Spontaneous breaking of Weyl quadratic gravity to Einstein action and inflation

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Phys.Rev.D101 (2020) 4, [1904.06596]; EPJC 80 (2020) 1147 [2003.08516]

- Plan of the talk:

  - From scale symmetry to Weyl conformal geometry
  - Weyl's original quadratic gravity: spontaneous breaking to Einstein action (no matter)
  - Similarities to Palatini gravity. Adding matter.
  - Testing Weyl gravity: inflation predictions.

• Symmetries beyond SM and GR?

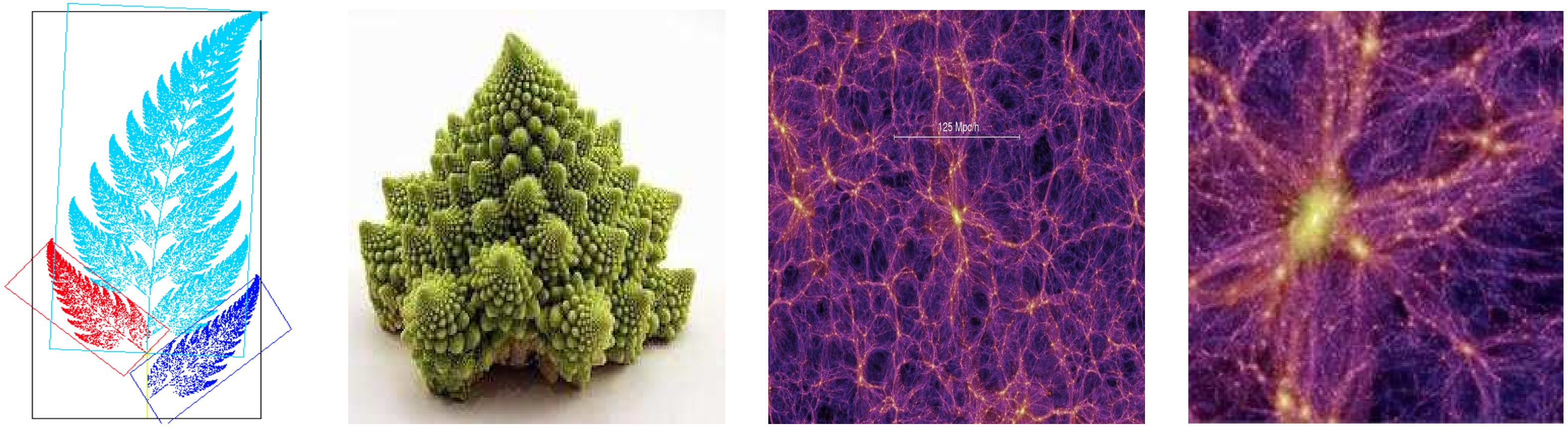
- Best model so far: SM and Higgs mechanism confirmed @ LHC; origin of EW scale.
[no SUSY @ TeV, why $m_h \ll M_P$?]. But what is the origin of Planck scale M_P?

• Scale symmetry: discrete/self-affinity, global, local, quantum; gauged scale symmetry [scales from vev's]
In this talk ⇒ gauged scale symmetry=Weyl gauge symmetry → Weyl conformal geometry [Weyl 1918]
→ Weyl quadratic gravity

• Why scale symmetry?

- scale symmetry may play a role in early cosmology; EFT at short distance may be scale inv/conformal
⇒ show: spontaneous breaking of Weyl theory [gauged scale symmetry] to Einstein action (no matter)
Transition: Weyl to Riemannian geometry; M_P generated! Add ϕ [=Higgs?], testable inflation predictions

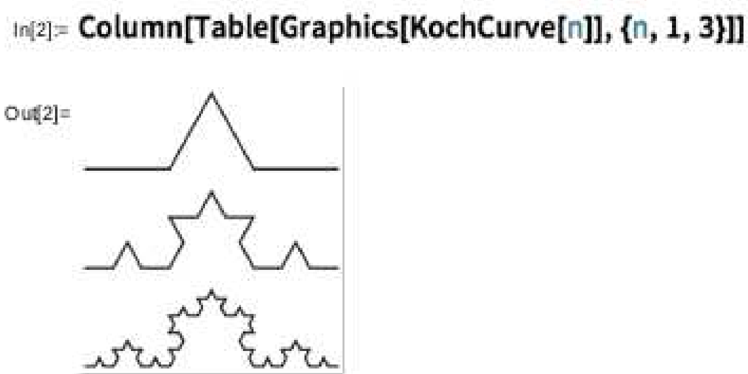
- Discrete scale invariance in Nature: : self-similarity/self-affinity across different scales [fractals]

