February 3, 2021 at the CAU Workshop
on Beyond Standard Model, Online

Toward waveform modeling for gravitational waves from black hole encounters

Gungwon Kang (Chung-Ang University)

Outline

I. Motivation

II. NR Results for GW Capture

III. Waveform modeling with general eccentricity

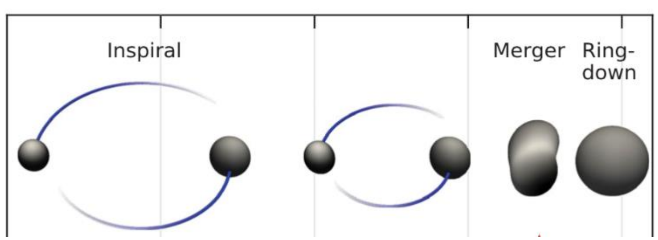
IV. Conclusion

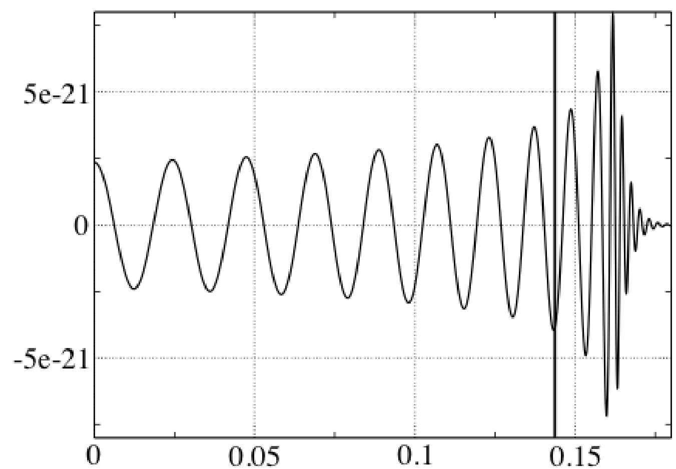
I. Motivation

✓ BBHs are important sources for GW observations

- **50 observations so far**
  - O1 (2015/09/12~2016/01/13): 3 BBH
  - O2 (2016/11/30~2017/08/25): 7 BBH +1 BNS
  - O3a (2019/04/01~2019/10/01): 36 BBH +3 others

- NR simulations and waveforms modeling have mostly focused on the last stage of coalescences, *e.g.,* quasi-circular orbits.

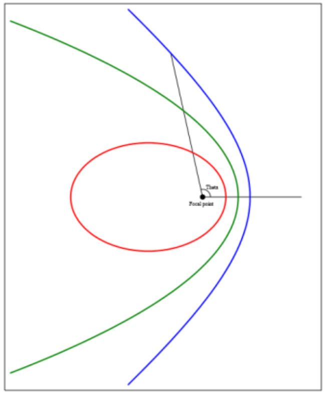


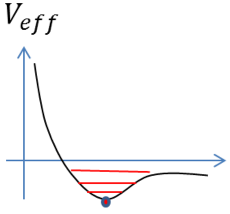


- In general,

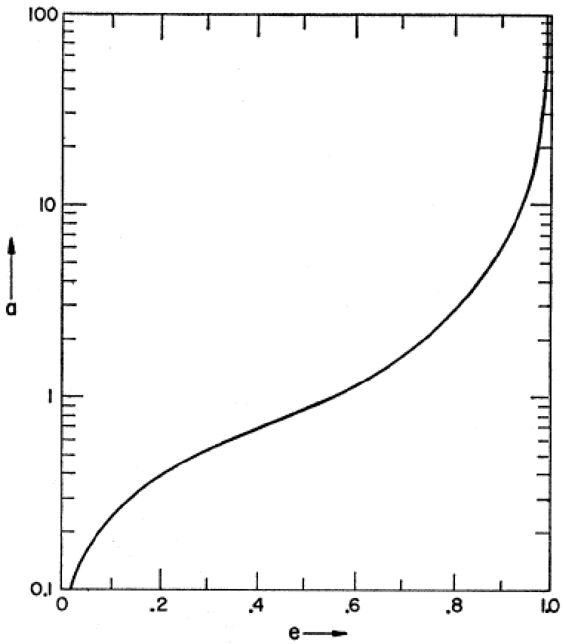
(E, L)

e: 0~1

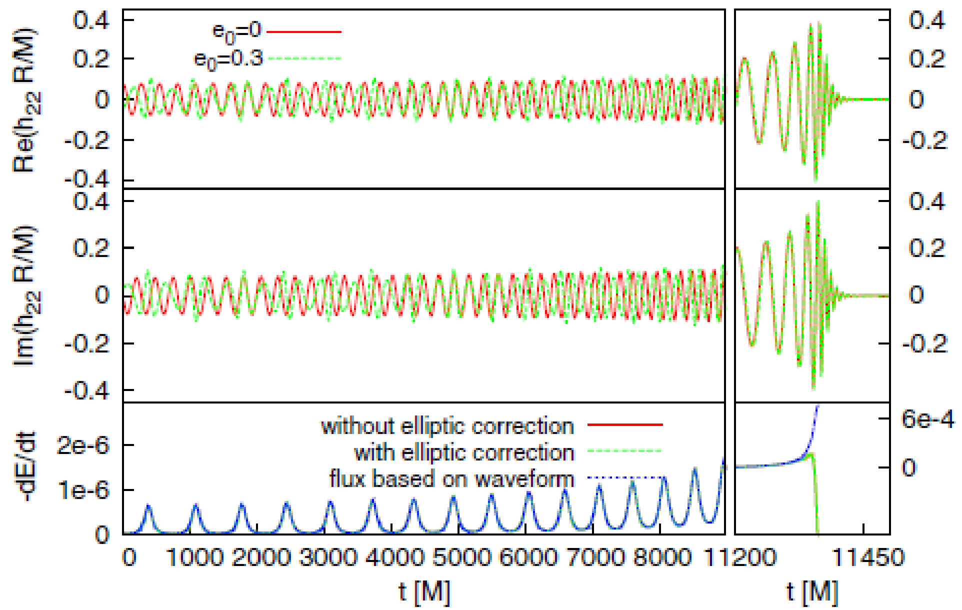




- "Circularization":

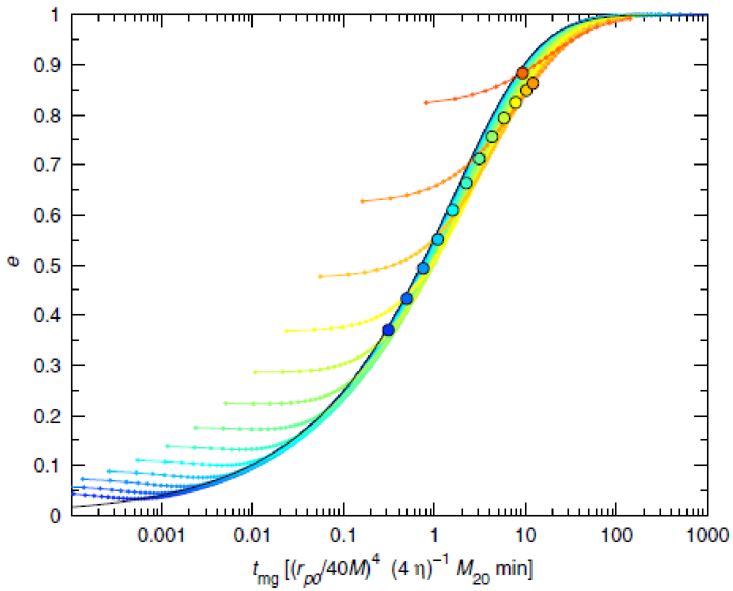


Peters (1964): Evolution of eccentricity ('2.5'PN)

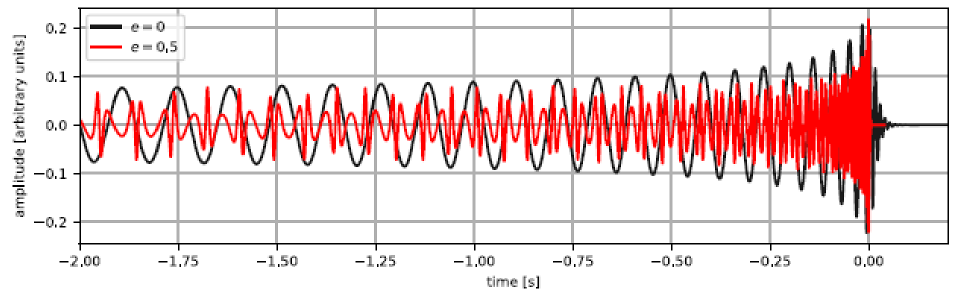


Cao & Han (2017)

• Higher-order (3.5) PN calculation: Kocsis & Levin ('12)



"**Modulated**" inspiral waveform



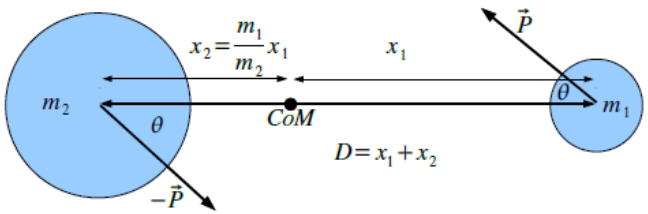
• So, eccentric BBH mergers (e.g., $e \neq 0$) might be relevant, and we may need to develop waveform models with a finite eccentricity.

• BBH waveforms with eccentricity for inspiral phase only:

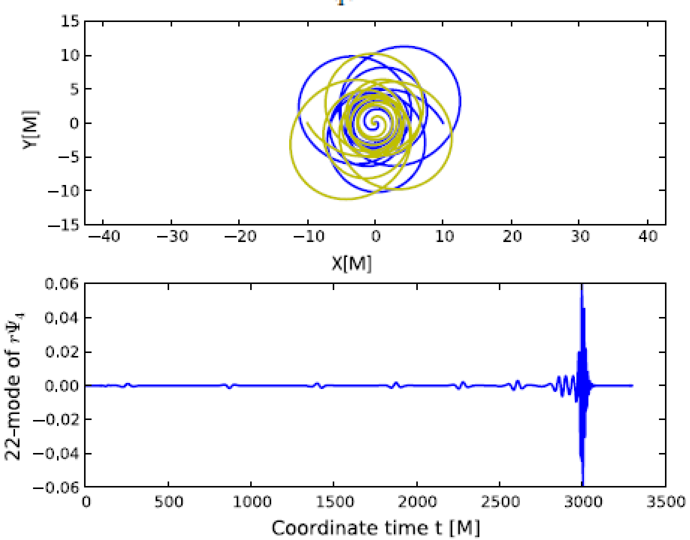
➔ TaylorF2ecc (e: $0.0001 \sim 0.2$) by C. Kim, J. Kim, H. Lee +, ...

