Quantum aspects of inflationary GWs

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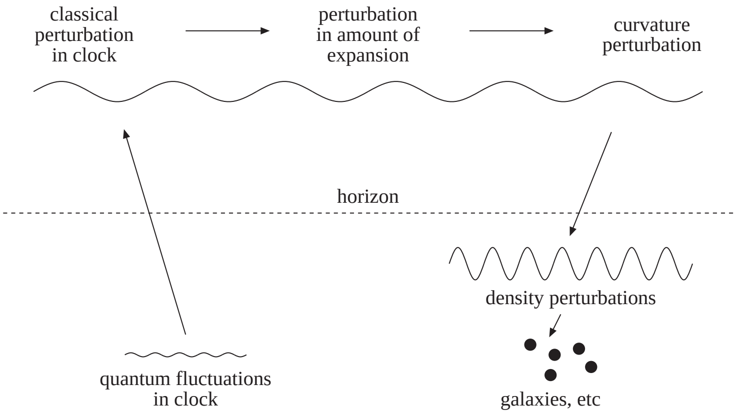
Outline

1. Introduction
2. Quantum non-linear evolution
   - Lindblad equation
   - Elements of the reduced density matrix
3. Quantum nature of inflationary GWs
4. Quantum origin of inflationary GWs
   - Black hole analogy
   - Instability of dS
5. Conclusions

Generation and evolution of perturbations



Everything seems to be clearly understood

Quantum aspects of perturbations?

If the inflationary picture is the case...

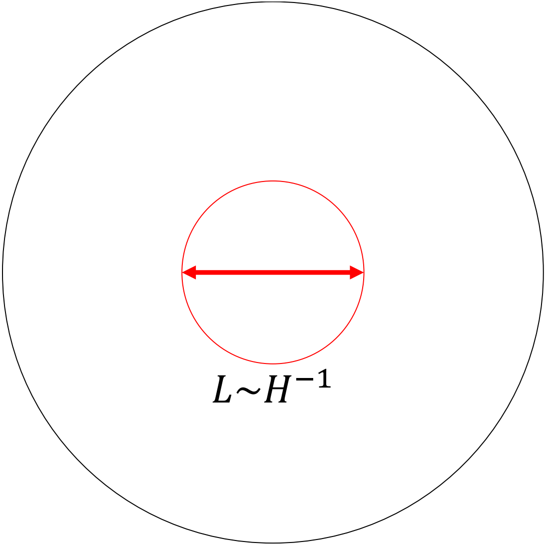
- Quantum origin of perturbations?
- Quantum-to-classical transition?
- Quantum signature of perturbations?

Important to test the inflationary paradigm

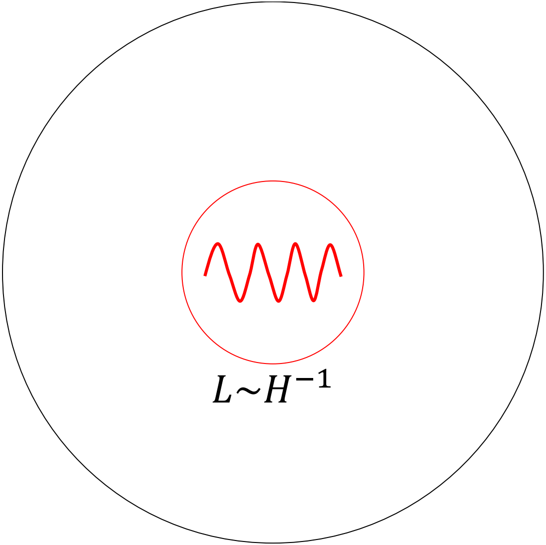
Why tensor perturbations?