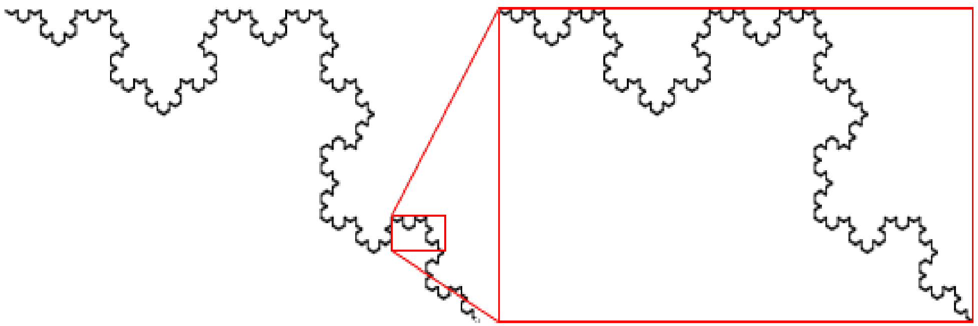


(a) (b) (c) (d)

(c), (d): cluster of galaxies, stars, DM. astro-ph/0504097

https://wwwmpa.mpa-garching.mpg.de/galform/millennium/





- self-repeating patterns at all length scales, new structure revealed.

Koch curve/snowflake

• Global scale invariance

$$x'_\mu = \rho\, x_\mu; \qquad \phi'(\rho\, x) = (1/\rho)\, \phi(x), \qquad \text{forbids} \quad \int d^4x\, m^2 \phi^2$$

- SM with Higgs ϕ of mass $m_\phi = 0$ is scale invariant [Bardeen 1995]

- scales generated by vev's, e.g. $M_P \sim \langle\sigma\rangle$. Broken by quantum corrections (μ scale of DR)

• Quantum scale invariance: replace $\mu \to \sigma$ (dilaton) $\Rightarrow$ scale invariance in $d=4-2\epsilon$; extra field!

- at one-loop [Englert et al 1976]; recently: [Shaposhnikov 0809.3406; D.G. 1508.00595] in SM [Z. Lalak, P. Olszewski, DG, 1612.09120]]

- two-loops [D.G., Z. Lalak, P. Olszewski, 1608.05336];

- three-loops [Gretsch, Monin 1308.3863; D.G. 1712.06024] protects a classical hierarchy of vevs $\phi \ll \sigma$.

- higher dimensional ops emerge ϕ^6/σ^2, ϕ^8/σ^4.... [D.G. 1508.00595, 1712.06024; D.G., Z. Lalak, P. Olszewski, 1612.09120]

- broken spontaneously; if σ decouples, usual results (breaking by DR) recovered.

Shortcomings....

- Gravity wanted $\Rightarrow$ fine tuning higgs selfcoupling $\beta_\lambda \sim \lambda(..)+\xi\,(...)$ from $\xi h^2 R$.

- Global symmetries broken by black-hole physics; global charges eaten by BH which subsequently evaporate

[Kallosh, Linde, Susskind, hepth/9502069; etc]

• Local scale invariance: L invariant under : $\hat{g}_{\mu\nu} = \Omega(x)^2\, g_{\mu\nu}, \quad \hat{\sigma} = \frac{\sigma}{\Omega(x)}$ (Ω real!)

[t'Hooft 1104.4543; 1410.6675; Bars, Steinhardt, Turok 1307.1848]

- Generating Einstein action:

$$L_0 = -\epsilon \frac{1}{2}\sqrt{g}\left[\frac{1}{6}\sigma^2 R + g^{\mu\nu}\,\partial_\mu\sigma\,\partial_\nu\sigma\right], \quad (\epsilon = 1) \qquad \Leftrightarrow \qquad L_0 = -\frac{1}{2}\sqrt{\hat{g}}\, M_P^2\, \hat{R} \text{ generated spontaneously}$$

$$\Omega^2 = \frac{\epsilon\,\sigma^2}{6\,M_P^2}, \quad M_P^2 \equiv \frac{\epsilon}{6}\langle\hat{\sigma}\rangle^2, \text{ "gauge fixing"}$$

Questions:

a) - negative kinetic term for σ ($\epsilon = 1$) or imaginary $\langle\sigma\rangle \sim \Omega$ ($\epsilon = -1$)

b) - Fake conformal symmetry? associated vanishing current [Jackiw, Pi 2015],

c) - Generating Planck scale requires adding a new scalar field in the spectrum $M_P \sim \langle\sigma\rangle$.

d) - Go to Einstein frame: $\langle\sigma\rangle$ fixed, σ decouples; d.o.f. changed (?)

$\Rightarrow$ We want to avoid a), b), c), d)....$\Rightarrow$ gauged scale invariance.

