

useful references :

- arXiv: 1509.01598
 - "Direct Detection of sub-GeV Dark Matter with semiconductor Targets"
- arXiv: 1904.07915
 - "TASI lectures on dark matter models and direct detection"

DM-nucleus scattering cross-section as

$$\frac{d\sigma}{dE_{nr}} = \frac{m_N}{2v^2 \mu_{XN}^2} \left[\Gamma_{SI} F_{SI}^2(q^2) + \Gamma_{SD} F_{SD}^2(q^2) \right]$$

spin-independent scattering

Helm Form Factor: $F(x) = \frac{3j_1(x)}{x} \exp\left[\frac{-(xs)^2}{2R_N^2}\right]$

$x = \frac{q}{RN}$ momentum transfer

$R_N \approx 1.2 A^{1/3}$ } nuclei dependent
 $s \approx 0.5 \text{ fm}^2$ }

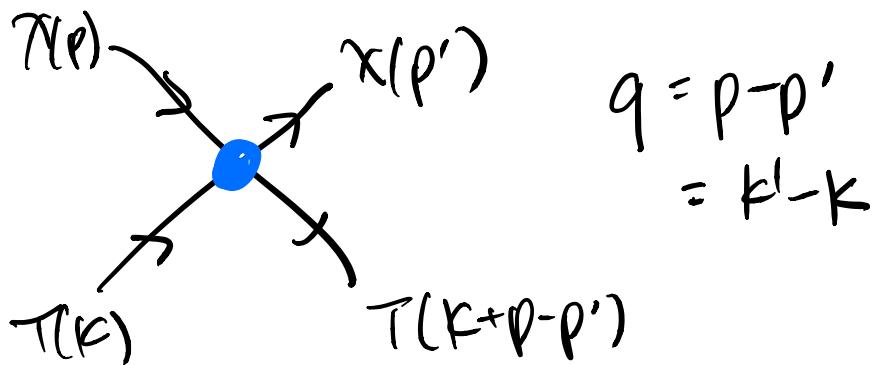
$$\Gamma_{SI} = \Gamma_n \frac{m_N^2}{m_X n^2} \left[\frac{f_p z + f_n (A-z)}{f_n^2} \right]^2$$

Total scattering rate:

$$\frac{dR}{dE_{nr}} = \frac{\rho_X}{m_X} \frac{M_I}{m_N} \int d^3v v g(\vec{v}) \frac{d\sigma}{dE_{nr}}$$

if we define $\eta(v_{min}) = \int_{v_{min}} d^3r \frac{g(\vec{v})}{v}$

$$\frac{dR}{dE_{nr}} = \frac{\rho_X}{m_X} \frac{M_I}{m_N} \frac{\Gamma_n m_N}{2 \mu_{XN} n^2} \left[\frac{f_p z + f_n (A-z)}{f_n} \right]^2 \times F^2(q^2) \cdot \eta(v_{min})$$



DM-nuclear scattering

can treat nucleus as a particle at rest $\rightarrow k=0$

Then initial energy and momentum are

$$E_i = \frac{|\vec{p}|^2}{2m_x} \quad \vec{p} = m_x \vec{v}_x$$

and final energy

$$E_f = \frac{|\vec{p}-\vec{q}|^2}{2m_x} + \frac{|\vec{q}|^2}{2m_N}$$

recoil energy of the nucleus is

$$E_{nr} = q^2 / 2m_N$$

conservation of energy

$$\frac{|\vec{p}| |\vec{q}| \cos\theta}{m_X} = \frac{q^2}{2M_X} \quad \mu_{XN} = \frac{m_X m_N}{m_X + m_N}$$

$$\hookrightarrow q_{\max} = 2 \frac{\mu_{XN} p}{m_X} = 2 \mu_{XN} v_X$$

$$E_R^{\max} = \frac{q_{\max}^2}{2m_N} = 2 \frac{\mu_{XN}^2 v_X^2}{m_N}$$
$$\approx 1 \text{ eV} \times \left(\frac{m_X}{100 \text{ MeV}} \right)^2 \left(\frac{20 \text{ GeV}}{m_N} \right)$$

