

Recap of day 1:

- review DM-n scattering
- kinematics \rightarrow
- we saw that searching sub-GeV DM using DM-n scattering was not viable
- looked DM-e⁺ scattering
- derived general form for the scattering rate

$$R_{1 \rightarrow 2} = \frac{p_F}{m_F} \frac{\overline{\sigma e}}{8\pi \mu_F e^2} \int d^3 q \frac{1}{q} \eta(v_{min}) |F_{DM}(q)|^2 \\ \times |f_{1 \rightarrow 2}(q)|^2$$

$$|M_{free}(q)|^2 = |M_{free}(dm_e)|^2 \times |F_{DM}(q)|^2$$

$$\overline{\sigma e} \equiv \mu_F e^2 |\overline{M_{free}(dm_e)}|^2$$

$$\eta(v_{min}) = \int_{v_{min}}^{\infty} \frac{d^3 v}{v} g_X(v)$$

$$v_{min} = \frac{\Delta E_{1 \rightarrow 2}}{q} + \frac{q}{2m_F}$$

Let's take a closer look at $f_{1 \rightarrow 2}(\vec{q})$:

$$f_{1 \rightarrow 2}(\vec{q}) = \int d^3x \psi_2^*(\vec{x}) \psi_1(\vec{k}) e^{i\vec{q} \cdot \vec{x}}$$

ex. For a free electron, $\psi_i(\vec{x})$ are plane waves

$$\begin{aligned} |f_{1 \rightarrow 2}(\vec{q})|^2 &= \left| \int d^3x \frac{e^{i\vec{k} \cdot \vec{x}}}{\sqrt{V}} \frac{e^{i\vec{k}' \cdot \vec{x}}}{\sqrt{V}} e^{-i\vec{q} \cdot \vec{x}} \right|^2 \\ &= \left| \frac{1}{\sqrt{V}} \int d^3x e^{i(\vec{k} - \vec{k}' - \vec{q}) \cdot \vec{x}} \right|^2 \\ &= \left| \frac{1}{\sqrt{V}} (2\pi)^3 \delta^{(3)}(\vec{k} - \vec{k}' - \vec{q}) \right|^2 \end{aligned}$$

$$\begin{aligned} (2\pi)^3 \delta^{(3)}(\vec{0}) &\equiv V \\ &= \frac{1}{V} (2\pi)^3 \delta^{(3)}(\vec{k} - \vec{k}' - \vec{q}) \end{aligned}$$

$$\Rightarrow V^2 |f_{1 \rightarrow 2}(\vec{q})|^2 = V \cdot (2\pi)^3 \delta^{(3)}(\vec{k} - \vec{k}' - \vec{q})$$

which reproduces the result for
free $2 \rightarrow 2$ scattering \therefore

$$R \sim N_T \frac{P_T}{m_\chi} \bar{\rho}_e V_0 \sim 50-100 \text{ events/kg/day}$$

$$\text{silicon: } N_T \sim 10^{26} / \text{kg}, \bar{\rho}_e = 10^{-37} \text{ cm}^2$$

$$M_x = 100 \text{ MeV}$$

Ionizing an isolated atom

- electrons are bound in isolated atomic potentials
 - * • spherical atomic potentials
 - * , filled shells

these approximations are accurate for low q

- full calculations require knowledge of the electron wavefunctions in a dense, disordered liquid \rightarrow can use DFT

take final state: continuum of positive-energy states where asymptotically

$$E_R = k'^2/2me$$

$$(k' l' m')$$

ionized electron phase space

$$\sum_{l', m'} \int \frac{k'^2 dk'}{(2\pi)^3} = \frac{1}{2} \sum_{l', m'} \frac{k'^3}{(2\pi)^3} d\ln E_R$$

$$\langle \tilde{\Psi}_{k'l'm'} | \tilde{\Psi}_{k'm'} \rangle = (2\pi)^3 \delta_{kk'} \delta_{mm'} + \frac{1}{k'^2} \delta(k-k')$$

$$\frac{dR_{\text{ion}}}{d\ln E_R} = \frac{p_F}{M_F} \frac{\bar{F}_e}{16\pi\mu_F c^2} \times v_{\min}(q, \Delta E)$$

$$\sum_{\text{occupied states}} \sum_{l'm'} \int \frac{k'^3}{(2\pi)^3} \frac{d^3 q}{q} \eta(v_{\min}) |F_{0l'm'}(q)|^2 |f_i \rightarrow f'l'm'(q)|^2$$

$$\Delta E = E_{B_i} + k'^2/2me \quad \text{where } E_{B_i} = \text{binding energy of initial state } i.$$