• Gauged scale invariance: $\hat{g}_{\mu\nu}(x) = \Omega(x)^2 \, g_{\mu\nu}(x), \quad \hat{\sigma}(x) = \dfrac{\sigma(x)}{\Omega(x)}, \quad \hat{\omega}_\mu(x) = \omega_\mu(x) - \partial_\mu \ln \Omega(x)^2 \quad (*)$

Weyl geometry: $(g_{\mu\nu}, \omega_\mu)$: $\tilde{\nabla}_\mu \, g_{\alpha\beta} = - \, \omega_\mu \, g_{\alpha\beta}$ Riemannian geometry: $\nabla_\mu \, g_{\alpha\beta} = 0$

$\Rightarrow \tilde{\Gamma}^\rho_{\mu\nu} = \Gamma^\rho_{\mu\nu} + (1/2) \left(\delta^\rho_\mu \, \omega_\nu + \delta^\rho_\nu \, \omega_\mu - g_{\mu\nu} \, \omega^\rho \right)$ inv of (*); $\Gamma^\rho_{\mu\nu} =$ Levi-Civita; ∇_μ with Γ

$\Rightarrow \tilde{R} = R - 3 \, \nabla_\mu \omega^\mu - 3/2 \, \omega^\mu \omega_\mu . \Rightarrow \hat{\tilde{R}} = \dfrac{\tilde{R}}{\Omega^2}$, covariant tr!

$\Rightarrow \tilde{D}_\mu \sigma = (\partial_\mu - 1/2 \, \omega_\mu) \, \sigma \; \Rightarrow \; \hat{\tilde{D}}_\mu \hat{\sigma} = (1/\Omega) \; \tilde{D}_\mu \sigma;$

$\Rightarrow F_{\mu\nu} = \tilde{\nabla}_\mu \, \omega_\nu - \tilde{\nabla}_\nu \, \omega_\mu = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu$ inv (*). Also $\omega_\mu = (1/2)(\tilde{\Gamma}_\mu - \Gamma_\mu)$ deviation from Levi-Civita.

$\Rightarrow$ if $\omega_\mu \to 0$: $\tilde{\Gamma} \to \Gamma$, Weyl geometry $\to$ Riemannian; $\tilde{R} \to R$, Weyl tensor $\tilde{C}_{\mu\nu\rho\sigma} \to C_{\mu\nu\rho\sigma}$

$\Rightarrow$ All invariants (*): $\sqrt{g} \, \tilde{R}^2, \; \sqrt{g} \, \sigma^2 \tilde{R}, \; \sqrt{g} \, F^2_{\mu\nu}, \; \sqrt{g} \, \tilde{C}^2_{\mu\nu\alpha\beta}; \; \sqrt{g} \, (\tilde{D}_\mu \sigma)^2$. no higher dim ops (no scale!)

[if no matter]

$\tilde{X}$ (X) notation in Weyl (Riemannian) geometry

• Weyl quadratic gravity: no matter

$$L_0 = \sqrt{g}\,\Big\{\frac{\xi_0}{4!}\,\tilde R^2 - \frac{1}{4}\,F_{\mu\nu}^2 + \frac{1}{\eta}\,\tilde C_{\mu\nu\rho\sigma}\,C^{\mu\nu\rho\sigma}\Big\}, \quad \xi_0>0, \qquad \text{Weyl (1919)} \qquad (**)$$

• Early critique:

- Non-metricity $\tilde\nabla_\mu\, g_{\alpha\beta} = \omega_\mu\, g_{\alpha\beta}$; [parallel transport: vector norm & clock's rate: path dependent]
- Einstein critique (1919) [massless ω_μ]. Weyl quadratic gravity abandoned [review: Scholz 1703.03187]
- only solution so far: $\omega_\mu = 0$ or $\partial_\mu(..)$, metric, Weyl integrable = Riemann geom/Einstein gravity & local scale invariance

• Previous works: [Dirac 1973] introduced new "Weyl gravity" linear in $\tilde R$ (no $\tilde R^2$) with extra matter ϕ: $\phi^2 \tilde R$, in this case ω_μ can become massive: [Smolin (1979)]. Subsequent studies followed this approach:

Cheng (1988), Nishino & Rajpoot (2009), Dreschler & Tann (1999); Ohanian (2016); DG & H. M. Lee (2018)]

⇒ Back to "quadratic" case (**), no matter: can ω_μ be massive, decouple ($\omega_\mu = 0$), so that metricity restored (below some scale) & critique avoided?