• **What kind of waveforms in general for two body encounters?**

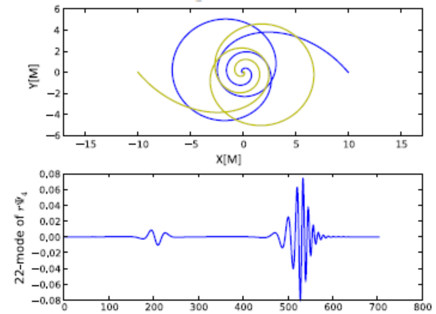
- **Gold & Brugmann ('13):**



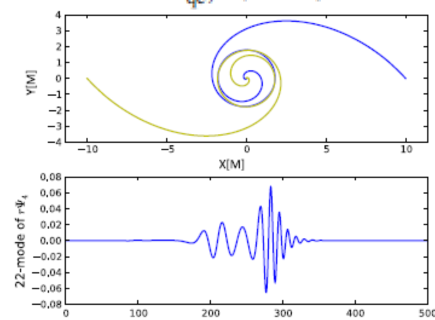
$P = P_{qc}, \quad \Theta = 60°$



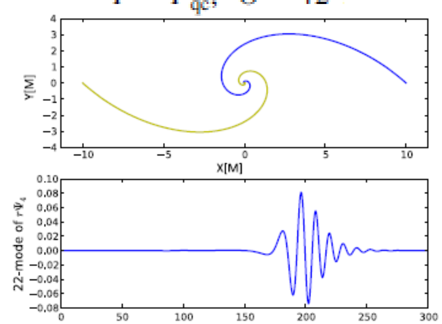
$P = P_{qc}, \quad \Theta = 50°$:



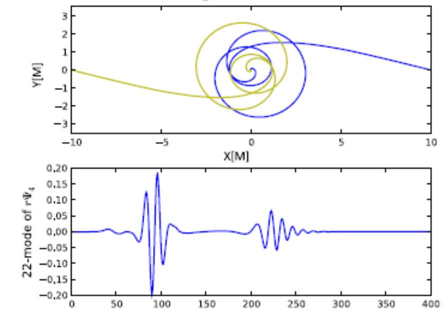
$P = P_{qc}, \quad \Theta = 48°$



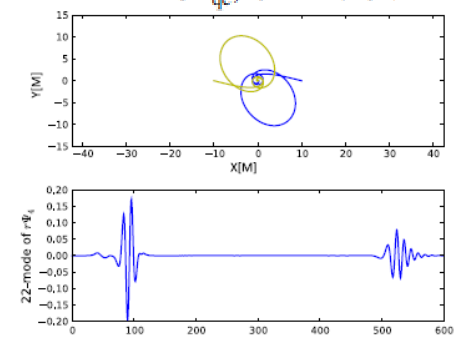
$P = P_{qc}, \quad \Theta = 42°$



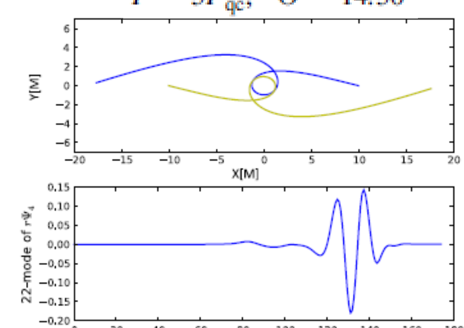
$P = 5P_{qc}, \Theta = 14.15°$



$P = 5P_{qc}, \Theta = 14.20°$:

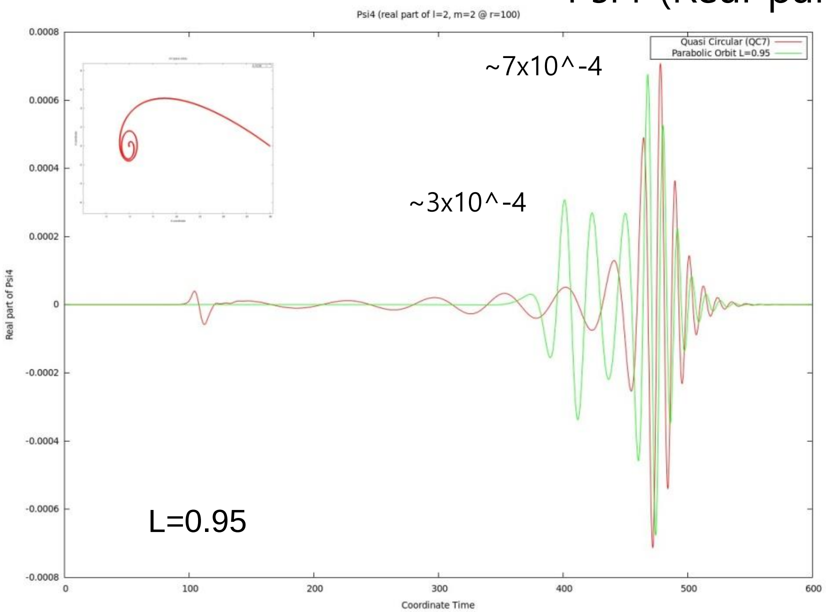


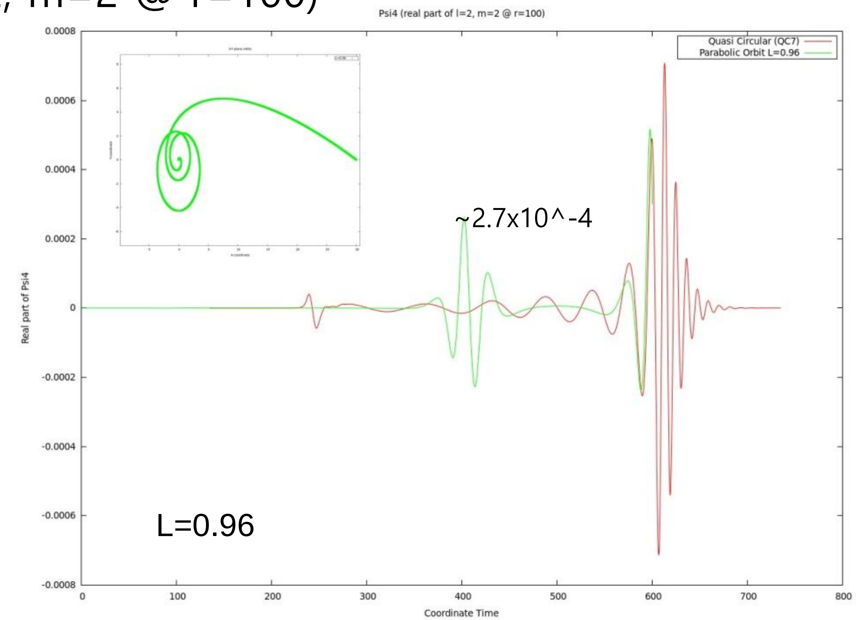
$P = 5P_{qc}, \quad \Theta = 14.30°$

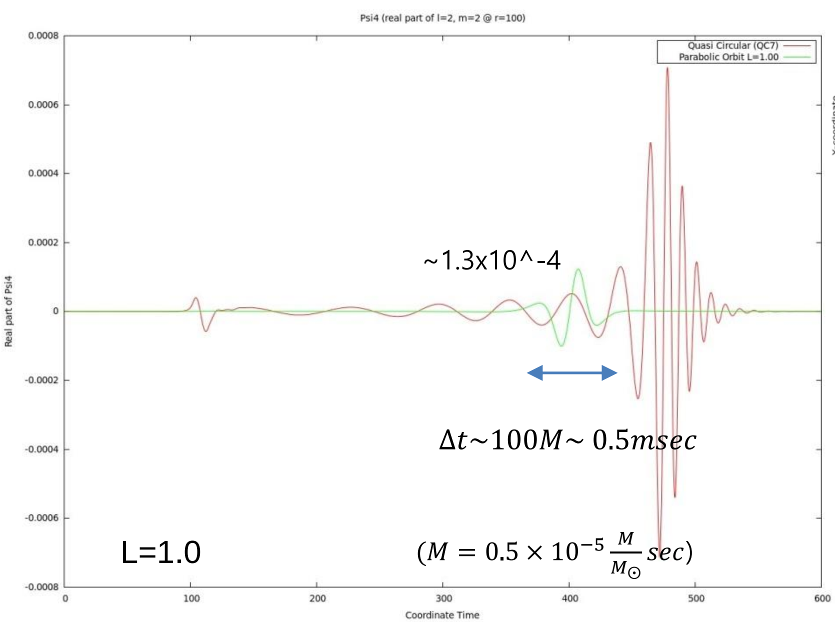


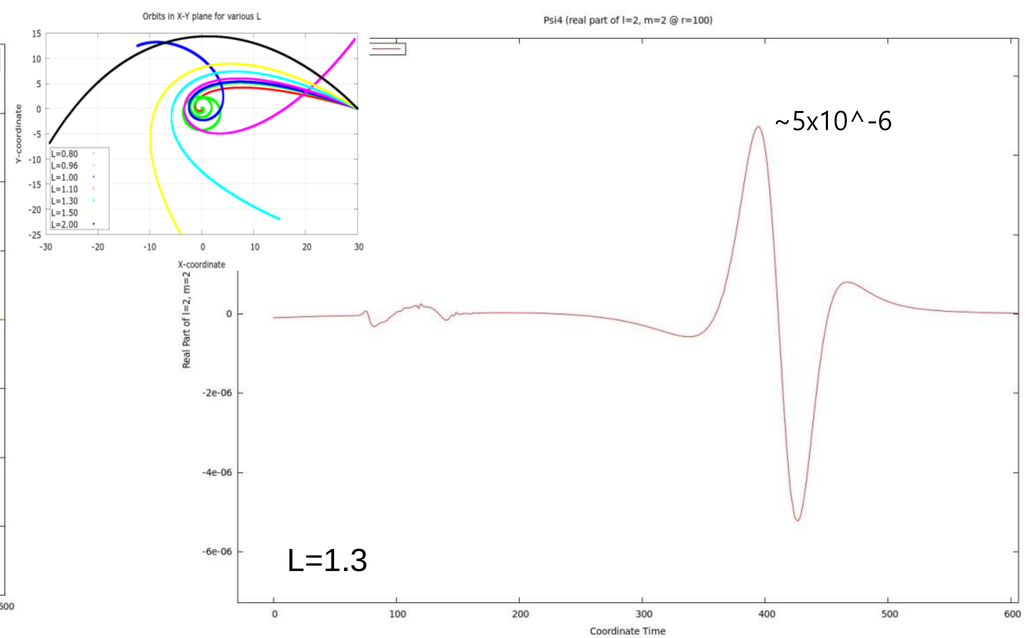
- **How weak?:** w/ J. Hansen, P. Diener, F. Loefler & H. Kim ('13)

Psi4 (Real part of l=2, m=2 @ r=100)