As a result of these assumptions

* **filled-shells:** we can sum $|f_i \rightarrow f'l'm'|^2$ over all initial and final angular momentum variables

* **spherical atomic potentials:** the result only depends on $|q|$

Define a dimensionless ionization form-factor

$$|f_{lm}(k', q)|^2 = \frac{2k'^3}{(2\pi)^3} \sum_{\text{occupied states}} \sum_{l'm'} |f_i \rightarrow f'l'm'(q)|^2$$

$$= \frac{2k'^3}{(2\pi)^3} \sum_i \sum_{l'm'} \left| \int d^3 x \Psi_i(\vec{x}) \Psi_{f'l'm'}^*(\vec{x}) e^{iq \cdot \vec{x}} \right|^2$$

$$\text{and take } d^3 q \rightarrow 4\pi q^2 dq$$

$$\frac{dR_{ion}}{d\ln E_R} = \frac{e}{m_e} \frac{\bar{v}_e}{8\pi r^2} \int q dq |F_{ion}(q)|^2 |f_{ion}(k', q)|^2 \times \gamma(v_{min})$$

ex. Hydrogen atom

initial: ground state

final: $\ell=0, m=0, k'$

$$|f_{ion}(k', q)|^2 = \frac{64 (k' a_0)^3}{\pi [a_0^4 (k'^2 - q^2)^2 + 2a_0^2 (k'^2 q^2) + 1]^2}$$

a_0 = bohr radius

ex. initial bound state: $\Psi_{nem} = R_{ne}(r) Y_{nlm}(\theta, \phi)$

final state: plane wave: $\Psi_{k' l' m'} \sim e^{ik' \cdot \vec{r}}$

$$|f_{ion}^{nl}(k', q)|^2 = \int_{(k'-q_1)}^{|k'+q_1|} \frac{(2l+1) k'^2 k dk}{4\pi^2 q} |\chi_{nl}(k')|^2$$

$$\chi_{nl}(k) = 4\pi (-i)^l \int r^2 dr R_{ne}(r) j_l(kr)$$

n.b. this form factor is enhanced by a Fermi-factor

$$\left| \frac{\Psi_{\text{exact}}(0)}{\Psi_{\text{free}}(0)} \right|^2 = F(k', z_{\text{eff}}) = \frac{2\pi\eta}{1 - e^{-2\pi\eta}}$$

$$\eta = z_{\text{eff}} \frac{a_0}{k'} = z_{\text{eff}} / (a_0 k')$$

z_{eff} = effective nuclear charge

a_0 = Bohr radius

Ex. initial bound state $\Psi_{lm} = R_{lm}(r) Y_{lm}(\theta, \phi)$

final state: $\Psi_{l'm'} = R_{l'm'}(r) \underline{Y_{l'm'}(\theta, \phi)}$

$$|\Psi_{l'm'}(k', q)|^2 = \frac{4E'}{(2\pi)^3} \sum_{l'l'm'} (2l+1)(2l'+1)(2L+1)$$