• Weyl quadratic gravity $\Rightarrow$ Einstein gravity + massive ω_μ [D.G. arXiv:1812.08613, 1904.06596, 2007.14733]

$$L_1 = \sqrt{g}\,\Big\{\,\frac{\xi_0}{4!}\,\tilde R^2 - \frac{1}{4q^2}\,F_{\mu\nu}^2\Big\} = \sqrt{g}\,\Big\{\,\frac{\xi_0}{4!}\,\big(-2\,\sigma^2\,\tilde R - \sigma^4\big) - \frac{1}{4q^2}\,F_{\mu\nu}^2\Big\}$$

eom: $\sigma^2 = -\tilde R$; "dilaton": $\ln\sigma$

Go to Riemannian notation: $\tilde R(\tilde\Gamma, g) = R(g) - 3\,\nabla_\mu\omega^\mu - 3/2\;\omega_\mu\omega^\mu$

$$L_1 = \sqrt{g}\,\Big\{-\frac{\xi_0}{2}\,\Big[\frac{1}{6}\,\sigma^2\,R + (\partial_\mu\sigma)^2\Big] - \frac{\xi_0}{4!}\sigma^4 + \frac{1}{8}\,\xi_0\,\underbrace{\sigma^2\,\big(\omega_\mu - \partial_\mu\ln\sigma^2\big)^2}_{\sim(\tilde D_\mu\sigma)^2} - \frac{1}{4\,q^2}\,F_{\mu\nu}^2\Big\}.$$

Use (*) of "gauge fixing": $\Omega^2 = \xi_0\,\sigma^2/(6M_P^2) \;\Rightarrow\; \hat g_{\mu\nu} = \Omega^2 g_{\mu\nu},\;\; \hat\sigma^2 = 6\,M_P^2/\xi_0,\;\; \hat\omega_\mu = \omega_\mu - \partial_\mu\ln\sigma^2$

$$L_1 = \sqrt{\hat g}\,\Big\{-\frac{1}{2}\,M_P^2\,\hat R - \frac{3\,M_P^4}{2\xi_0} + \underbrace{\frac{3}{4}\,q^2\,M_P^2\,\hat\omega_\mu\,\hat\omega^\mu}\, - \frac{1}{4}\,\hat F_{\mu\nu}^2\Big\}. \qquad M_P^2 \equiv \frac{\xi_0}{6}\langle\hat\sigma\rangle^2$$

$\Rightarrow$ Stueckelberg mechanism: σ eaten by $\omega_\mu \rightarrow$ massive ω_μ; dof=3 conserved! no ghost.

$\Rightarrow$ Einstein-Proca action of ω_μ; mass of $m_\omega \propto q\,M_P$, decouples; $\Rightarrow$ metricity restored below m_ω!

$\Rightarrow$ Einstein action: "low energy"/broken phase of Weyl's theory. No scalar in spectrum.

- Weyl quadratic gravity $\Rightarrow$ Einstein gravity + massive ω_μ [D.G. arXiv:1812.08613, 1904.06596, 2007.14733]

- Previous result remains true for most general Weyl quadratic gravity (below) which is $L_1 + L_C$: since Weyl tensor $\tilde{C}_{\mu\nu\rho\sigma}$, $(C_{\mu\nu\rho\sigma})$ invariant under previous transformations.

$$L_1 = \sqrt{g}\,\Big\{\frac{\xi_0}{4!}\,\tilde{R}^2 - \frac{1}{4q^2}\,F_{\mu\nu}^2\Big\}$$

$$L_C = \frac{\sqrt{g}}{\eta}\tilde{C}_{\mu\nu\rho\sigma}\,\tilde{C}^{\mu\nu\rho\sigma} = \frac{\sqrt{g}}{\eta}\Big[C_{\mu\nu\rho\sigma}\,C^{\mu\nu\rho\sigma} + \frac{3}{2}\,F_{\mu\nu}^2\Big]$$

Weyl geometry Riemannian geometry

$\Rightarrow$ After Weyl gauge symmetry breaking ω_μ massive, decouples. Weyl geometry $\Rightarrow$ Riemannian

$\Rightarrow$ Planck scale generation is a geometric transition from Weyl to Riemannian geometry!

$\Rightarrow$ $\sigma \to \langle\sigma\rangle \sim M_P$, dynamically in FRW universe: [GG Ross et al 1801.07676]

non-trivial, conserved current $\partial^\mu J_\mu = 0$, if ω_μ dynamical: $\partial^\alpha(F_{\alpha\mu}\sqrt{g}) + \underbrace{\frac{1}{2}\sqrt{g}\,\xi_0\,\sigma\,\big[\partial_\mu - \frac{q}{2}\,\omega_\mu\big]\,\sigma}_{=J_\mu} = 0$

$\Rightarrow$ Weyl quadratic gravity: an embedding of Einstein gravity. Renormalizable? [K. Stelle 1979]