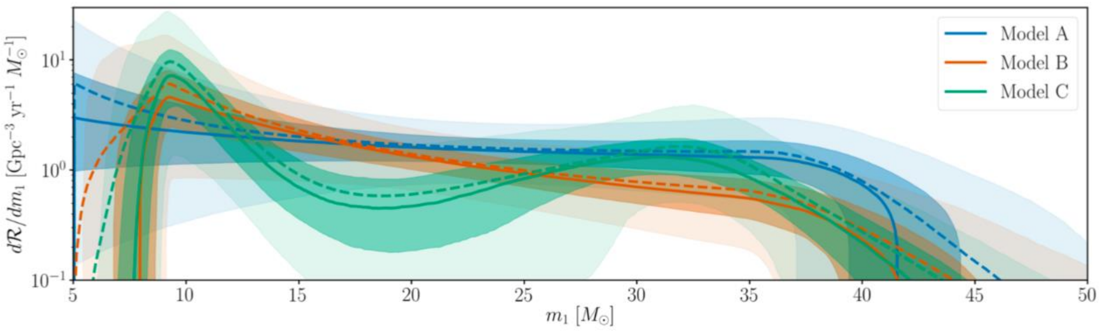




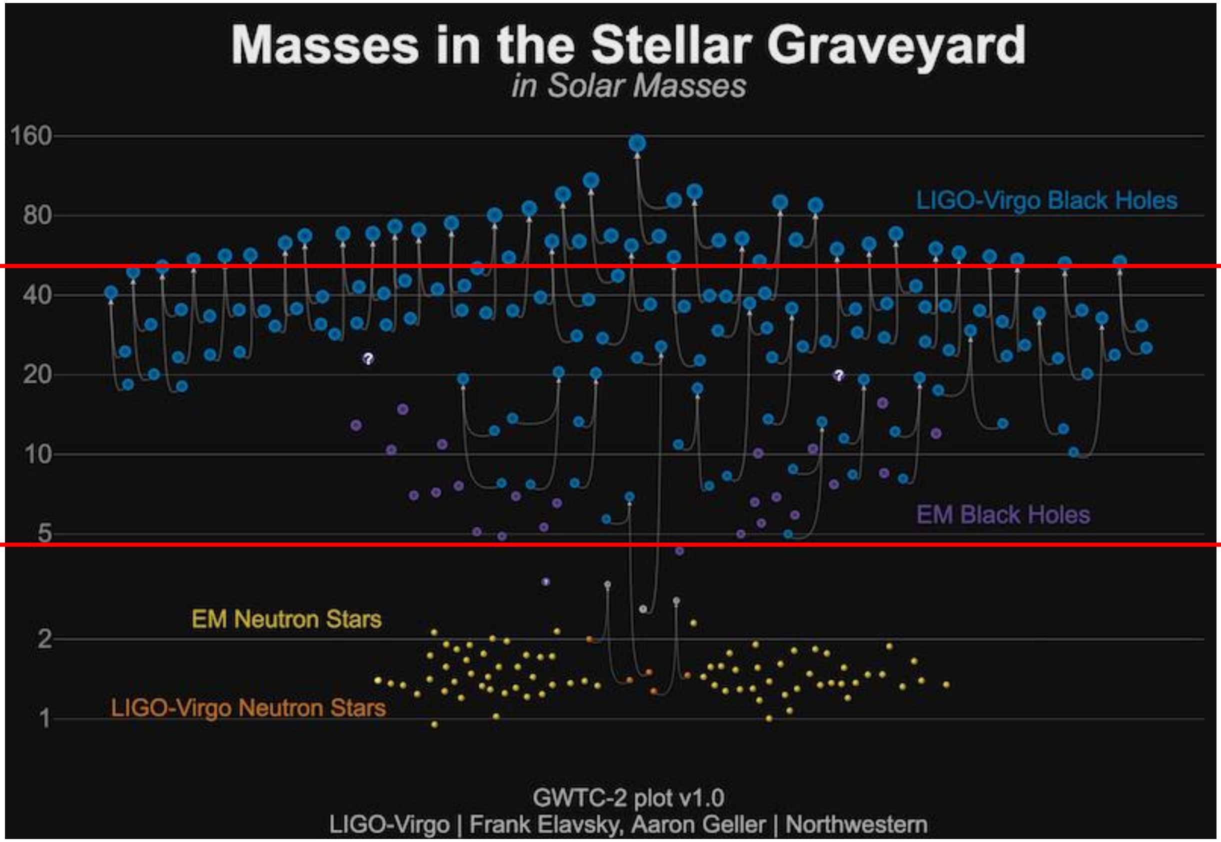




✓ Masses in BBHs:



Abbott+19
(arXiv:1811.12940)



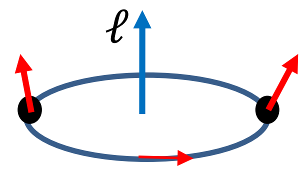
- No BBH of stellar origin $> 50M_\odot$?
- Similarly, no BBH $< 5M_\odot$?

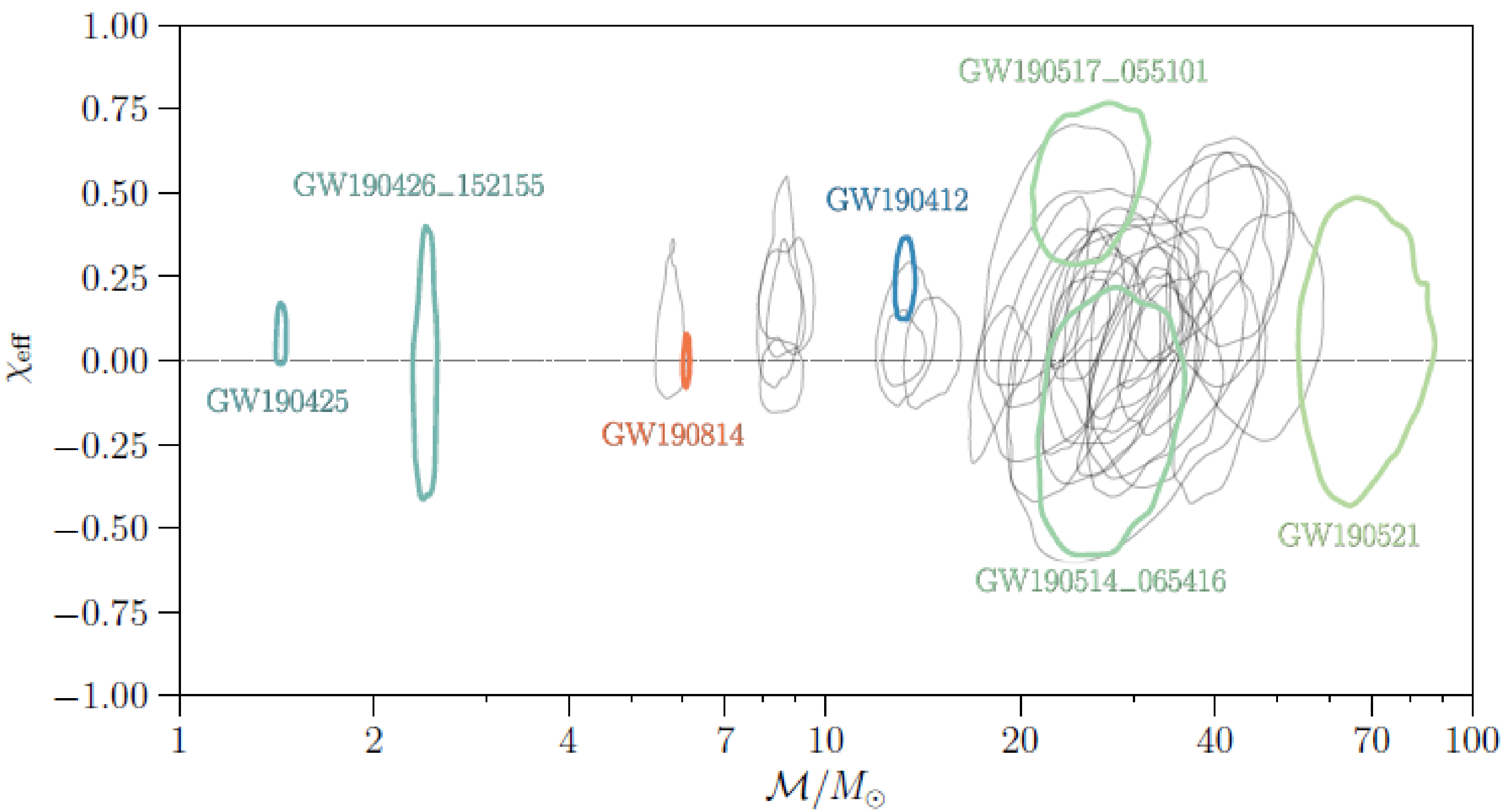
➔ **"Mass Gap" (?):**

Belczynski+2011,
Sathya+2019

✓ Effective spins of binary objects:

$$\chi_{\text{eff}} = \frac{(m_1\vec{\chi}_1 + m_2\vec{\chi}_2)\cdot\hat{L}_N}{M}$$





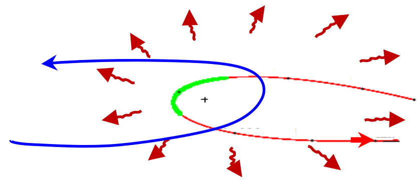
- So, some GWs from BH encounters could be detected in the future detectors, e.g., cosmic explorer or Einstein telescope!
  ➔ We need to prepare waveform models for highly eccentric BBH mergers.
- What are the origins of non-stellar binary black holes?

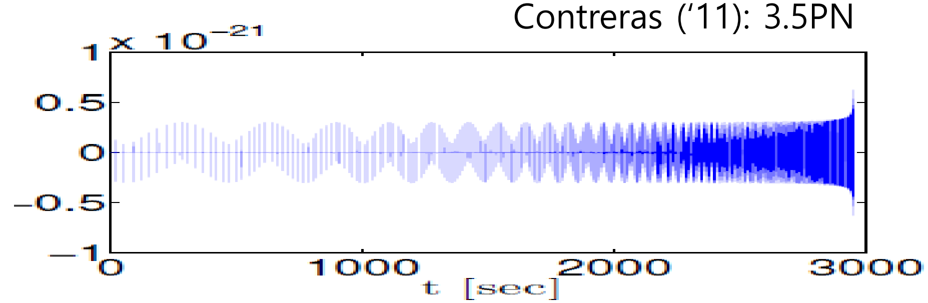
➔ **What is the whole life of a two-body system and the evolution of the waveform associated?**

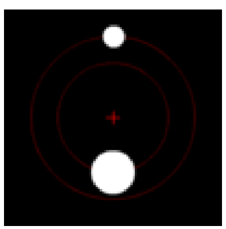
✓ The whole life of a BBH system:

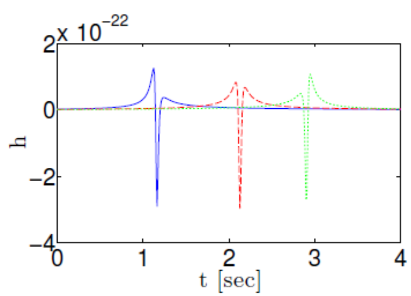
- “Inspiral-Merger-Ringdown” is just a tiny part at the last moment of binary coalescences!

Levin, McWilliams & Contreras (‘11): 3.5PN



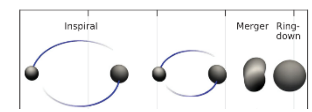


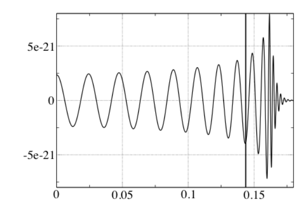




. . .

. . .





Formation of binary:

Unbound ➔ Bound

(Hyperbolic ➔ Elliptic)

Encounters

Precessions

Coalescence-IMR

“Construct a waveform model covering all of it, in particular, highly eccentric phases!”

✓ Binary formation through gravitational radiation capture

- Formation of compact binaries:

  - Primordial binaries (Postnov & Yungelson 06)
  - Three-body interactions (Aarseth & Heggie '76, Bae et al. 14)
  - **<u>Gravitational radiation capturing processes</u>**

- **Gravitational radiation captures:**
  - Hansen '72, Quinlan & Shapiro '87, '89; Lee '95, O'Leary et al. 09, Hong & Lee 15
  - All in the context of Post-Newtonian theory

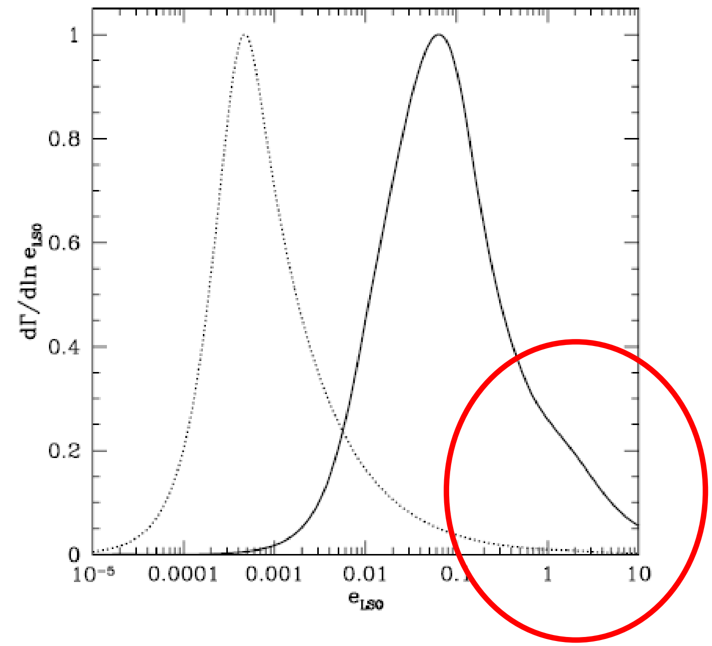
  - How good are the Post-Newtonian results? When do they break down?