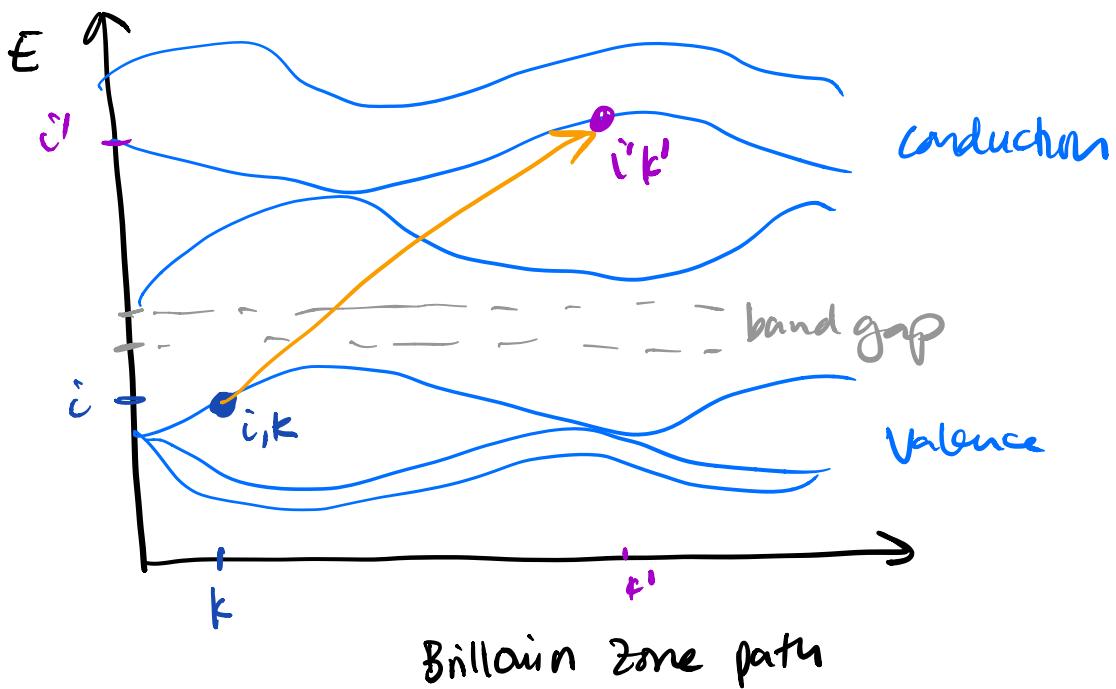
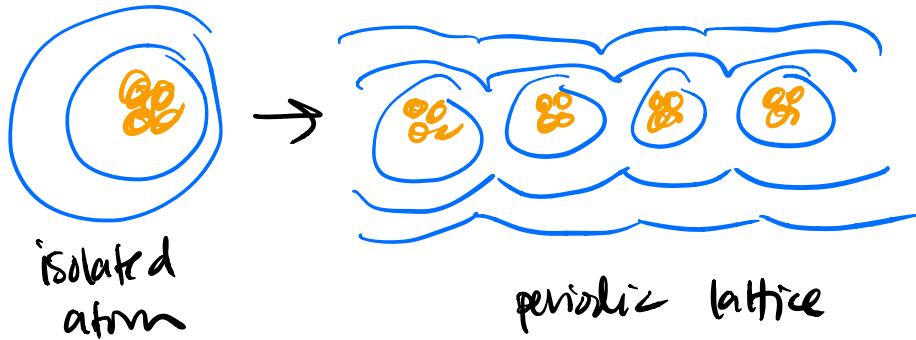
$$\times \begin{pmatrix} l & l' & L \\ 0 & 0 & 0 \end{pmatrix}^2$$

$$\times \left| \int r^2 dr R_{l'm'}(r) R_{lm}(r) j_{l'm'}(qr) \right|^2$$

$R_{l'm'}(r)$ is the solution to the radial Schrödinger equation with potential $z_{\text{eff}}(r)/r$

Excitations in a crystal

ex. silicon, germanium, NaI, GaAs



Bloch electrons form the solutions to the single-electron Schrödinger eq. with a periodic lattice potential:

$$\Psi_{i\vec{k}}(\vec{x}) \rightarrow \frac{1}{\sqrt{V}} \sum_{\vec{G}} U_i(\vec{k} + \vec{G}) e^{i(\vec{E} + \vec{G}) \cdot \vec{x}}$$

↑
band index ↑ wave vector
 ↑ crystal volume

Bloch Waves
reciprocal lattice vector

with normalization $\sum_{\vec{G}} |U_i(\vec{k} + \vec{G})|^2 = 1$

$$\Psi_i \vec{k} = \Psi_i(\vec{E} + \vec{G})$$

$$|f_{i\vec{k} \rightarrow i'\vec{k}'}(q)|^2 =$$

$$= \left| \sum_{\vec{G}, \vec{G}'} \frac{(2\pi)^3 \delta^{(3)}(\vec{k} + \vec{q} - \vec{k}' - \vec{G}')}{\sqrt{V}} U_{i'}(\vec{k}' + \vec{q} + \vec{G}') U_i(\vec{k} + \vec{G}) \right|^2$$

$$= \sum_{\vec{G}, \vec{G}'} \frac{(2\pi)^3 \delta^{(3)}(\vec{k} + \vec{q} - \vec{k}' - \vec{G}')}{\sqrt{V}} \underbrace{\left| \sum_{\vec{G}} U_{i'}(\vec{k}' + \vec{q} + \vec{G}') U_i(\vec{k} + \vec{G}) \right|^2}_{f[i\vec{k} i' \vec{k}' \vec{G}']}$$

