

Soft nuclear recoils (phonons)

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Some cats are bosons

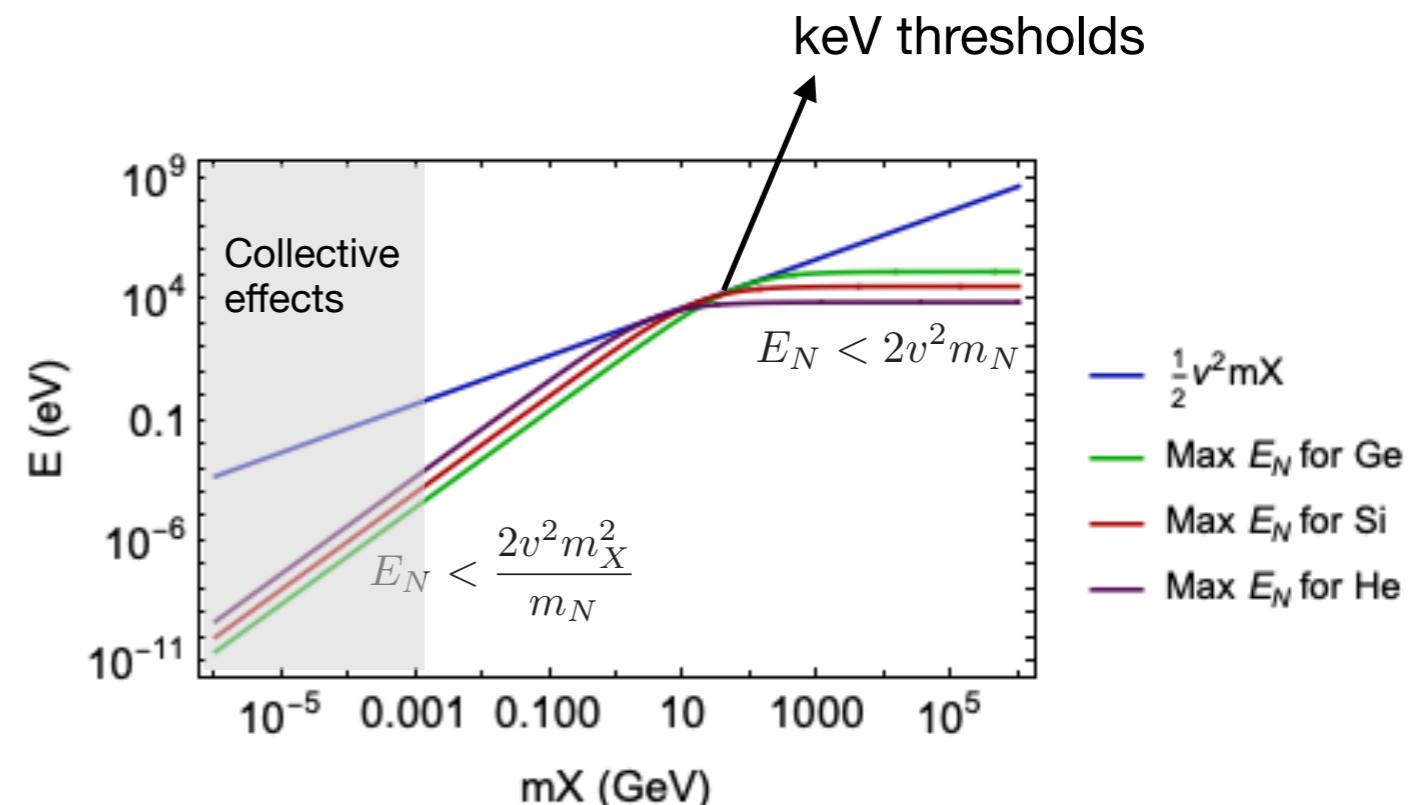


Elastic nuclear recoil kinematics

Momentum conservation implies that for elastic nuclear recoils we have

$$E_N < \frac{(2v\mu_{XN})^2}{2m_N}$$

For $m_X \ll m_N$, we are not accessing the vast majority of the kinetic energy of the dark matter

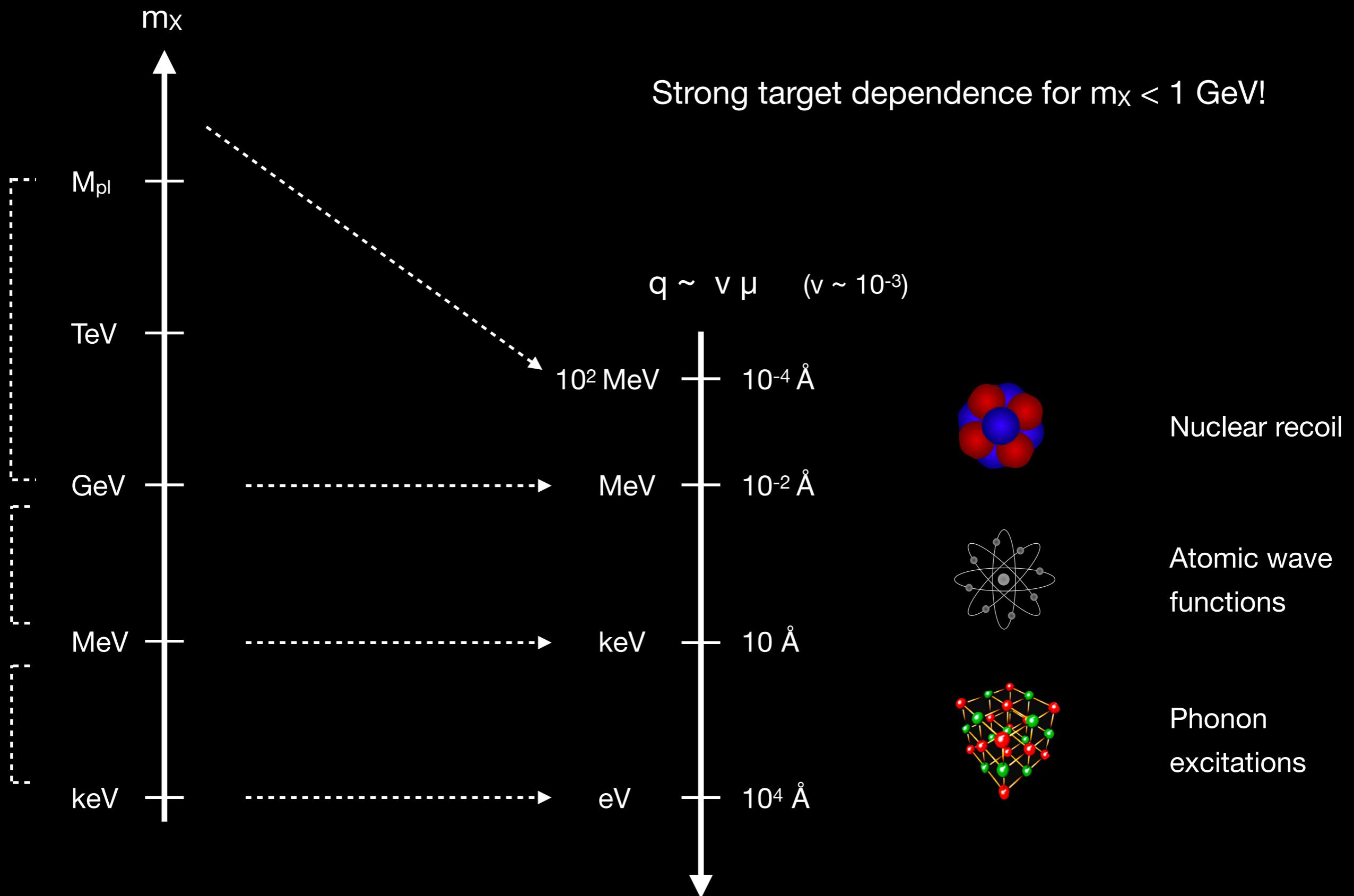


Two questions:

1. When does the elastic billiard ball picture break?
(Today)

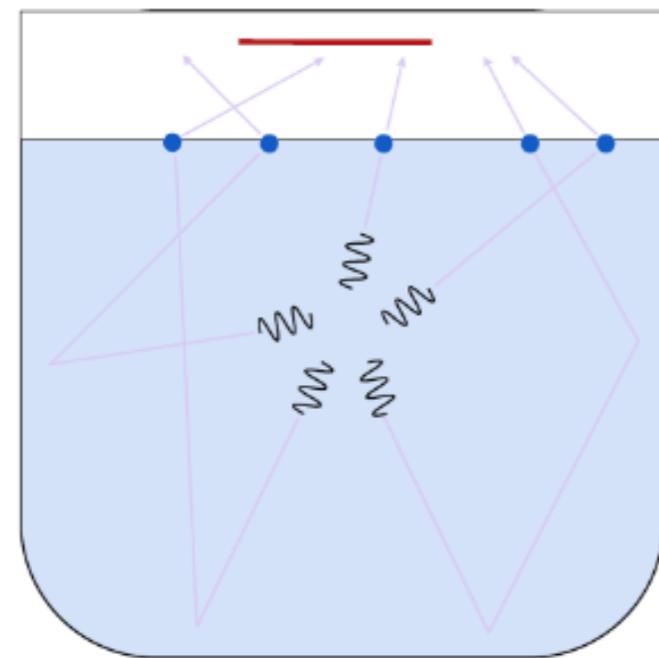
2. What about *inelastic* recoils?
(Yesterday)

Relevant length scales

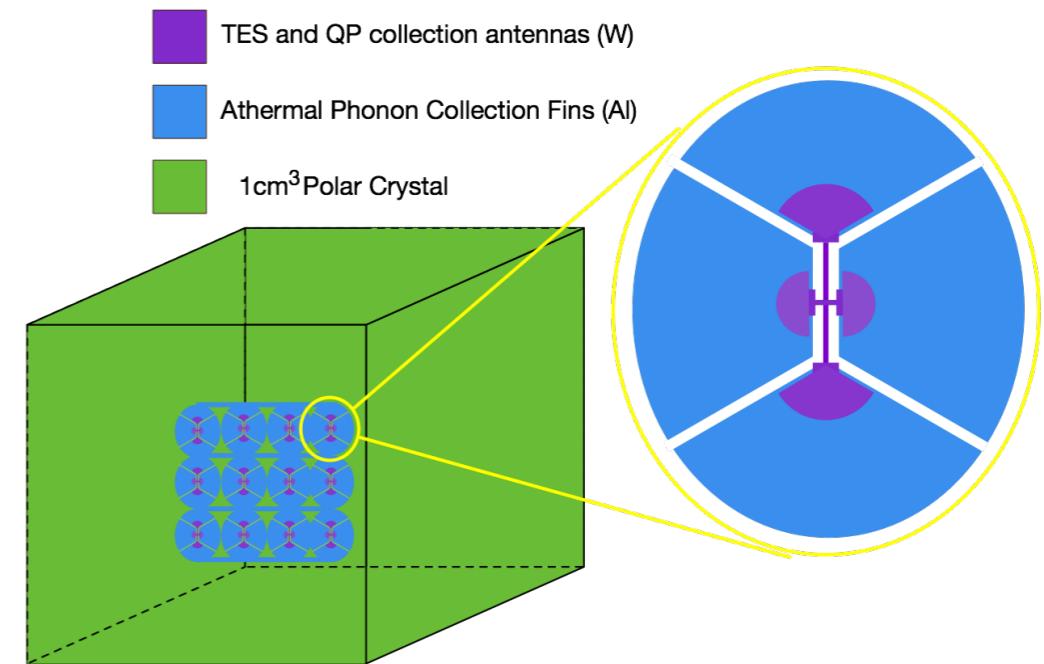


Experiments in R&D phase

HERALD
(superfluid He)



SPICE
(GaAs, sapphire)



W. Guo, D. McKinsey: 1302.0534

M. Pyle et. al.

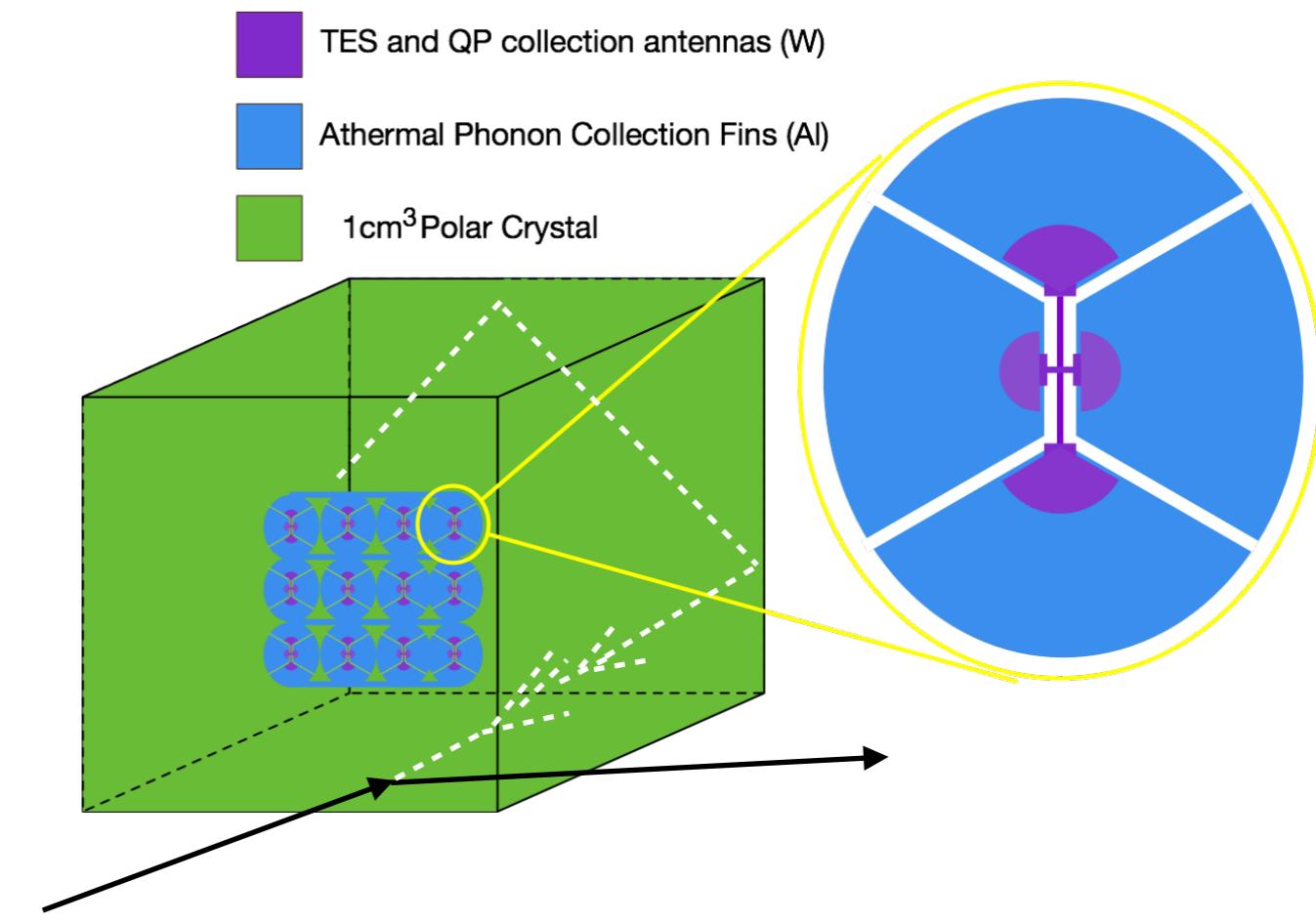
Ultimately ~ 1 meV energy threshold

Ultimately ~ 10-20 meV energy threshold

Towards single phonon detection

Strategy

- Very small, ultra cold detector (1 cm^3)
- No bias voltage
- Develop low threshold TES sensor



Phonon collection

1. DM creates athermal phonon(s)
2. phonon decays rapidly to soft ($\sim 1 \text{ meV}$) athermal phonons
3. phonons bounce ballistically
4. Collection fins guide phonon energy to Transition Edge Sensor (TES)

Projected sensitivity to a single $\sim 30 \text{ meV}$ phonon

Theory input



Outline

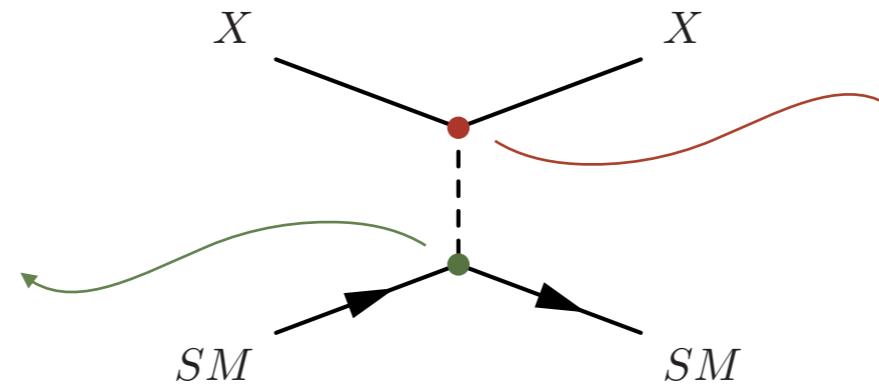
1. Models: Constraints on the mediator
2. Beyond “billiard ball” scattering
 - Dark Matter - phonon effective theory
 - Dark Matter with coupling to charge
 - Dark Matter with coupling to nuclei
3. Summary and outlook

Strong model & material dependence of the scattering rate

Models: Executive summary

Constraints

Stellar cooling
 5th force
 meson factories
 BBN / CMB
 ...
 ...



Dark Matter self-interactions
(e.g. bullet cluster)

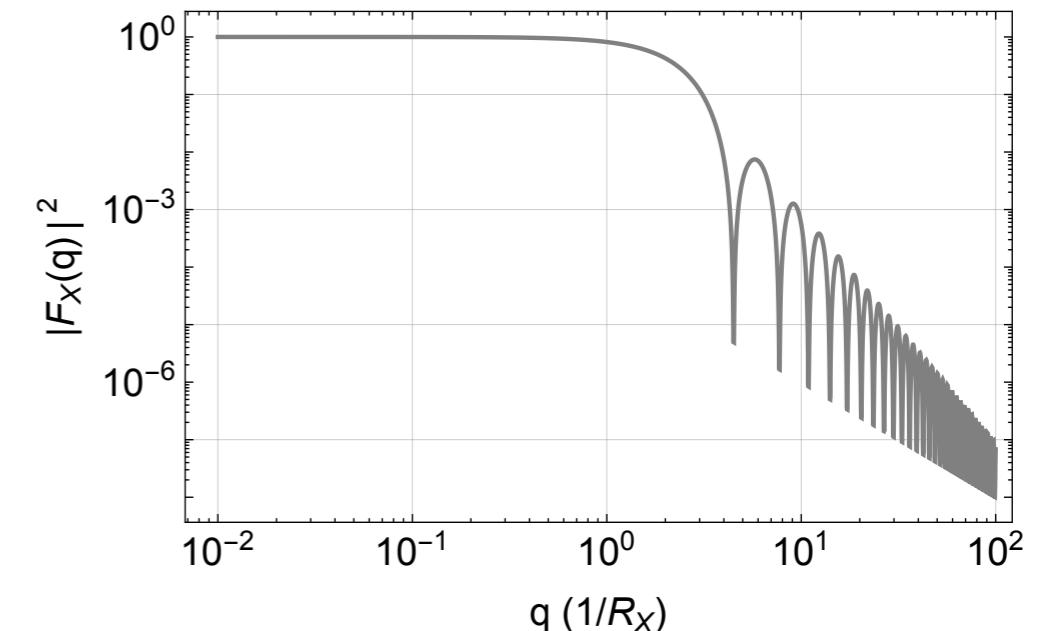
Models

If $m_X < 1$ MeV:

- (✓) Coupling to nuclei
(if subcomponent of DM density)
- ✗ Coupling to electrons
- ✓ Coupling to Charge
(Dark photon mediator)

We won't go into this today, but the moral here is that you need to be **very careful** with DM lighter than \sim MeV!

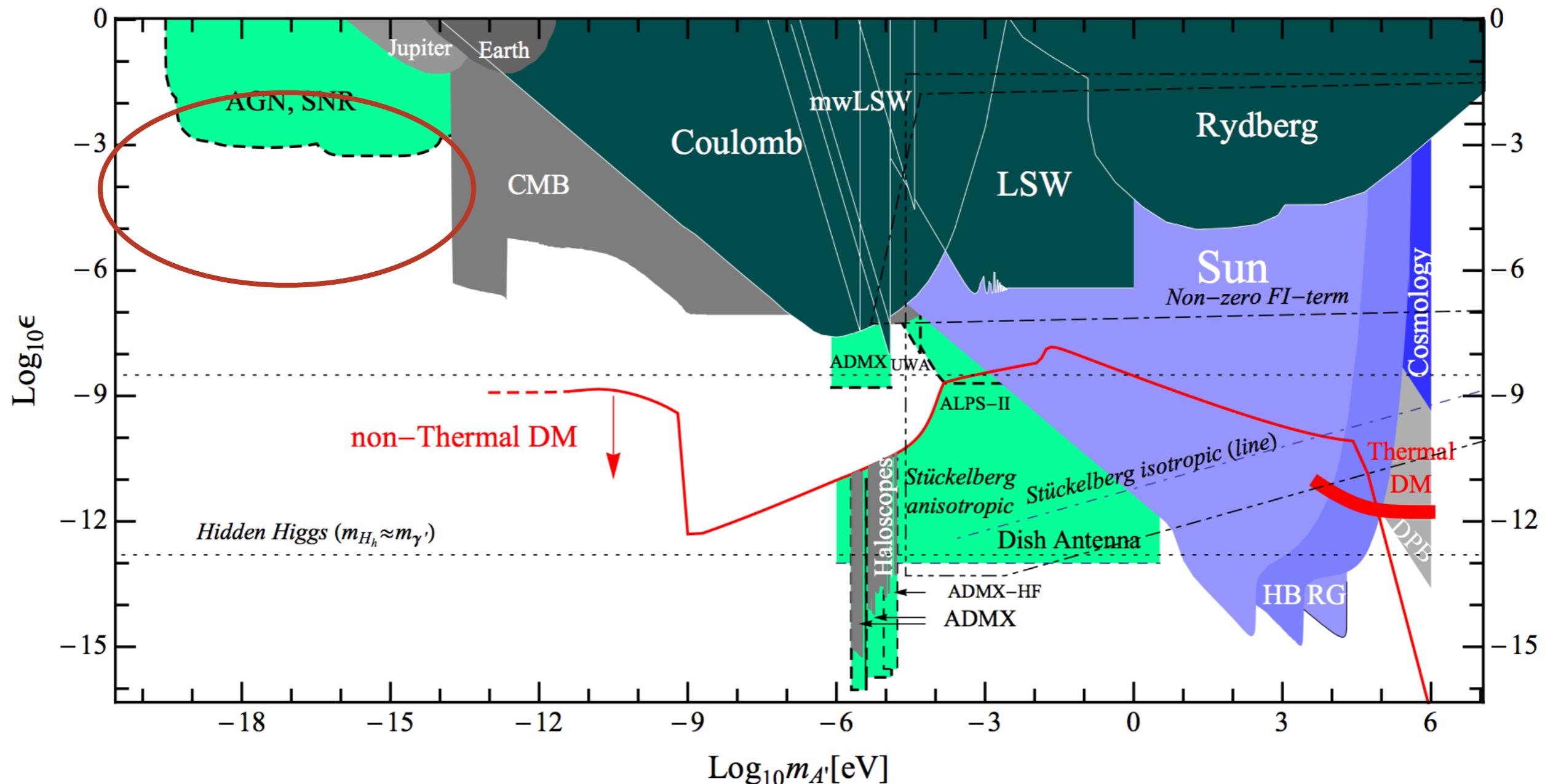
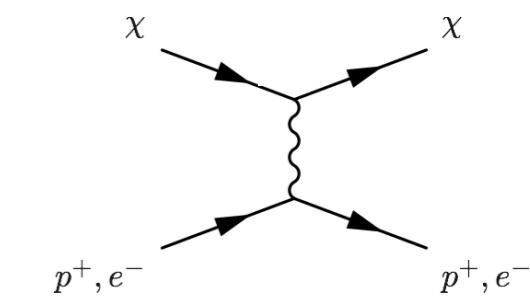
Composite dark matter