➔ **Numerical studies at the level of full general relativity**

- How often such captures occur?
  - Direct capture could be rare: 0.02 ~ 5 yr^{-1} Gpc^{-3} (Hong & Lee '15)
  - How many binaries would be formed at a galaxy center w/ a SMBH?: O'Leary, Kocsis, Loeb ('09)

➔ Eccentricity distributions:

GC



$$\Gamma_{1GN} = \int_{r_{min}}^{r_{max}} dr\, 4\pi r^2 \int_{M_{min}}^{M_{max}} dM \int_{M_{min}}^{M} dm \times \iint_{x_m, x_M > 10, J > J_{LC}} d^3 v_m d^3 v_M f_m(r, v_m) f_M(r, v_M) \sigma_{cs} w,$$

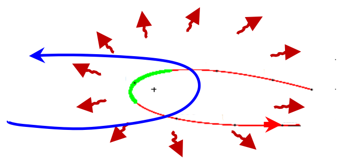
$\sim 10^{-8}$ and 10^{-10} yr^{-1}

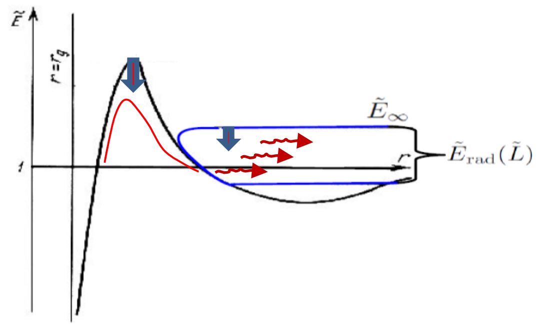
➔ 1~1000/yr at aLIGO!

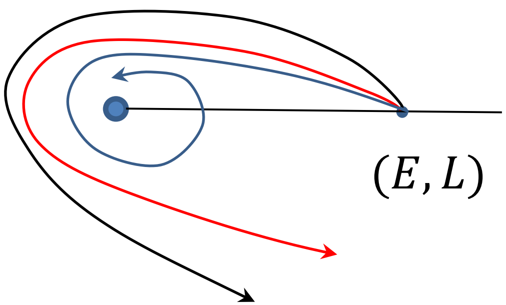
N-body simulations (Hong & Lee '13): ~ 0.02~20/yr

✓ NR simulations

- **Gravitational Radiation Capture:** “through GW emissions”







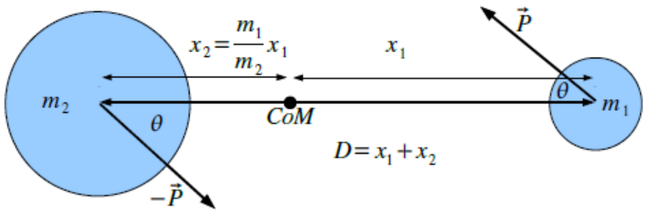
- The marginal capturing gives

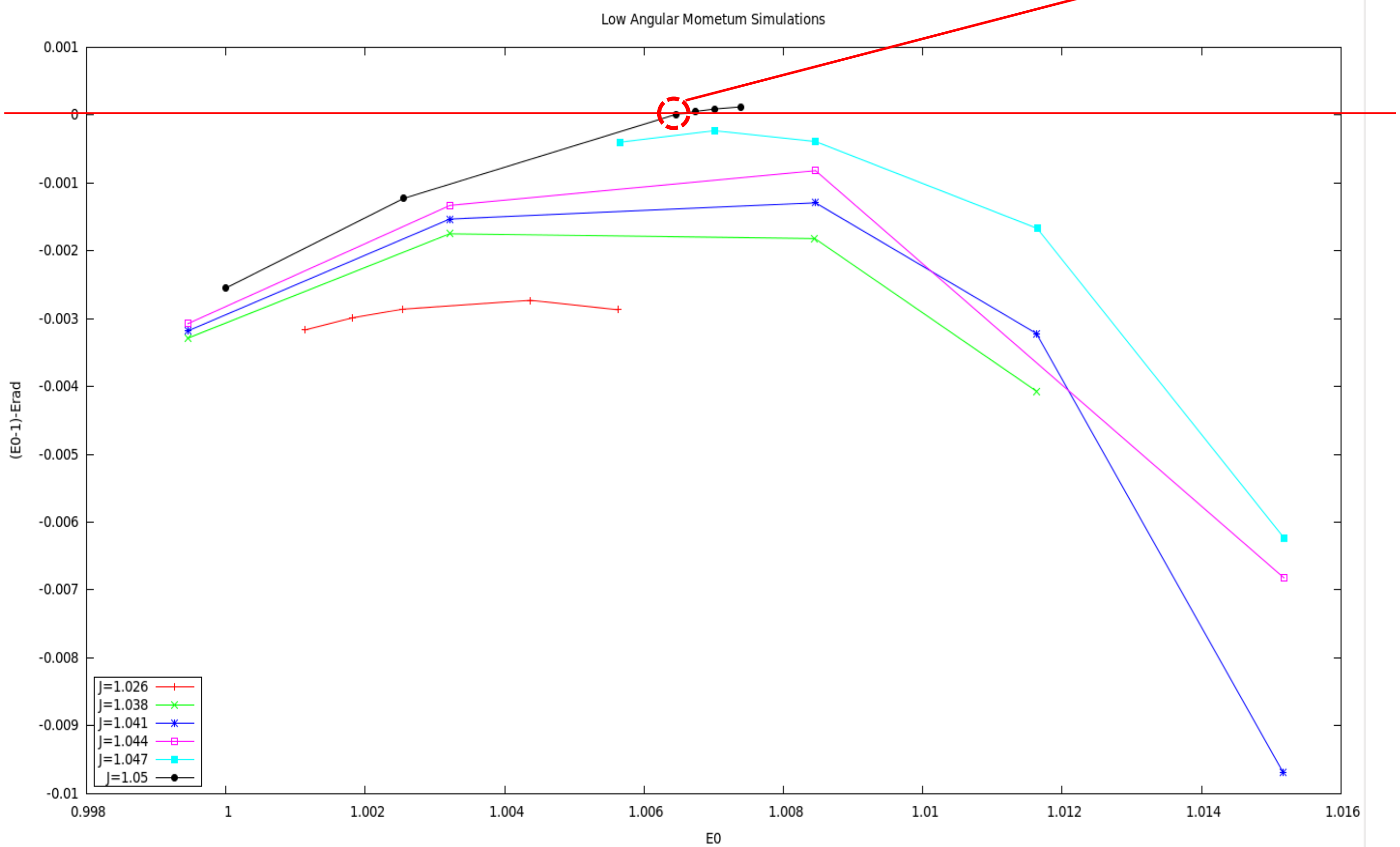
$$b_{\rm max} = \frac{L_{\rm cr}(E)}{\mu v_\infty} = \frac{L_{\rm cr}(E)}{\sqrt{2\mu E}}$$

$$\sigma_{\rm cap} = \pi b_{\rm max}^2$$

“Capturing Cross-section”

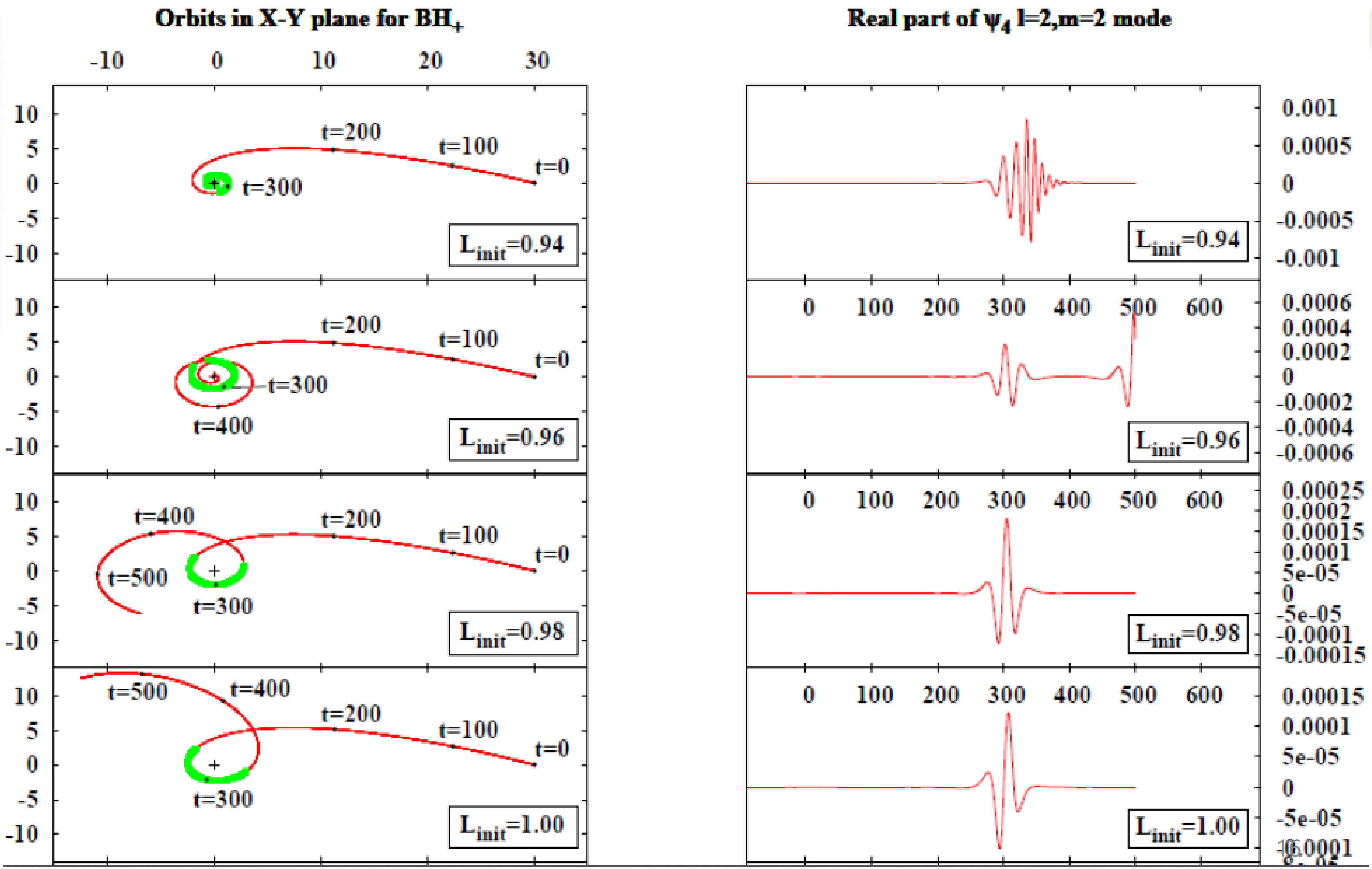
✓ Two non-spinning equal mass black holes:
w/ J. Hansen, P. Diener, F. Loefler & H. Kim ('13)





Identifying this point is computationally very expensive!