We can now write down the rate of scattering for an electron to go from $\{i\vec{k}\}$ to $\{i'\vec{k}'\}$

$$R_{i\vec{k} \rightarrow i'\vec{k}'} = \frac{e^2}{m_e} \frac{e^2 \pi^2}{\mu_{Fe}^2} \frac{1}{V} \sum_{\vec{q}} \frac{1}{q} \eta(v_{min}) |F_{Dm}(q)|^2$$

$$\times |f_{i\vec{k} i' \vec{k}' \vec{q}}|^2_{q=(\vec{k}' + \vec{q} - \vec{k})}$$

Final state phase space:

$$\sum_{i'} \int_{BZ} V \frac{d^3 \vec{k}'}{(2\pi)^3}$$

initial state phase space:

$$2 \sum_i \int_{BZ} V \frac{d^3 \vec{k}}{(2\pi)^3}$$

↑
electron spin

$$R_{\text{crystal}} = 2 \sum_{i,i'} \int_{BZ} V^2 \frac{d^3 \vec{k}'}{(2\pi)^3} \frac{d^3 \vec{k}}{(2\pi)^3} R_{i\vec{k} \rightarrow i'\vec{k}'}$$

$$\vec{k}_e = \vec{E}_i \vec{p}_i - \vec{E}_{i'} \vec{p}_{i'} \quad \text{and} \quad q = |\vec{k}' + \vec{q}' - \vec{k}|$$

$$R_{\text{crystal}} = \frac{\rho_e}{m_e} \frac{2\pi^2 \bar{v}_e}{\mu_e e^2} V \int E_e d\ln E_e$$

$$* \int \frac{d^3 q}{q^2} \eta(v_{min}) |F_{0m}(q)|^2$$

$$* \sum_{i,i'} \int \frac{d^3 k}{(2\pi)^3} \frac{d^3 k'}{(2\pi)^3} \delta(E_e - E_{i\vec{k}} + E_{i'\vec{k}'})$$

$$* |f_{i\vec{k} \rightarrow i'\vec{k}'}|^2 \sum_{\vec{q}} \delta^{(3)}(\vec{q} - \vec{q}' - \vec{k} + \vec{k}')$$

$$|f_{\text{crystal}}(q, Ee)|^2 = \frac{2\pi^2 (\lambda m_e^2 V_{\text{cell}})^{-1}}{Ee}$$

$$\times \sum_{\vec{k}_i} \int B_{\vec{k}_i} \frac{V_{\text{cell}} d^3 p_i}{(2\pi)^3} \frac{V_{\text{cell}} d^3 k}{(2\pi)^3} Ee S(Ee - E_i' + \vec{k}_i \cdot \vec{E})$$

$$\times \sum_{\vec{q}} q \delta(q - |\vec{k}' + \vec{G} - \vec{k}|) |f_{\vec{k}' \vec{k} \vec{G} \vec{G}'}|^2$$

$$\left. \begin{aligned} \frac{dR_{\text{crystal}}}{d\text{rate}} &= \frac{\rho_x}{m_x} N_{\text{cell}} \frac{\bar{\epsilon}_c \lambda m_e^2}{m_x c^2} \int d\text{length} \frac{Ee}{q} \eta(V_{\text{min}}) \\ &\times (F_{\text{DM}}(q))^2 |f_{\text{crystal}}(q, Ee)|^2 \end{aligned} \right\}$$

$$V = N_{\text{cell}} \cdot V_{\text{cell}} \quad \text{where}$$

N_{cell} = number of cells

V_{cell} = volume of crystal's unit cell

$$N_{\text{cell}} = N_{\text{target}} / M_{\text{cell}}$$

$$M_{\text{cell}} = \begin{cases} 2 \times m_{Si} = 82.33 \text{ GeV} \\ 2 \times m_{Ge} = 135.33 \text{ GeV} \end{cases}$$

$$Si, \rho_x = 0.4 \text{ GeV/cm}^3, m_x = 100 \text{ MeV}, \bar{\epsilon}_c \approx 3.6 \times 10^{-37} \text{ cm}^2$$

$$29 \text{ events/kg/day}$$