Form factor suppresses high energy recoils

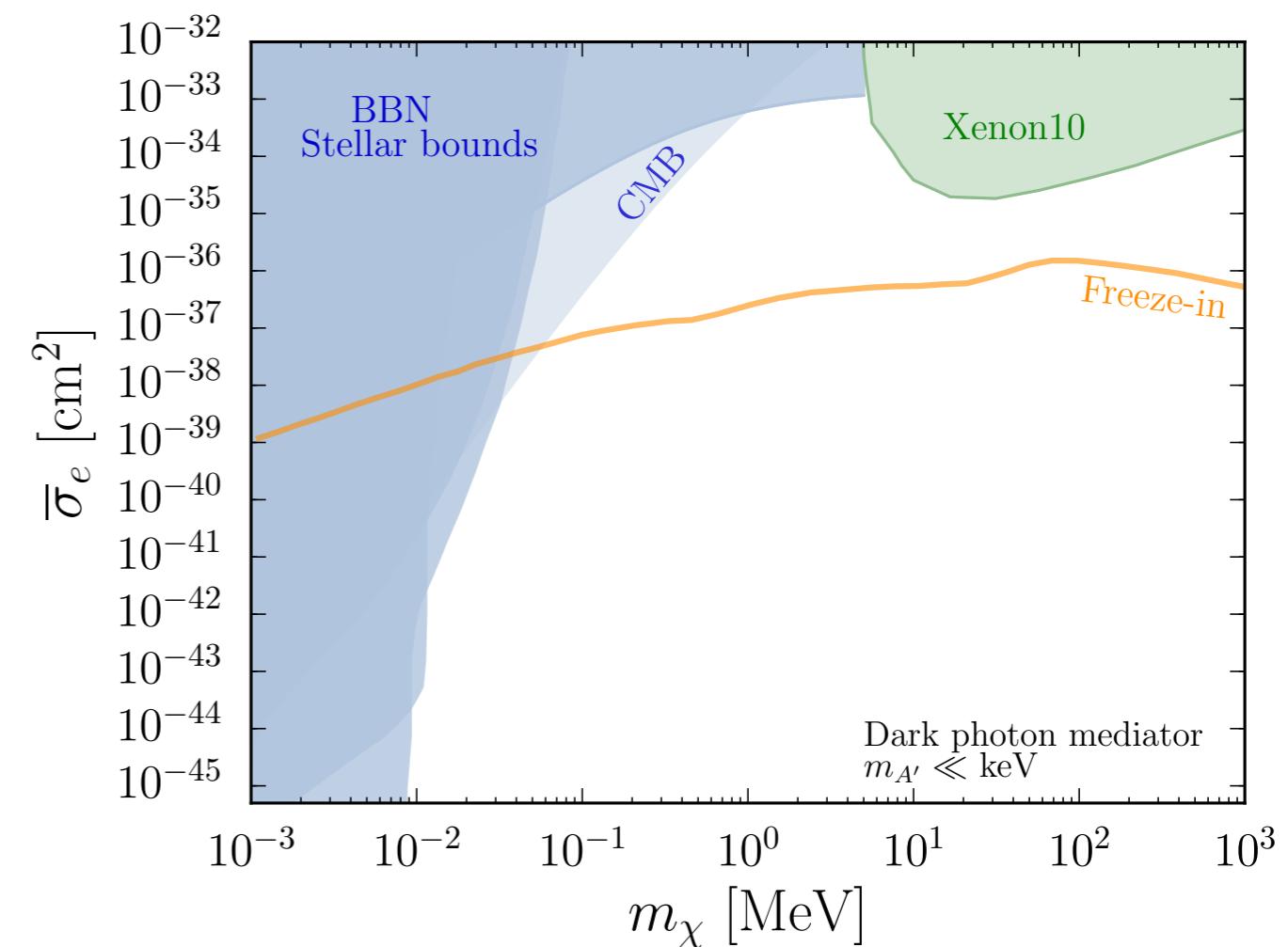
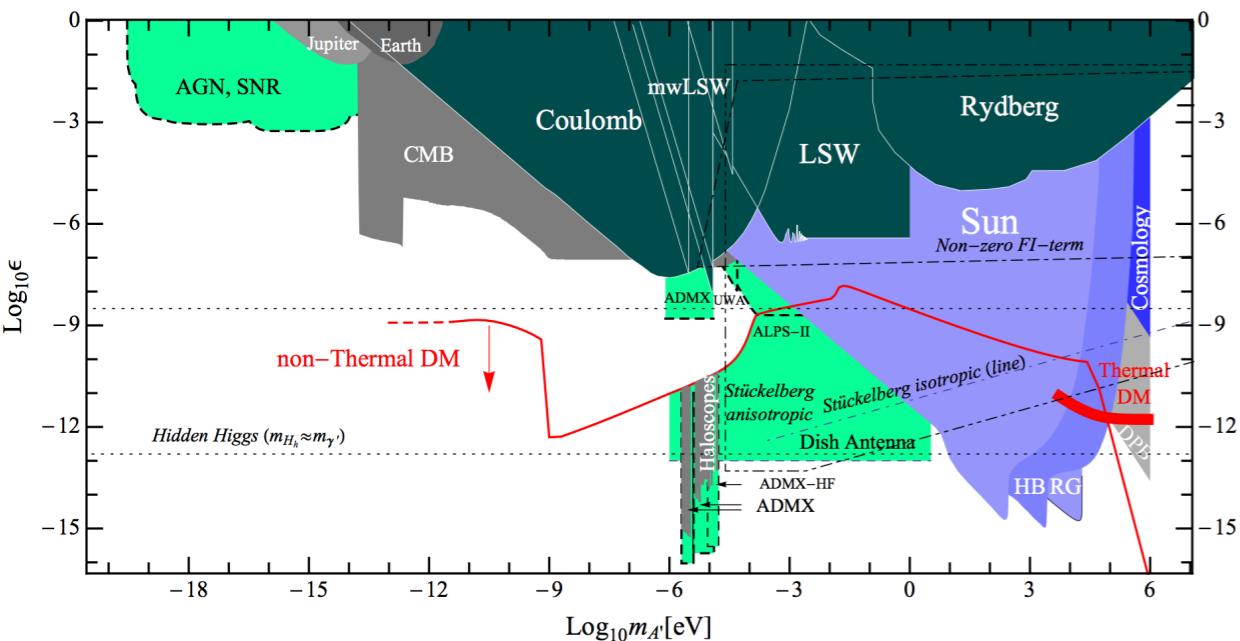
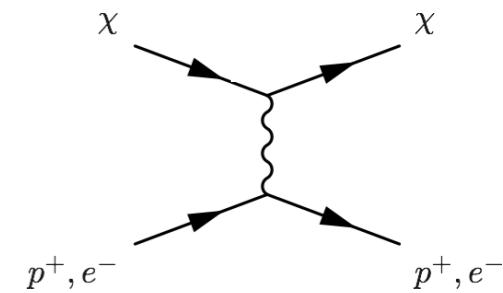
Dark photon mediator

Very light dark photon mediator



Dark photon mediator

Very light dark photon mediator



Dark Matter effectively “nano-charged”

Theory input

Outline



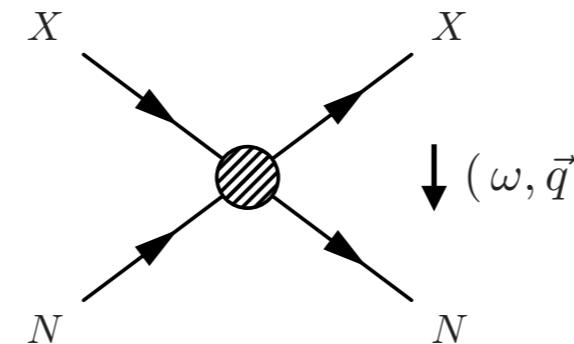
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Effective theory overview

Nuclear recoil

$$\omega = \frac{q^2}{2m_N}$$

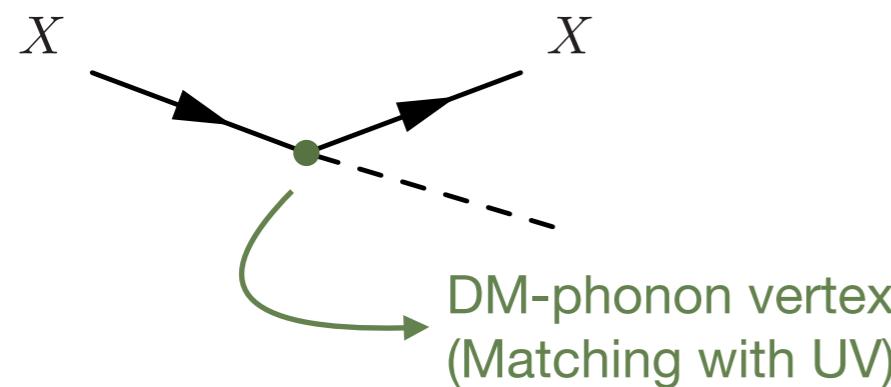


Phonon regime

$q \ll \sqrt{2m_N\omega} \rightarrow$ Momentum exchange is a good expansion parameter

Phonons are the goldstones of spontaneous breaking of translation invariance: They must be derivatively coupled!

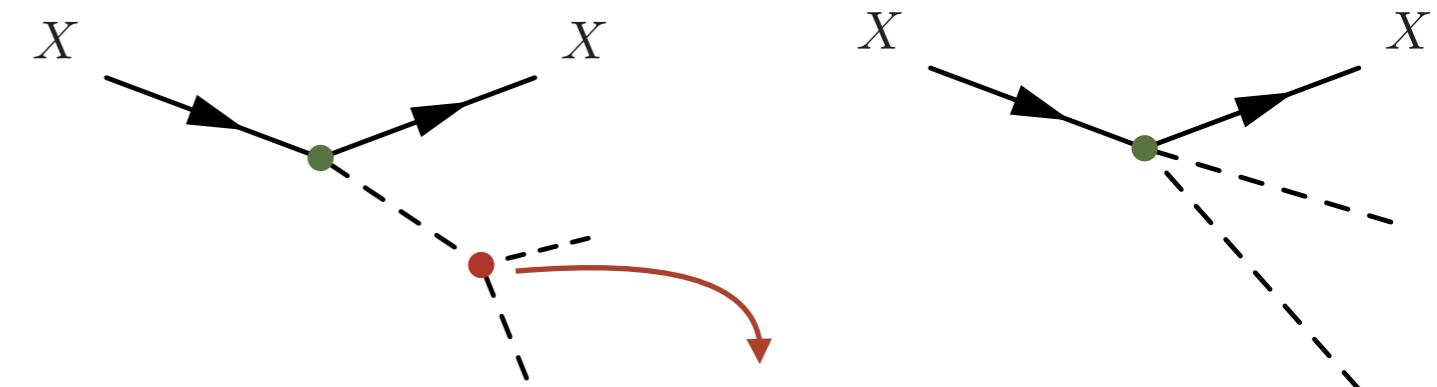
Leading order



$\mathcal{O}(q)$, $\mathcal{O}(q^2)$ or $\mathcal{O}(q^4)$

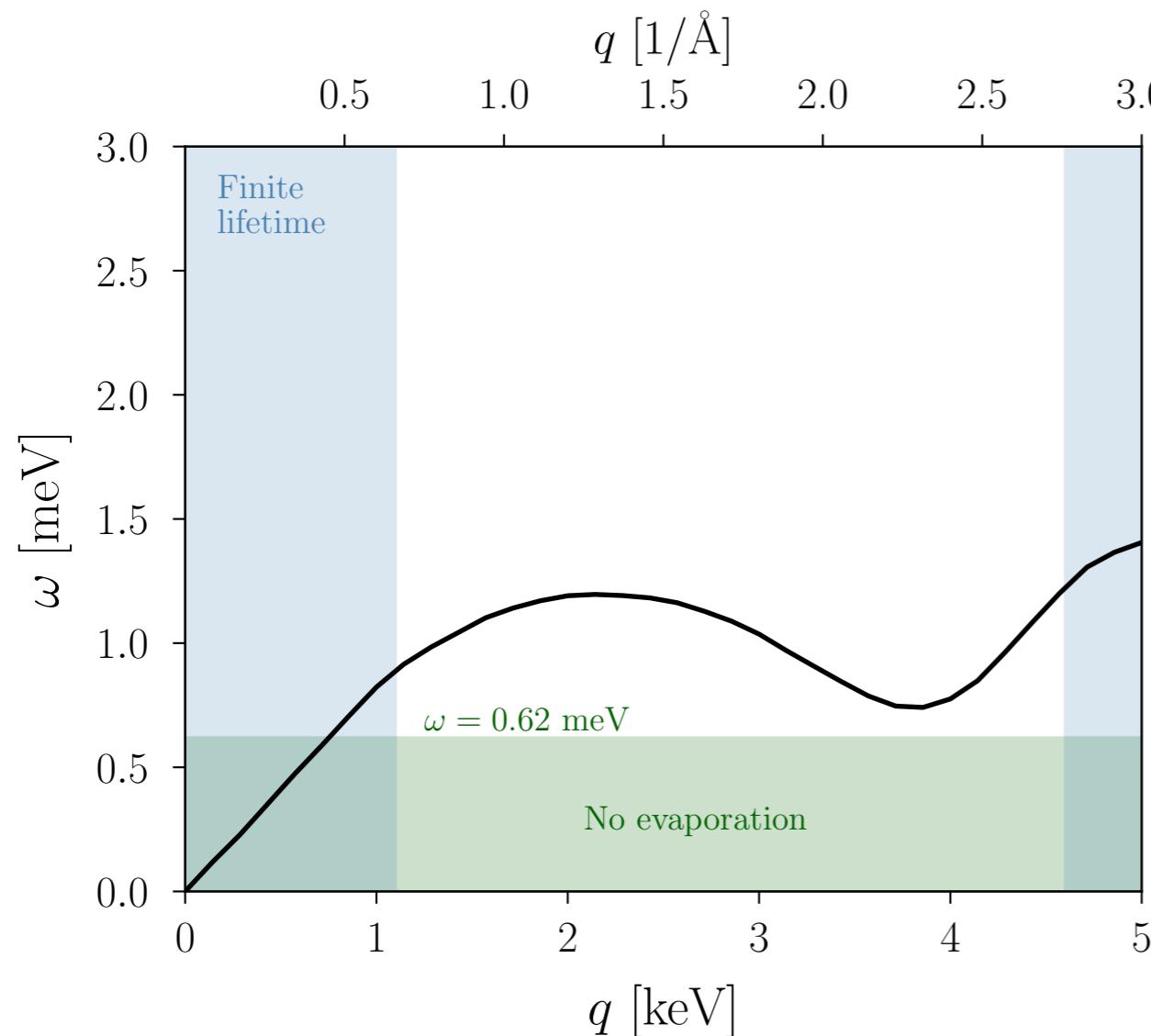
(Depends on DM model & phonon branch)

Next-to-leading order

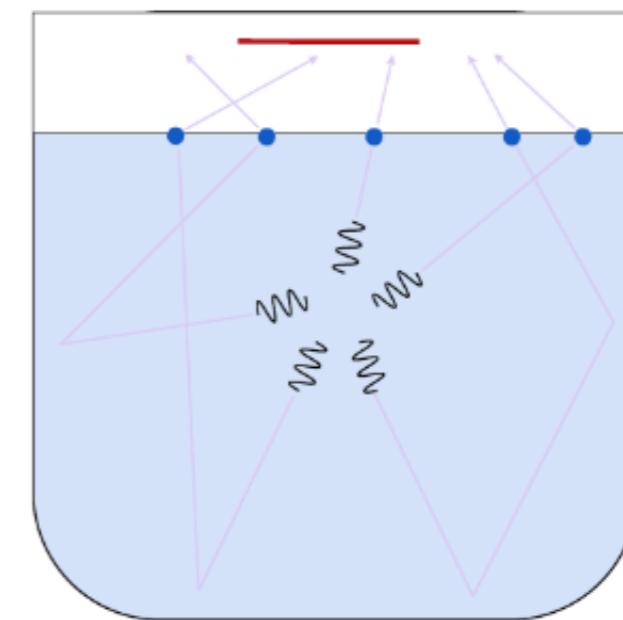


$\mathcal{O}(q^4)$

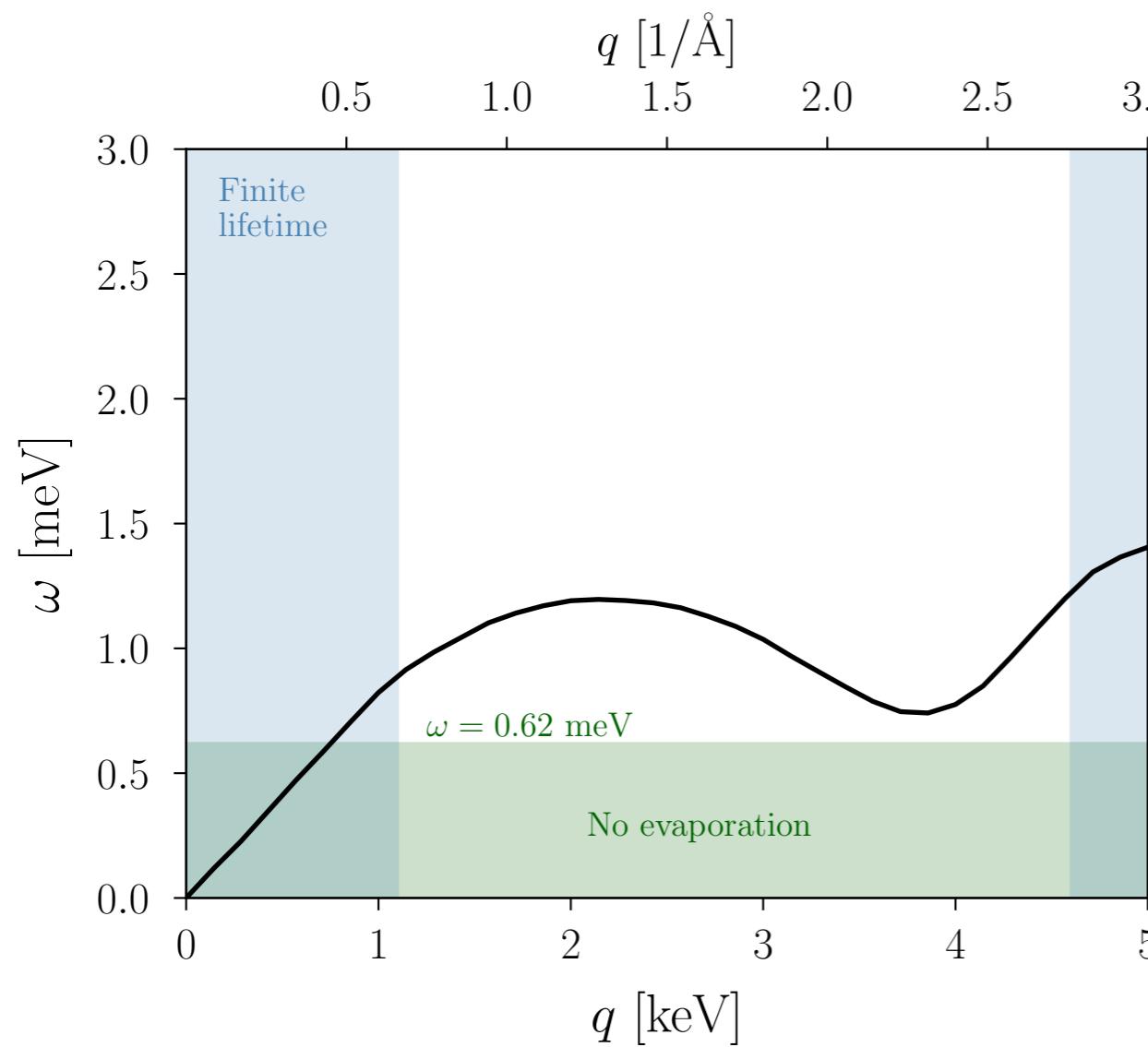
Kinematics: superfluid helium



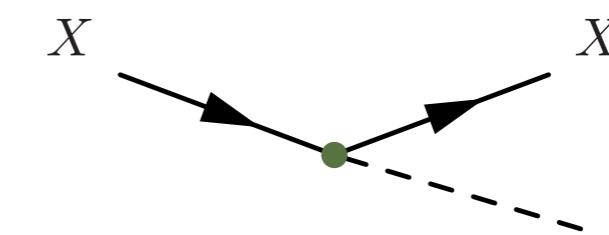
Detection principle



Kinematics: superfluid helium

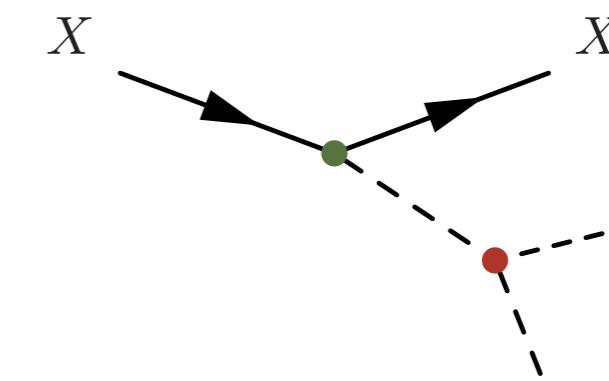


Leading order



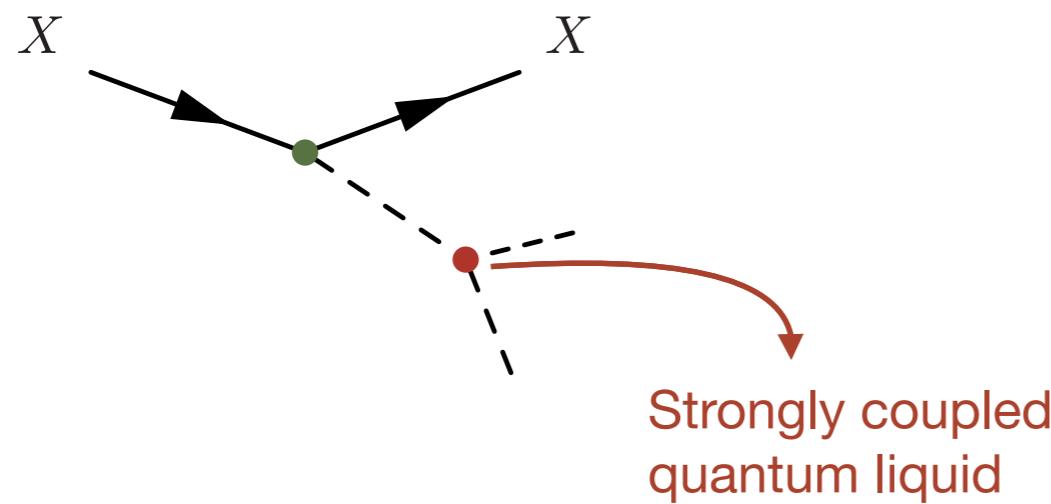
Only accessible for $m_X \gtrsim 1$ MeV

Next-to-leading order



Always accessible for back-to-back phonons.

Multi-phonons in superfluid helium



There is no straightforward expansion parameter for the multiphonon vertex

But we experimentally know that the multi-phonon expansion converges (slowly)

We must use an effective theory, impose some symmetries and match unknown parameters to data

Quantum hydrodynamics

$$H = \int d^3\mathbf{r} \left(\frac{1}{2} m_{\text{He}} \mathbf{v} \cdot n \mathbf{v} + \mathcal{V}(n) \right)$$

Unknown, but drops out to leading order

Need to renormalize the ground state, which involves using an ansatz for the counter term

K. Schutz, K. Zurek: 1604.08206

T. Lin, K. Zurek: 1611.06228

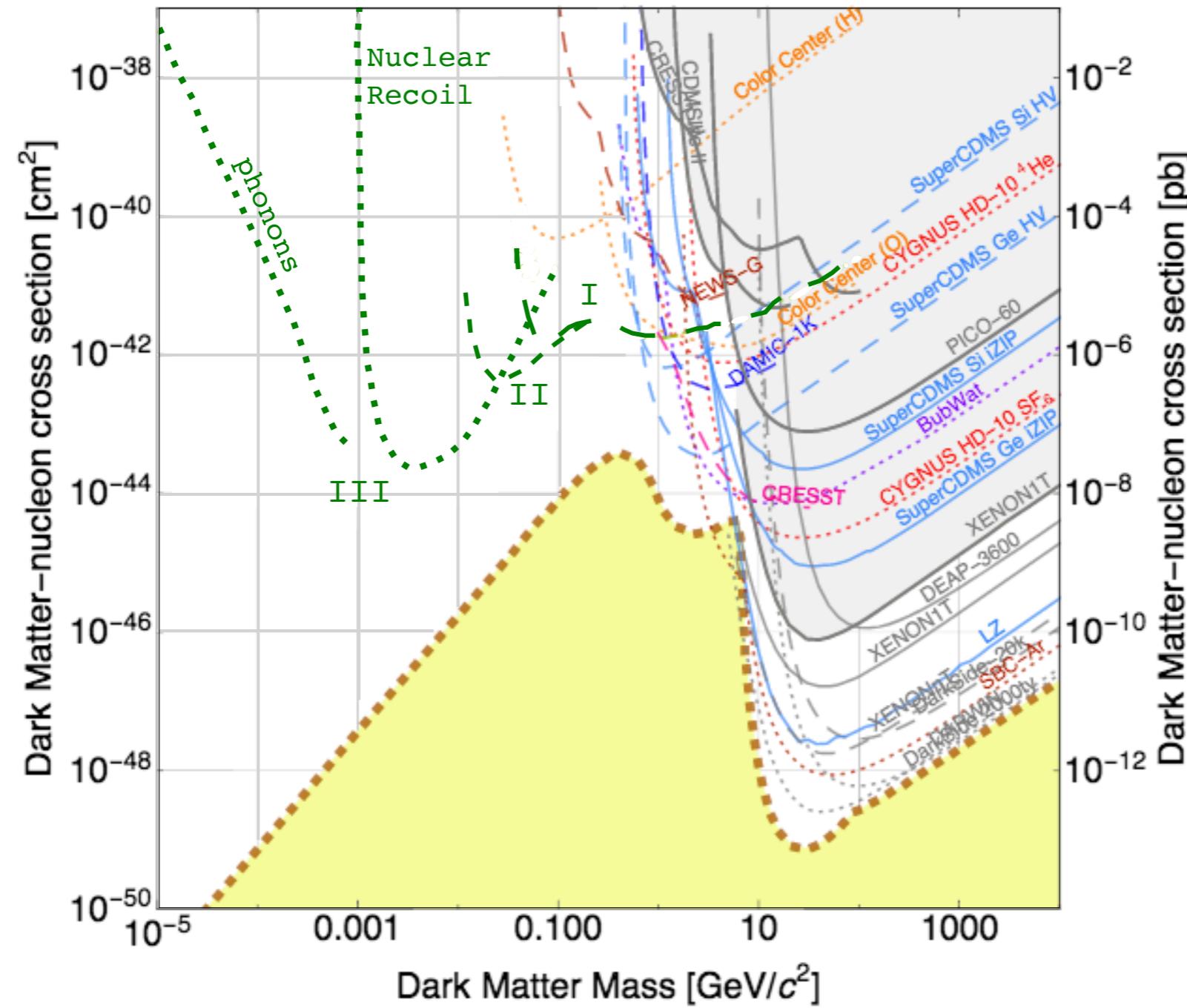
Superfluid effective field theory

$$\begin{aligned} S_{\text{bulk}} \supset & \frac{\bar{n}}{\mu c_s^2} \int d^4x \left[\frac{1}{2} \dot{\pi}^2 - \frac{c_s^2}{2} (\nabla \pi)^2 \right. \\ & \left. + \lambda_3 \dot{\pi} (\nabla \pi)^2 + \lambda'_3 \dot{\pi}^3 \right] \end{aligned}$$

Extract from pressure dependence of sound speed
(We'll do this later for crystals, which is simpler)

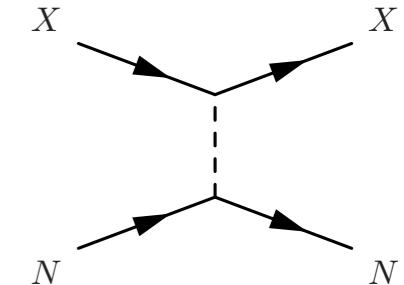
- F. Acanfora, A. Esposito, A. Polosa: 1902.02361
- A. Caputo, A. Esposito, A. Polosa: 1907.10635
- A. Caputo, et. al.: 1911.04511
- A. Caputo, et. al.: 2012.01432

