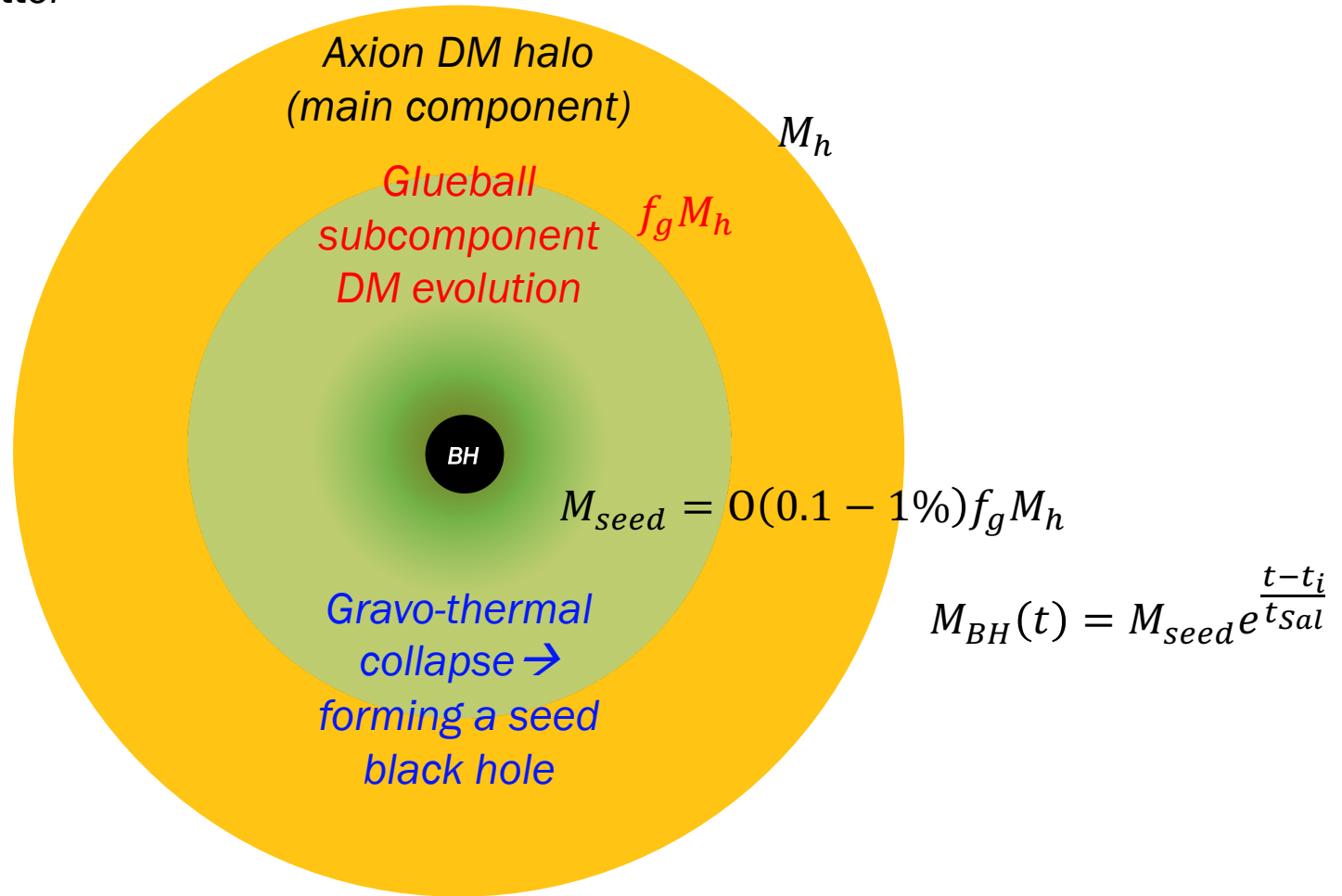1812.05088 Choquette, Cline, Cornell, N-body simulation up to f$_{sub}$=0.1