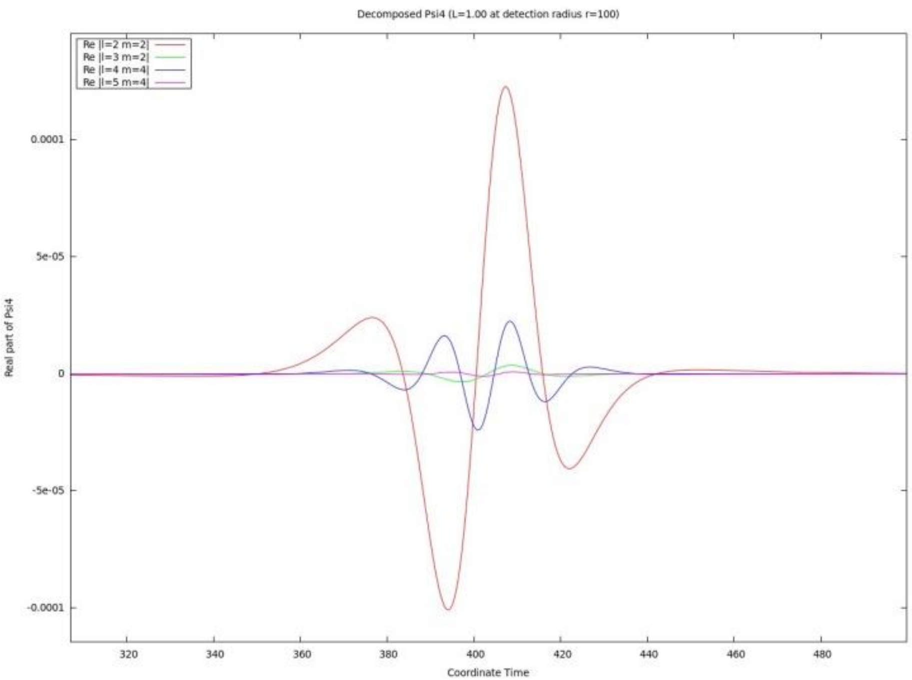
- **Features of orbits and waveforms:**

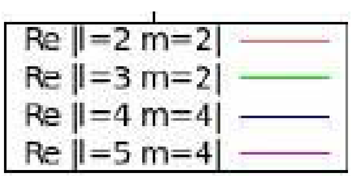


- **Eccentric orbits ➔ Non-negligible multi mode contributions?**

$$\Psi_4 = \sum_{l=2}^{\infty} \sum_{m=-l}^{l} A^{l,m} \left({}_{-2}Y^{l,m}(\theta, \phi) \right)$$

Decomposed Psi4 (L=1.00 at detection radius r=100)





$$\frac{dE}{dt} = \lim_{r\to\infty} \frac{r^2}{16\,\pi} \oint \left| \int_{-\infty}^{t} \Psi_4 \, dt' \right|^2 d\Omega$$

$$= \lim_{r\to\infty} \frac{r^2}{16\,\pi} \sum_{l,m} \left| \int_{-\infty}^{t} A^{l,m} \, dt' \right|^2$$

Energy budget (L=1.00):

l=2: ~98.9%
l=3: ~0.1%
l=4: ~1%
Rest: less than 0.01%

- **Maximum impact parameter or capturing cross-section:**

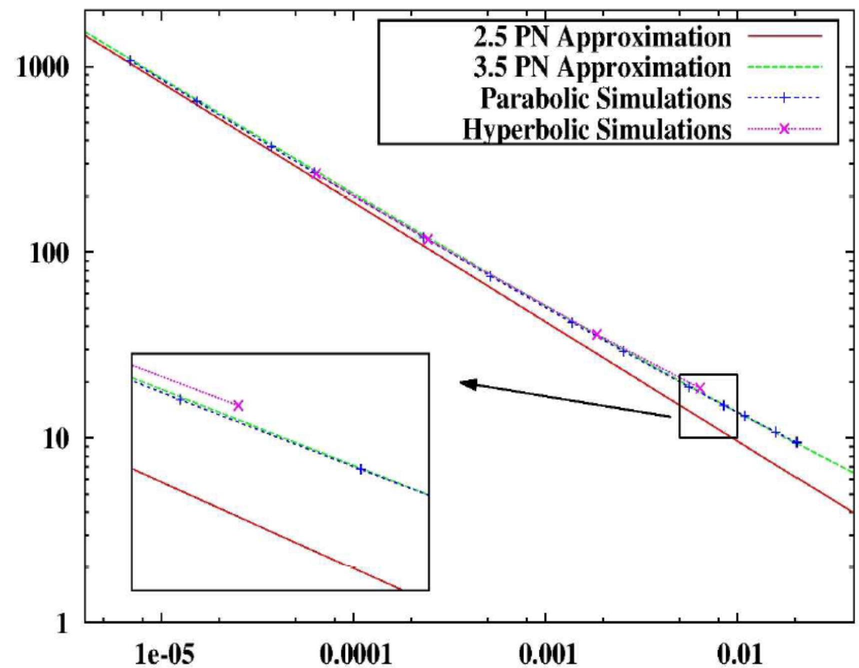
  - Less capturing for large initial energies

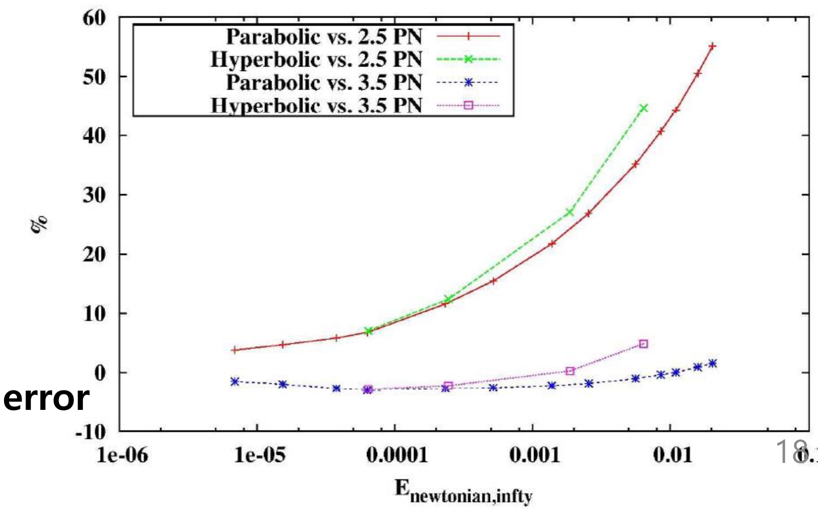
  - 2.5PN deviates from NR as E increases: ~40% maximally

  - But, 3.5PN is still in good agreement with error less than ~4%

  - For any given energy, the GR result gives the strongest capturing.

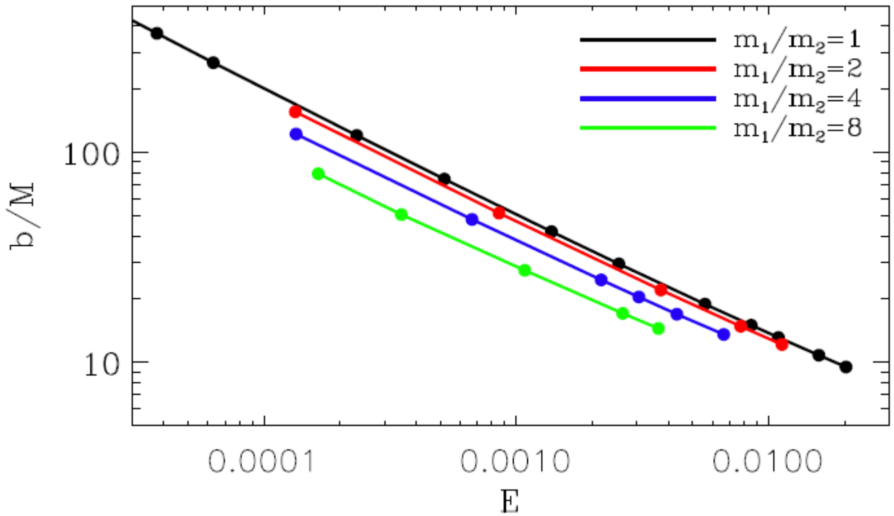
b_{max}

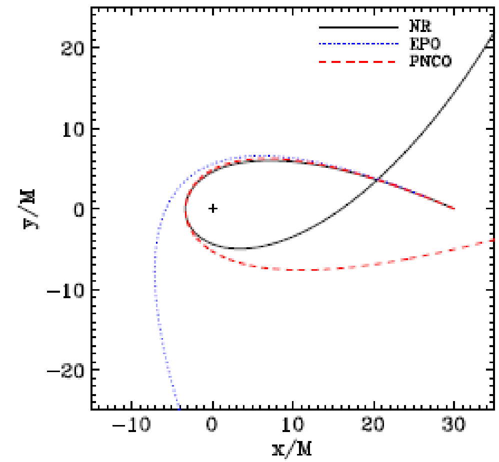




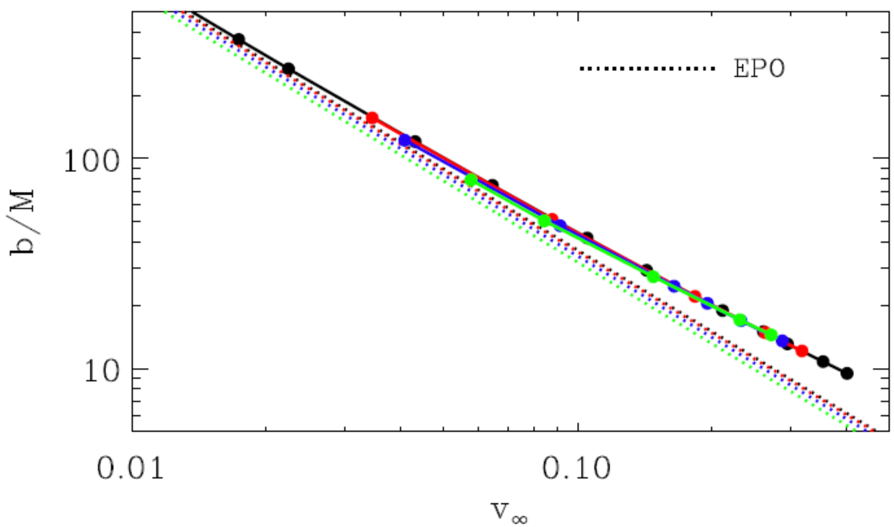
Percentage error

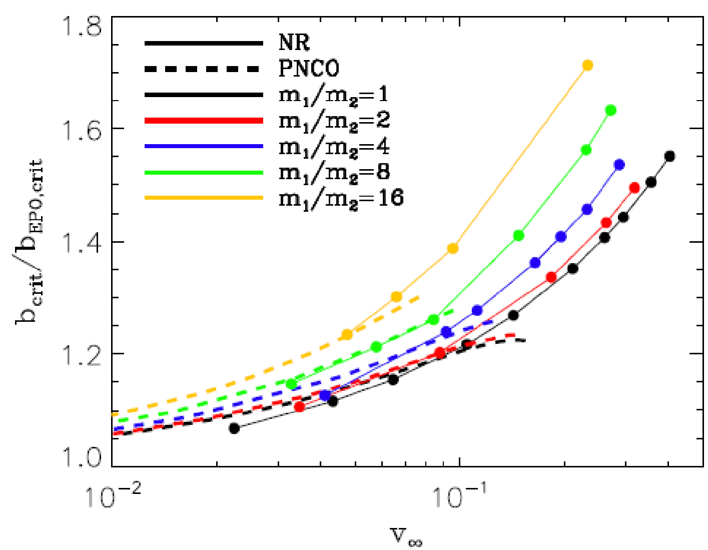
✓ Un-equal masses without spin: w/ Y. Bae, H. Lee & J. Hansen ('17)