Reach



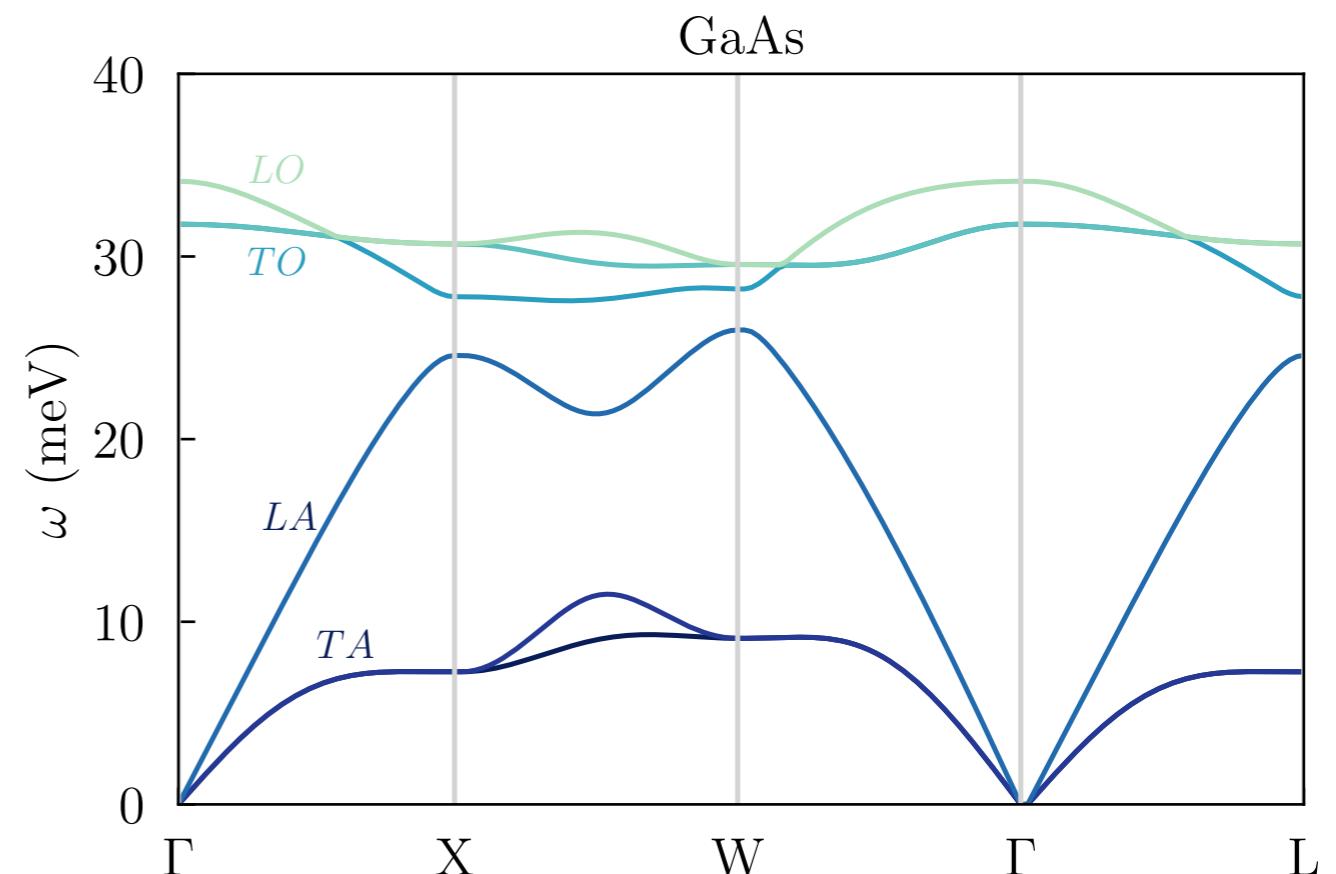
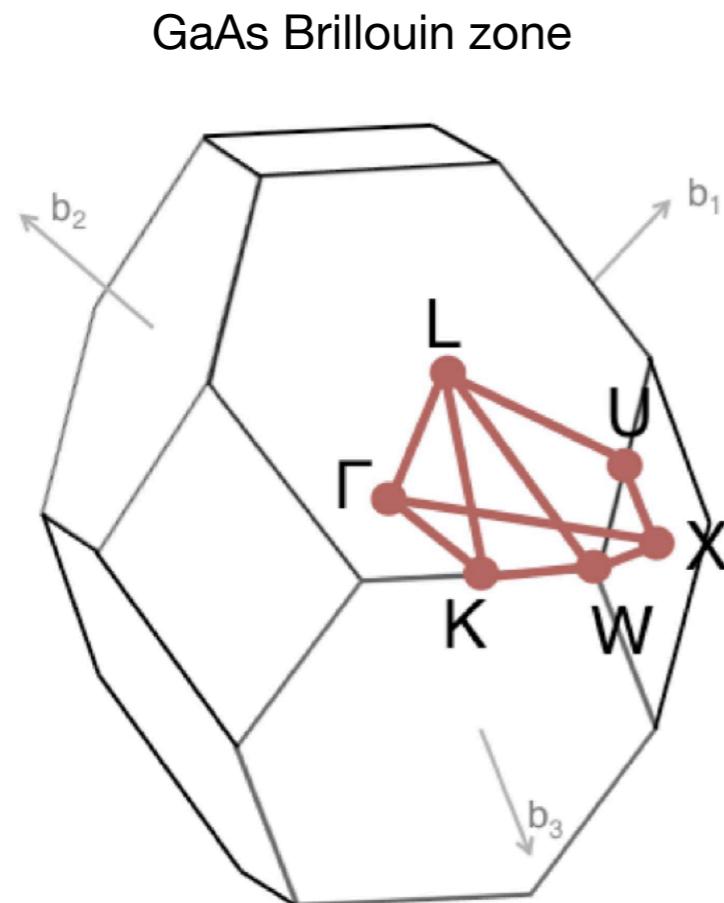
Threshold:

- I: $E > 2.5 \text{ eV}$
- II: $E > 0.25 \text{ eV}$
- III: $E > 1 \text{ meV}$



Superfluid helium can be sensitive down to $m_X \sim 10 \text{ keV}$

Kinematics: crystals



3 gapless modes (acoustic)
3 n - 3 gapped modes (optical)

$n = \#$ atoms in primitive cell

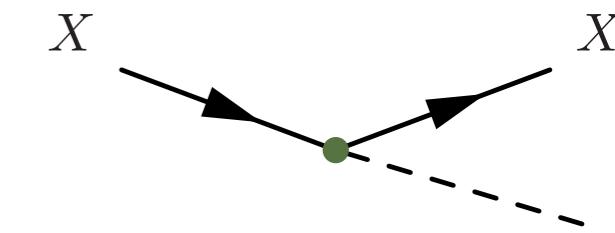
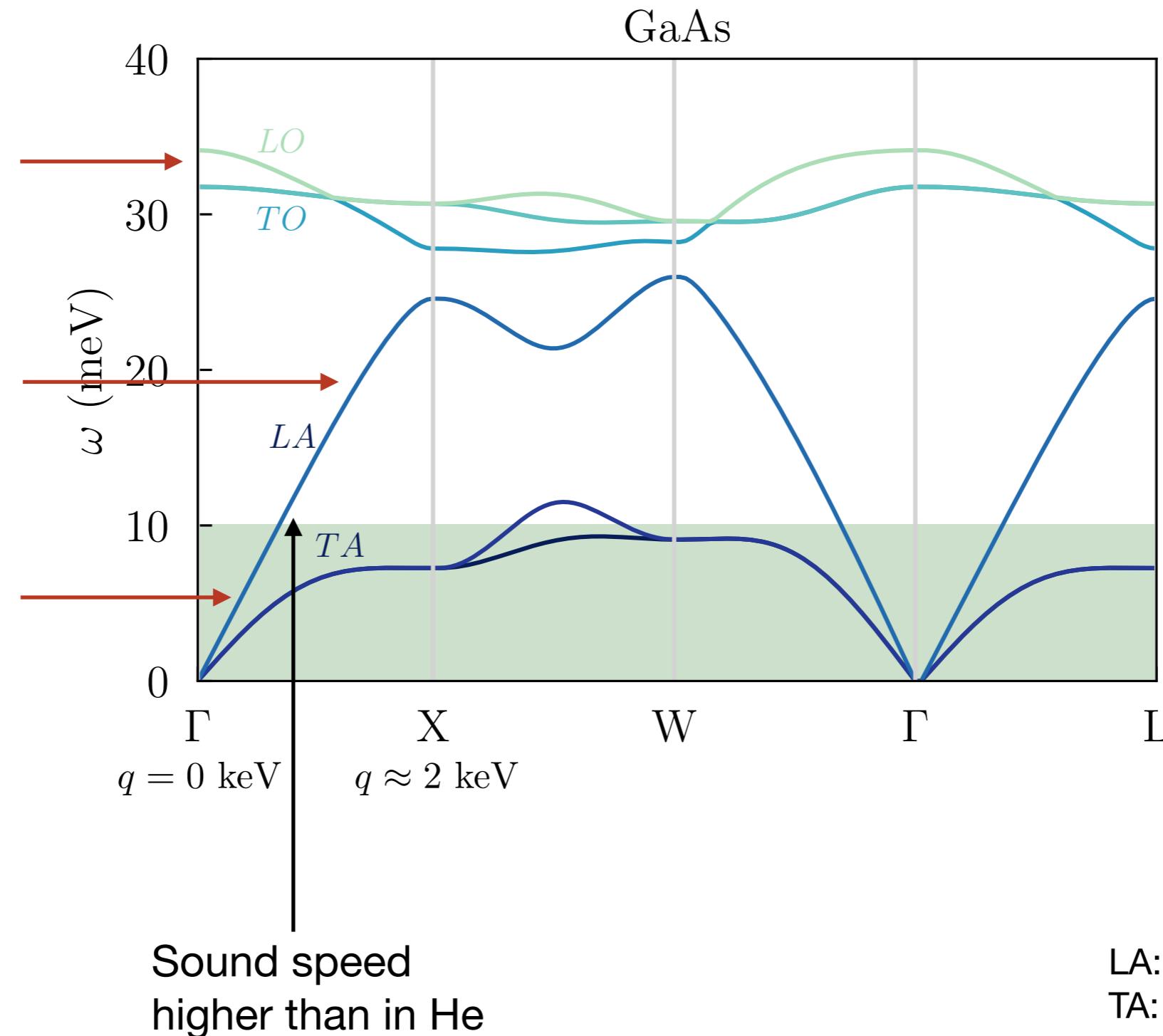
Kinematics: crystals

Recall $q \sim v m_x$

Always accessible

Accessible for high m_x only

Not accessible



LA: Longitudinal Acoustic
TA: Transverse Acoustic

LO: Longitudinal Optical
TO: Transverse Optical

Degrees of freedom

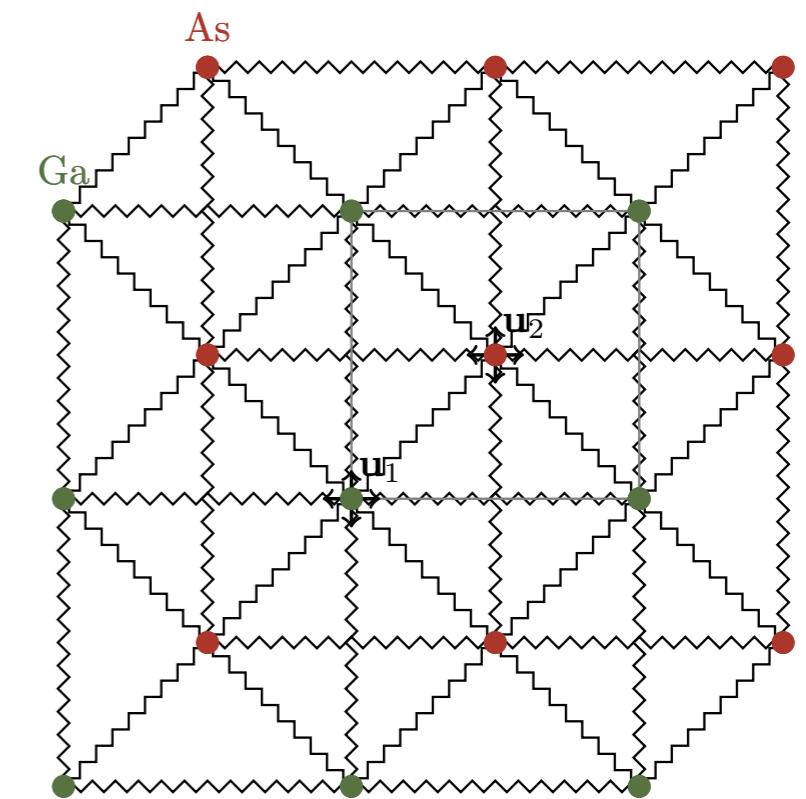
Deviations from equilibrium position

$$\mathbf{u}_d = \sum_{\nu}^{3n} \sum_{\mathbf{q}} \sqrt{\frac{1}{2N m_d \omega_{\nu, \mathbf{q}}}} \left(\mathbf{e}_{d, \nu, \mathbf{q}} a_{\nu, \mathbf{q}} e^{i(\mathbf{q} \cdot \mathbf{r}_d^0 - \omega_{\nu, \mathbf{q}} t)} + \text{h.c.} \right)$$

Atom mass Eigenvector Annihilation operator Equilibrium position
 Eigen-frequency

Indices:

- ν : labels phonon branches
- d : labels atoms in primitive cell
- \mathbf{q} : momentum over 1th Brioullin zone
- i, j : spacial indices (x, y, z)



Degrees of freedom

Deviations from equilibrium position:

$$\mathbf{u}_d = \sum_{\nu}^{\text{3n}} \sum_{\mathbf{q}} \sqrt{\frac{1}{2Nm_d\omega_{\nu,\mathbf{q}}}} \left(\mathbf{e}_{d,\nu,\mathbf{q}} a_{\nu,\mathbf{q}} e^{i(\mathbf{q} \cdot \mathbf{r}_d^0 - \omega_{\nu,\mathbf{q}} t)} + \text{h.c.} \right)$$

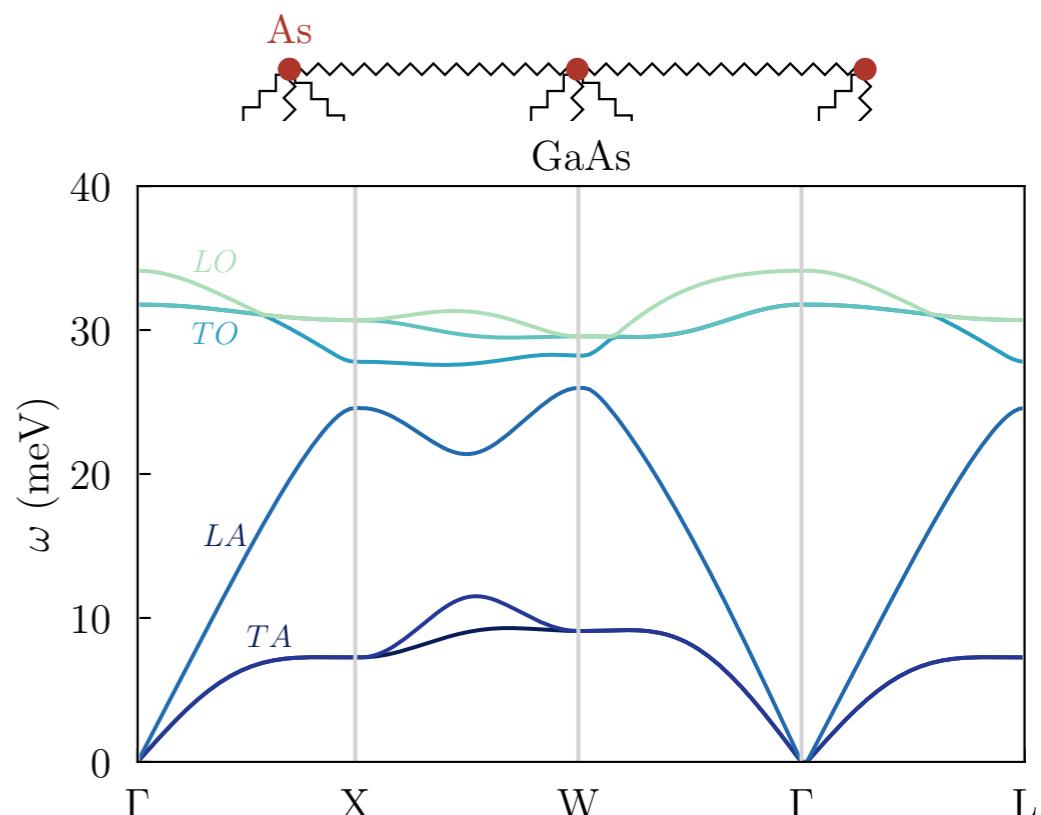
Eigenvalue problem:

$$\sum_{d'} \mathbf{D}_{\mathbf{q},d,d'} \cdot \mathbf{e}_{\nu,d',\mathbf{q}} = \omega_{\nu,\mathbf{q}}^2 \mathbf{e}_{d,\nu,\mathbf{q}}$$

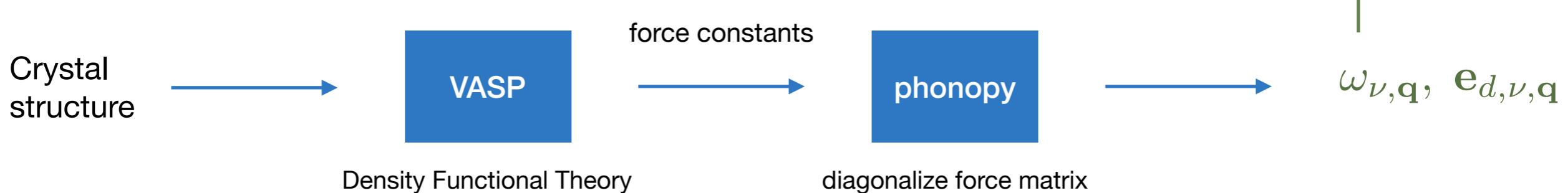
with

$$\mathbf{D}_{\mathbf{q},d,d'} \equiv \frac{\mathcal{V}_{d,d'}^{(2)}}{\sqrt{m_d m_{d'}}} e^{i\mathbf{q} \cdot (\mathbf{r}_{d'}^0 - \mathbf{r}_d^0)}.$$

Quadratic part of electrostatic potential
aka “force constants”



Numerical Calculation:



Theory input



Outline

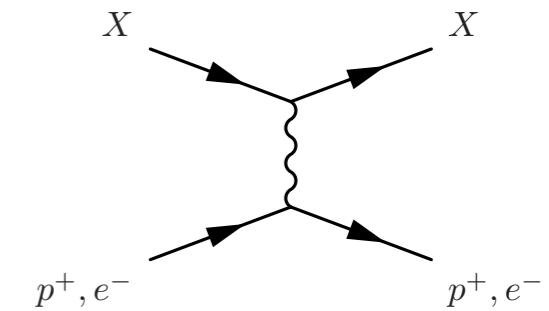
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Dark Matter - phonon coupling: Charge

Model:

“Nano-charged dark matter” aka “light dark photon mediator”



Born effective charges:

Atoms in the primitive cell can have an “effective” charge tensor

$$\mathbf{Z}_{ij} \equiv \frac{1}{e} \frac{\partial \mathbf{P}_i}{\partial \mathbf{u}_j}$$

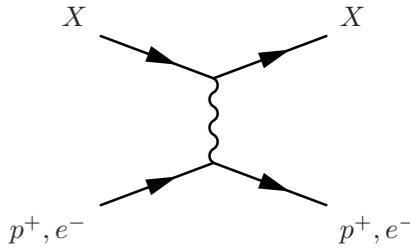
Example: GaAs

$$\mathbf{Z}_{\text{Ga}} = \begin{pmatrix} 2.27 & & \\ & 2.27 & \\ & & 2.27 \end{pmatrix}$$

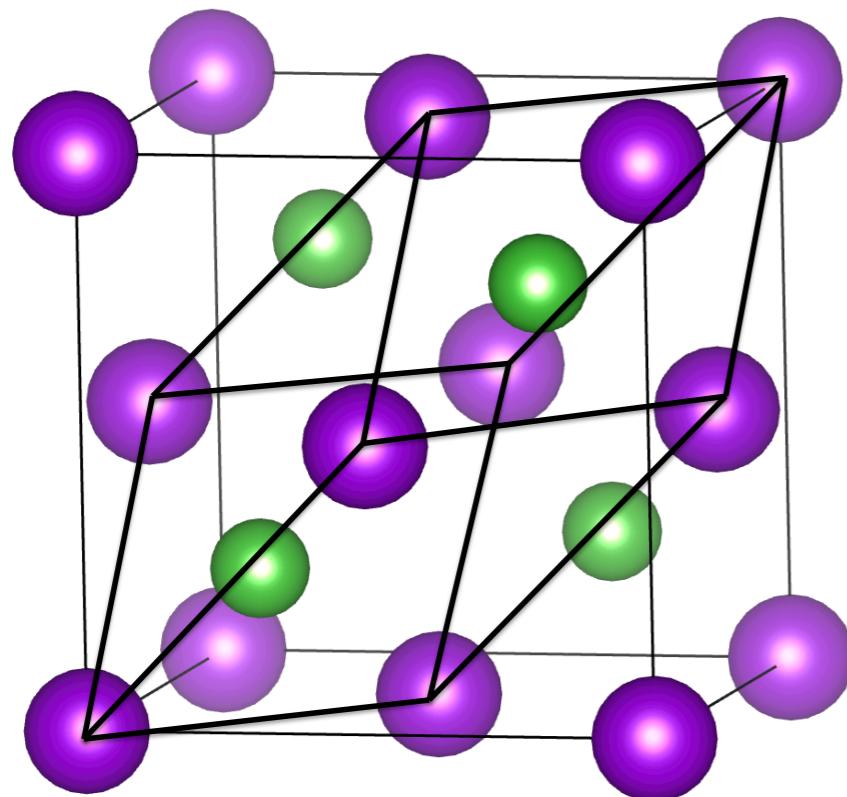
$$\mathbf{Z}_{\text{As}} = \begin{pmatrix} -2.27 & & \\ & -2.27 & \\ & & -2.27 \end{pmatrix}$$

Materials for which $\mathbf{Z} \neq 0$ are called “polar materials”

Examples of Polar Materials

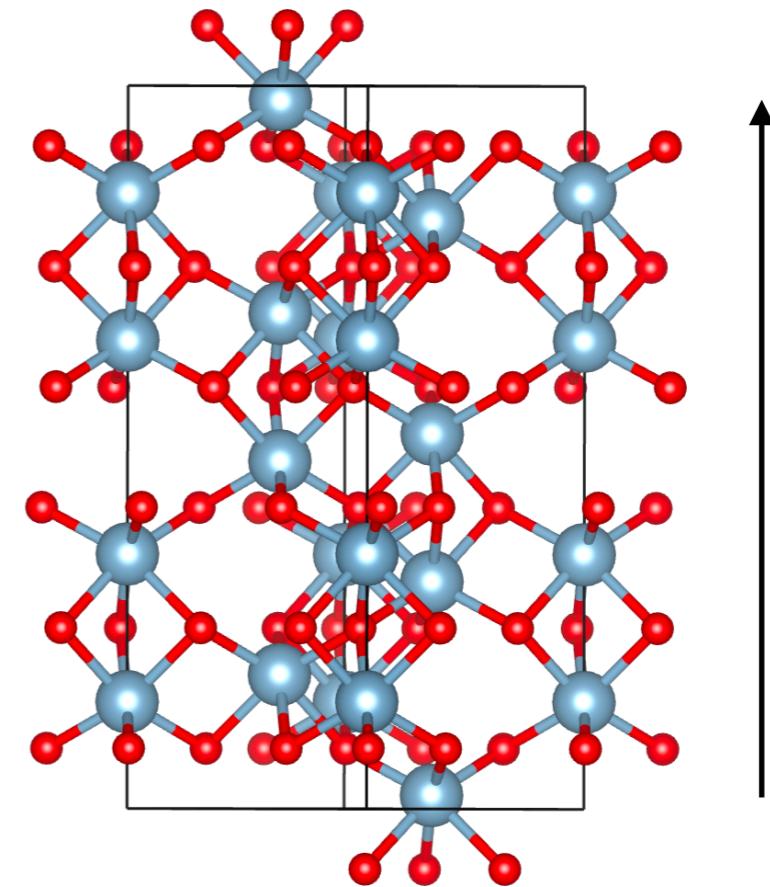


GaAs



2 atoms in primitive cell

Al_2O_3 (Sapphire)



Primary
crystal axis

At least two *different* atoms in the unit cell

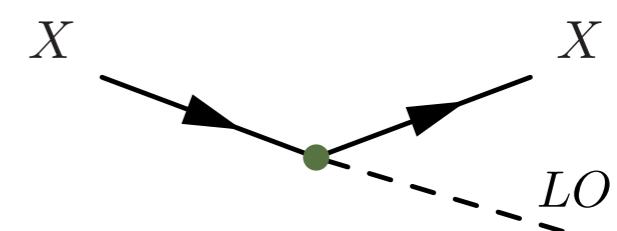
Frölich Hamiltonian

Each unit cell contains an electric dipole



Electric dipole interacting with test charge

$$H \sim i e' \sum_{\mathbf{q}} \frac{\mathbf{q} \cdot \mathbf{P}}{|\mathbf{q}|^2} e^{i\mathbf{q} \cdot \mathbf{r}}$$



Same thing, in a periodic lattice

$$H = i \frac{\kappa e e'}{V} \sum_{d,\nu,\mathbf{q}} \sum_{\mathbf{G}} \frac{1}{\sqrt{2N m_d \omega_{\nu,\mathbf{q}}}} \frac{(\mathbf{q} + \mathbf{G}) \cdot \mathbf{Z}_d \cdot \mathbf{e}_{d,\nu,\mathbf{q}}}{(\mathbf{q} + \mathbf{G}) \cdot \epsilon_{\infty} \cdot (\mathbf{q} + \mathbf{G})} a_{\nu,\mathbf{q}}^\dagger e^{i(\mathbf{q} + \mathbf{G}) \cdot \mathbf{r}} + \text{h.c.}$$