- Persistent
- Well defined even during dS
- (For pure tensor) free from gauge

System and environment

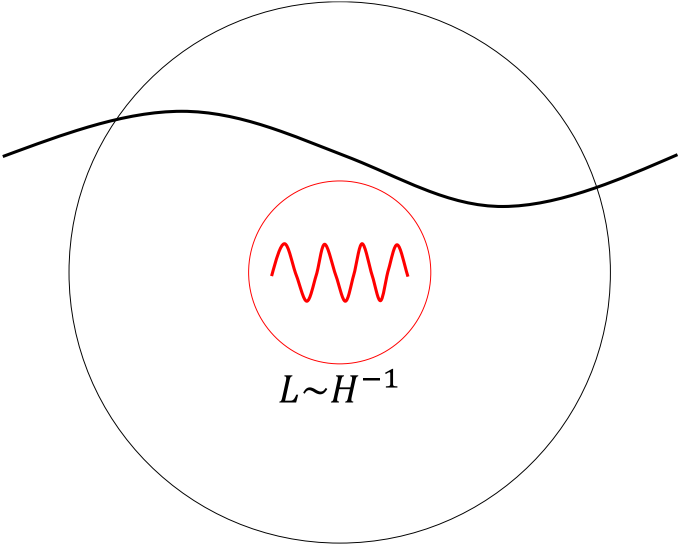


System and environment



$L \sim H^{-1}$

System and environment



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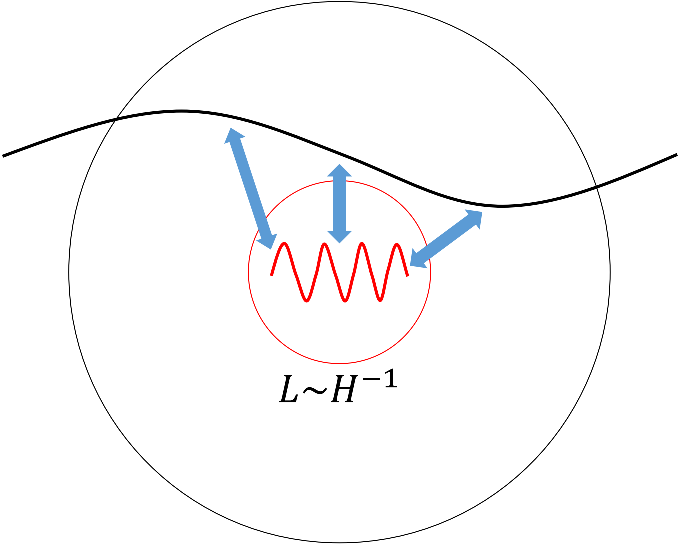

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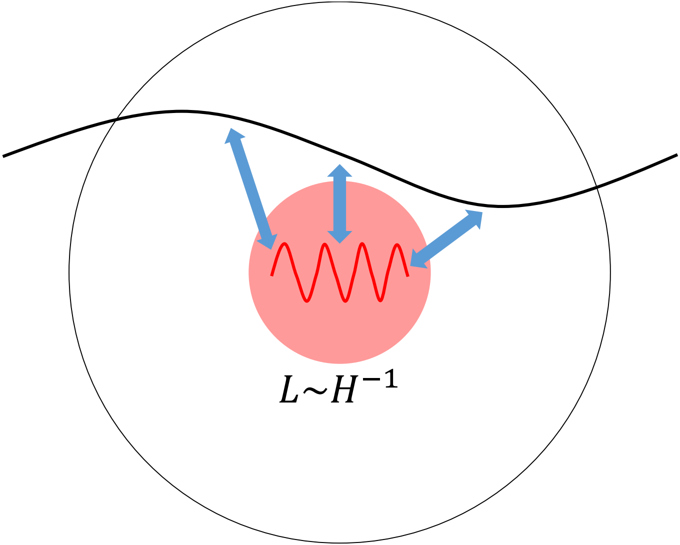

Lindblad equation

$$\frac{d\rho_{\rm red}}{d\tau} = -i[H, \rho_{\rm red}] - \frac{1}{2}\sum\left(L_\mu^\dagger L_\mu \rho_S + \rho_S L_\mu^\dagger L_\mu - 2L_\mu \rho_S L_\mu^\dagger\right)$$

cf. Talk by C. Burgess today

Lindblad equation

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- Unitary evolution: von Neumann equation

cf. Talk by C. Burgess today

Lindblad equation

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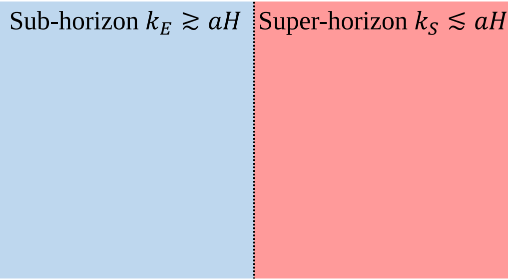
- Unitary evolution: von Neumann equation
- Non-unitary evolution: Lindblad operators
  1. Due to the interaction between system and environment

$$L_\mu \sim \left\langle \mathcal{E}_f \middle| H_{\rm int} \middle| \mathcal{E}_i \right\rangle$$