$$m_X = 100 \text{ GeV} \quad m_N \sim 131 \text{ GeV} \quad v_X \sim 10^{-3}$$

$$E_R \sim 49 \text{ keV}$$

$$m_X = 100 \text{ MeV} \quad m_N \sim 131 \text{ GeV} \quad v_X \sim 10^{-3}$$

$$E_R \sim 0.1 \text{ eV}$$

example thresholds

- XENON1T $E_{\text{thr}}^{\text{thresh}} \sim \text{keV}$
- CRESST-II $E_{\text{thr}}^{\text{thresh}} \sim 30 \text{ eV}$

DM-electron scattering

cannot take $\mathbf{k} \approx 0$

$$\Delta E_e = E_e(\mathbf{k}') - E_e(\mathbf{k})$$

$$= \frac{\vec{p} \cdot \vec{q}}{m_X} - \frac{q^2}{2m_X}$$

Energy conservation:

$$\Delta E_e = -\Delta E_p - \Delta E_N$$

$$= -\frac{|\mathbf{m}_X \vec{v} - \vec{q}|^2}{2m_X} + \frac{1}{2} m_X v^2 - \frac{q^2}{2m_N}$$

$$= \vec{q} \cdot \vec{v} - \frac{q^2}{2m_N}$$

In practice $\Delta E_N \ll 1$: which allows us to replace $m_{XN} \rightarrow m_X$

★
$$\boxed{\Delta E_e = \vec{q} \cdot \vec{v} - \frac{q^2}{2m_X}}$$

maximizing wrt \vec{q} : $\Delta E_e^{\text{max}} = \frac{1}{2} m_{XN} v^2 \approx \frac{1}{2} eV \tau \left(\frac{m_F}{100 \text{ MeV}} \right)$

↳ All of the DM kinetic energy

is available to excite the electron.

Q. What are some typical values for $q, \Delta E_e$?

A. Here, the electron is both the lightest and fastest particle $v_e \sim z_{\text{eff}} \alpha \sim 10^{-2}$

Then $q_{\text{typ}} \approx M_x e v_{\text{rel}} \approx M_x v_e \sim z_{\text{eff}} \alpha M_x$

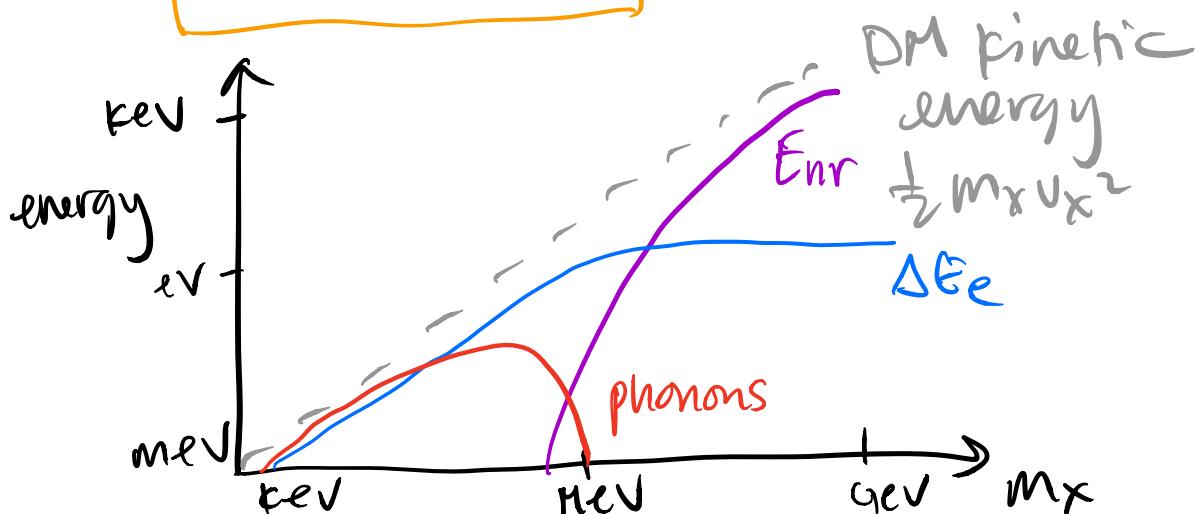
$$q_{\text{typ}} \approx z_{\text{eff}} \times 4 \text{ keV}$$