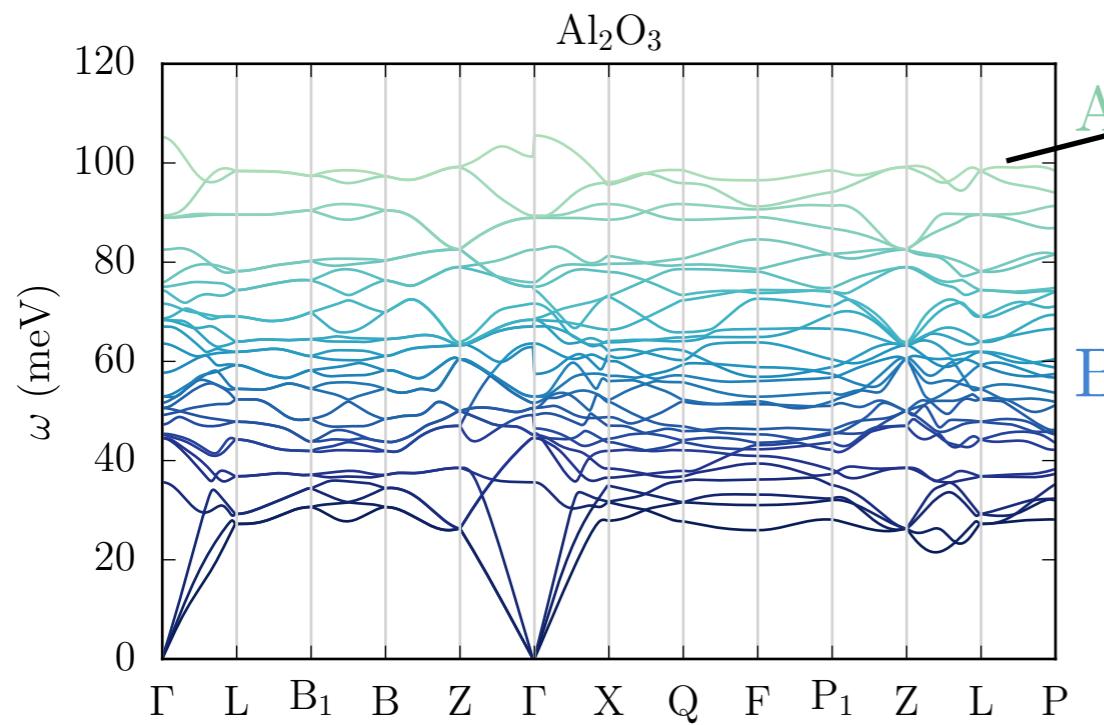
Reciprocal lattice vectors

H. Frölich, 1954

C. Verdi, F. Giustino, Phys. Rev. Lett. 115, 176401 (2015)

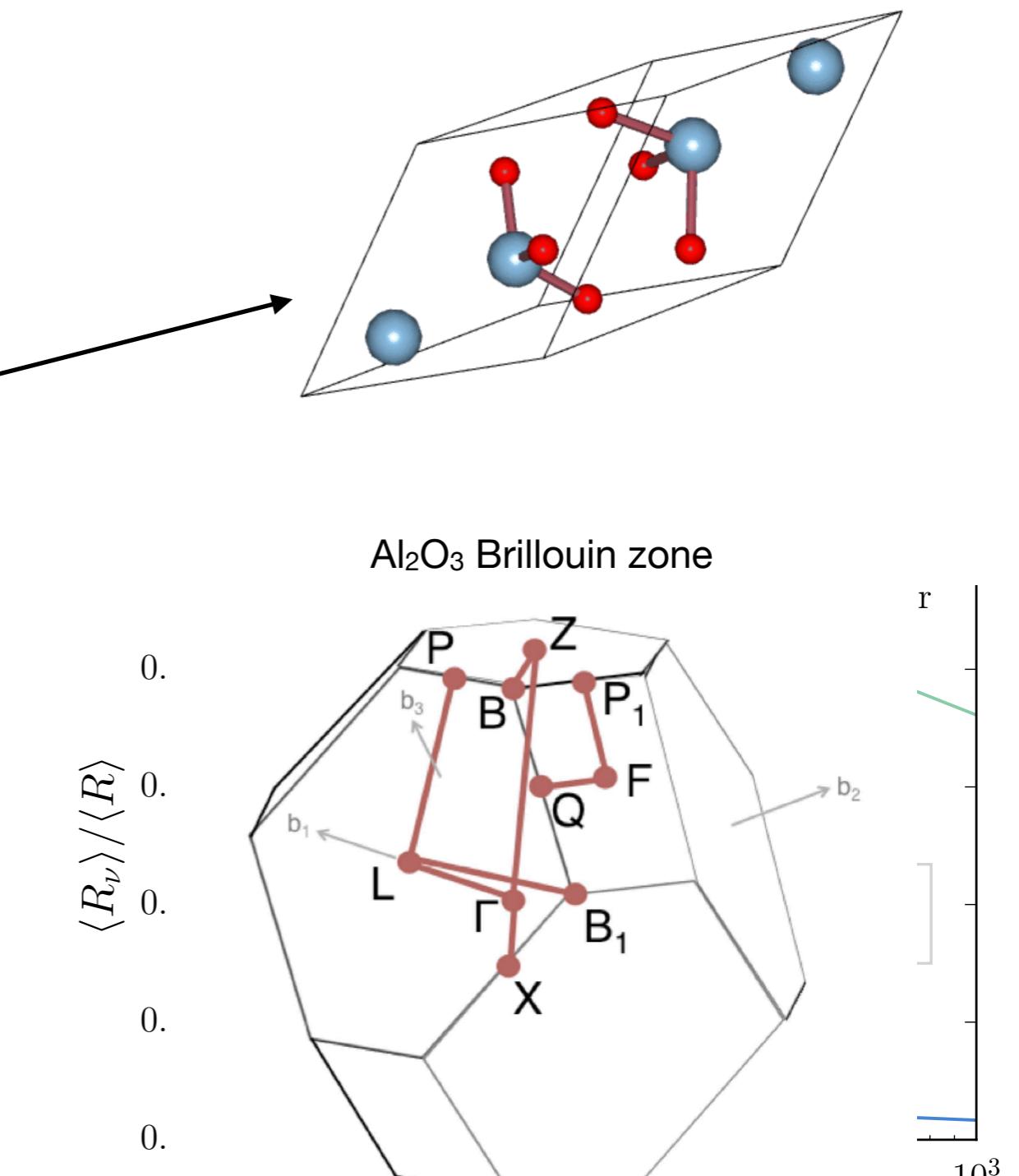
Sapphire in more detail

Most energetic mode dominates



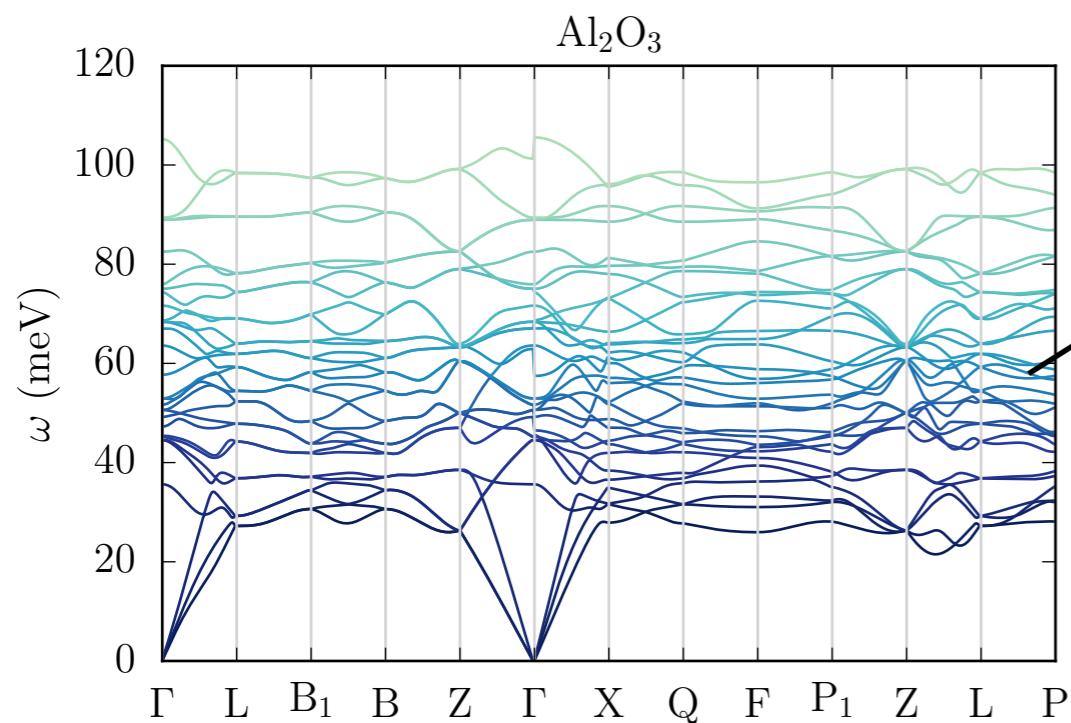
Aluminum atoms move in phase

Large dipole

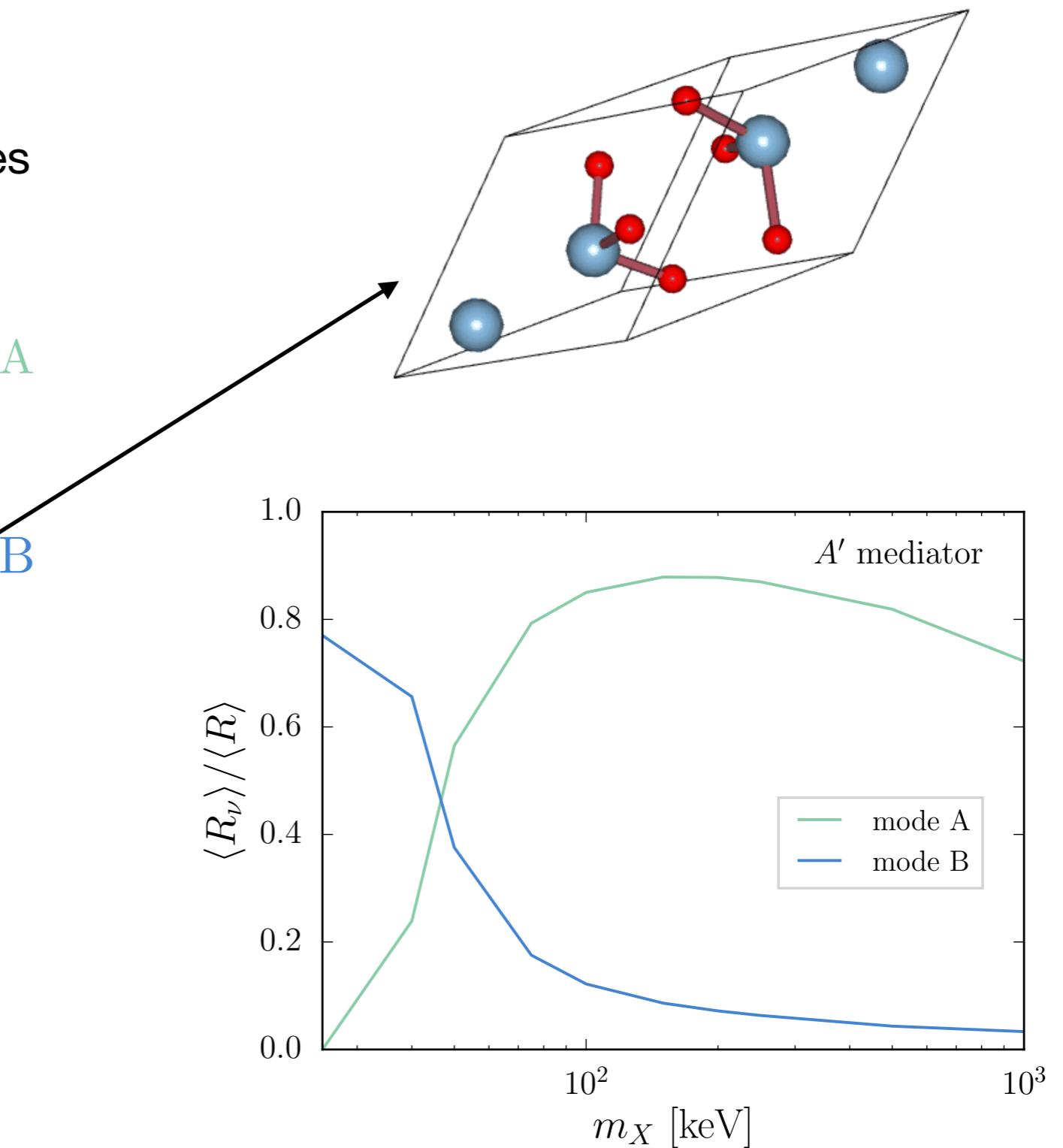


Sapphire in more detail

For low m_X , a lower mode dominates

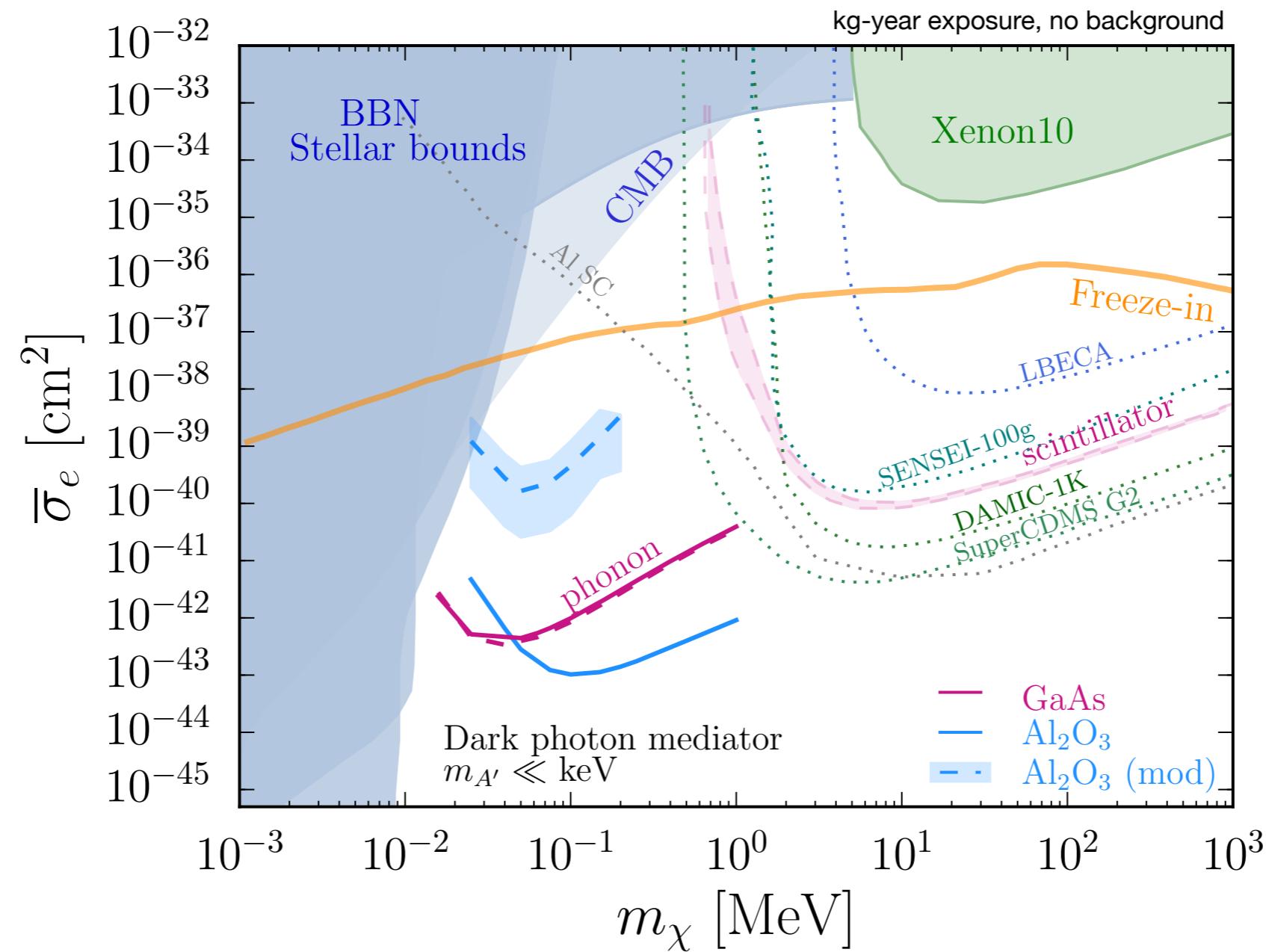
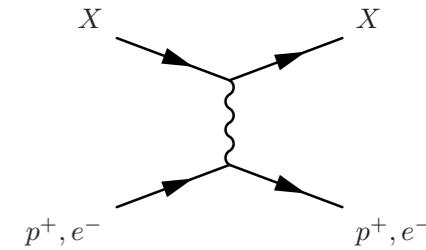


Aluminum atoms still move in phase, but smaller amplitude



Reach

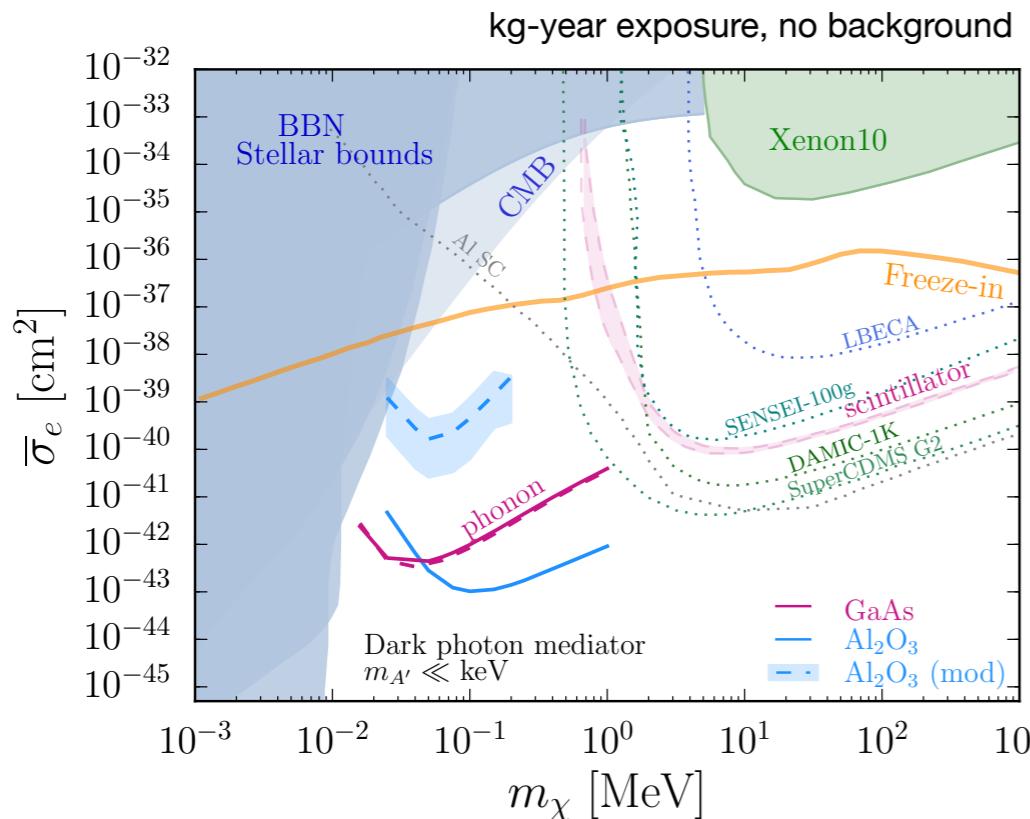
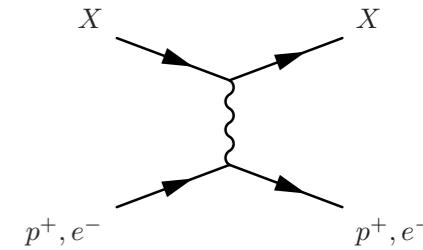
Both GaAs and Sapphire probe Dark Matter masses as low as 10 keV



Probe the new parameter space with gram-minute **green** exposure

Sidenote: how to read these plots?

Both GaAs and Sapphire probe Dark Matter masses as low as 10 keV



These are *not* “projected limit” plots:

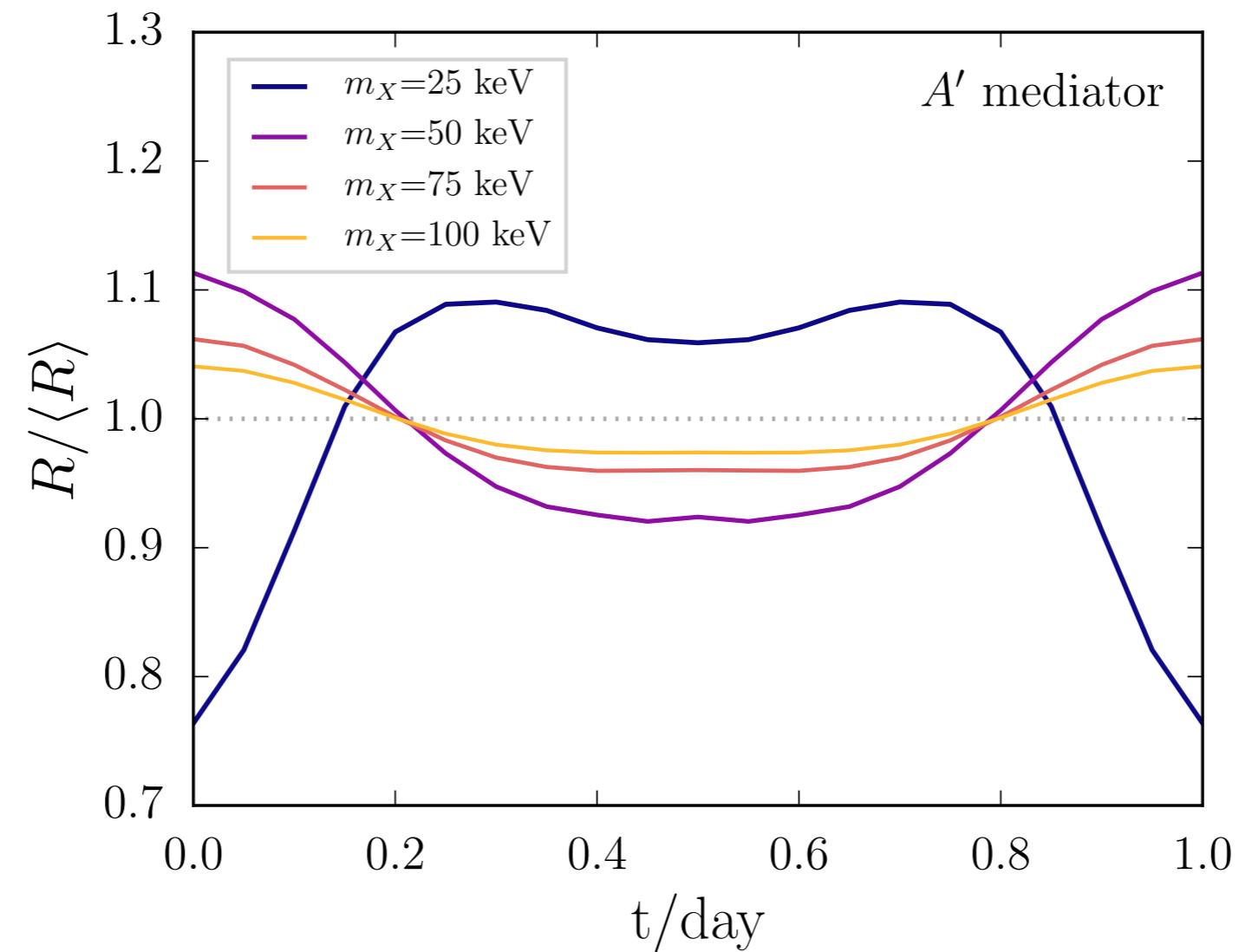
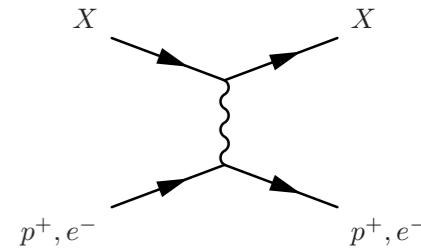
- Backgrounds in many future detectors are not yet known
- Eventual exposure and thresholds are not yet known

At some point, it became sort of a *convention* to plot cross section curves for 3 events with kg-year exposure

This is quite useful to quickly compare materials, proposals etc, but should not be taken overly literal

You need to exercise your own judgement and evaluate the status of the experiments: Some have data, some have pilot data, some are still in R&D phase

Daily modulation

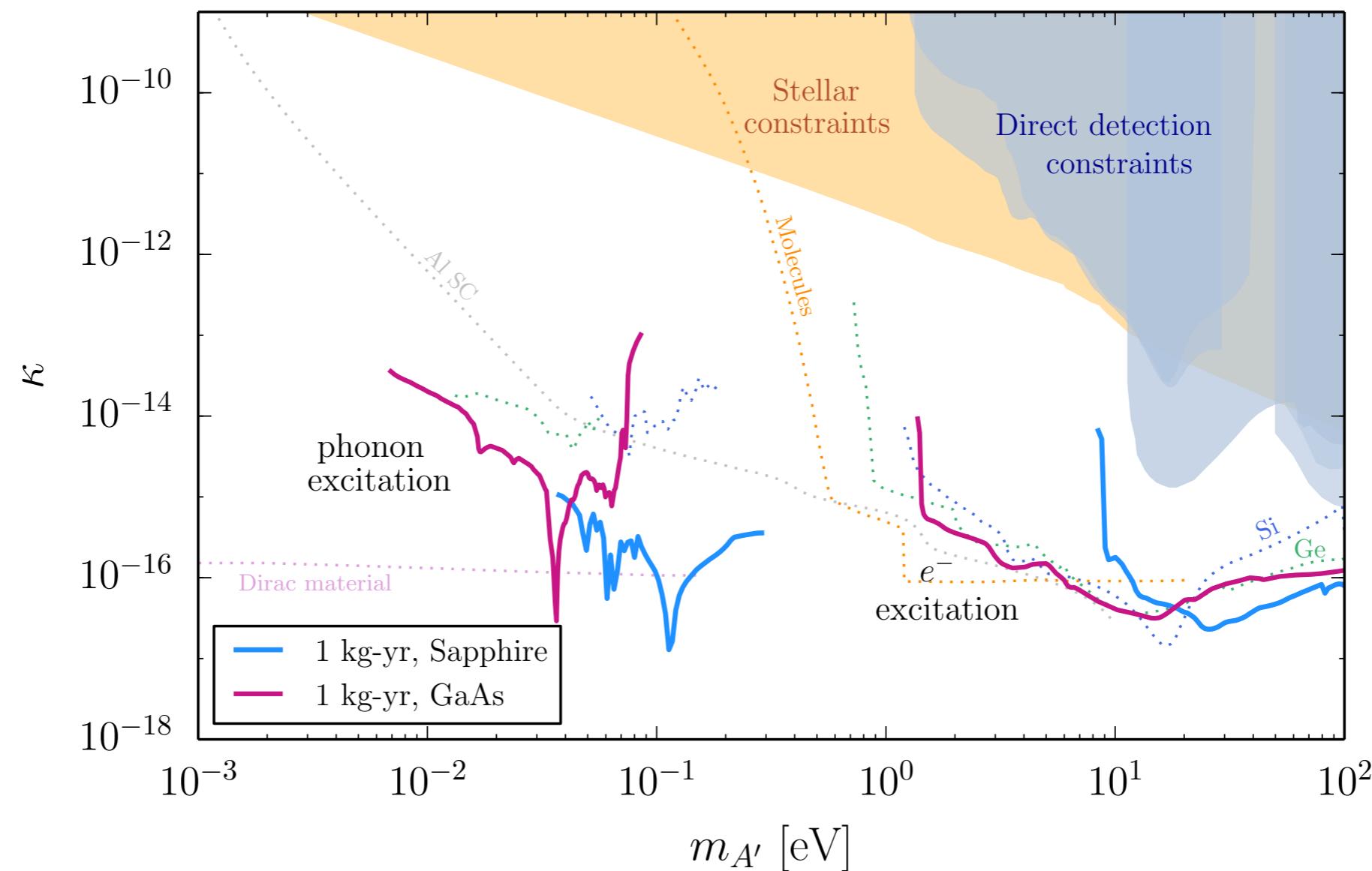


Amplitude and pattern depends on Dark Matter mass

Dark photon absorption

Very light, bosonic dark matter can be **absorbed** on the target

Example: Dark photon dark matter: $\mathcal{L} \supset -\frac{\kappa}{2} F'_{\mu\nu} F^{\mu\nu}$