  2. Exponential decay of (some components of) $\rho_{\rm red}$
  3. Effective theory description

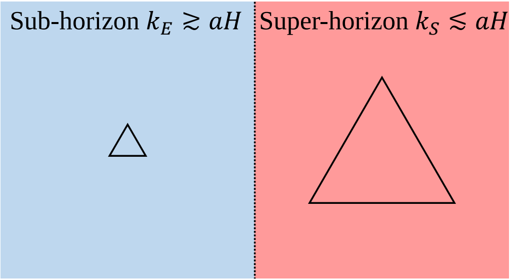
cf. Talk by C. Burgess today

Triangular contributions from cubic interactions



$$L = 1/(aH)$$

Triangular contributions from cubic interactions



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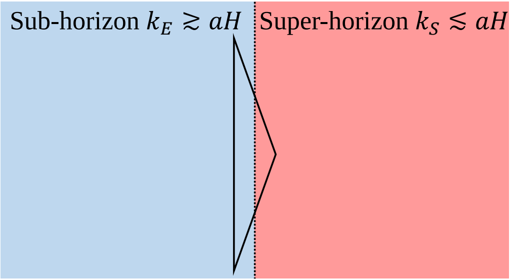
- All modes are in the environment or system sector

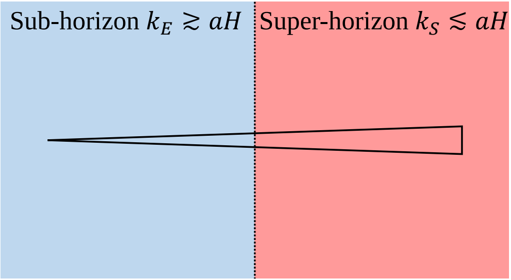
Triangular contributions from cubic interactions



$L = 1/(aH)$

- All modes are in the environment or system sector
- 2 system and 1 environment: $\boldsymbol{k}_1 \approx \boldsymbol{k}_2$ and $|\boldsymbol{k}_3| \approx 2|\boldsymbol{k}_1|$

Triangular contributions from cubic interactions



$L = 1/(aH)$

- All modes are in the environment or system sector
- 2 system and 1 environment: $\boldsymbol{k}_1 \approx \boldsymbol{k}_2$ and $|\boldsymbol{k}_3| \approx 2|\boldsymbol{k}_1|$
- 1 system and 2 environment: $\boldsymbol{k}_1 \approx -\boldsymbol{k}_2$ and $k_3 \ll k_1 \approx k_2$

Matrix notation of Lindblad equation

$$\rho_{\text{red}}|_{ab} = \langle a|\rho_{\text{red}}|b\rangle = \left(\begin{array}{ccc|c} 1-\rho_{00} & 0 & \rho_{02} & \\ 0 & \rho_{11} & 0 & 0_{3\times4} \\ \rho_{20} & 0 & 0 & \\ \hline & 0_{4\times3} & & 0_{4\times4} \end{array}\right)$$

- ρ_{red} is reduced from 1 (non-unitary evolution)
- Probability of keeping the pure squeezed state is reduced
- (Classical) probability for other processes involving 2 excitation emerges

Number of e-folds for decoherence

The e-fold for $\rho_{\text{red}}|_{00}$ to change by e^{-1}

$$\Delta N_{\text{dec}} \approx \frac{1}{3}\log\left[\frac{(2\pi)^2}{\Delta^2_{\mathscr{R}}}\frac{9}{2}\left(r\mathscr{C}_{\mathscr{S}\mathscr{E}}\right)^{-1}\right] \approx 8.38689 - \frac{1}{3}\log\left(r\mathscr{C}_{\mathscr{S}\mathscr{E}}\right)$$

$$\left(\mathscr{C}_{\mathscr{S}\mathscr{E}} \sim \log\varepsilon\,,\ \varepsilon \ll 1\right)$$

Typically $5 \lesssim \Delta N_{\text{dec}} \lesssim 10$ for a wide range of r and $\mathscr{C}_{\mathscr{S}\mathscr{E}}$