Away from threshold, 1st term of \star

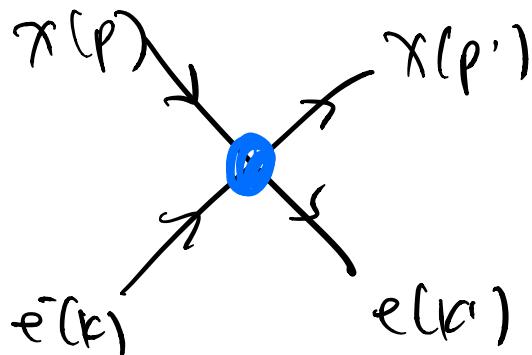
dominates so

$$q \gtrsim \frac{\Delta E_e}{\sqrt{v}} = \frac{\Delta E_e}{4 z_{\text{eff}} \text{ eV}} \cdot q_{\text{typ}}$$

$$\Delta E_e^{\text{typ}} \sim \text{eV}$$



DM-electron scattering rates



$$\vec{q} = \vec{p} - \vec{p}' \\ = \vec{k} - \vec{k}'$$

Let's start with free 2-2

DM-electron scattering

$$DN_{\text{free}} = \frac{1}{4E'_x E'_e} \int \frac{d^3 q}{(2\pi)^3} \frac{d^3 k'}{(2\pi)^3} \frac{1}{4E_x E_e} \\ \times (2\pi)^4 \delta(E_i - E_f) \delta^{(3)}(\vec{p} + \vec{q} - \vec{k}') \\ \times \overline{|M_{\text{free}}(\vec{q})|^2}$$

If the electrons are unbound, then the non-relativistic scattering amplitude is

$$\langle \bar{\chi}_p \bar{q}, \bar{e}_k | H_{\text{int}} | \bar{\chi}_p \bar{q}, \bar{e}_k \rangle =$$

$$C M_{\text{free}}(\vec{q}) \times (2\pi)^3 \delta^{(3)}(\vec{k} - \vec{q} - \vec{E'})$$

However for bound-electrons, we must take into account the electron wavefunctions

$$\langle \chi_{\vec{p}-\vec{q}, e_2} | H_{\text{int}} | \chi_{\vec{p}, e_1} \rangle =$$

$$\left[\int \sqrt{V} \frac{d^3 k'}{(2\pi)^3} \hat{\psi}_2^*(\vec{k}') \langle \chi_{\vec{p}', e_{\vec{k}'}} | \right] H_{\text{int}}$$

$$\left[\int \sqrt{V} \frac{d^3 k}{(2\pi)^3} \hat{\psi}_1(\vec{k}) | \chi_{\vec{p}, e_{\vec{k}}} \rangle \right]$$

$$= C M_{\text{free}}(\vec{q}) \times \int V \frac{d^3 k}{(2\pi)^3} \hat{\psi}_2^*(\vec{k} + \vec{q}) \hat{\psi}_1(\vec{k})$$

a note: $\hat{\psi}_i$ are the unit-normalized momentum-space wavefunctions.