Theory input

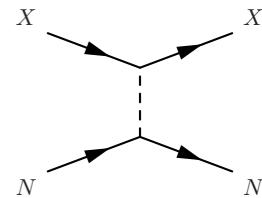


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Dark Matter - phonon coupling: Nuclei



Model

Coupling proportional to nucleon mass: e.g. scalar mediator

$$\mathcal{L} \supset \frac{A}{\Lambda^2} X \bar{X} N \bar{N}$$

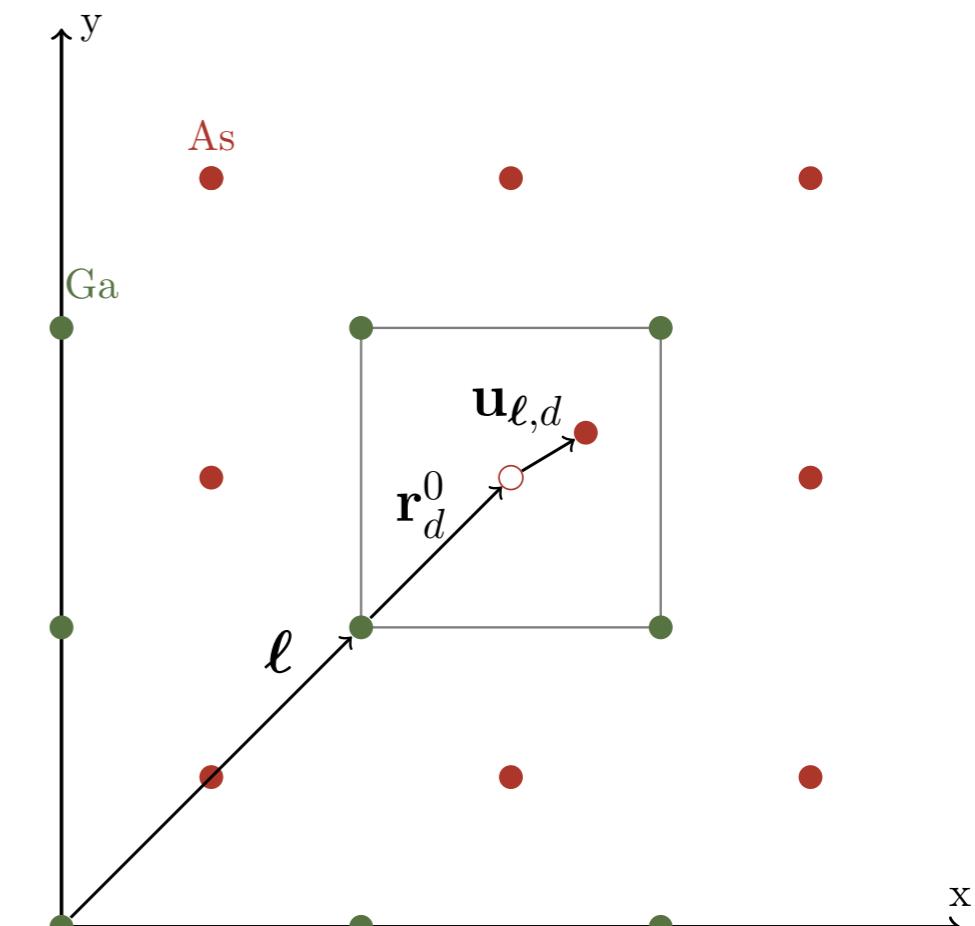
Atomic mass number

Potential felt by DM in crystal

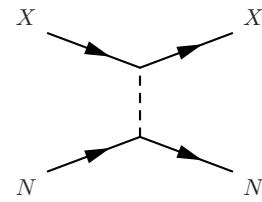
$$\mathcal{V}(\mathbf{r}) = \frac{1}{\Lambda^2} \sum_{\ell} \sum_d A_d \delta(\ell + \mathbf{r}_d^0 + \mathbf{u}_{\ell,d} - \mathbf{r})$$



$$\mathcal{V}(\mathbf{q}) = \frac{1}{\Lambda^2} \sum_{\ell} \sum_d A_d e^{i\mathbf{q} \cdot (\ell + \mathbf{r}_d^0 + \mathbf{u}_{\ell,d})}$$



Dark Matter - phonon coupling: Nuclei



$$\mathcal{V}(\mathbf{q}) = \frac{1}{\Lambda^2} \sum_{\ell} \sum_d A_d e^{i\mathbf{q} \cdot (\ell + \mathbf{r}_d^0 + \mathbf{u}_{\ell,d})}$$

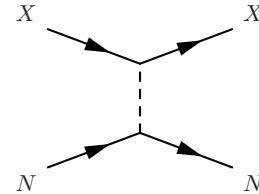
$$\downarrow \quad \mathbf{q} \cdot \mathbf{u}_{\ell,d} \ll 1$$

$$\mathcal{V}(\mathbf{q}) = \frac{1}{\Lambda^2} \sum_{\ell} \sum_d A_d e^{i\mathbf{q} \cdot (\ell + \mathbf{r}_d^0)} \left[i\mathbf{q} \cdot \mathbf{u}_{\ell,d} - \frac{1}{2}(\mathbf{q} \cdot \mathbf{u}_{\ell,d})^2 + \dots \right]$$

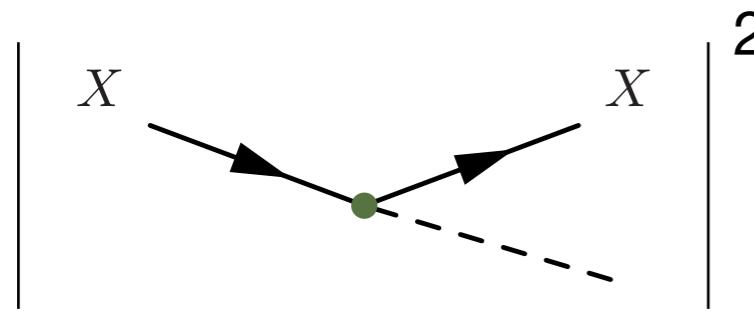


Relative coefficients of both operators are fixed by the UV completion

Single phonon rate



Matrix element



$$|\mathcal{M}|^2 = \frac{1}{\Lambda^2} \left| \langle \mathbf{q} | \sum_{\ell} \sum_d A_d e^{i\mathbf{q} \cdot (\ell + \mathbf{r}_d^0)} \mathbf{q} \cdot \mathbf{u}_{\ell,d} | 0 \rangle \right|^2$$

Bloch decomposition +
some index wrangling

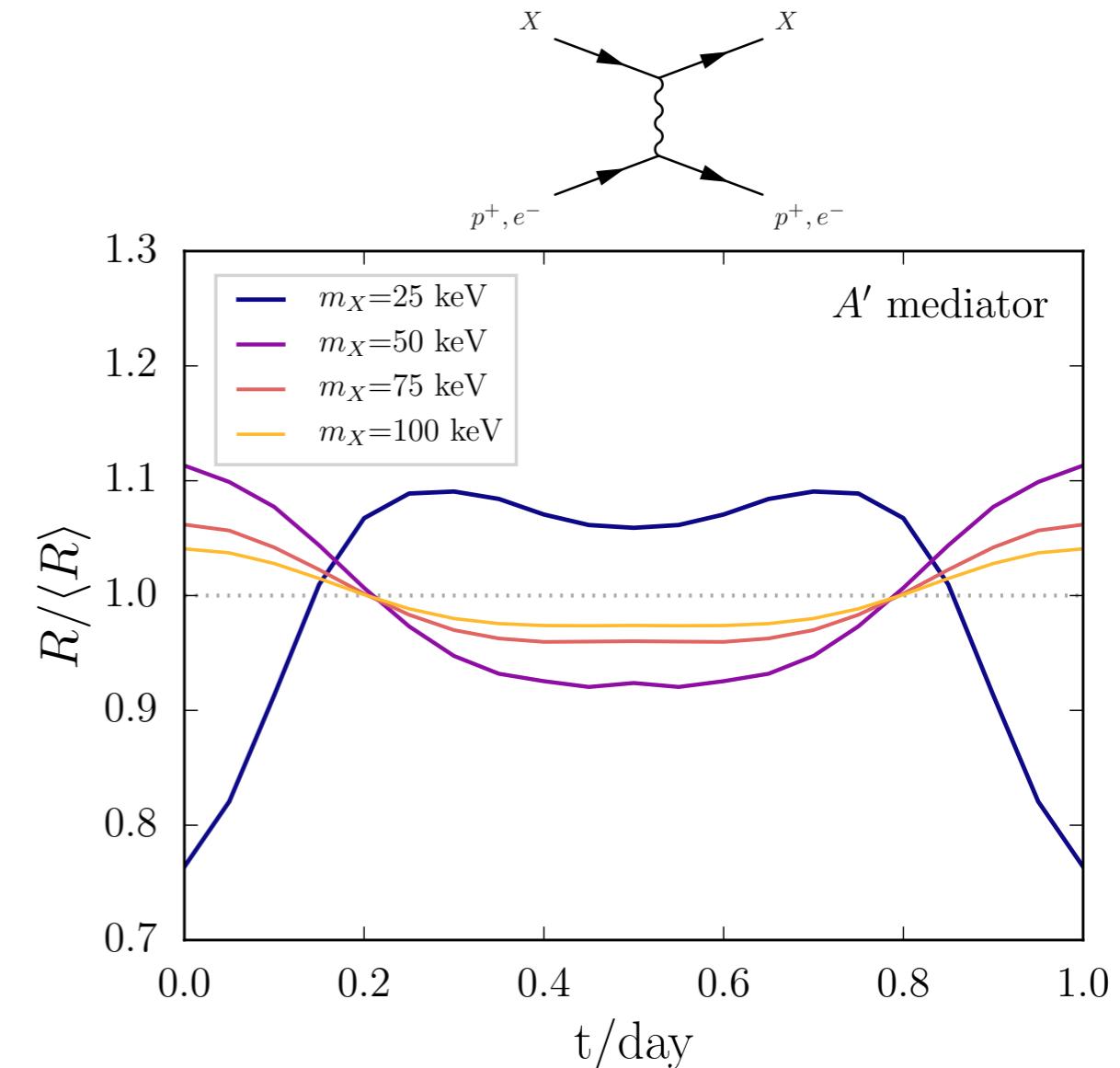
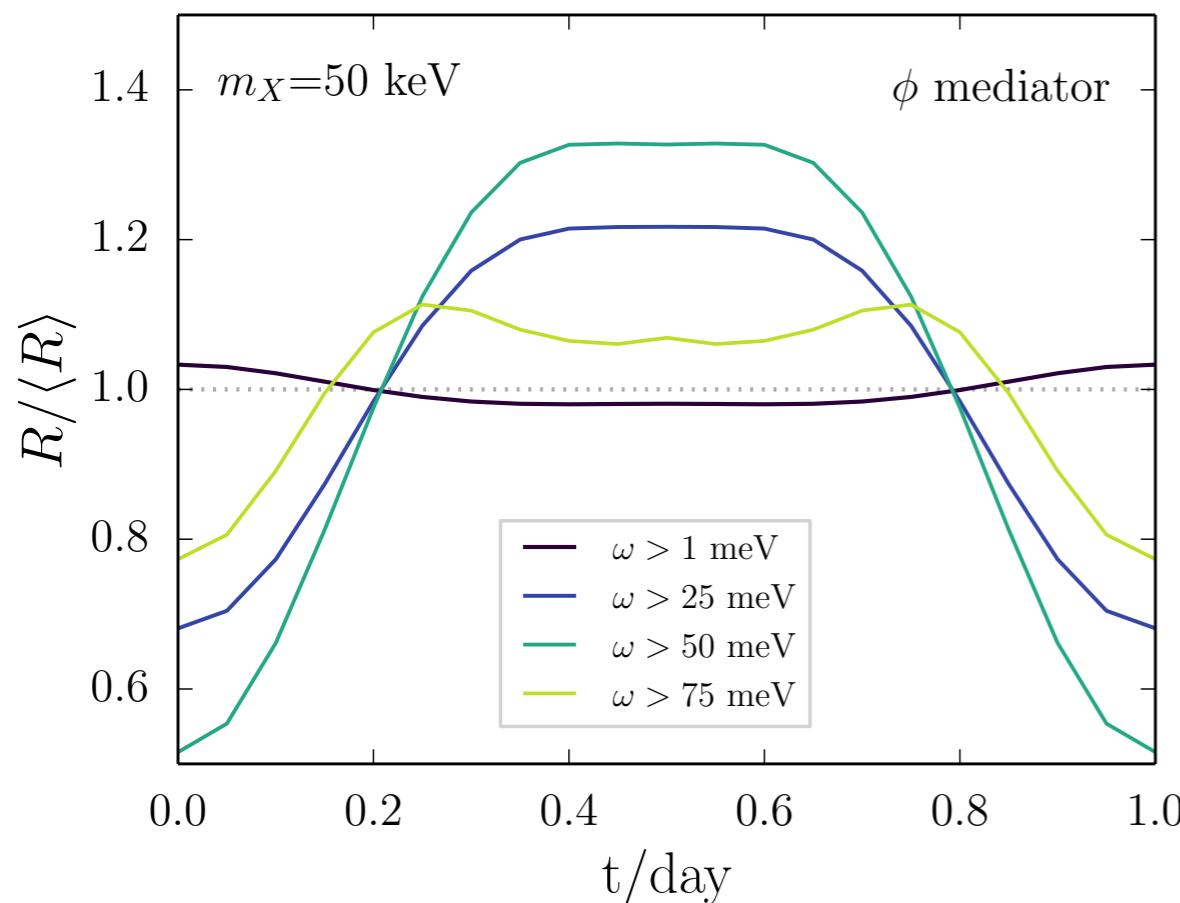
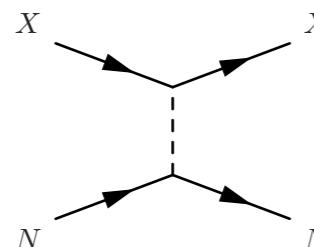
Form factor

$$|F_{\nu}(\mathbf{q})|^2 = \left| \sum_d \frac{A_d}{\sqrt{m_d}} e^{-W_d(\mathbf{q})} \mathbf{q} \cdot \mathbf{e}_{d,\nu,\mathbf{q}} e^{-i\mathbf{q} \cdot \mathbf{r}_d^0} \right|^2$$

Debye-Waller factor
(Measures the delocalization of
the nucleus)

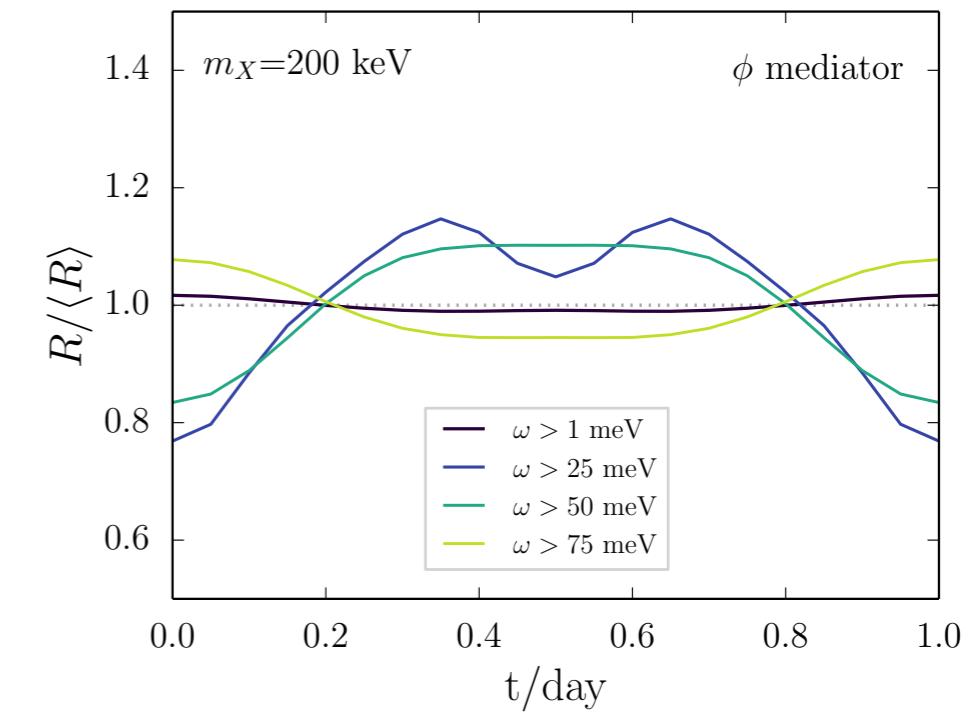
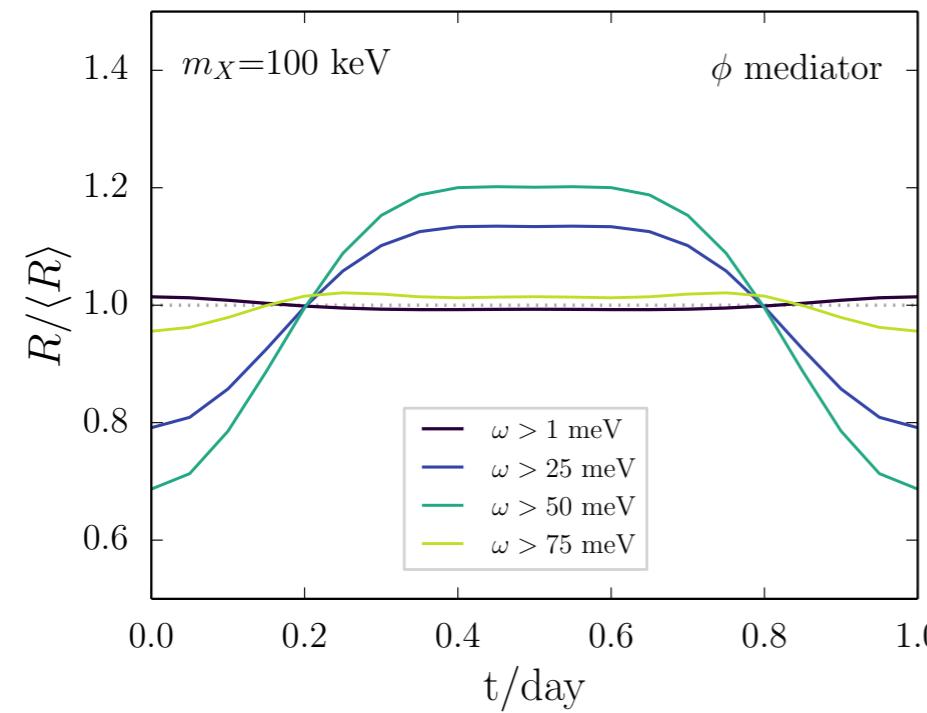
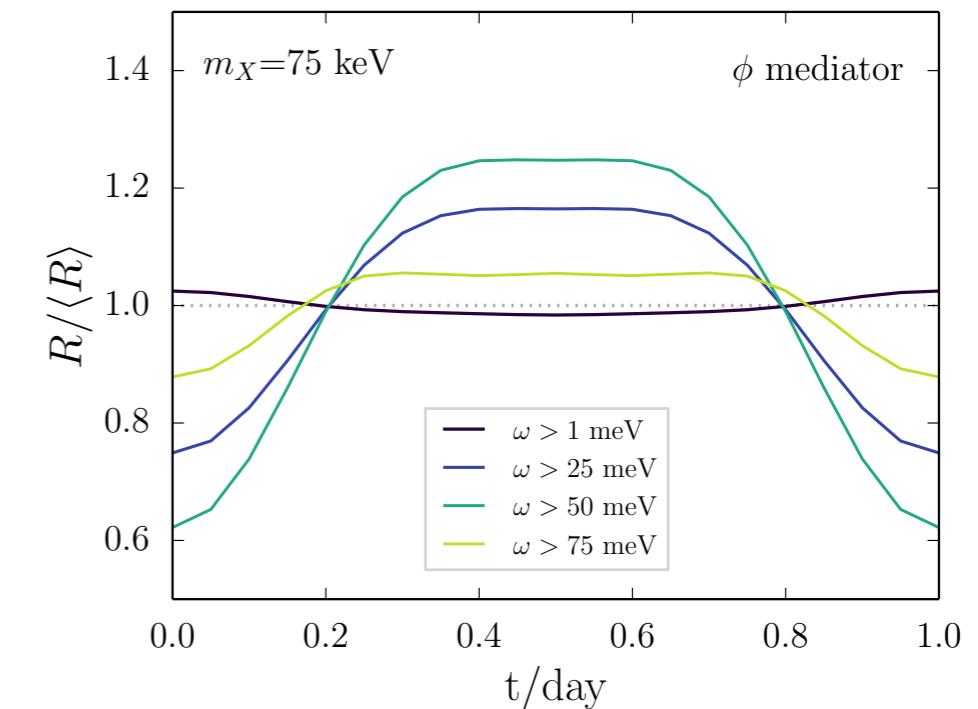
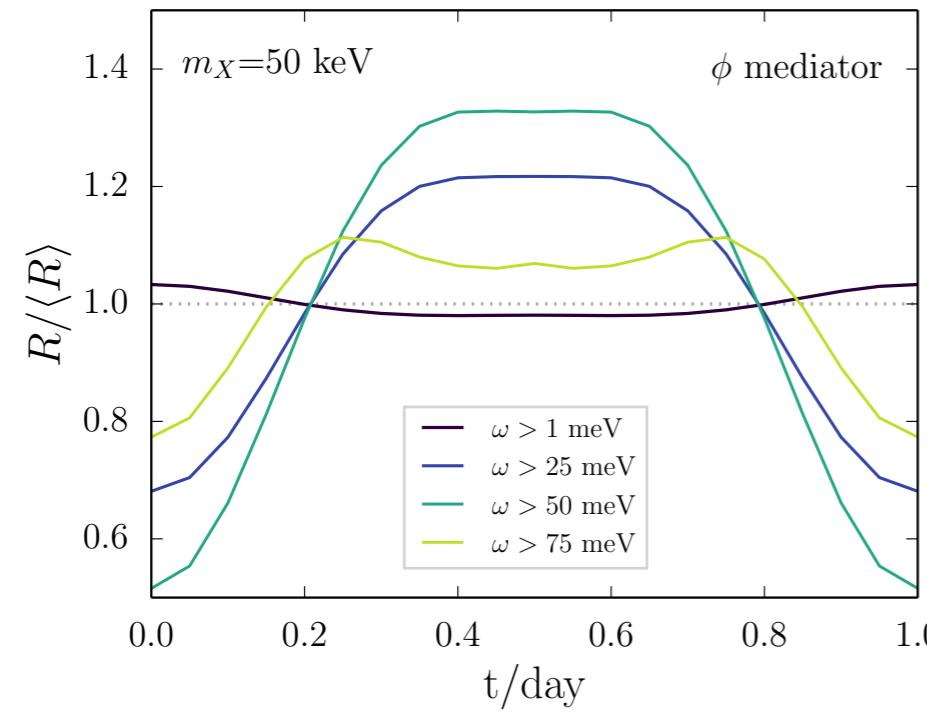
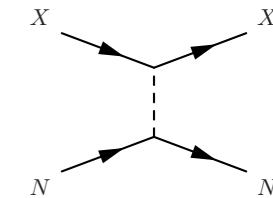
Daily modulation!

Daily modulation (Sapphire)



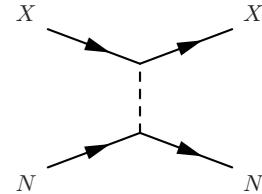
Opposite phase from dark photon mediator!

Daily modulation (Sapphire)

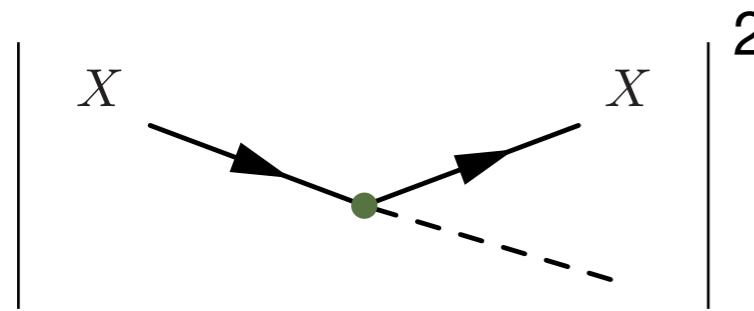


Modulation pattern depends on the DM mass

Single phonon rate



Matrix element



$$|\mathcal{M}|^2 = \frac{1}{\Lambda^2} \left| \langle \mathbf{q} | \sum_{\ell} \sum_d A_d e^{i\mathbf{q} \cdot (\ell + \mathbf{r}_d^0)} \mathbf{q} \cdot \mathbf{u}_{\ell,d} | 0 \rangle \right|^2$$

Bloch decomposition +
some index wrangling

Form factor

$$|F_{\nu}(\mathbf{q})|^2 = \left| \sum_d \frac{A_d}{\sqrt{m_d}} e^{-W_d(\mathbf{q})} \mathbf{q} \cdot \mathbf{e}_{d,\nu,\mathbf{q}} e^{-i\mathbf{q} \cdot \mathbf{r}_d^0} \right|^2$$

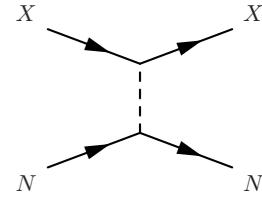


Debye-Waller factor
(Measures the delocalization of
the nucleus)

LA: constructive interference
LO: destructive interference
TA,TO: vanish

Daily modulation!