Take-home message #1

Non-linearity is essential for classicalization

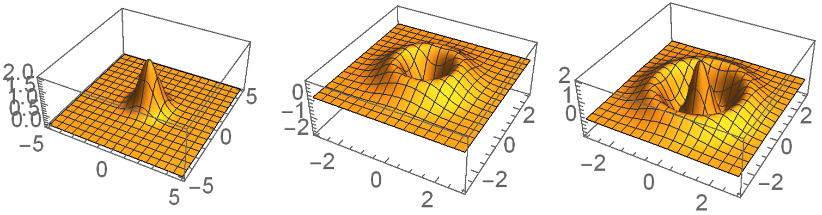
Wigner function

- Analogue of phase space distribution
- "Classical" probability distribution
- Function of *both* position and momentum

$$W(q,p) = \int_{-\infty}^{\infty} ds e^{-ips} \left\langle q + \frac{s}{2} \middle| \rho \middle| q - \frac{s}{2} \right\rangle$$

Wigner function for harmonic oscillator

$W(q, p)$ is positive definite only for a Gaussian wavefunction



N.B. Positive $W(q, p) \neq$ no quantum effect (e.g. Bell inequality)

Wavefunction for tensor perturbations

$$\widehat{U}_{\boldsymbol{k}}(\tau,\tau_0) = \overbrace{\widehat{R}_{\boldsymbol{k}}(\tau,\tau_0)}^{\text{“Rotation” operator, only phase shift}} \underbrace{\widehat{S}_{\boldsymbol{k}}(\tau,\tau_0)}_{\text{“Squeezing” operator, responsible for 2-mode squeezing}}$$

The state $|\Psi\rangle$ evolved from $|0\rangle$ at τ_0: $|\Psi\rangle = \widehat{U}(\tau,\tau_0)|0\rangle$

$$\Psi(q_{\boldsymbol{k}}, q_{-\boldsymbol{k}}) = \frac{e^{A(r_k,\varphi_k)\left(q_{\boldsymbol{k}}^2+q_{-\boldsymbol{k}}^2\right)-B(r_k,\varphi_k)q_{\boldsymbol{k}}q_{-\boldsymbol{k}}}}{\cosh(r_k/2)\sqrt{\pi}\sqrt{1-e^{4i\varphi_k}\tanh^2(r_k/2)}}$$

Wigner function for tensor perturbations

$$W\left(q_{\mathbf{k}},q_{-\mathbf{k}};p_{\mathbf{k}},p_{-\mathbf{k}}\right)=\int dxdy\,e^{-ip_{\mathbf{k}}x}e^{-ip_{-\mathbf{k}}y}\times\left\langle q_{\mathbf{k}}+\frac{x}{2},q_{-\mathbf{k}}+\frac{y}{2}\right|\rho\left|q_{\mathbf{k}}-\frac{x}{2},q_{-\mathbf{k}}-\frac{y}{2}\right\rangle$$

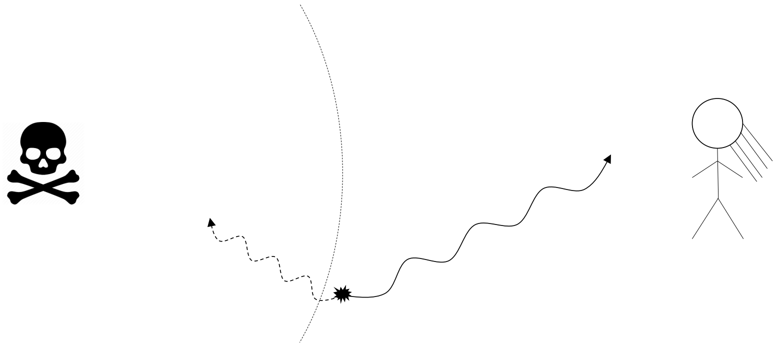
- $W_{00}=4(1-\rho_{00})w_{\mathbf{k}}\geq 0$
- $W_{11}=4\rho_{11}w_{\mathbf{k}}(-1+\cdots)$, negative for small $\left(q_{\mathbf{k}},q_{-\mathbf{k}},p_{\mathbf{k}},p_{-\mathbf{k}}\right)$
- $W_{20}=2\rho_{20}w_{\mathbf{k}}(\cdots)e^{-3i\theta_{\mathbf{k}}}$