We use the plane-wave normalization for both free and bound-states:

$$\langle e_{\vec{k}} | e_{\vec{k}} \rangle = \langle e_i | e_i \rangle = (2\pi)^3 \delta^{(3)}(\vec{q}) \equiv \sqrt{V}$$

From this, we see that we can write the bound-state scattering amplitude by substituting $V(2\pi)^3 \delta^{(3)}(\vec{k} - \vec{q} - \vec{k}') |M_{\text{free}}|^2$

$$\rightarrow |M_{\text{free}}|^2 \times V^2 |f_{1 \rightarrow 2}(\vec{q})|^2$$

where $f_{1 \rightarrow 2}(\vec{q})$ is the atomic form-factor

$$f_{1 \rightarrow 2}(\vec{q}) = \int \frac{d^3 k}{(2\pi)^3} \tilde{\psi}_2(\vec{k} + \vec{q}) \tilde{\psi}_1(\vec{k})$$

* Calculating $f_{1 \rightarrow 2}$ and the $\tilde{\psi}_i$ is the primary challenge of DM- e^- scattering.

When we consider electron transitions from $1 \rightarrow 2$ there is only one final-state electron so we can remove the final-state phase-space integral

free-electron phase space =

$$\sqrt{\int \frac{d^3 k}{(2\pi)^3}} \rightarrow 1$$

\therefore we have

$$N_{1 \rightarrow 2} = \frac{1}{4\epsilon_1^i \epsilon_i^f} \int \frac{d^3 q}{(2\pi)^3} \cdot \frac{1}{4\epsilon_F^f \epsilon_F^i} \times 2\pi \delta(E_i^i - E_F^F) \times \overline{|M_{\text{free}}(\vec{q})|^2} \times |f_{1 \rightarrow 2}(\vec{q})|^2$$

We can parameterize the microphysics that governs the DM-e⁻ interactions via

$$|\overline{M_{\text{free}}(\vec{q})}|^2 = |\overline{M_{\text{free}}(dM_e)}|^2$$

$$\times |F_{\text{DM}}(q)|^2$$

$$\overline{\rho_e} \approx \frac{M_{\text{Fe}}^2 |\overline{M_{\text{free}}(dM_e)}|^2}{(8\pi M_F^2 m_e)^2}$$

\uparrow
model-dependent

$$\begin{aligned} \mathcal{N}_{1 \rightarrow 2} &: \frac{\overline{\rho_e}}{M_{\text{Fe}}^2} \int \frac{d^3 q}{4\pi} \delta(E_i - E_f) \\ &\times |F_{\text{DM}}(q)|^2 \times |f_{1 \rightarrow 2}(\vec{q})|^2 \end{aligned}$$

The rate is given by

$$R_{1\rightarrow 2} = \frac{\rho_x}{m_x} \int d^3v g(v) \sigma v_{1\rightarrow 2}$$

For spherically-symmetric system

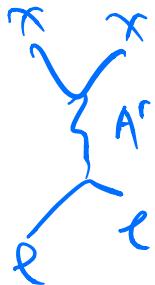
$$R_{1\rightarrow 2} = \frac{\rho_x \overline{\rho_e}}{m_x 8\pi \mu_x e^2} \int d^3q \frac{1}{q} \eta(v_{min}) \times |F_{OM}(q)|^2 \cdot |f_{1\rightarrow 2}(q)|^2$$

$$\eta(v_{min}) = \int_{v_{min}}^{\infty} \frac{d^3v}{v} g_x(v)$$

$$v_{min}(q, \Delta E_{1\rightarrow 2}) = \frac{\Delta E_{1\rightarrow 2}}{q} + \frac{q}{2m_x}$$

Exercises

Q₁: In the dark photon portal model the interactions of the DM with SM are mediated by a dark photon A' with SM interactions governed by $\mathcal{L} \supset \frac{e}{2c_W} F_y^{\mu\nu} F_{\mu\nu}'$.



What is the spin-averaged matrix element for DM-e scattering? $|\overline{M}_{\text{tree}}|^2$

Q₂: For the above model, what is $\overline{\rho}_e$ and $|F_{\text{DM}}(q)|^2$?

Q₃: What is the minimum mass accessible for Si ? Xe ? in DM-electron scattering?

$$\Delta E_e^{\min}, \text{Si} = 1.2 \text{ eV}$$

$$\Delta E_e^{\min}, \text{Xe} \approx 12 \text{ eV}$$