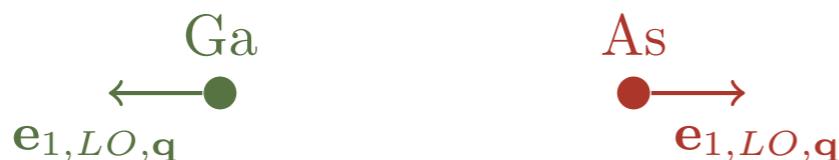
Destructive interference



$$|F_\nu(\mathbf{q})|^2 = \left| \sum_d \frac{A_d}{\sqrt{m_d}} e^{-W_d(\mathbf{q})} \mathbf{q} \cdot \mathbf{e}_{d,\nu,\mathbf{q}} e^{-i\mathbf{q} \cdot \mathbf{r}_d^0} \right|^2$$

LA:  $\rightarrow |F_{LA}(q)|^2 \sim \mathcal{O}(q^2)$

$$\begin{matrix} \mathbf{q} \\ \longrightarrow \end{matrix}$$

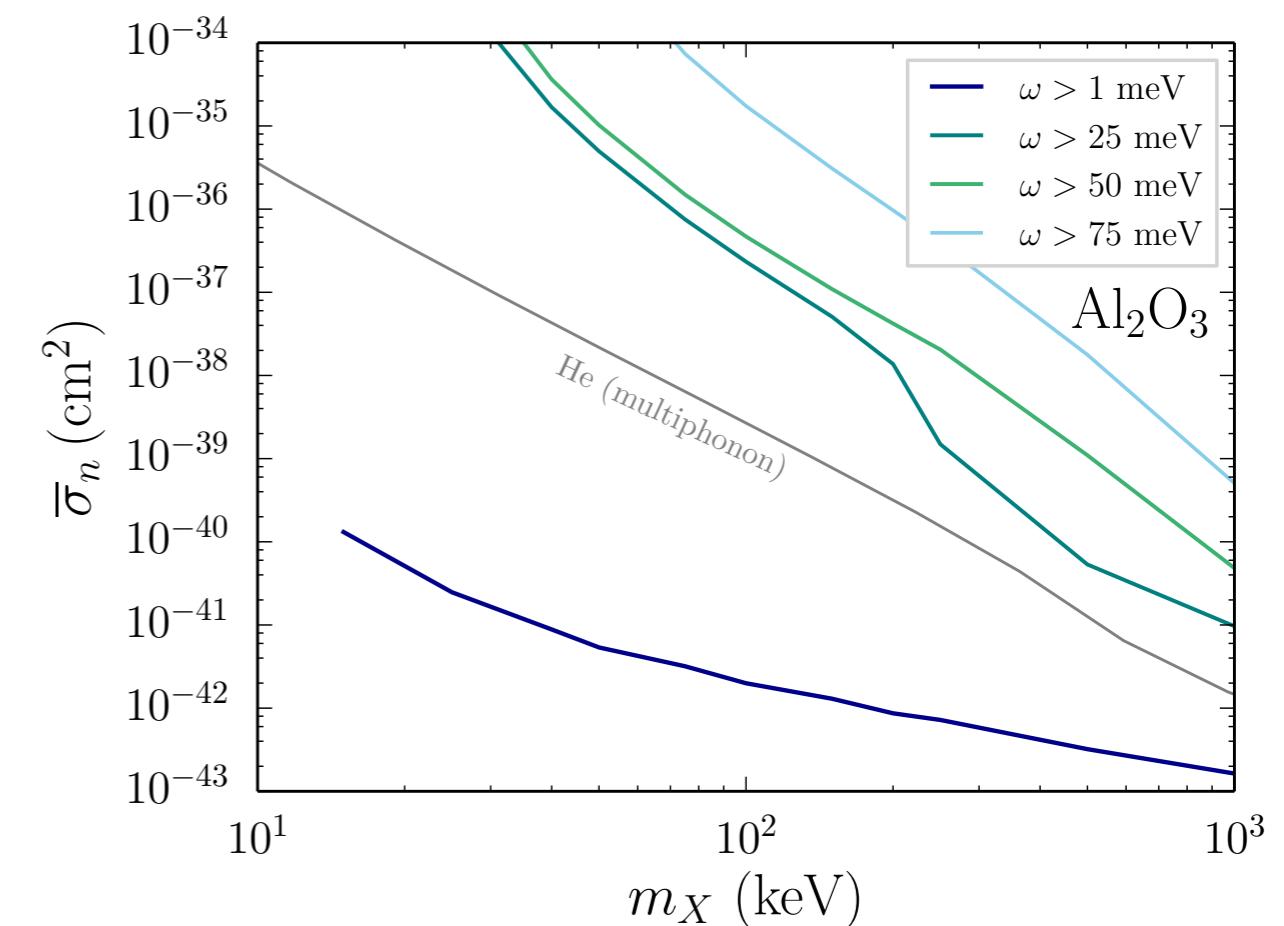
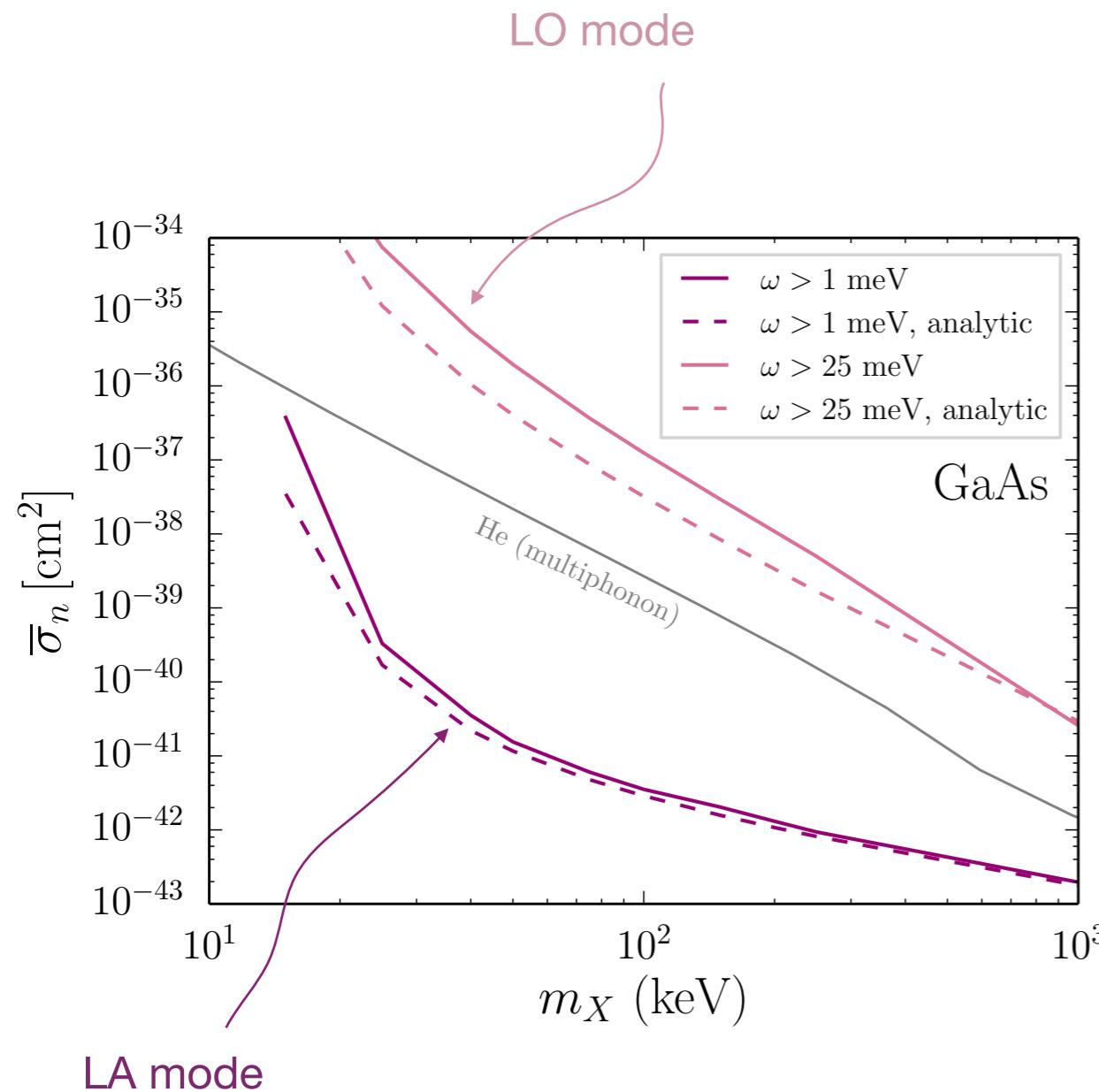
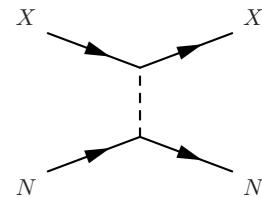
LO:  $\rightarrow |F_{LO}(q)|^2 \sim \mathcal{O}(q^4)$

Contributions for the LO mode add up with the opposite sign



Strong suppression of LO mode

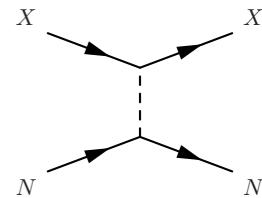
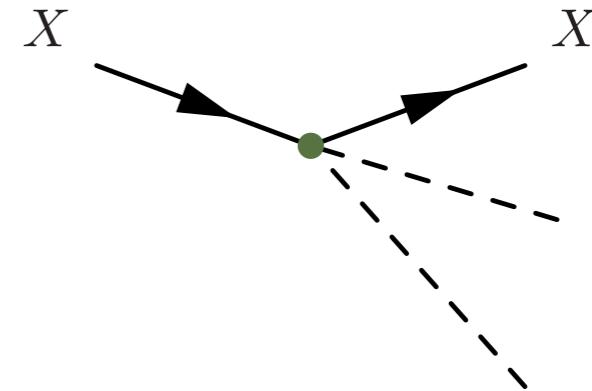
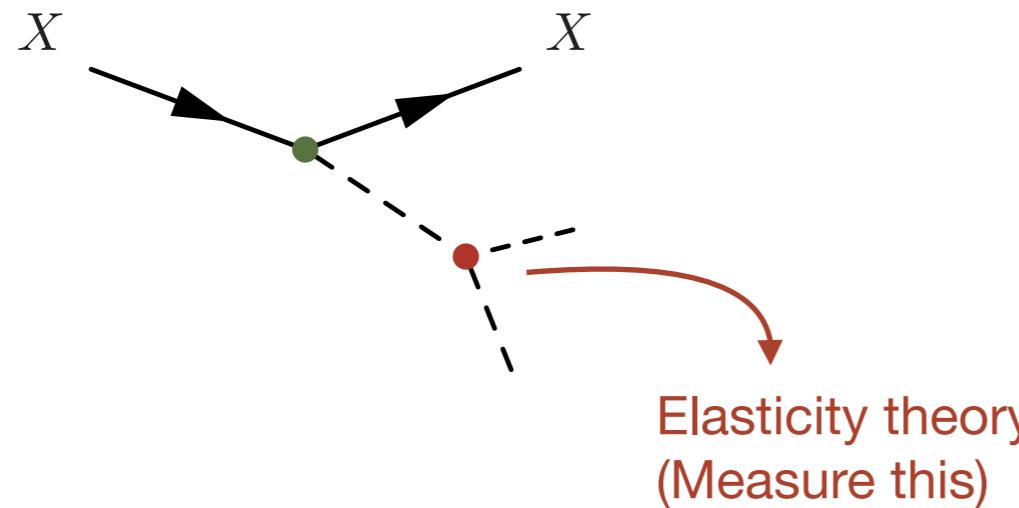
Result (single phonon)



Is the next-to-leading order (multi-phonon) process important?

Multi-phonon processes

Contributions



Four channels

- LA-LA
- LA-TA
- TA-TA (both in momentum plane)
- TA-TA (both orthogonal to momentum plane)

Some elasticity theory

Elastic strain tensor

$$\eta_{ij} \equiv \partial_i u_j + \partial_j u_i + \sum_k \partial_k u_i \partial_k u_j$$



Local deformations of the solid



Identify this with acoustic phonons, after averaging over the primitive cell

Hooke's law

$$\sigma_{ij} = \sum_{k,\ell} C_{ijkl} \eta_{kl}$$



Second order elasticity parameters

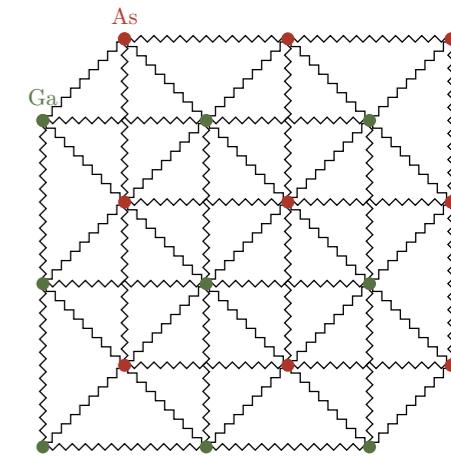
Stress tensor

In Hamiltonian form

$$H = \frac{1}{2} C_{ijkl} \eta_{ij} \eta_{kl} + \frac{1}{3!} \mathcal{C}_{ijklmn} \eta_{ij} \eta_{kl} \eta_{mn} + \sigma_{ij} \eta_{ij}$$



Third order elasticity parameters

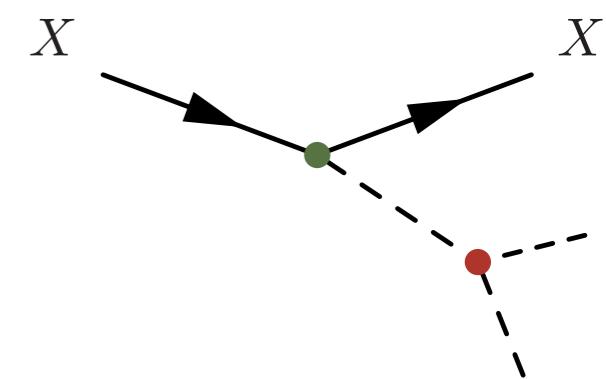


Phonon self-interactions

$$H = \frac{1}{2} C_{ijkl} \eta_{ij} \eta_{kl} + \frac{1}{3!} C_{ijklmn} \eta_{ij} \eta_{kl} \eta_{mn} + \sigma_{ij} \eta_{ij}$$

Independent components

	C_{ijkl}	C_{ijklmn}
general	21	56
cubic	3	6
isotropic	2	3



Phonon self-interactions in the isotropic approximation

$$H = \frac{1}{2} (\beta + \lambda) u_{ii} u_{jk} u_{jk} + (\gamma + \mu) u_{ij} u_{ki} u_{kj} + \frac{\alpha}{3!} u_{ii} u_{jj} u_{kk} + \frac{\beta}{2} u_{ii} u_{jk} u_{kj} + \frac{\gamma}{3} u_{ij} u_{jk} u_{ki}$$

with $u_{ij} \equiv \partial_i u_j$

Couplings map to the elasticity parameters

λ, μ \leftrightarrow Bulk modulus & Young's modulus

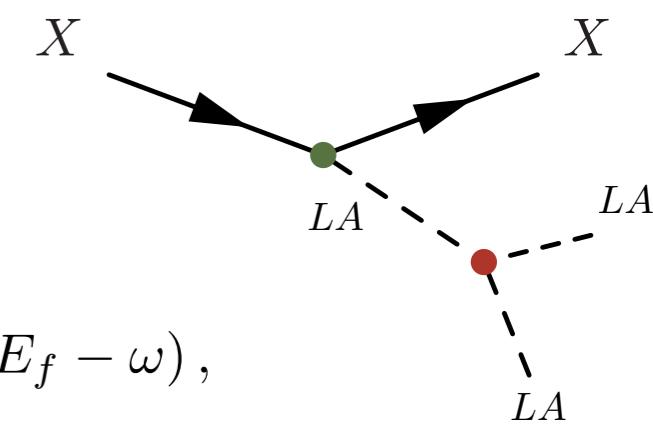
α, β, γ \leftrightarrow Third order elastic constants

Can be measured or calculated from first principles!

Computing the scattering rate

Dynamical structure function

$$\frac{d^2\sigma}{dq d\omega} = \frac{q}{2v^2 m_X^2} \sigma_n S(q, \omega) \quad \text{with} \quad S(\mathbf{q}, \omega) \equiv \sum_f |\langle f | \mathcal{V}(\mathbf{q}) | 0 \rangle|^2 \delta(E_f - \omega),$$



Example:

$$S_{LALA}^{(anh)}(q, \omega) = \frac{\sum_d A_d \mathbf{q}^2 \omega^4}{64\pi^2 c_{LA}^9 m_p \rho^3 [(\omega - c_{LA}q)^2 + \Gamma_{LA,q}^2/4]} g_{LALA}^{(anh)}\left(\frac{qc_{LA}}{\omega}\right) \theta(\omega - c_{LA}q)$$

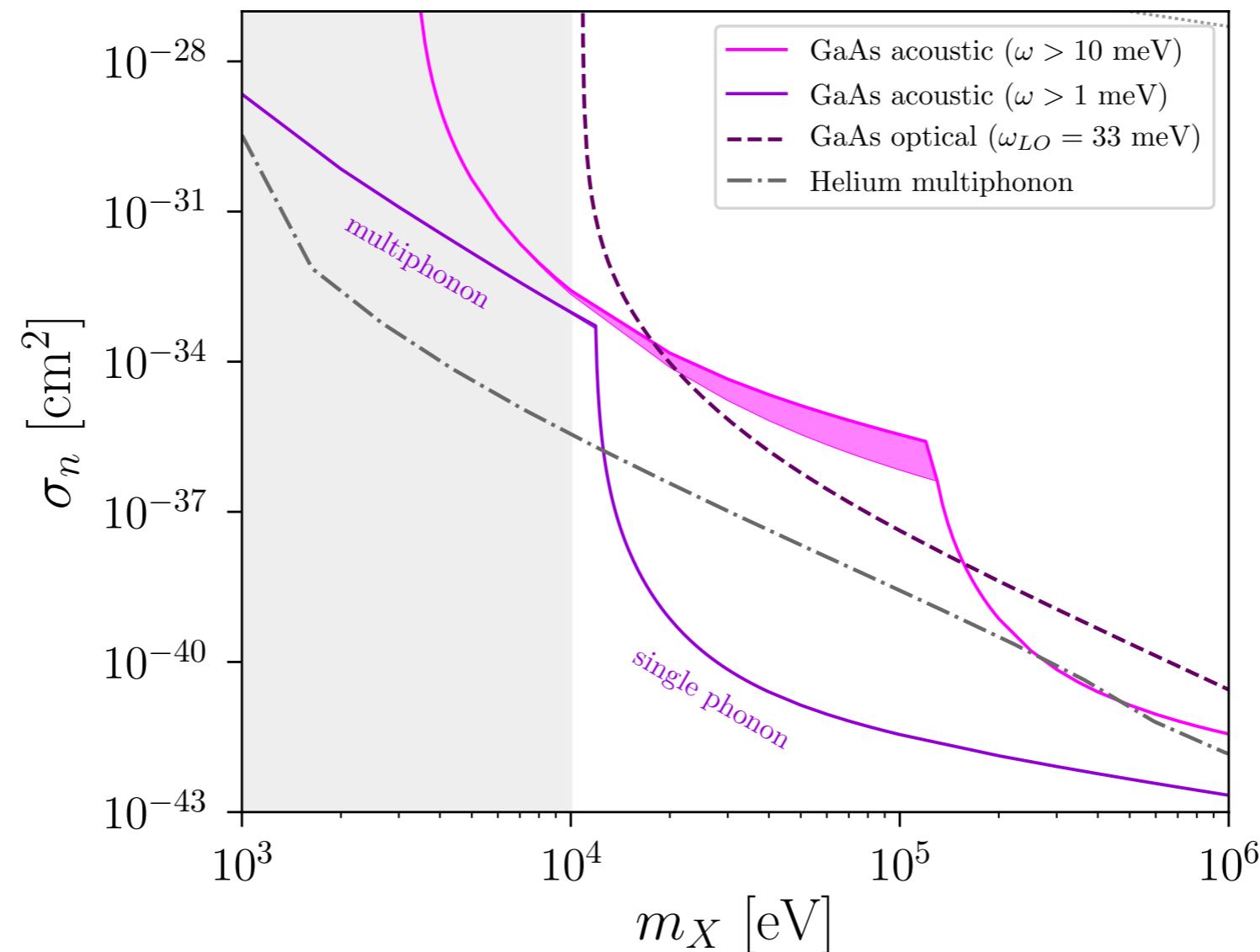
with

$$\begin{aligned} g_{LALA}^{(anh)}(x) &\equiv (2\beta + 4\gamma + \lambda + 3\mu)^2 \frac{(x^2 - 1)^3}{2x^5} (x^6 + 3x^4 + 7x^2 + 5) \left(\tanh^{-1}(x) - \frac{x^3}{3} - x \right) \\ &+ a_{10}x^{10} + a_8x^8 + a_6x^6 + a_4x^4 + a_2x^2 + a_0 \\ a_0 &\equiv \frac{1}{240} \left(15\alpha^2 + 10\alpha(10\beta + 8\gamma + 5\lambda + 6\mu) + 668\beta^2 + 4\beta(568\gamma + 167\lambda + 426\mu) \right. \\ &\left. + 2112\gamma^2 + 1136\gamma\lambda + 3168\gamma\mu + 167\lambda^2 + 852\lambda\mu + 1188\mu^2 \right) \end{aligned}$$

...

Results

Reach comparison



Tends to be subdominant compared to the LO mode

Magnons

We won't go into any detail here, but spin-dependent DM can interact with collective *spin-wave excitations* in the crystal (**Magnons**)

$$\mathcal{L} = g_\chi \bar{\chi} \chi \phi + g_e \bar{e} i \gamma^5 e \phi \quad \hat{O}_\chi^\alpha = -\frac{g_\chi g_e}{q^2 m_e} i q^\alpha \mathbb{1}_\chi$$

The treatment is fairly analogous but we now need to quantize the **spin operator** (see Trickle et.al. 1905.13744 for details)

Heisenberg hamiltonian:

$$H = \frac{1}{2} \sum_{l,l'=1}^N \sum_{j,j'=1}^n J_{ll'jj'} \mathbf{S}_{lj} \cdot \mathbf{S}_{l'j'} \xrightarrow{\text{Can be diagonalized}} H = \sum_{\nu=1}^n \sum_{\mathbf{k} \in \text{BZ}} \omega_{\nu,\mathbf{k}} \hat{b}_{\nu,\mathbf{k}}^\dagger \hat{b}_{\nu,\mathbf{k}}$$

Given existing constraints on these models, this avenue seems more challenging than the more “vanilla” spin-independent case

For **axion absorption** see Mitridate et. al. 2005.10256 and Chigusa et. al. 2001.10666

Summary

For $m_x < \text{MeV}$ the deBoglie wavelength exceeds the inter-particle spacing

This means that we must integrate out the atoms and figure out how the DM couples to **collective degrees of freedom**, such as phonons, magnons etc

We do this by

1. Matching the SM operator to a **macroscopic property** e.g. number density:

$$\bar{N}N \rightarrow \sum_i \delta^{(3)}(\mathbf{r} - \mathbf{r}_i)$$

2. **Canonically quantizing** the fluctuations of the displacement operator

$$\mathbf{u}_d = \sum_{\nu}^{\text{3n}} \sum_{\mathbf{q}} \sqrt{\frac{1}{2N m_d \omega_{\nu, \mathbf{q}}}} \left(\mathbf{e}_{d, \nu, \mathbf{q}} a_{\nu, \mathbf{q}} e^{i(\mathbf{q} \cdot \mathbf{r}_d^0 - \omega_{\nu, \mathbf{q}} t)} + \text{h.c.} \right)$$

3. **Match** the various parameters of the effective theory to either data or a first principle DFT calculation, e.g. from elasticity parameters

$$H = \frac{1}{2}(\beta + \lambda)u_{ii}u_{jk}u_{jk} + (\gamma + \mu)u_{ij}u_{ki}u_{kj} + \frac{\alpha}{3!}u_{ii}u_{jj}u_{kk} + \frac{\beta}{2}u_{ii}u_{jk}u_{kj} + \frac{\gamma}{3}u_{ij}u_{jk}u_{ki}$$

4. **Compute the amplitude** & evaluate the rate. (This can involve a nasty set of integrals)

Summary

Different materials can have very different phonon excitations!

Which phonon branch the DM dominantly couples to **depends on your model!**

Rule of thumb:

Coupling to charge → LO mode

Anything else → LA mode

↳ “Astrophysically challenged” anyways...

The LA mode in most materials will likely be too low energy to low to detect, in this case we likely need multi-phonon production OR live with the suppression of the destructive interference in the LO mode

Questions?



Additional References

Model constraints: 1709.07882

First calculations: 1604.08206 (He), 1712.06598 (crystals)

Comprehensive theory discussion: 1807.10291, 2009.13534

Also: Many other works by Kathryn Zurek's group (Caltech)
(e.g. comparative studies of different targets etc.)

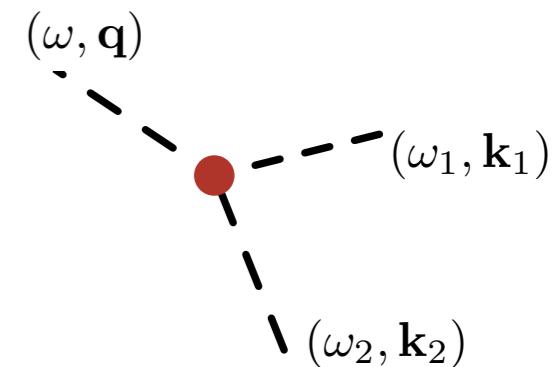
Thank you!