- EPO (Exact parabolic orbit)
- PNCO (PN corrected orbit)
- NR orbit





✓ Effects of spin with equal masses: w/ Y. Bae & H. Lee ('20)

- How to design the parameter space?:
  - Energy flux emitted at 2.5PN order

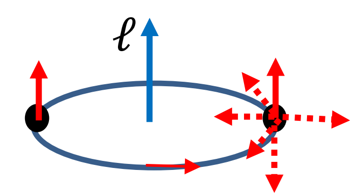
$$\mathcal{F} = \frac{G^3}{c^5}\left\{ f_{\rm NS} + f_{\rm SO} + f_{\rm SS} + \mathcal{O}\left(\frac{1}{c^6}\right)\right\}$$

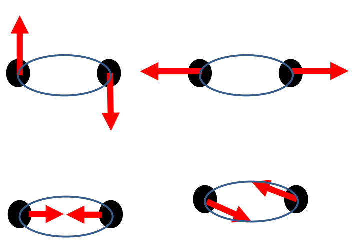
$$\mathcal{F}_{\rm SO} = AM\chi_{\rm eff}^{(+)} + B\delta m \chi_{\rm eff}^{(-)} + \cdots$$

$$\boldsymbol{S}_i = m_i^2 \boldsymbol{\chi}_i$$

$$\delta m = m_1 - m_2,$$

$$\chi_{\rm eff}^{(\pm)} = \boldsymbol{\chi}_\pm \cdot \boldsymbol{\ell} = \left(\frac{m_2}{M}\boldsymbol{\chi}_2 \pm \frac{m_1}{M}\boldsymbol{\chi}_1\right)\cdot \boldsymbol{\ell},$$

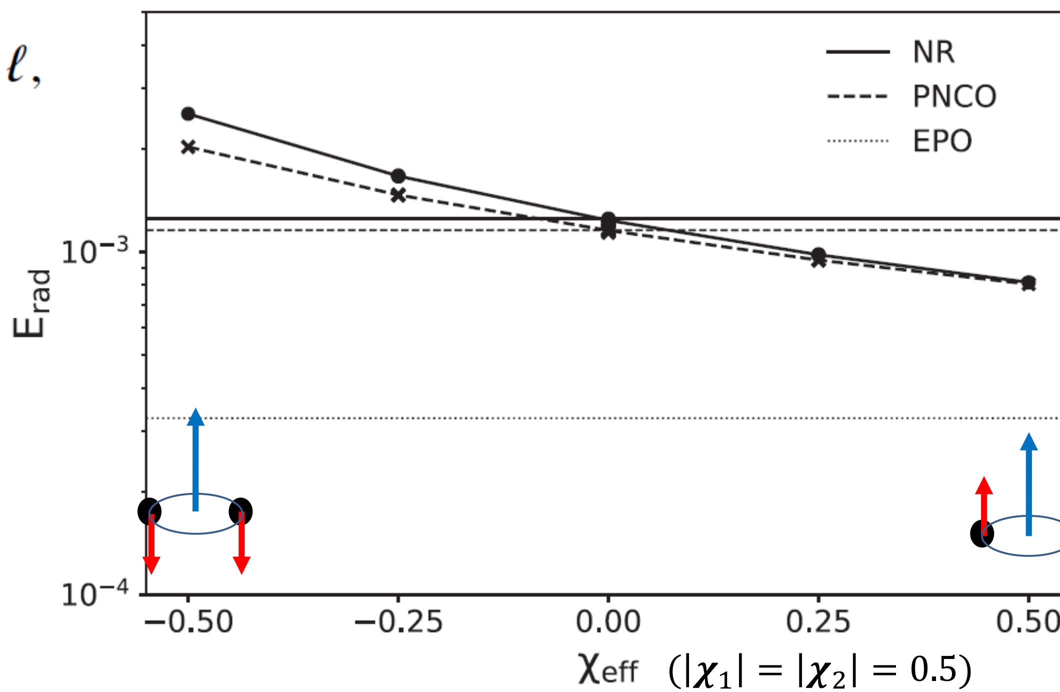


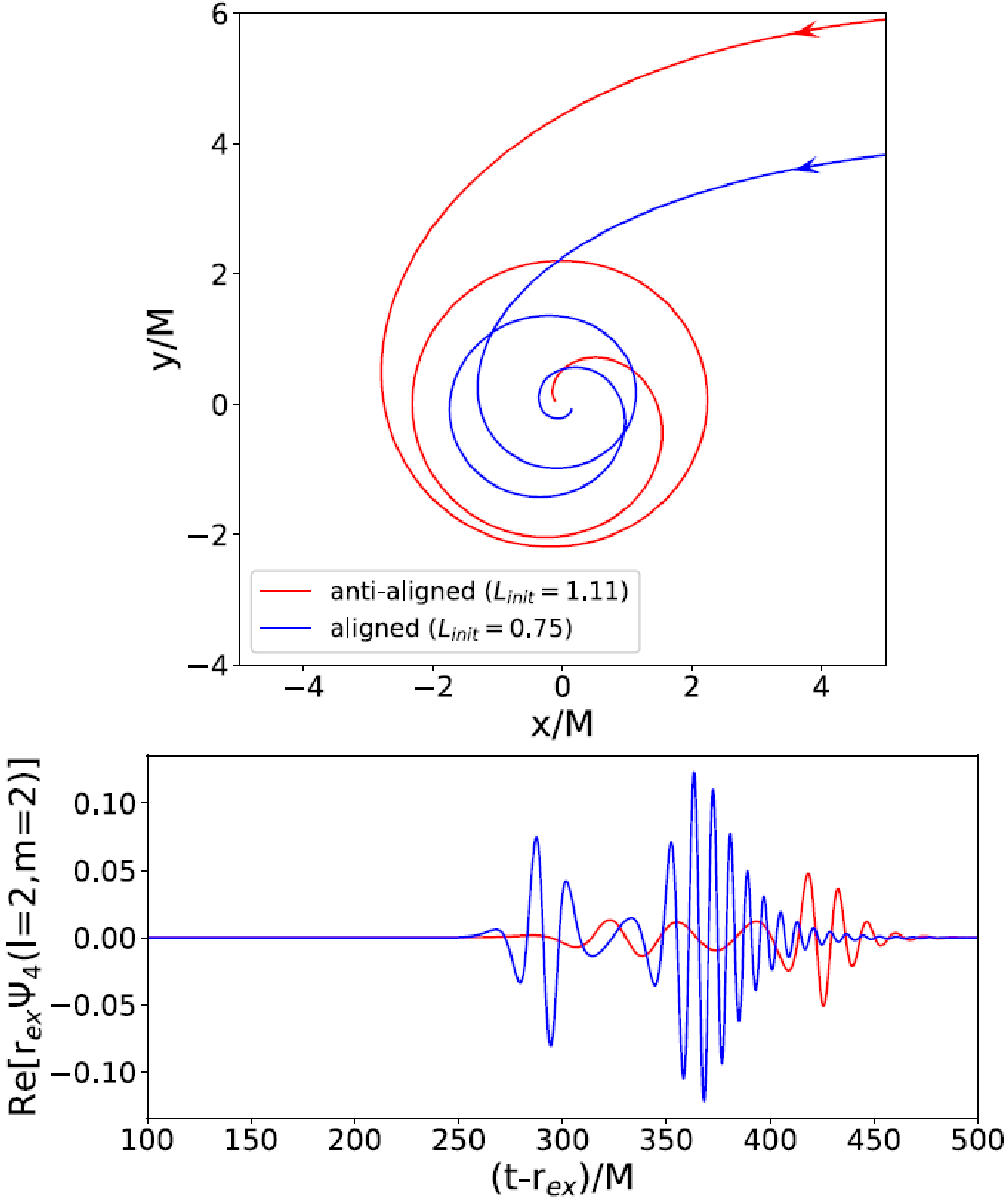


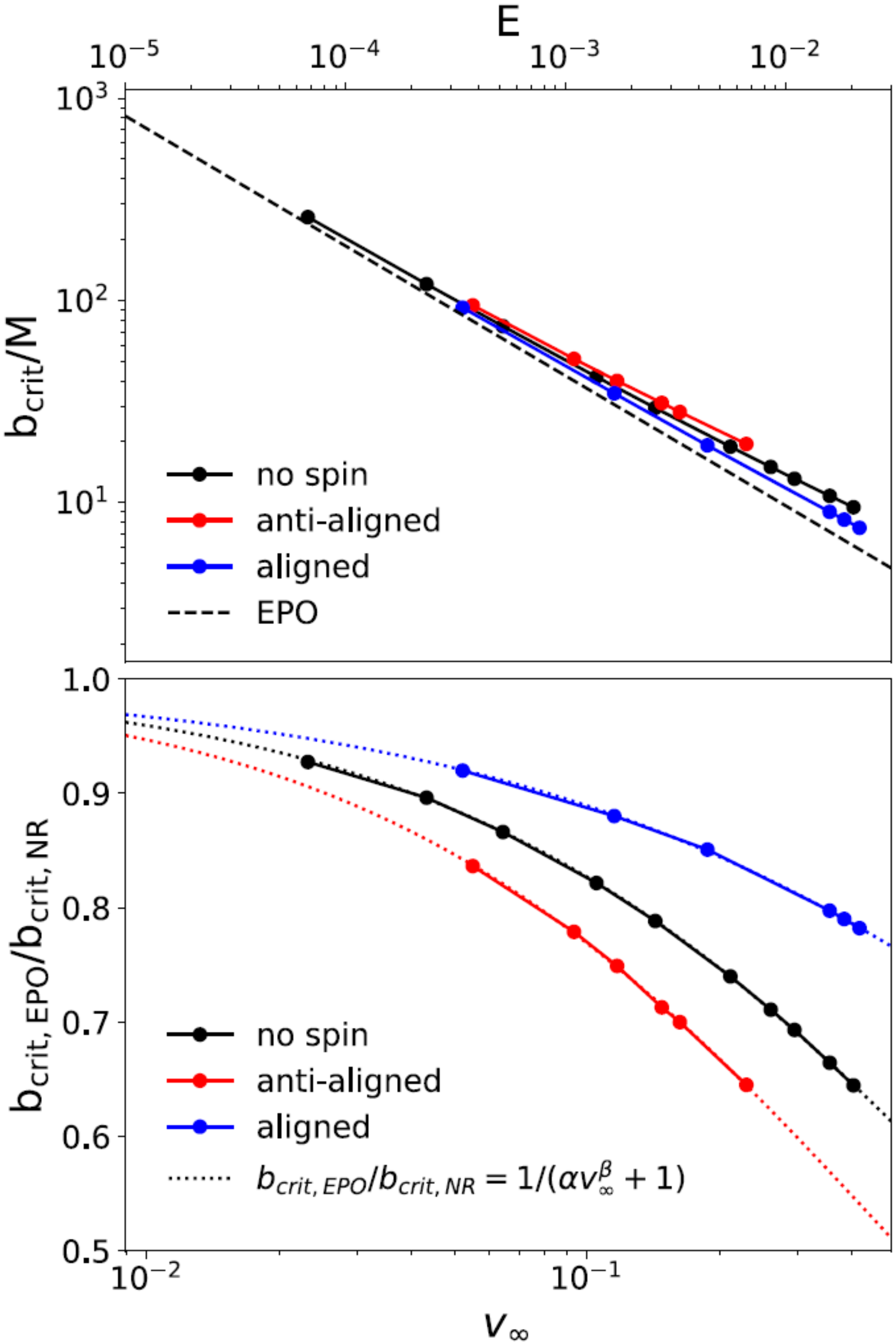
$$\chi_{eff}^{(+)} = \chi_{eff} = 0$$

...