In order to explain the SMBH at z=7, ($\Delta t_{col} < t(z_{col}) < t(z = 7)$)

$$f_{sub} \sigma_{el} / m_{subDM} \sim 1 - 10 \text{ cm}^2/\text{g}$$

$$f_{sub}^3 \sigma_{el} / m_{subDM} \sim 1 - 10 \text{ cm}^2/\text{g}$$

Seed Black Hole Formation in Our Model

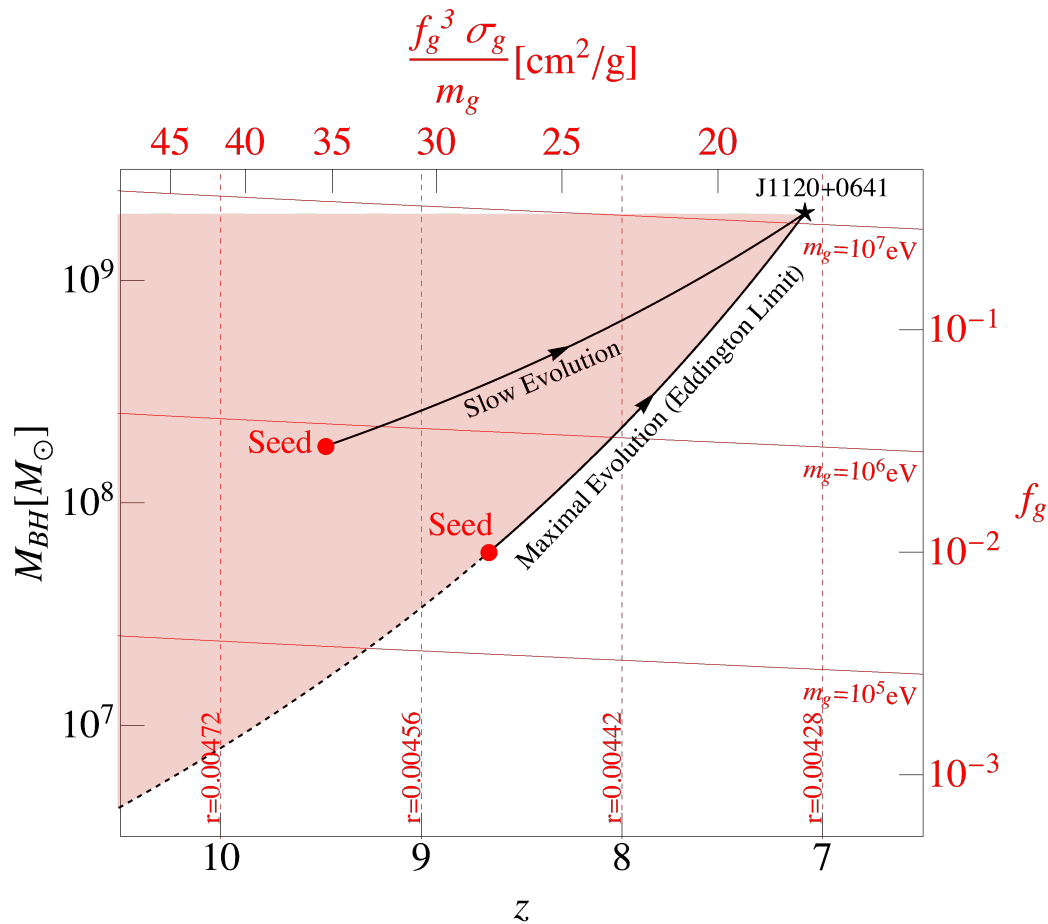*The large seed black hole can be made by the gravo-thermal collapse of the subcomponent glueball dark matter*

$$\Delta t_{col} \simeq \frac{480}{f_g^2}\, t_{relax}(t_i), \qquad M_{seed} \simeq 0.006 f_g M_h$$

$$t_{relax}(t_i) = \frac{m_g}{\sigma_g\, f_g\, \rho_s(t_i) v_s(t_i)}$$

SMBH at High z for an Isolated Host Halo

The large seed black hole can be made by the gravo-thermal collapse of the subcomponent glueball dark matter