Comment on superfluid helium

Phonon 3-points function in crystal

$$\langle \mathbf{q} | \mathbf{k}_1, \mathbf{k}_2 \rangle \sim (\mathbf{q} \cdot \mathbf{e}) (\mathbf{k}_1 \cdot \mathbf{e}_1) (\mathbf{k}_2 \cdot \mathbf{e}_2) + \dots \sim q$$



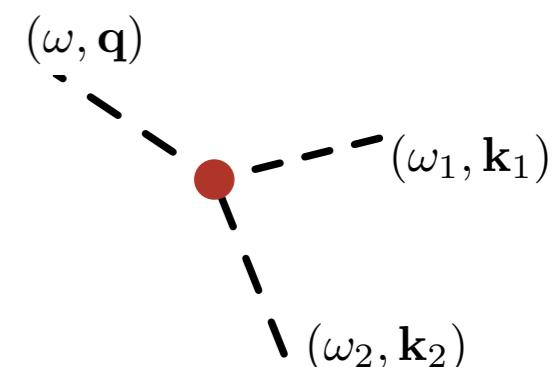
Phonon 3-points function in superfluid He

$$\langle \mathbf{q} | \mathbf{k}_1, \mathbf{k}_2 \rangle \sim a_1 (\omega_2 \mathbf{q} \cdot \mathbf{k}_1 + \omega_1 \mathbf{q} \cdot \mathbf{k}_2) + a_2 \omega \mathbf{k}_1 \cdot \mathbf{k}_2 + \underbrace{a_3 \omega \omega_1 \omega_2}_{\text{red line}} \sim q^2$$

Mysterious cancellation: Why does the 3-points function vanish when $\mathbf{q} \rightarrow 0$?

Comment on superfluid helium

$$\langle \mathbf{q} | \mathbf{k}_1, \mathbf{k}_2 \rangle \sim a_1 (\omega_2 \mathbf{q} \cdot \mathbf{k}_1 + \omega_1 \mathbf{q} \cdot \mathbf{k}_2) + a_2 \cancel{\omega \mathbf{k}_1 \cdot \mathbf{k}_2} + a_3 \omega \omega_1 \omega_2$$



Superfluid He has a spontaneously broken U(1) symmetry

$$q_\mu \langle 0 | J^\mu | \mathbf{k}_1, \mathbf{k}_2 \rangle = 0 \quad \xrightarrow{\mathbf{q} \rightarrow 0, \omega \neq 0} \quad \omega \langle 0 | J^0 | \mathbf{k}_1, \mathbf{k}_2 \rangle = 0$$

The charge of the conserved current is the particle number density: $J^0(\mathbf{q}) = n(\mathbf{q})$

$$\langle 0 | J^0 | \mathbf{k}_1, \mathbf{k}_2 \rangle = \langle 0 | n(\mathbf{q}) | \mathbf{k}_1, \mathbf{k}_2 \rangle = \langle \mathbf{q} | \mathbf{k}_1, \mathbf{k}_2 \rangle$$

The Ward identity ensures that 3-points function vanishes if $\mathbf{q} \rightarrow 0$ *

* discussions with A. Esposito

Phonons in superfluid Helium

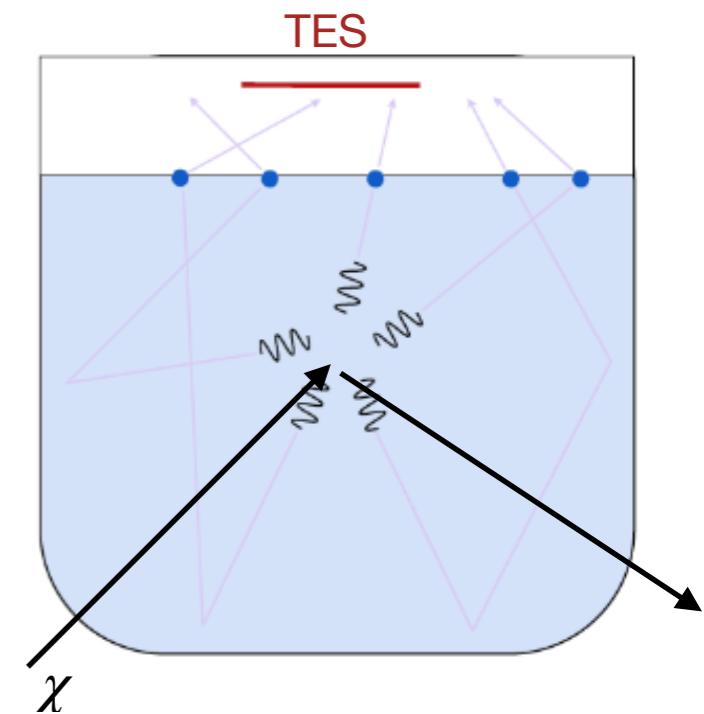
Superfluid helium allows for efficient heat collection

- Transition Edge Sensor (TES) readout
- No E-field needed: lower threshold

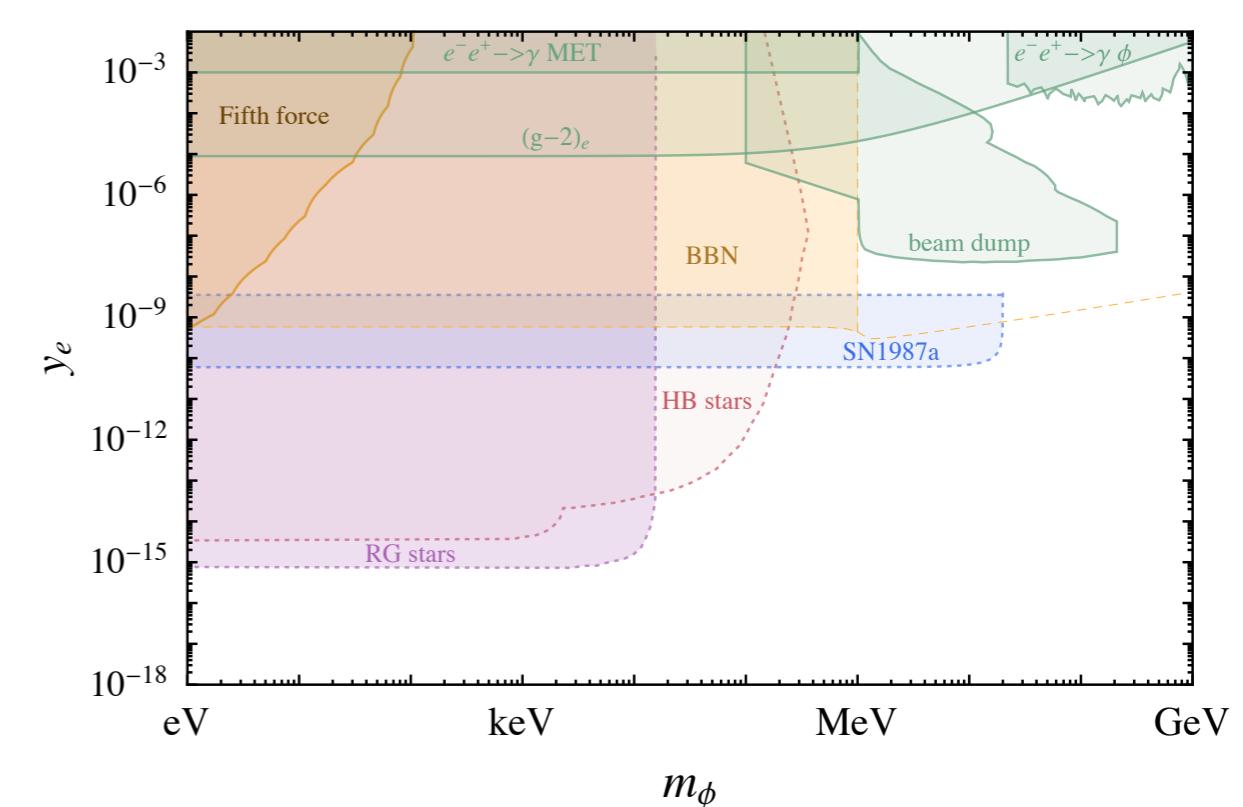
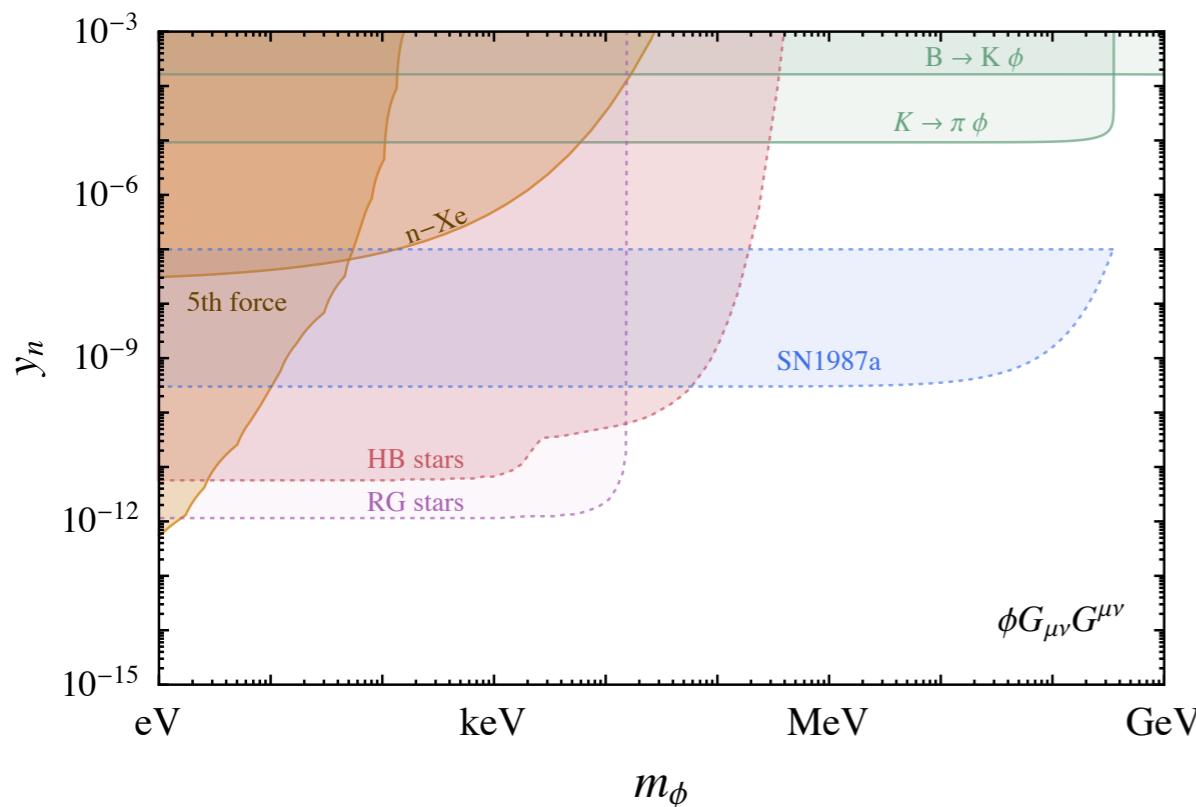
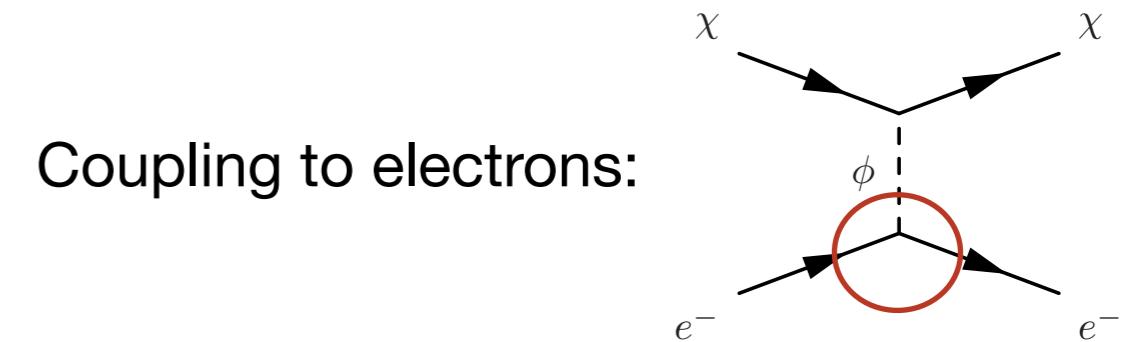
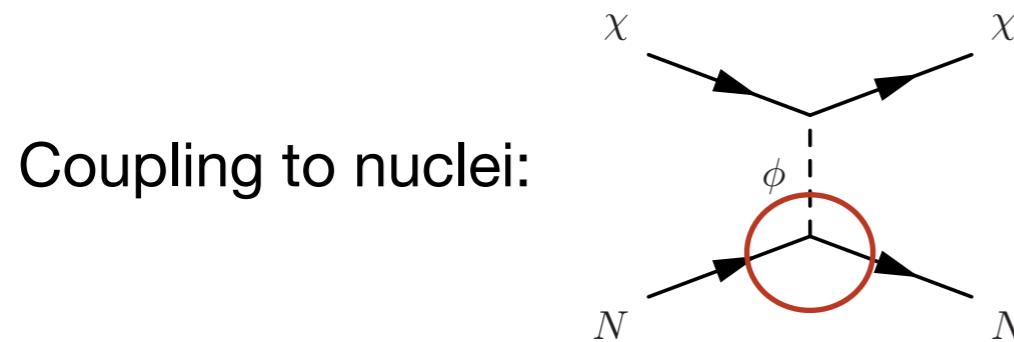
Look for phonons instead of ionization

Phonon collection

1. DM creates phonons/rotons
2. phonon/rotон bounces **ballistically**
3. phonon/rotон evaporates He atom
(gives 10-40 meV boost)
4. Helium atom is detected on Transition Edge Sensor (TES)

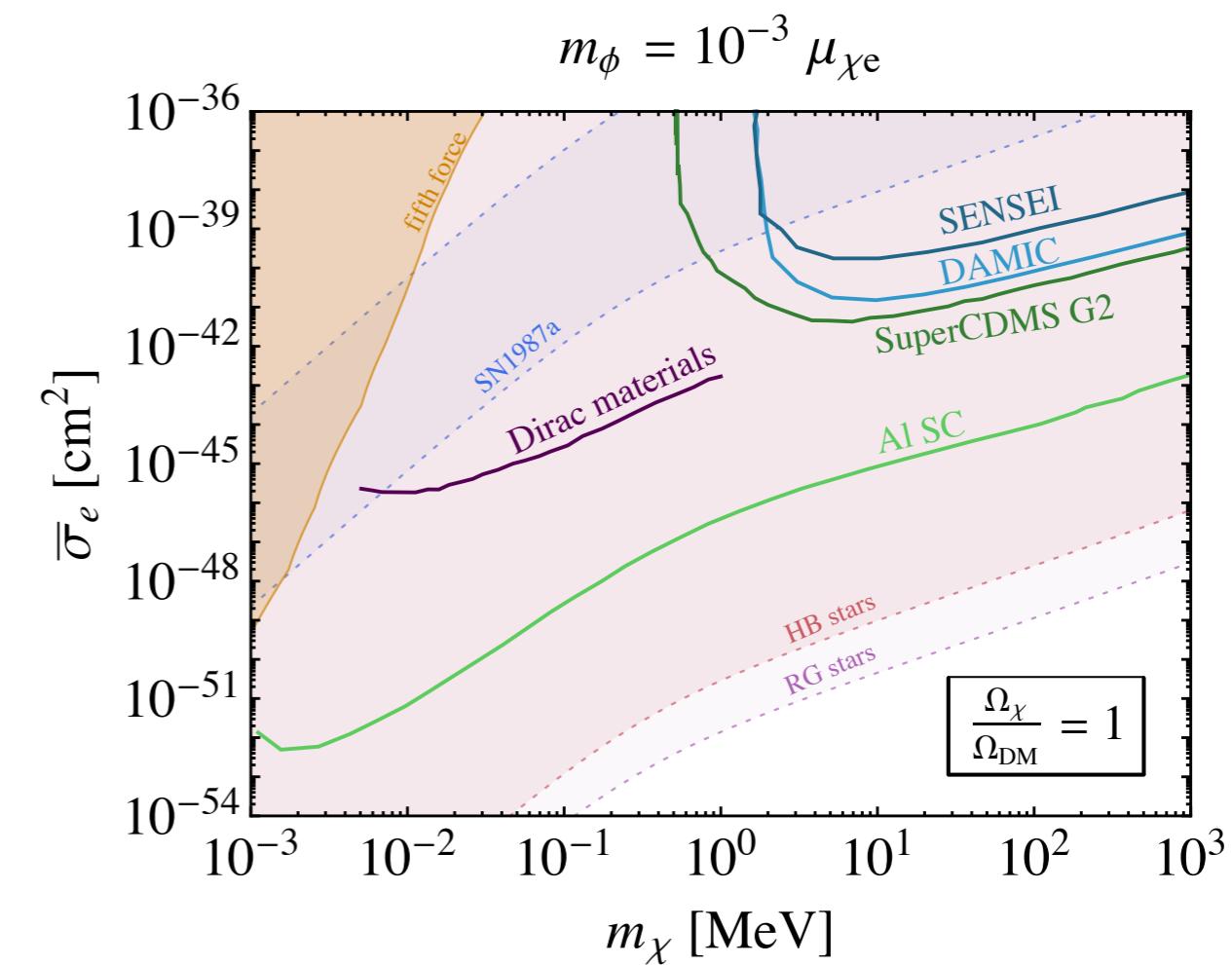
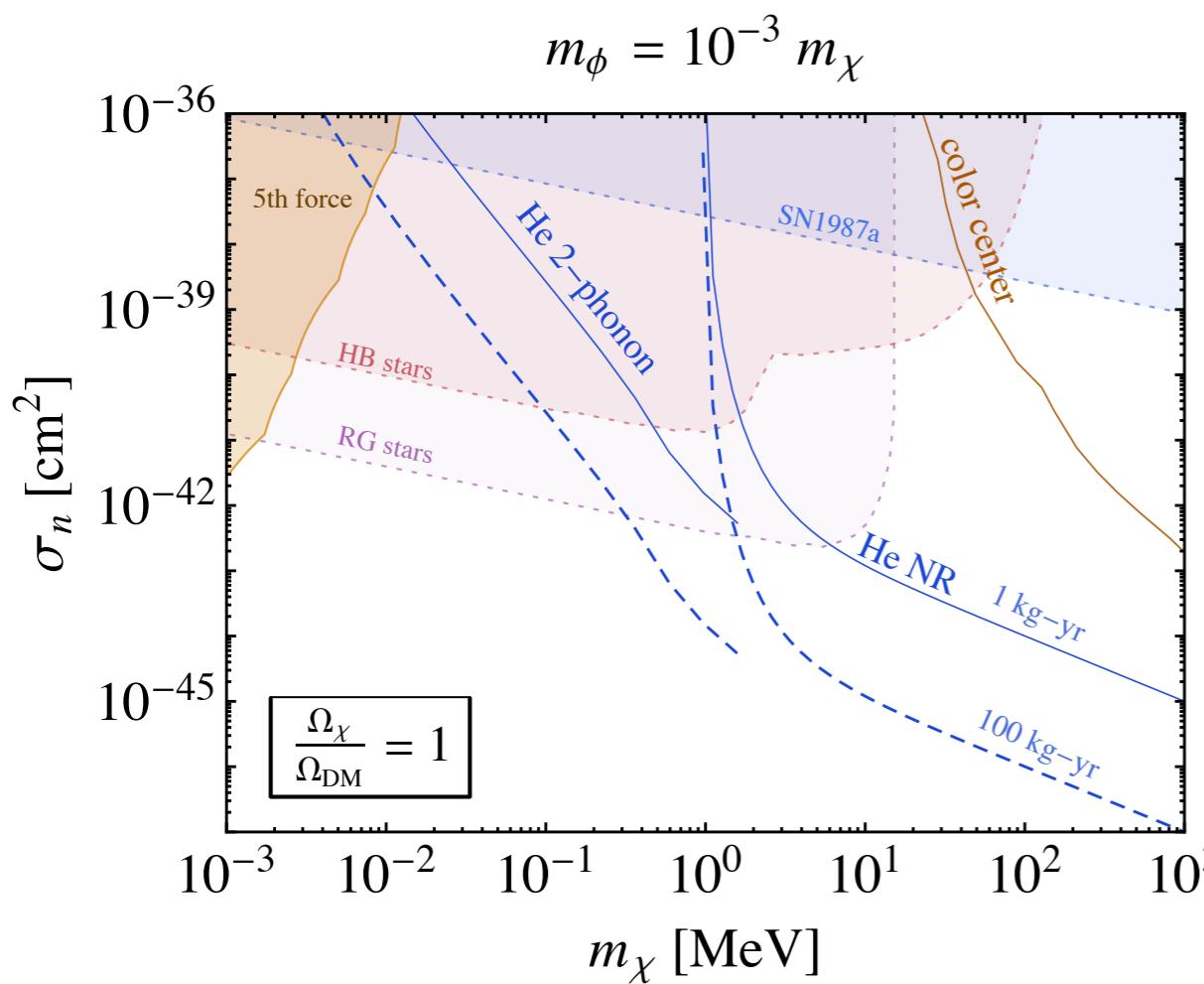
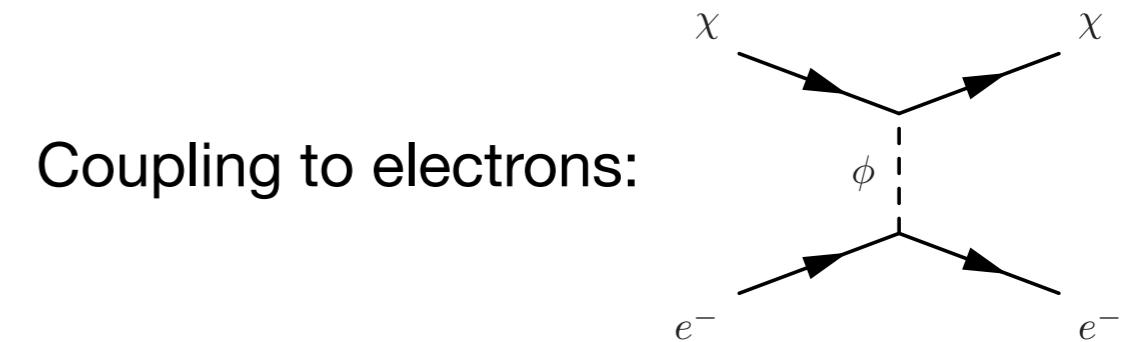
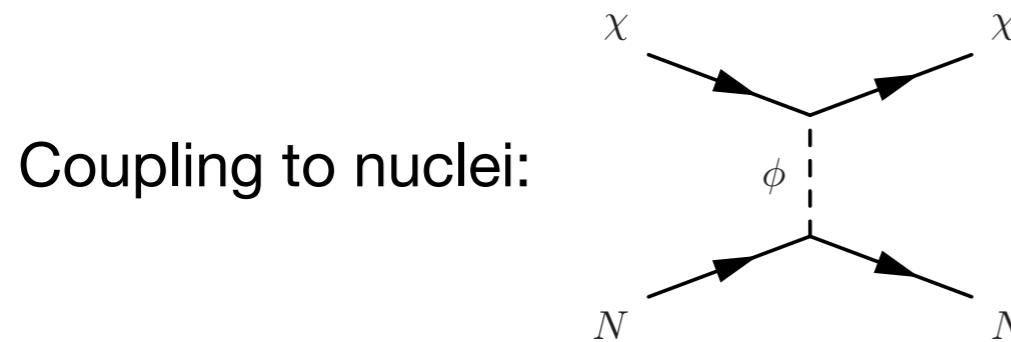


Mediators matter!



Strong astrophysical & terrestrial constraints

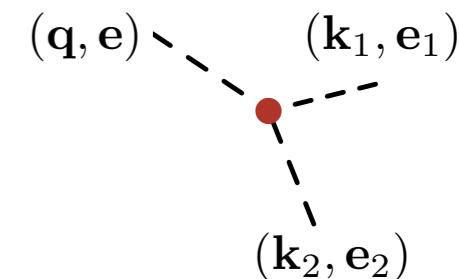
Mediators matter!