• Weyl gravity: adding matter ϕ- scalar field, higgs-like, inflaton, etc

$$L_2 = \sqrt{g}\,\Big\{\frac{\xi_0}{4!}\,\tilde R^2(\tilde\Gamma,g) - \frac{1}{4q^2}\,F_{\mu\nu}^2\;-\;\frac{1}{12}\,\xi_1\,\phi^2\,\tilde R(\tilde\Gamma,g)\;+\;\frac{1}{2}(\tilde D_\mu\phi)^2 - \frac{\lambda}{4!}\phi^4\Big\},\quad \tilde R^2 \to -2\sigma^2\,\tilde R - \sigma^4$$

$$= \sqrt{g}\,\Big\{-\frac{1}{12}\,(\xi_0\,\sigma^2 + \xi_1\,\phi^2)\,\tilde R(\tilde\Gamma,g) - \frac{1}{4\,q^2}\,F_{\mu\nu}^2 + \frac{1}{2}(\tilde D_\mu\phi)^2 - \frac{1}{4!}\,(\lambda\,\phi^4 + \xi_0\,\sigma^4)\Big\},\quad \rho^2 = \frac{1}{6}\,(\xi_0\sigma^2 + \xi_1\,\phi^2)$$

Riemannian language ($\tilde\Gamma \to \Gamma + \cdots$)

$$L_2 = \sqrt{g}\,\Big\{-\frac{1}{2}\Big[\rho^2\,R + 6\,(\partial_\mu\rho)^2\Big] + \frac{3}{4}\,\rho^2\,\big(\omega_\mu - \partial_\mu\ln\rho^2\big)^2 - \frac{1}{4\,q^2}\,F_{\mu\nu}^2 + \frac{1}{2}(\tilde D_\mu\phi)^2 - V(\phi,\rho)\Big\}.$$

Stueckelberg: radial direction $\ln\rho$ eaten by ω_μ. "Gauge fixing" (*): $\Omega = \dfrac{\rho^2}{M_P^2},\ \hat\rho = M_P,\ \hat\omega_\mu = \omega_\mu - \partial_\mu\ln\rho^2$

number d.o.f. conserved (2+1) vs (3)

$\Rightarrow$Einstein-Proca action (ω_μ) : $\quad L_2 = \sqrt{\hat g}\,\Big\{-\dfrac{1}{2}\,M_P^2\,\hat R + \dfrac{3}{4}\,M_P^2\,\hat\omega_\mu\hat\omega^\mu - \dfrac{1}{4\,q^2}\hat F_{\mu\nu}^2 + \dfrac{1}{2}\,(\hat{\tilde D}_\mu\hat\phi)^2 - V\Big]\Big\},$

[D.G. arXiv:1812.08613, 1904.06596, 2007.14733]

• The action, "unitarity gauge": [D.G. arXiv:1812.08613, 1904.06596, 2007.14733]

$\hat\phi \to M\sqrt{6}\sinh\varphi/(M\sqrt{6})$, $\quad \hat\omega_\mu \to \hat\omega_\mu + \partial_\mu \ln\cosh^2\varphi/(M\sqrt{6})$:

$$\Rightarrow \; L_2 = \sqrt{\hat g}\,\Big\{ -\frac{1}{2}\,M_P^2\,\hat R + \frac{3}{4}\,M_P^2\,\hat\omega_\mu\hat\omega^\mu\,\cosh^2\frac{\varphi}{M\sqrt{6}} - \frac{1}{4\,q^2}\hat F_{\mu\nu}^2 + \frac{1}{2}(\partial_\mu\varphi)^2 - V(\varphi)\Big]\Big\},$$

• The potential

$$V(\varphi) \;=\; \frac{3}{2}\frac{M_P^4}{\xi_0}\Big\{\Big[1-\xi_1\sinh^2\frac{\varphi}{M_P\sqrt{6}}\Big]^2 + \lambda\xi_0\,\sinh^4\frac{\varphi}{M_P\sqrt{6}}\Big\},$$
$$=\; \frac{3M_P^4}{2\,\xi_0}\Big[1-\frac{\xi_1\varphi^2}{6\,M_P^2}\Big]^2 + \frac{\lambda}{4!}\,\varphi^4 + \mathcal{O}\Big(\frac{\varphi^2}{M_P^2}\Big), \quad \varphi \ll M_P.$$

$\Rightarrow$ gravitational higgs mechanism, if $\varphi =$ higgs. Also $m_\phi^2 = (-\xi_1/\xi_0)\,M_P^2$.

$\Rightarrow$ with M_P phase transition scale: $\phi \geq M_P$ natural.

$\Rightarrow$ Weyl inflation? similarities to Starobinsky inflation, but note $\varphi^2\,\omega^\mu\omega_\mu$ coupling.