Radiated energy through a single encounter




anti-aligned ($L_{init} = 1.11$)
aligned ($L_{init} = 0.75$)
y/M
x/M
6
4
2
0
−2
−4
−4
−2
0
2
4
Re[$r_{ex}\Psi_4$(l=2,m=2)]
(t-r_{ex})/M
0.10
0.05
0.00
−0.05
−0.10
100
150
200
250
300
350
400
450
500



- Two BHs with anti-aligned spins will be captured or form a binary more easily.

- The EPO approximation underestimates the capture cross-section.

- Error for the EPO approximation goes about 5% at $v_\infty = 0.01\,c$ ~ about 35% at $v_\infty = 0.2\,c$ with anti-aligned spins.

III. Waveform modeling with general eccentricity

- IMR waveforms with eccentricity:
  - Cao & Han 2017; Hinderer & Babak 2017; Hinder et al. 2018; Huerta et al. 2018; Ireland et al. 2019
  - Klein+2018, Tiwari+2019 (PN), Chiaramello & Nagar 2020 (EOB)

- Ex) Cao & Han 2017: works **up to $e \sim 0.2$** with overlap factor $\gtrsim 0.98$, compared to NR simulations. Near circular orbit though…

$$\dot{\vec{r}} = \frac{\partial H}{\partial \vec{p}}, \qquad \dot{\vec{p}} = -\frac{\partial H}{\partial \vec{r}} + \vec{\mathcal{F}}.$$

$$H = M\sqrt{1 + 2\eta\left(\frac{H_{\rm eff}}{M\eta} - 1\right)},$$

- Conservative part
- Same as SEOBNR's

$$\vec{\mathcal{F}} = \frac{1}{M\eta\omega_\Phi|\vec{r}\times\vec{p}|}\frac{dE}{dt}\vec{p}, \qquad -\frac{dE}{dt} = \frac{1}{16\pi}\sum_{\ell}\sum_{m=-\ell}^{\ell}|\dot{h}_{\ell m}|^2.$$

l=2, 3, …, 8

$$h_{22}^{\rm insp-plun} = h_{22}^{(C)} + h_{22}^{(PNE)}, \qquad h_{22}^{(PNE)} = h_{22} - h_{22}|_{\dot{r}=0},$$

for a circular orbit ↓ ($h_{22}^{(C)}$) circular + eccentricity ↓ (h_{22})

$$h_{\ell m}^{\rm merger-RD} = \sum_{n=0}^{N-1} A_{\ell mn} e^{-i\sigma_{\ell mn}(t - t_{\rm match}^{\ell m})}$$

$$M_{\rm final} = M[1 + 4(m^0 - 1)\eta + 16m^1\eta^2(\chi_1 + \chi_2)],$$

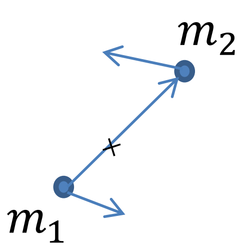
$$\chi_{\rm final} \equiv \frac{a_{\rm final}}{M_{\rm final}} = \chi^0 + \eta\chi^0(t_4\chi^0 + t_5\eta + t_0) + \eta(2\sqrt{3} + t_2\eta + t_3\eta^2),$$

SEOBNRE (Spinning EOBNR Eccentric)

✓ 3PM Hamiltonian:

- Recently, Bern et al (PRL, '19) have obtained the Hamiltonian at the third post-Minkowskian (3PM) order describing the scattering amplitude for two massive spinless particles in the context of effective field theory.

$$H(\boldsymbol{p},\boldsymbol{r}) = \sqrt{\boldsymbol{p}^2 + m_1^2} + \sqrt{\boldsymbol{p}^2 + m_2^2} + V(\boldsymbol{p},\boldsymbol{r}), \qquad V(\boldsymbol{p},\boldsymbol{r}) = \sum_{i=1}^{\infty} c_i(\boldsymbol{p}^2)\left(\frac{G}{|\boldsymbol{r}|}\right)^i,$$



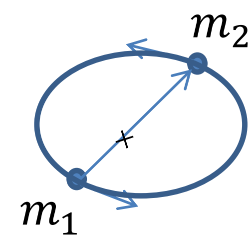
$$c_1 = \frac{\nu^2 m^2}{\gamma^2 \xi}(1-2\sigma^2), \qquad c_2 = \frac{\nu^2 m^3}{\gamma^2 \xi}\left[\frac{3}{4}(1-5\sigma^2) - \frac{4\nu\sigma(1-2\sigma^2)}{\gamma\xi} - \frac{\nu^2(1-\xi)(1-2\sigma^2)^2}{2\gamma^3\xi^2}\right],$$

$$c_3 = \frac{\nu^2 m^4}{\gamma^2 \xi}\left[\frac{1}{12}(3-6\nu+206\nu\sigma-54\sigma^2+108\nu\sigma^2+4\nu\sigma^3) - \frac{4\nu(3+12\sigma^2-4\sigma^4)\,\mathrm{arcsinh}\sqrt{\frac{\sigma-1}{2}}}{\sqrt{\sigma^2-1}}\right.$$
$$- \frac{3\nu\gamma(1-2\sigma^2)(1-5\sigma^2)}{2(1+\gamma)(1+\sigma)} - \frac{3\nu\sigma(7-20\sigma^2)}{2\gamma\xi} - \frac{\nu^2(3+8\gamma-3\xi-15\sigma^2-80\gamma\sigma^2+15\xi\sigma^2)(1-2\sigma^2)}{4\gamma^3\xi^2}$$
$$\left. + \frac{2\nu^3(3-4\xi)\sigma(1-2\sigma^2)^2}{\gamma^4\xi^3} + \frac{\nu^4(1-2\xi)(1-2\sigma^2)^3}{2\gamma^6\xi^4}\right],$$

$$E_{1,2} = \sqrt{\boldsymbol{p}^2 + m_{1,2}^2}, \quad m = m_1 + m_2, \quad \nu = m_1 m_2/m^2 \quad E = E_1 + E_2, \quad \xi = E_1 E_2/E^2,$$
$$\gamma = E/m, \quad \sigma = p_1 \cdot p_2/m_1 m_2.$$

- An EOB Hamiltonian for a system of two BHs in general

$$H^{\mathrm{EOB}} = M\sqrt{1 + 2\nu\left(\frac{H^{\mathrm{eff}}}{\mu} - 1\right)},$$



For $\nu = \frac{m_1 m_2}{(m_1+m_2)^2} \to 0$, i.e., $m_1 \gg m_2$, we know the answer

$$H_{\mathrm{S}}^2 = \left(1 - \frac{2GM}{r}\right)\left[\mu^2 + \frac{L^2}{r^2} + \left(1 - \frac{2GM}{r}\right)p_r^2\right]$$

✓ **Antonelli, Buonanno+ 2019:**

Then, the post-Schwazschild EOB Hamiltonian at 3PM would be

$$(\hat{H}^{\mathrm{eff,PS}})^2 = \hat{H}_{\mathrm{S}}^2 + (1 - 2u)[u^2 q_{\mathrm{2PM}} + u^3 q_{\mathrm{3PM}} + \mathcal{O}(G^4)] \qquad \hat{H}^{\mathrm{eff}} = \frac{H^{\mathrm{eff}}}{\mu}, \quad \cdots$$

$$q_{\mathrm{2PM}} = \frac{3}{2}(5\hat{H}_{\mathrm{S}}^2 - 1)\left(1 - \frac{1}{\sqrt{1 + 2\nu(\hat{H}_{\mathrm{S}} - 1)}}\right), \quad q_{\mathrm{3PM}} = -\frac{2\hat{H}_{\mathrm{S}}^2 - 1}{\hat{H}_{\mathrm{S}}^2 - 1} q_{\mathrm{2PM}} + \frac{4}{3}\nu\hat{H}_{\mathrm{S}}\frac{14\hat{H}_{\mathrm{S}}^2 + 25}{1 + 2\nu(\hat{H}_{\mathrm{S}} - 1)}$$

$$+ \frac{8\nu}{\sqrt{\hat{H}_{\mathrm{S}}^2 - 1}}\frac{4\hat{H}_{\mathrm{S}}^4 - 12\hat{H}_{\mathrm{S}}^2 - 3}{1 + 2\nu(\hat{H}_{\mathrm{S}} - 1)}\sinh^{-1}\sqrt{\frac{\hat{H}_{\mathrm{S}} - 1}{2}}.$$

Angular momentum: $l \equiv |\boldsymbol{L}|/(GM\mu).$

- PN/PM corrected:

$$H^{eff} = G^0\left(\cdots + \left(\frac{v}{c}\right)^{2n} + \cdots\right) + G\left(\cdots + \left(\frac{v}{c}\right)^{2n} + \cdots\right) + G^2\left(\cdots + \left(\frac{v}{c}\right)^{2n} + \cdots\right) + G^3\left(\cdots + \left(\frac{v}{c}\right)^{2n} + \cdots\right) + G^4\left(\cdots + \left(\frac{v}{c}\right)^{2n} + \cdots\right) + \cdots$$

$$H^{\mathrm{EOB}} = M\sqrt{1 + 2\nu\left(\frac{H^{\mathrm{eff}}}{\mu} - 1\right)}, \qquad H_S^2 = \left(1 - \frac{2GM}{r}\right)\left[\mu^2 + \frac{L^2}{r^2} + \left(1 - \frac{2GM}{r}\right)p_r^2\right]$$

$$\hat{H}^{\mathrm{eff}} = \frac{H^{\mathrm{eff}}}{\mu}, \quad \hat{H}_S = \frac{H_S}{\mu}, \quad u = \frac{GM}{r}, \quad \hat{p}_r = \frac{p_r}{\mu}, \quad l \equiv \hat{p}_\phi = \frac{L}{GM\mu},$$

$$[\hat{H}^{\mathrm{eff,PS}}(u, \hat{p}_r, l)]^2 = \hat{H}_S^2 + (1 - 2u)\hat{Q}^{\mathrm{PS}}(u, \hat{H}_S, \nu)$$

$$\hat{H}_S^2 = (1 - 2u)[1 + l^2u^2 + (1 - 2u)\hat{p}_r^2]$$

$$\hat{Q}^{\mathrm{PS}} = u^2 q_{\mathrm{2PM}}(\hat{H}_S, \nu) + u^3 q_{\mathrm{3PM}}(\hat{H}_S, \nu) + \Delta_{\mathrm{3PN}}(u, \hat{H}_S, \nu) + \Delta_{\mathrm{4PN}}(u, \hat{H}_S, \nu) + \mathcal{O}(\mathrm{5PN})$$

$$q_{\mathrm{2PM}} = \frac{3}{2}(5\hat{H}_S^2 - 1)\left(1 - \frac{1}{\sqrt{1 + 2\nu(\hat{H}_S - 1)}}\right)$$