Caveats: History of the Host Halo Mass

The assumption of the isolated host halo with a mass $M_h = 10^{12} M_\odot$ for $z \geq 10$ is not realistic: the problem of SMBH → the problem of supermassive dark matter halo.
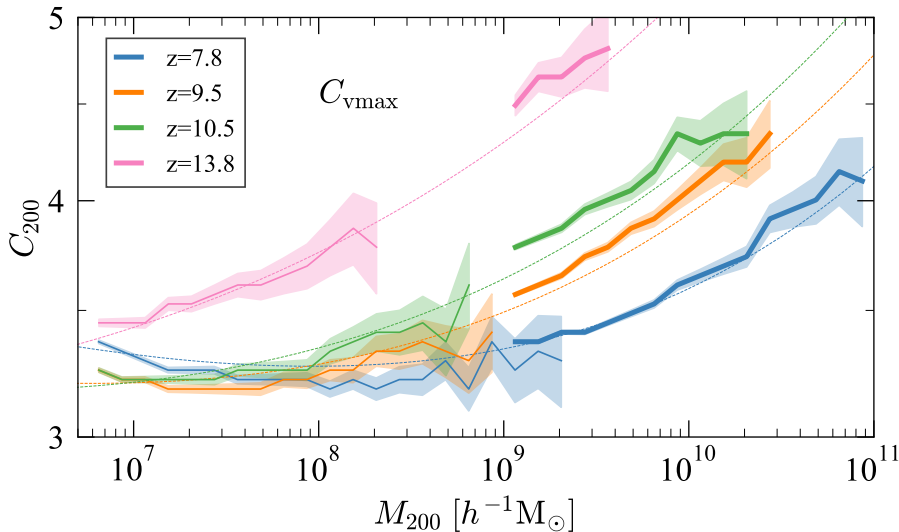
The merger history of the host halo (e.g. the history of heaviest halo) and its profile should be considered: $\{M_h(z), \rho_s(z), c(M_h, z), v_s(z)\}$

Our approach: at $z = z_i$, a seed black hole forms with $M_{seed} = 0.006\, f_g\, M_h(z_i)$

Both the black hole and the host halo grow such that $M_h(z = 7) \gtrsim 10^{12} M_\odot$ and
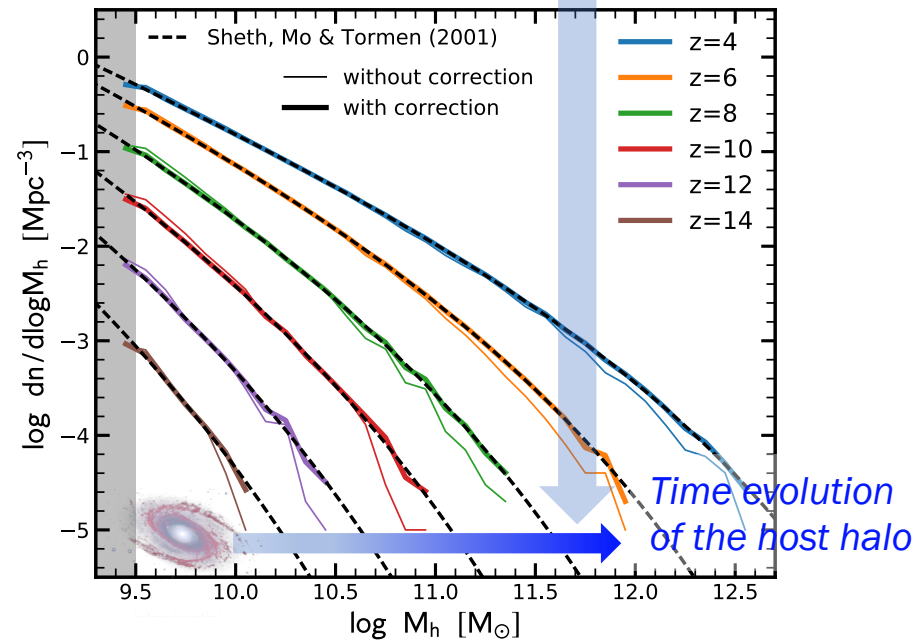
$$M_{BH}(z = 7) = M_{seed}\, e^{\frac{t(7)-t(z_i)}{t_{Sal}}} \sim 10^9 M_\odot$$