• Weyl vs Palatini quadratic gravity [D.G. arxiv:2003.08516; 2007.14733]

- Palatini approach to gravity [Einstein 1925]: $\tilde\Gamma$ unknown, fixed by eqs of motion (action).

- apriori $\tilde\Gamma$ independent of $g_{\mu\nu}$ $\Rightarrow$ invariant under (*); define $\omega_\mu = (1/2)(\tilde\Gamma_\mu - \Gamma_\mu)$.

- in previous Weyl action, replace $\tilde\Gamma$ of Weyl $\rightarrow$ $\tilde\Gamma$ of Palatini (still Weyl gauge inv):

$$L_2 = \sqrt{g}\,\Big\{\frac{\xi_0}{4!}\,\tilde R^2(\tilde\Gamma, g) - \frac{1}{4q^2}\,F_{\mu\nu}^2(\tilde\Gamma) \;-\; \frac{1}{12}\,\xi_1\,\phi^2\,\tilde R(\tilde\Gamma, g) \;+\; \frac{1}{2}(\tilde D_\mu\phi)^2 - \frac{\lambda}{4!}\phi^4\Big\},$$

Solve for $\tilde\Gamma$ (difficult!) $\Rightarrow$ $\tilde\nabla_\lambda\, g_{\mu\nu} = (-2)(g_{\mu\nu}\,\omega_\lambda - g_{\mu\lambda}\,\omega_\nu - g_{\nu\lambda}\,w_\mu)$ non-metricity $\neq$ Weyl geometry.

$\Rightarrow$ Onshell $\tilde\Gamma$: Stueckelberg breaking, same steps as before, etc:

$$L_2 = \sqrt{g}\,\Big\{-\frac{1}{2}\Big[\rho^2\,R + 6\,(\partial_\mu\rho)^2\Big] + \frac{3}{4}\,\theta\,\rho^2\,\big(\omega_\mu - \partial_\mu \ln\rho^2\big)^2 - \frac{1}{4\,q^2}\,F_{\mu\nu}^2 + \frac{1}{2}(\tilde D_\mu\phi)^2 - V(\phi,\rho)\Big\}.$$

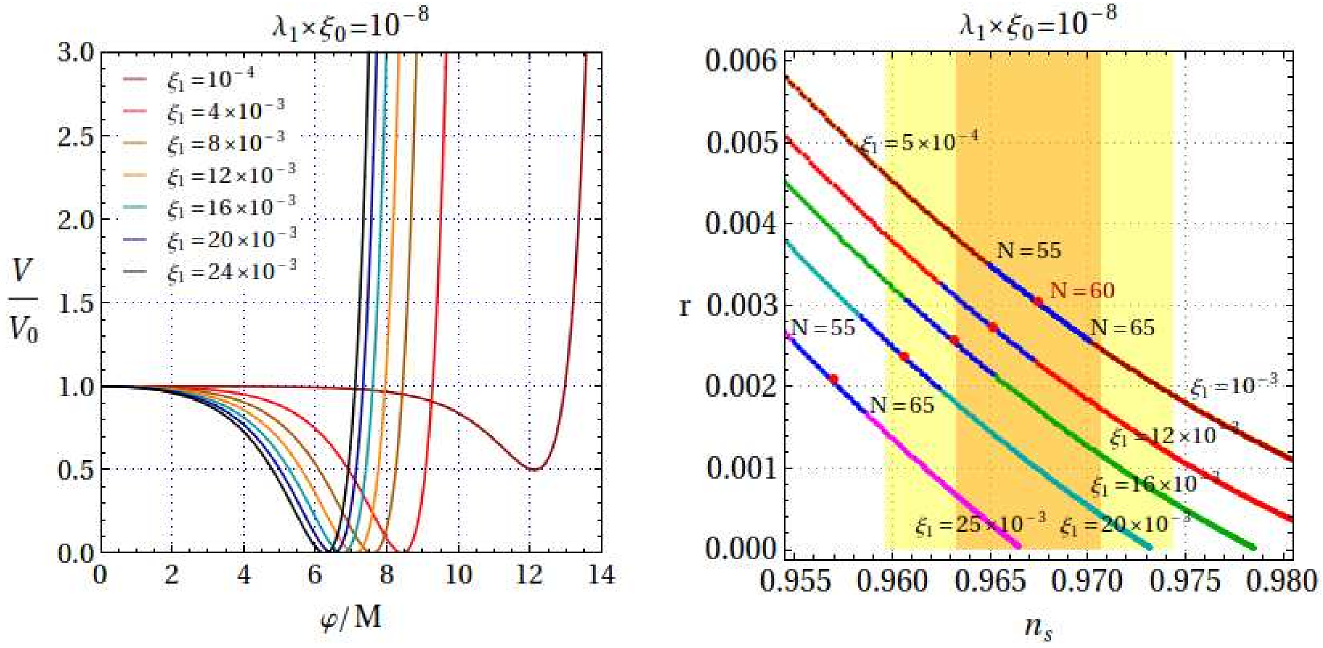
$\Rightarrow$ onshell, gauge fixing: again Einstein-Proca action, similar to Weyl theory but $\theta = 4$ (Weyl: $\theta = 1.$).