$$q_{\mathrm{3PM}} = -\frac{2\hat{H}_S^2 - 1}{\hat{H}_S^2 - 1} q_{\mathrm{2PM}} + \frac{4}{3}\nu\hat{H}_S \frac{14\hat{H}_S^2 + 25}{1 + 2\nu(\hat{H}_S - 1)} + \frac{8\nu}{\sqrt{\hat{H}_S^2 - 1}} \frac{4\hat{H}_S^4 - 12\hat{H}_S^2 - 3}{1 + 2\nu(\hat{H}_S - 1)} \sinh^{-1}\sqrt{\frac{\hat{H}_S - 1}{2}}$$

$$\Delta_{\mathrm{3PN}} = \left(\frac{175}{3}\nu - \frac{41\pi^2}{32}\nu - \frac{7}{2}\nu^2\right)u^4$$

$$\Delta_{\mathrm{4PN}} = \sum_{n=2}^{5} \alpha_{4n} u^n (\hat{H}_S^2 - 1)^{5-n} + (\alpha_{44,\ln} u^4 (\hat{H}_S^2 - 1) + \alpha_{45,\ln} u^5) \ln u$$

$$\alpha_{42} = \left(-\frac{1027}{12} - \frac{147432}{5}\ln 2 + \frac{1399437}{160}\ln 3 + \frac{1953125}{288}\ln 5\right)\nu,$$

$$\alpha_{43} = \left(-\frac{78917}{300} - \frac{14099512}{225}\ln 2 + \frac{14336271}{800}\ln 3 + \frac{4296875}{288}\ln 5\right)\nu,$$

$$\alpha_{44} = \left(-\frac{43807}{225} + \frac{296\gamma_{\rm E}}{15} - \frac{33601\pi^2}{6144} - \frac{9771016}{225}\ln 2 + \frac{1182681}{100}\ln 3 + \frac{390625}{36}\ln 5\right)\nu + \left(-\frac{405}{4} + \frac{123}{54}\pi^2\right)\nu^2 + \frac{13}{2}\nu^3,$$

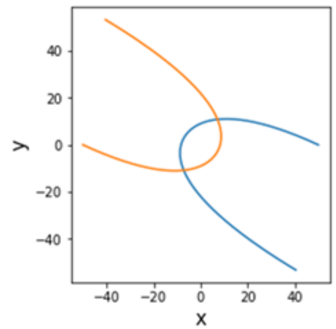
$$\alpha_{45} = \left(-\frac{34499}{1800} + \frac{136}{3}\gamma_{\rm E} - \frac{29917}{6144}\pi^2 - \frac{254936}{25}\ln 2 + \frac{1061181}{400}\ln 3 + \frac{390625}{144}\ln 5\right)\nu + \left(-\frac{2387}{24} + \frac{205}{64}\pi^2\right)\nu^2 + \frac{9}{4}\nu^3,$$

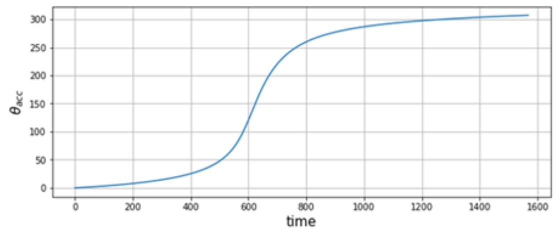
and

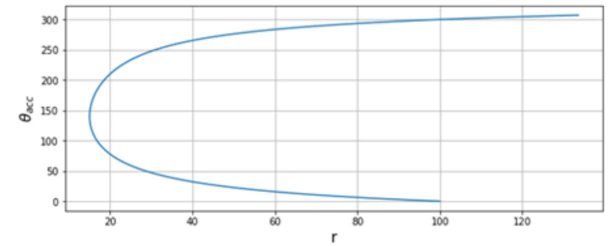
$$\alpha_{44,\ln} = \frac{148}{15}\nu, \qquad \alpha_{45,\ln} = \frac{68}{3}\nu.$$

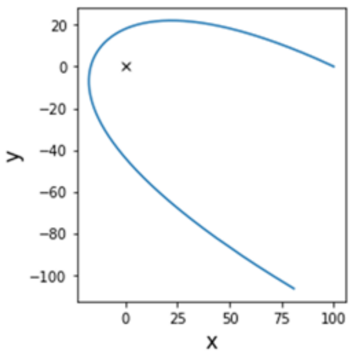
γ_E = 0.57721... is the Euler-Mascheroni constant.

- **Scattering angles:** NR vs Newtonian vs EOB vs 3PM









$$H(r, p_r, L) = E$$

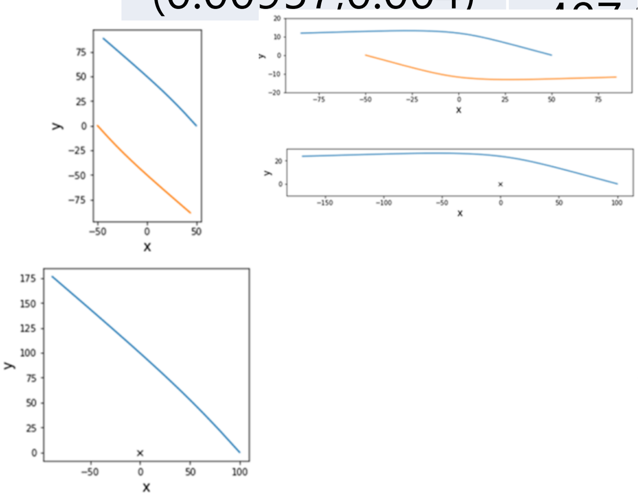
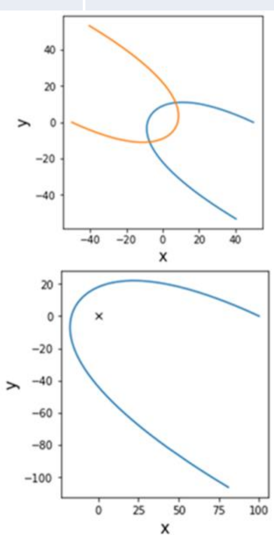
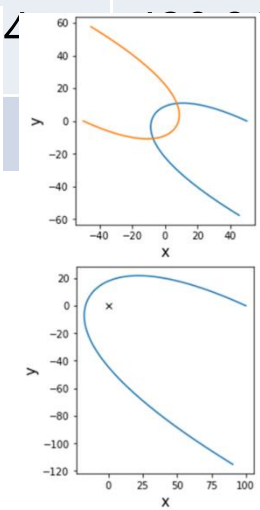
$$p_r(r, E, L)$$

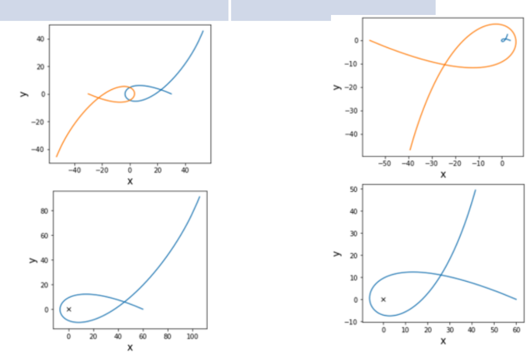
$$\left(\frac{\partial H}{\partial L}\right)\left(r, p_r, \frac{\partial p_r}{\partial L}, L\right) = 0$$

$$\Delta\phi = \pi + \chi(E, L) = -2\int_{r_{\min}}^{\infty} dr \frac{\partial}{\partial L} p_r(r, E, L)$$

- Results:

Case: $\vec{P} = (P_x, P_y)$	NR	Newtonian	3PM	EOB	$\Delta\chi/\chi$ (%)
(0.090, 0.099)	90.13	87.97	90.34	89.78	-0.388
$(0.214, 0.058)$	165.11°	155.00°	165.73°	165.30	0.115
$(0.0326, 0.015)$	300.11°	258.51	299.98	300.52	0.137
(0.0331, 0.015)	299.08	257.63	299.12	299.67	0.197
(0.0376, 0.012)	356.05	273.65	370.20	359.91	1.084
(0.043, 0.0185)	376.41	262.64	359.44	384.72	2.208
(0.00957,0.004)	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]



✓ Design the parameter space in which we can check the validity of our EOB Hamiltonian:

- High velocity and strong interaction

b	θ	px=pr	py	J	$E_{5\,\text{PNlog}}$ →	$\chi^{\text{Heob}}_{5\,\text{PNlog}}$	$\chi^{\text{Heob}}_{3\,\text{PM+4 PN}}$	$\chi^{\text{Heob}}_{3\,\text{PM}}$	$\chi^{\text{H}}_{3\,\text{PM}}$	$\chi_{4\,\text{PN}}$	$\chi_{3\,\text{PN}}$	$\chi^{\text{Heob}}_{4\,\text{PN}}$	$\chi^{\text{Heob}}_{3\,\text{PN}}$	$\chi^{\text{Heob}}_{2\,\text{PN}}$	$\chi^{\text{Heob}}_{1\,\text{PN}}$
3.75	2.15	−0.666	0.025	2.5	1.377699	0	0	0	0	163.4	126.4	0	0	0	0
4.	2.29	−0.666	0.0267	2.67	1.3777	0	0	0	0	145.5	112.2	0	0	0	0
4.25	2.44	−0.666	0.0283	2.83	1.377701	0	0	0	0	130.9	100.6	0	0	0	0
4.5	2.58	−0.666	0.03	3.	1.377702	177.2	169.7	0	0	118.7	91.14	219.	0	0	0
4.75	2.72	−0.666	0.0317	3.17	1.377703	119.7	114.9	247.3	0	108.4	83.21	125.8	255.7	0	0
5.	2.87	−0.666	0.0333	3.33	1.377704	98.1	94.7	131.5	0	99.69	76.5	100.8	133.1	0	0
5.25	3.01	−0.666	0.035	3.5	1.377705	85.08	82.68	104.	0	92.15	70.76	86.58	103.8	206.3	0
5.5	3.15	−0.666	0.0367	3.67	1.377706	75.89	74.28	88.66	0	85.61	65.8	76.82	87.66	126.2	162.3
5.75	3.3	−0.666	0.0383	3.83	1.377707	68.87	67.9	78.32	0	79.87	61.48	69.48	76.88	99.79	117.1
6.	3.44	−0.665	0.04	4.	1.377708	63.24	62.78	70.68	0	74.8	57.68	63.66	68.96	84.57	95.91
6.25	3.58	−0.665	0.0417	4.17	1.377709	58.59	58.54	64.69	0	70.3	54.31	58.89	62.82	74.24	82.58
6.5	3.73	−0.665	0.0433	4.33	1.377711	54.66	54.93	59.83	0	66.27	51.32	54.88	57.87	66.62	73.15
6.75	3.87	−0.665	0.045	4.5	1.377712	51.27	51.79	55.77	0	62.65	48.63	51.44	53.76	60.68	66.01

- We have finished NR simulations for these parameters.

- The analysis and comparisons with the predictions of the known waveform models, which are valid in the limited regimes, are currently work in progress.

IV. Conclusion

- Gravitational radiation capture processes for two BHs have been analyzed numerically.

- Effects of unequal masses (upto $m_1 : m_2 = 1:16$) and spin configurations ($\chi_{eff}^{(+)}$) are shown.

- Scattering angles for BH encounters with arbitrary eccentricities are calculated in NR.

- A 3PM/4PN EOB Hamiltonian for arbitrary eccentricities has been constructed and various tests with NR simulations are work in progress. (w/ Y.-H. Hyun & Y.-B. Bae)

THANKS!