Concentration parameter for given M_h, z

halo mass function (z)

Ishiyama et al. 2007.14720

Time evolution of the host halo

Tacchella et al. 1806.03299

Caveats: Evolution of the Glueballs inside the Core

As the core becomes extremely dense after the gravo-thermal collapse accelerates, the temperature of the core also increases as $T_c \propto \rho_c^{1/3}$. This could provide nontrivial effects for the evolution of DM (e.g. formation of BEC, $T_{BEC} \propto \rho_c^{2/3}$).

For the glueball DM case, the number-changing interaction is also crucial for $m_g \lesssim$ keV → prevent the gravo-thermal collapse

$$\Delta t_{col} \left(\frac{1}{T_g} \frac{dT_g}{dt} \right)_{3 \to 2} \simeq 0.06 \left(\frac{10^{-3}}{f_g} \right) \left(\frac{10^{-3}}{v_s(t_i)} \right)^2 \left(\frac{\text{keV}}{m_g} \right)^4 \left(\frac{\rho_s(t_i)}{10^{12} M_\odot / \text{kpc}^3} \right)$$
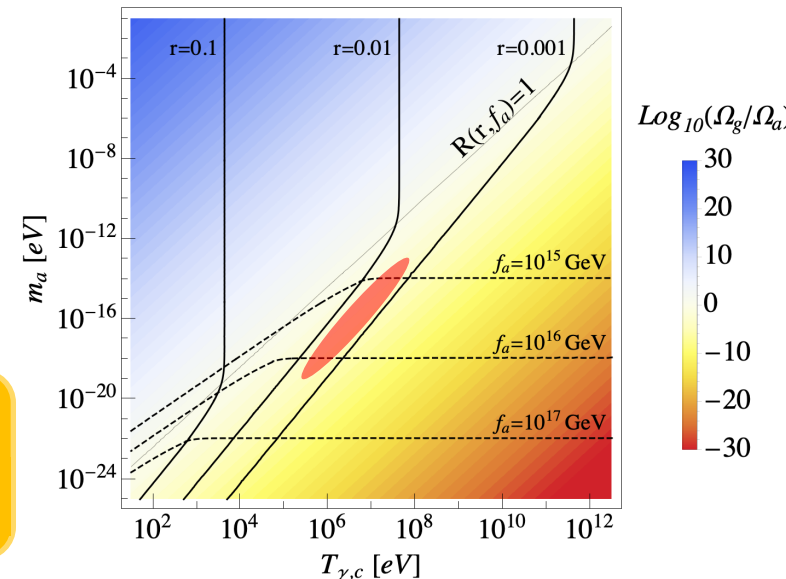
Even for $m_g \gg$ keV, it becomes gradually important ($\Gamma_{3 \to 2} \propto \rho_c^2$) during the gravo-thermal collapse. Its effect on the final formation of the BH is not clear yet.

From the cosmological history, the mass of the glueball is also bounded by its effective free-streaming length.

If $m_g \lesssim 100$ eV, the glueball DM will not form a halo because it is too warm

$$m_g \sim 0.01 - 1 \text{ MeV}$$
$$m_a \sim 10^{-18} - 10^{-14} \text{ eV}$$

Summary

The origin of the lightness of the scalar is directly related to its cosmological evolution. Two well-known mechanisms provide opposite behavior for the relation between the interaction strength and the mass of the dark matter.

We studied the minimal nontrivial model of the dark sector that comprises the coupled scalar dark matters: dark axion and dark glueball. Some nontrivial features are clarified.

Strongly interacting subcomponent glueball dark matter can provide a hint on the origin of supermassive black holes at high redshifts.

The possible observations of black hole superradiance by the dark axion & glueball bose stars and other substructures can provide the complementary hints