$\Rightarrow$ similar structure of V, θ different (due to different non-metricity)

In Palatini quadratic gravity: additional operators can be present...

• Weyl R^2-inflation ($\theta = 1$)

$$V = V_0 \left\{ \left[1 - \theta\, \xi_1 \sinh^2 \frac{\varphi}{2\, M_P\sqrt{6\,\theta}}\right]^2 + \lambda \xi_0\, \theta^2 \sinh^4 \frac{\varphi}{2 M_P \sqrt{6\,\theta}} \right\}$$



$$\lambda\,\xi_0 \ll \xi_1^2 \ll 1: \quad \epsilon = \frac{M_P^2}{2}\frac{V'^2}{V^2} = \frac{\xi_1^2}{3}\,\theta \sinh^2\frac{2\phi}{M_P\sqrt{6\theta}} + \mathcal{O}(\xi_1^3); \quad \eta = M_P^2\,\frac{V''}{V} = -\frac{2\xi_1}{3}\cosh\frac{2\phi}{M_P\sqrt{6\theta}} + \mathcal{O}(\xi_1^2)$$

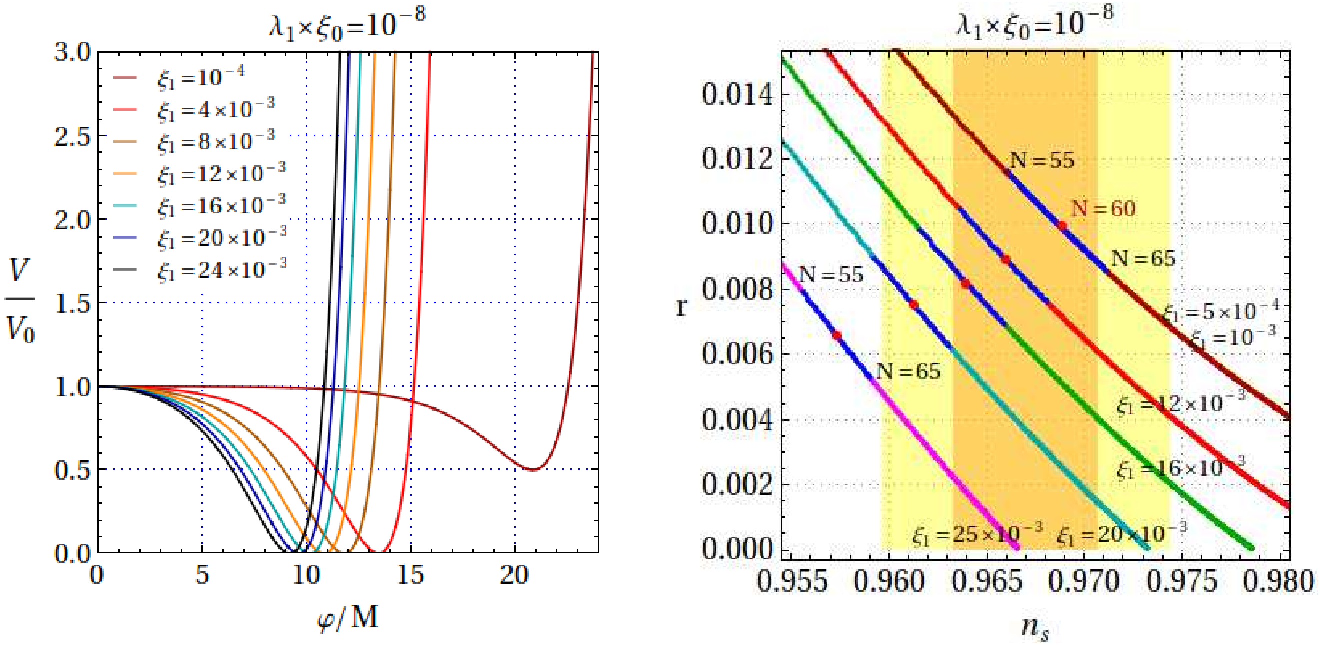
$$n_s = 1 + 2\eta - 6\epsilon = 1 - \frac{4}{3}\xi_1 \cosh\frac{2\phi_*}{M_P\sqrt{6\theta}} + \mathcal{O}(\xi_1^2); \quad \Rightarrow \quad r = 3\theta\,(1 - n_s)^2 + \mathcal{O}(\xi_1^2)$$