N.B. $W(0,0;0,0)=4(1-2\rho_{00})$, IR divergent

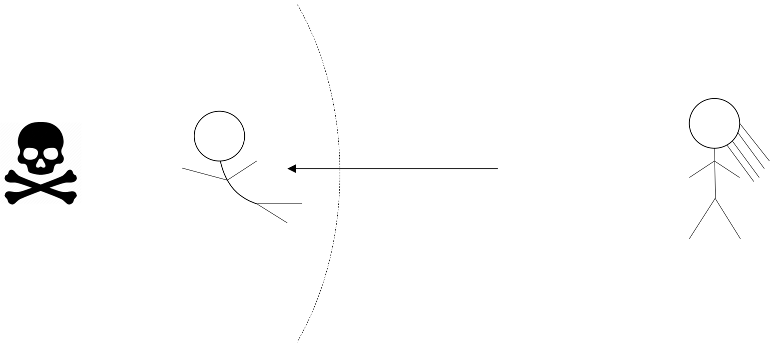
Take-home message #2

Classicality includes quantum nature

Hawking radiation in dS



Hawking radiation in dS



Hawking radiation in dS

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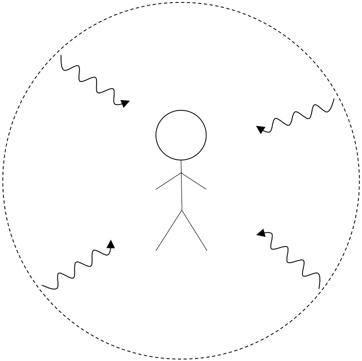
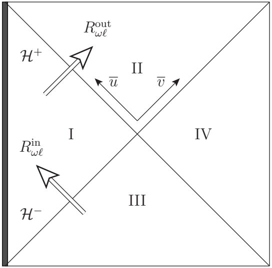


Radiation from dS horizon

$$ds^2 = -\left(1 - H^2 r_s^2\right) dt_s^2 + \frac{dr_s^2}{1 - H^2 r_s^2} + r_s^2 d\Omega$$



+-frequency modes w.r.t. t_s and $\overline{u}$ are related by Bogoliubov x-form

$$\underbrace{b^{\text{out}}_{\omega\ell m}}_{\text{for } t_s} \sim e^{\pi\omega/(2H)} \underbrace{a^{\text{out}}_{\omega\ell m}}_{\text{for } \overline{u}} + \text{h.c.}$$

cf. Talk by Y. Tsai today

Vacuum states and horizon thermal flux

Different sets of operators define different vacua

1. Annihilated by $b^{\text{in}}_{\omega\ell m}$ and $b^{\text{out}}_{\omega\ell m}$: $|B\rangle$
2. Annihilated by $a^{\text{in}}_{\omega\ell m}$ and $a^{\text{out}}_{\omega\ell m}$: $|H\rangle$
3. Annihilated by $a^{\text{in}}_{\omega\ell m}$ and $b^{\text{out}}_{\omega\ell m}$ (by $a^{\text{out}}_{\omega\ell m}$ and $b^{\text{in}}_{\omega\ell m}$): $|U\rangle$ ($|U'\rangle$)

"Luminorsity" L at r_s: thermal flux from / into the surface at r_s

$$\langle T_{t_s r_*}\rangle = -\frac{L}{4\pi r_s^2} \quad \rightarrow \quad L(r_s = 1/H) = \begin{cases} 0 & \text{for } |B\rangle \text{ and } |H\rangle \\ \mp \dfrac{H^2}{12}|Y_{\ell m}|^2 & \text{for } |U\rangle \text{ and } |U'\rangle \end{cases}$$

Only $|U\rangle$ and $|U'\rangle$ allow a non-zero flux from thermal radiation

Raychaudhuri equation and evolution of dS

Change of the horizon area $\mathscr{A}$ can be found from Raychaudhuri eq

$$\epsilon = \mp \frac{H^2}{384\pi^2 m_{\rm Pl}^2} \text{ for } |U\rangle \text{ and } |U'\rangle \quad \text{(s-wave contribution)}$$

- $\Delta S_{\rm rad} = \pm 1/24$ per one e-fold
- Adiabaticity demands $\Delta S_{\rm dS} = \mp 1/24$ (cf. $\Delta S_{\rm dS} \gg 1$ for SR inf)
- Quantum break-time: dS evolution due to thermal radiation

$$H(t) = H_0 \left(1 \pm \frac{1}{128\pi^2} \frac{H_0^3}{m_{\rm Pl}^2} t \right)^{-1/3}$$

Take-home message #3

Quantum fluctuations destabilize dS geometry

Conclusions

- Studying quantum origin may be relevant
- Pure tensor perturbations are of physical interest
- Non-linear evolution allows system-environment interactions
    1. Generation of classical probability for other states
    2. Classicality inherits quantum nature
    3. dS is unstable due to quantum fluctuations
- Many more questions to be answered