$0.002567 \le r \le 0.00303$ if $n_s = 0.9670 \pm 0.0037$ ($N = 60$). Upper limit on r: Starobinsky: ($n_s \approx 0.968$)

[D.G. arxiv:2007.14733, 1906.11572; G. Ross, C. Hill, P, Ferreira, J. Noller 1906.03415]

• Palatini R^2-Inflation $(\theta = 4)$

$$V = V_0 \left\{ \left[1 - \theta\, \xi_1 \sinh^2 \frac{\varphi}{2\, M_P \sqrt{6\, \theta}} \right]^2 + \lambda \xi_0\, \theta^2 \sinh^4 \frac{\varphi}{2 M_P \sqrt{6\, \theta}} \right\}$$



$$\lambda\, \xi_0 \ll \xi_1^2 \ll 1 : \quad \epsilon = \frac{M_P^2}{2} \frac{V'^2}{V^2} = \frac{\xi_1^2}{3}\, \theta \sinh^2 \frac{2\phi}{M_P \sqrt{6\theta}} + \mathcal{O}(\xi_1^3); \quad \eta = M_P^2 \frac{V''}{V} = -\frac{2\xi_1}{3} \cosh \frac{2\phi}{M_P\sqrt{6\theta}} + \mathcal{O}(\xi_1^2)$$

$$n_s = 1 + 2\eta - 6\epsilon = 1 - \frac{4}{3} \xi_1 \cosh \frac{2\phi_*}{M_P \sqrt{6\theta}} + \mathcal{O}(\xi_1^2); \quad \Rightarrow \quad r = 3\theta\, (1 - n_s)^2 + \mathcal{O}(\xi_1^2)$$

$0.00794 \le r \le 0.01002$ if $n_s = 0.9670 \pm 0.0037$; $(N = 60)$; $r(n_s)$ different from Starobinsky: $(n_s \approx 0.968)$

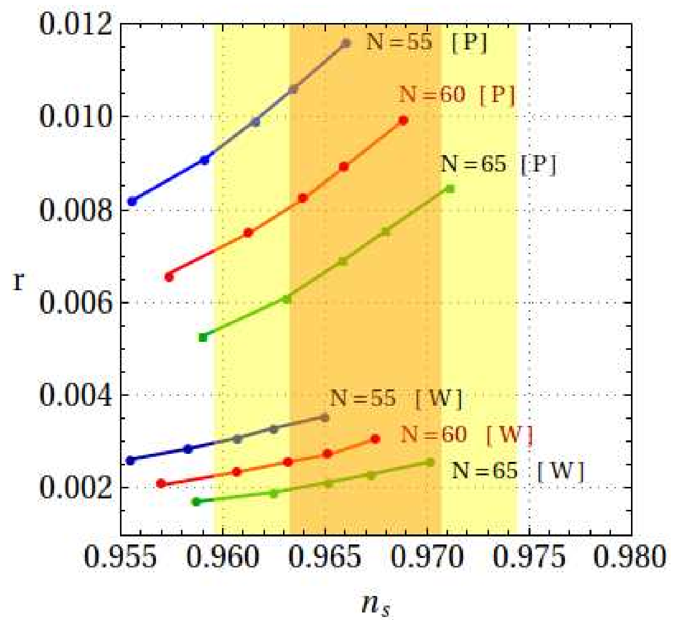
[D.G. arxiv:2007.14733, 2003.08516]

• Weyl versus Palatini: testing inflation predictions

[D.G. arXiv:2007.14733]

- tensor-to-scalar ratio r versus spectral index n_s with yellow (orange) values of n_s at 68% (95%) CL.
- the difference (θ) due to different non-metricity of these theories.
- such values of r reachable by future CMB experiments (0.0005 precision; LiteBIRD, CMB-S4).

⇒ One will be able test and discriminate between these models



- Conclusions:

- In the absence of matter: Weyl quadratic gravity broken via Stueckelberg to Einstein-Proca for ω_μ
- $M_\omega \sim q M_P \sim q\langle\sigma\rangle$, decouples; Einstein action: "low energy"/broken phase of Weyl quadratic gravity
- M_P generation has geometric interpretation: transition Weyl $\Rightarrow$ Riemann geometry

- non-metricity criticism of Weyl/Palatini quadratic gravity: avoided, metricity restored below $m_\omega \propto q M_P$
  Non-metricity bounds (m_ω) are low ($\sim$TeV).
- Weyl's quadratic theory of gravity: viable theory! renormalizable?

- In the presence of scalars (Higgs-like), successful inflation, similar to Starobinsky
  Weyl gravity/inflation: testable prediction: $0.00257 \leq r \leq 0.00303$ ($N = 60$, n_s= measured 68%CL).