Experimentally viable if subcomponent DM

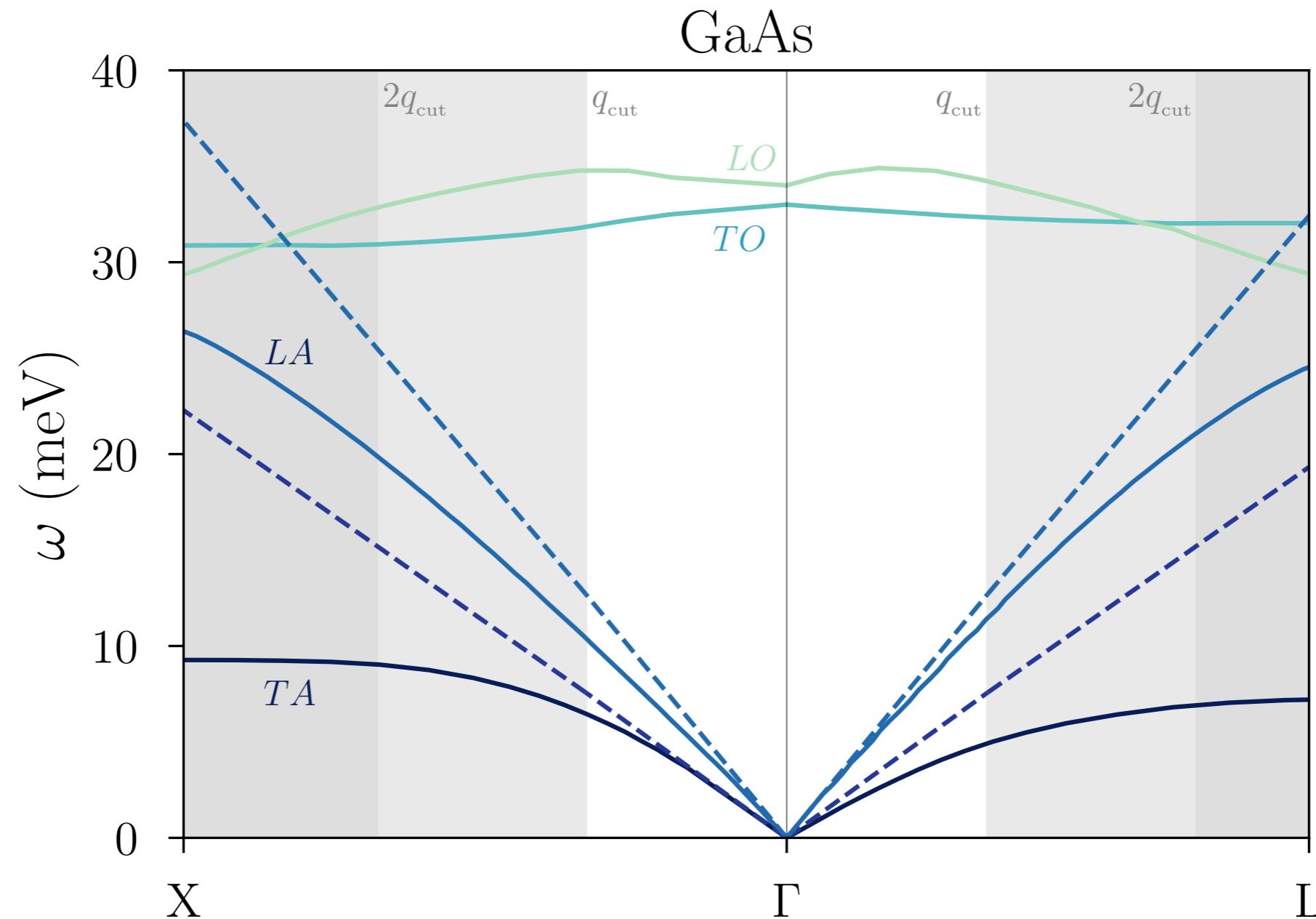


Phonon 3-points function



$$\begin{aligned}
 \widetilde{\mathcal{M}} = & (\beta + \lambda) \left[(\mathbf{q} \cdot \mathbf{e})(\mathbf{k}_1 \cdot \mathbf{k}_2)(\mathbf{e}_1 \cdot \mathbf{e}_2) + (\mathbf{k}_1 \cdot \mathbf{e}_1)(\mathbf{q} \cdot \mathbf{k}_2)(\mathbf{e} \cdot \mathbf{e}_2) + (\mathbf{k}_2 \cdot \mathbf{e}_2)(\mathbf{k}_1 \cdot \mathbf{q})(\mathbf{e}_1 \cdot \mathbf{e}) \right] \\
 & + (\gamma + \mu) \left[(\mathbf{q} \cdot \mathbf{k}_2) \left[(\mathbf{k}_2 \cdot \mathbf{e}_1)(\mathbf{e}_2 \cdot \mathbf{e}) + (\mathbf{k}_2 \cdot \mathbf{e})(\mathbf{e}_2 \cdot \mathbf{e}_1) \right] \right. \\
 & \quad + (\mathbf{k}_2 \cdot \mathbf{k}_1) \left[(\mathbf{q} \cdot \mathbf{e}_1)(\mathbf{e}_2 \cdot \mathbf{e}) + (\mathbf{q} \cdot \mathbf{e}_2)(\mathbf{e} \cdot \mathbf{e}_1) \right] \\
 & \quad \left. + (\mathbf{q} \cdot \mathbf{k}_2) \left[(\mathbf{k}_1 \cdot \mathbf{e}_2)(\mathbf{e}_1 \cdot \mathbf{e}) + (\mathbf{k}_1 \cdot \mathbf{e})(\mathbf{e}_1 \cdot \mathbf{e}_2) \right] \right] \\
 & + \alpha(\mathbf{q} \cdot \mathbf{e})(\mathbf{k}_1 \cdot \mathbf{e}_1)(\mathbf{k}_2 \cdot \mathbf{e}_2) \\
 & + \beta \left[(\mathbf{k}_1 \cdot \mathbf{e}_1)(\mathbf{q} \cdot \mathbf{e}_2)(\mathbf{k}_2 \cdot \mathbf{e}) + (\mathbf{q} \cdot \mathbf{e})(\mathbf{k}_1 \cdot \mathbf{e}_2)(\mathbf{k}_2 \cdot \mathbf{e}_1) + (\mathbf{k}_2 \cdot \mathbf{e}_2)(\mathbf{q} \cdot \mathbf{e}_1)(\mathbf{k}_1 \cdot \mathbf{e}) \right] \\
 & + \gamma \left[(\mathbf{q} \cdot \mathbf{e}_1)(\mathbf{k}_1 \cdot \mathbf{e}_2)(\mathbf{k}_2 \cdot \mathbf{e}) + (\mathbf{q} \cdot \mathbf{e}_1)(\mathbf{k}_1 \cdot \mathbf{e})(\mathbf{k}_2 \cdot \mathbf{e}_1) \right]
 \end{aligned}$$

EFT cut off

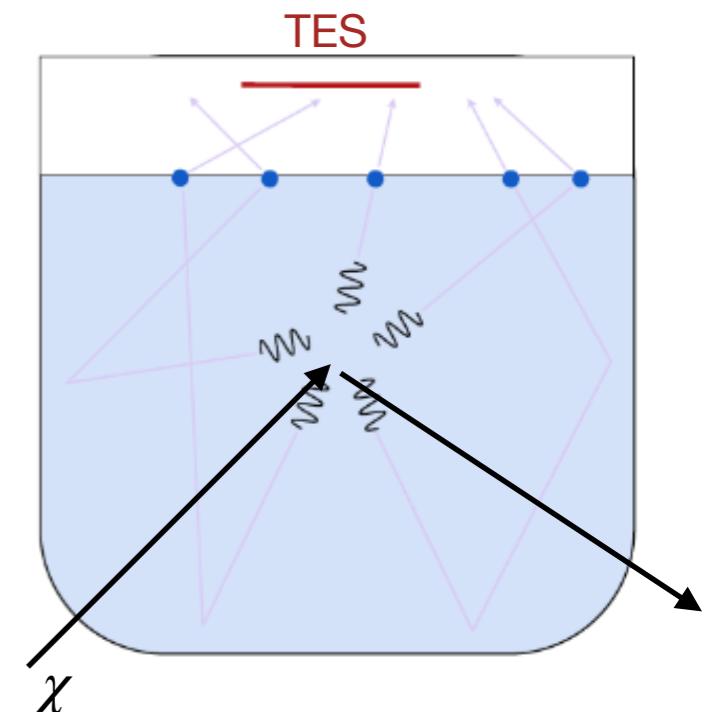


Phonons in superfluid Helium

Superfluid helium allows for efficient heat collection

- Transition Edge Sensor (TES) readout
- No E-field needed: lower threshold

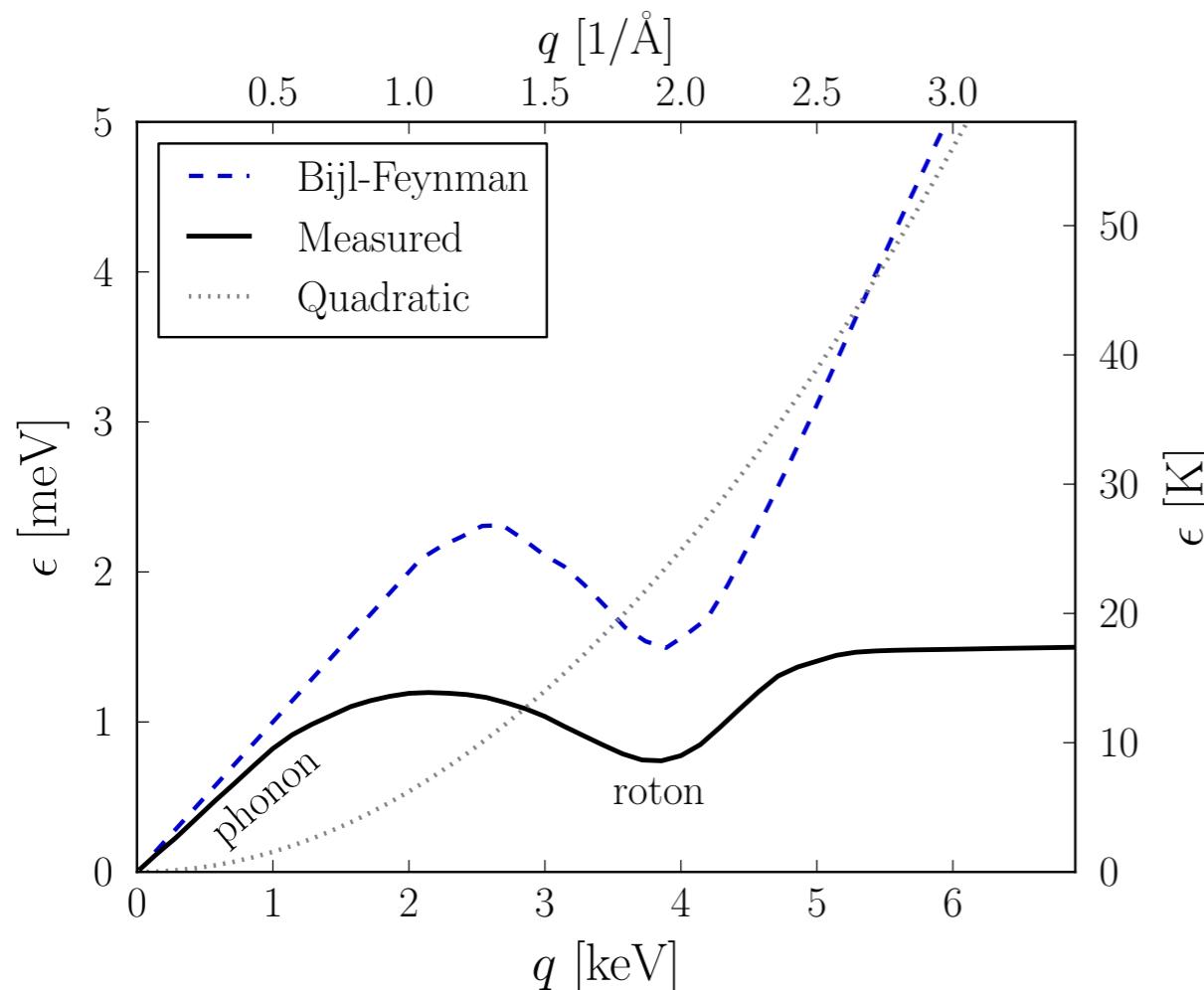
Look for phonons instead of ionization



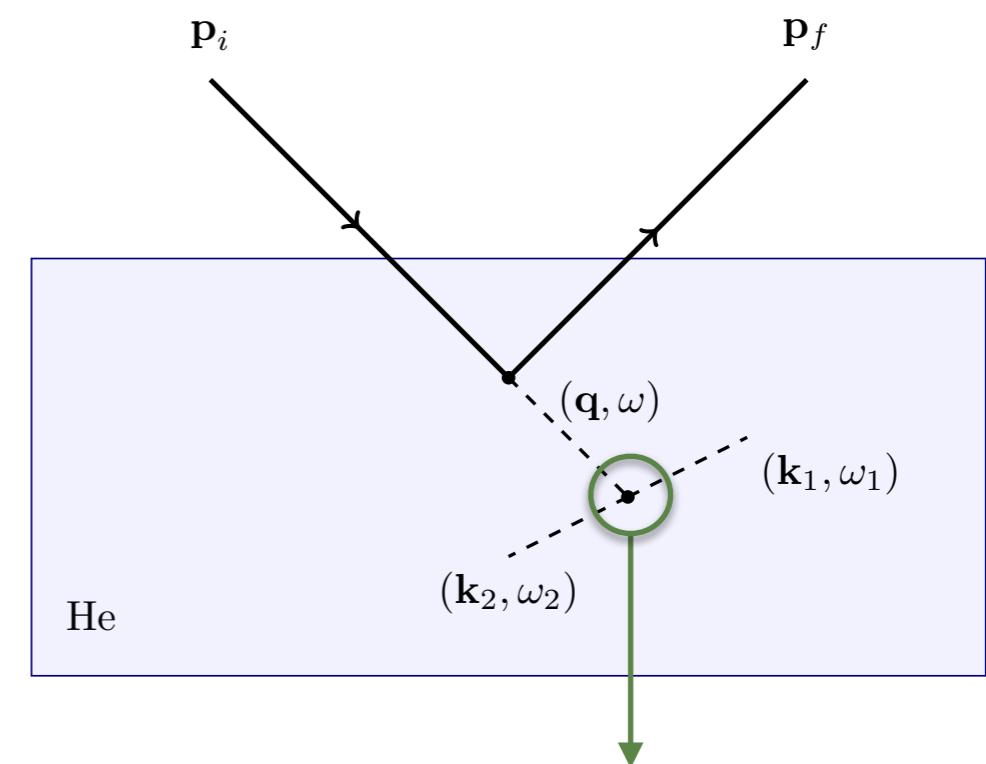
Phonon collection

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4. Helium atom is detected on Transition Edge Sensor (TES)

Phonons & rotons in superfluid He



Final state: two hard, back-to-back phonons



Issue: speed of Dark Matter \gg speed of sound



Cannot scatter against single, on shell excitation

Calculate the 3-excitation matrix element

R. Feynman, 1954
H. W. Jackson, E. Feenberg, 1962
E. Feenberg, 1969
M. J. Stephen, 1969

Basis of states

$$|\mathbf{q}\rangle^0 \equiv \frac{1}{\sqrt{n_0 S(\mathbf{q})}} n_{\mathbf{q}} |\Psi_0\rangle$$

$$|\mathbf{q}_1, \mathbf{q}_2\rangle^0 \equiv \frac{1}{\sqrt{n_0 S(\mathbf{q}_1)}} \frac{1}{\sqrt{n_0 S(\mathbf{q}_2)}} n_{\mathbf{q}_1} n_{\mathbf{q}_2} |\Psi_0\rangle$$

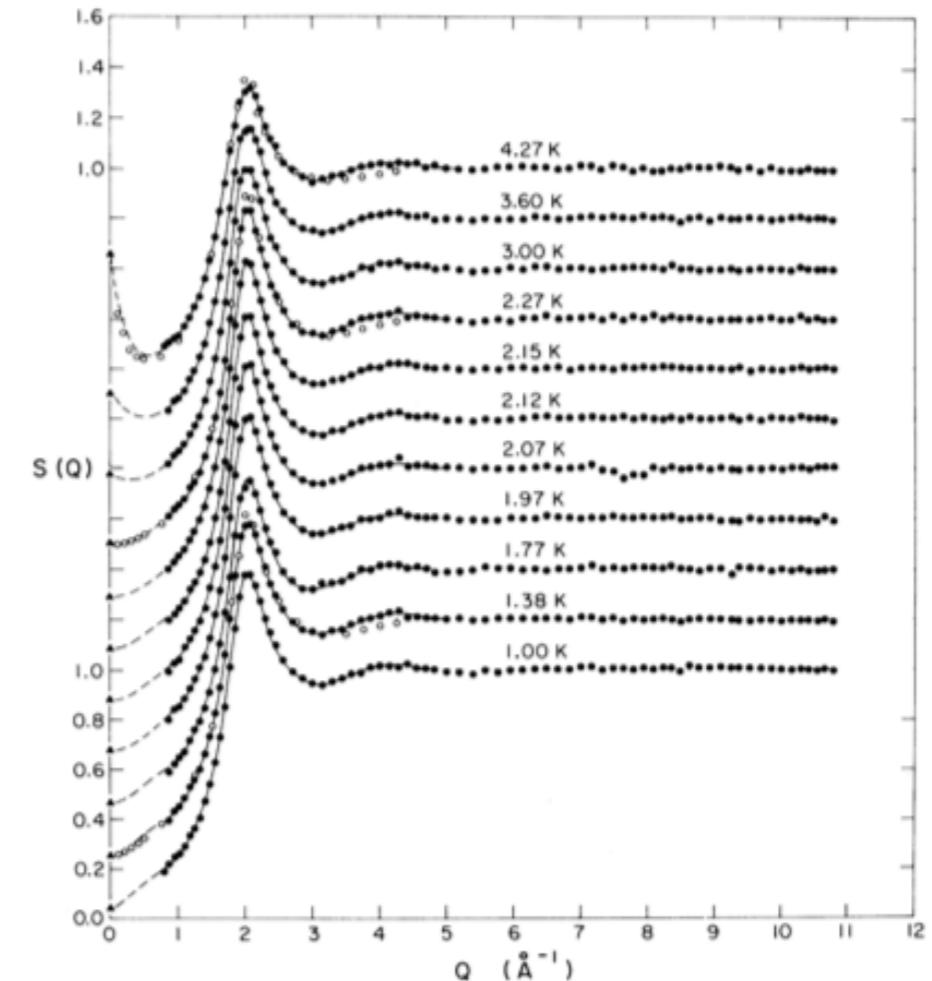
with $n_{\mathbf{q}} \equiv \frac{1}{\sqrt{V}} \sum_{i=1}^N \exp(i\mathbf{q} \cdot \mathbf{r}_i)$

Static structure function

$$S(\mathbf{q}) \equiv \frac{1}{n_0} \langle \Psi_0 | n_{-\mathbf{q}} n_{\mathbf{q}} | \Psi_0 \rangle$$

Problems

- Ground state extremely complicated
- $|\mathbf{q}\rangle^0$ and $|\mathbf{q}_1, \mathbf{q}_2\rangle^0$ are not orthogonal



E. C. Svensson, et. al. (1980)

SK, T. Lin, K. Zurek: 1611.06228

Calculation

Step 1: Gram-Schmidt orthogonalization

$$|\mathbf{q}\rangle \equiv |\mathbf{q}\rangle^0$$

$$|\mathbf{q}_1, \mathbf{q}_2\rangle \equiv |\mathbf{q}_1, \mathbf{q}_2\rangle^0 - \sum_{\mathbf{q}'} \langle \mathbf{q}' | \mathbf{q}_1, \mathbf{q}_2 \rangle^0 |\mathbf{q}'\rangle$$



Fix overlap term with ansatz

Step 2: Specify Hamiltonian

Quantum hydrodynamics

or

Microscopic formulation

$$H = \int d^3\mathbf{r} \left(\frac{1}{2} m_{\text{He}} \mathbf{v} \cdot n \mathbf{v} + \mathcal{V}(n) \right)$$

(+ continuity equation)

$$H = \sum_i \left(-\frac{\nabla_i^2}{2m_{\text{He}}} \right) + \mathcal{V}(\{\mathbf{r}_i\})$$

Step 3: Compute the matrix element

All dependence on the unknown ground state and potential ends up in overlap term

Step 4: Validate against modern simulation data

A Modern Simulation

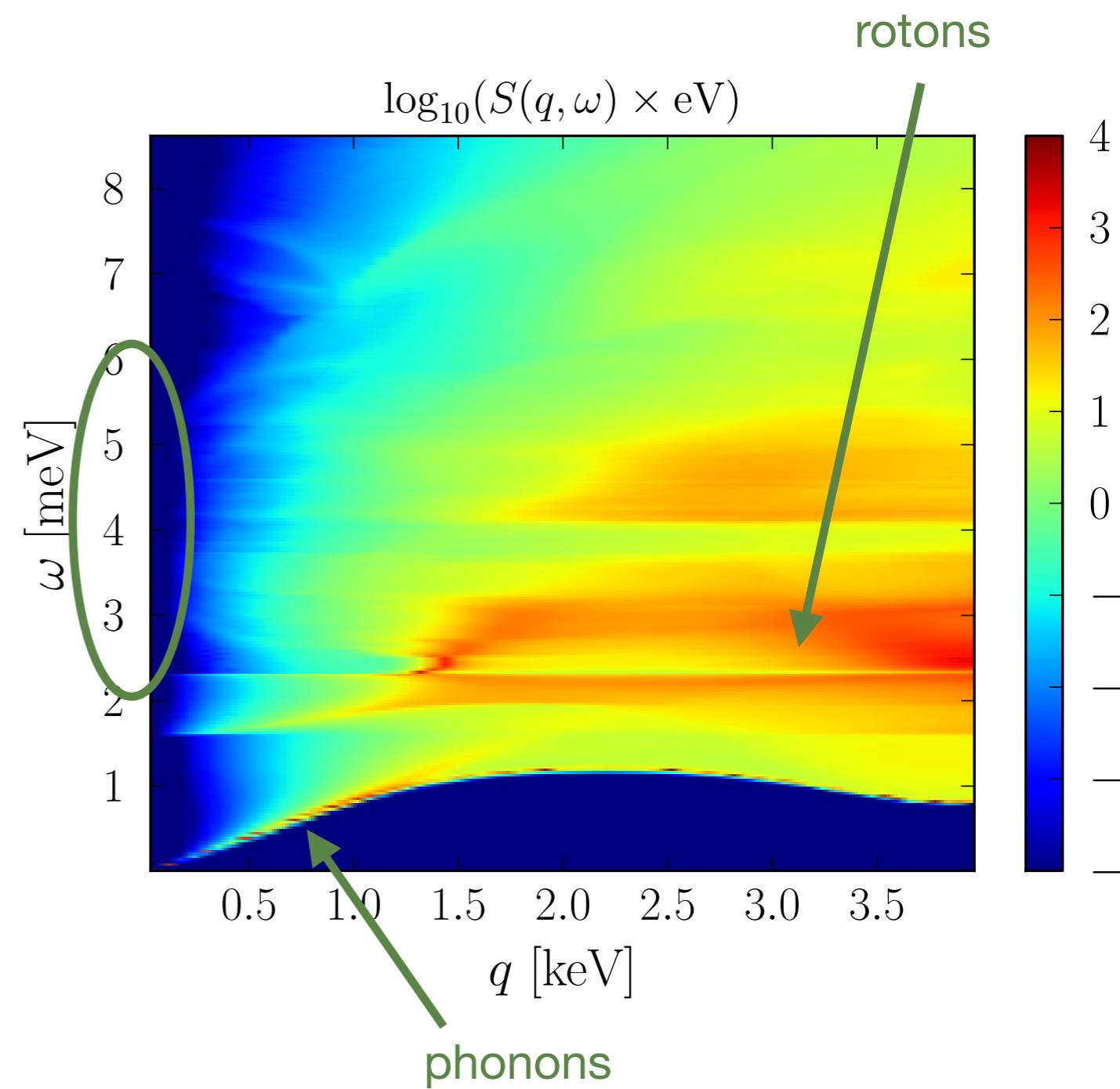
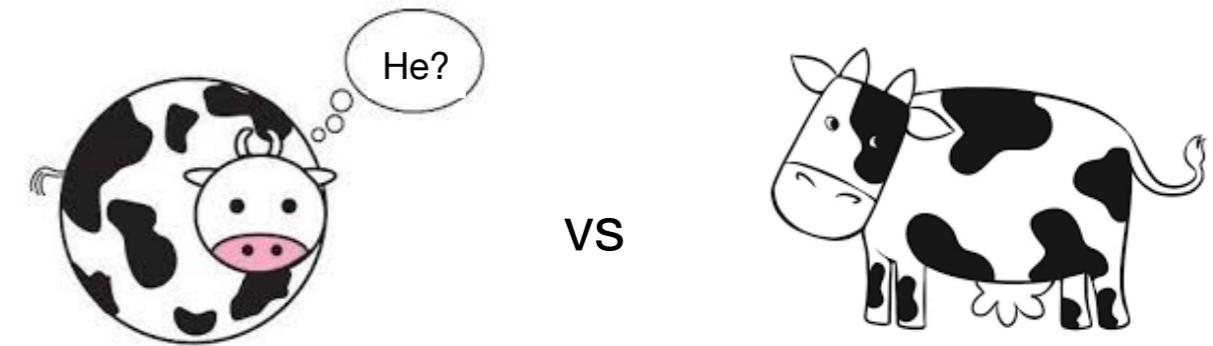
Combination of standard perturbation theory & dynamic multiparticle fluctuations theory

- More sophisticated ansatz for the potential
- Resummed self-energies

No resolution for low momentum transfer

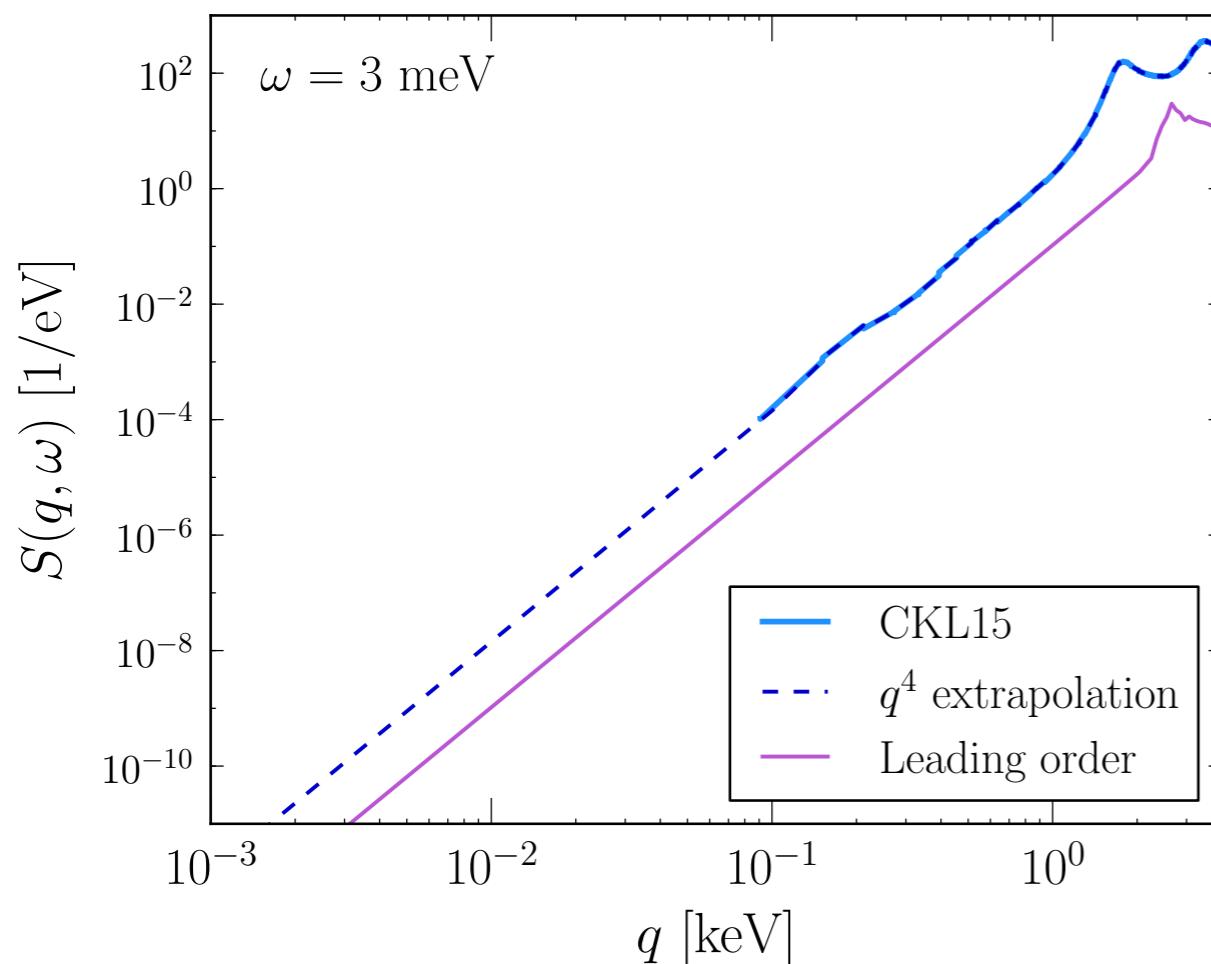


Use our analytic expressions to extrapolate

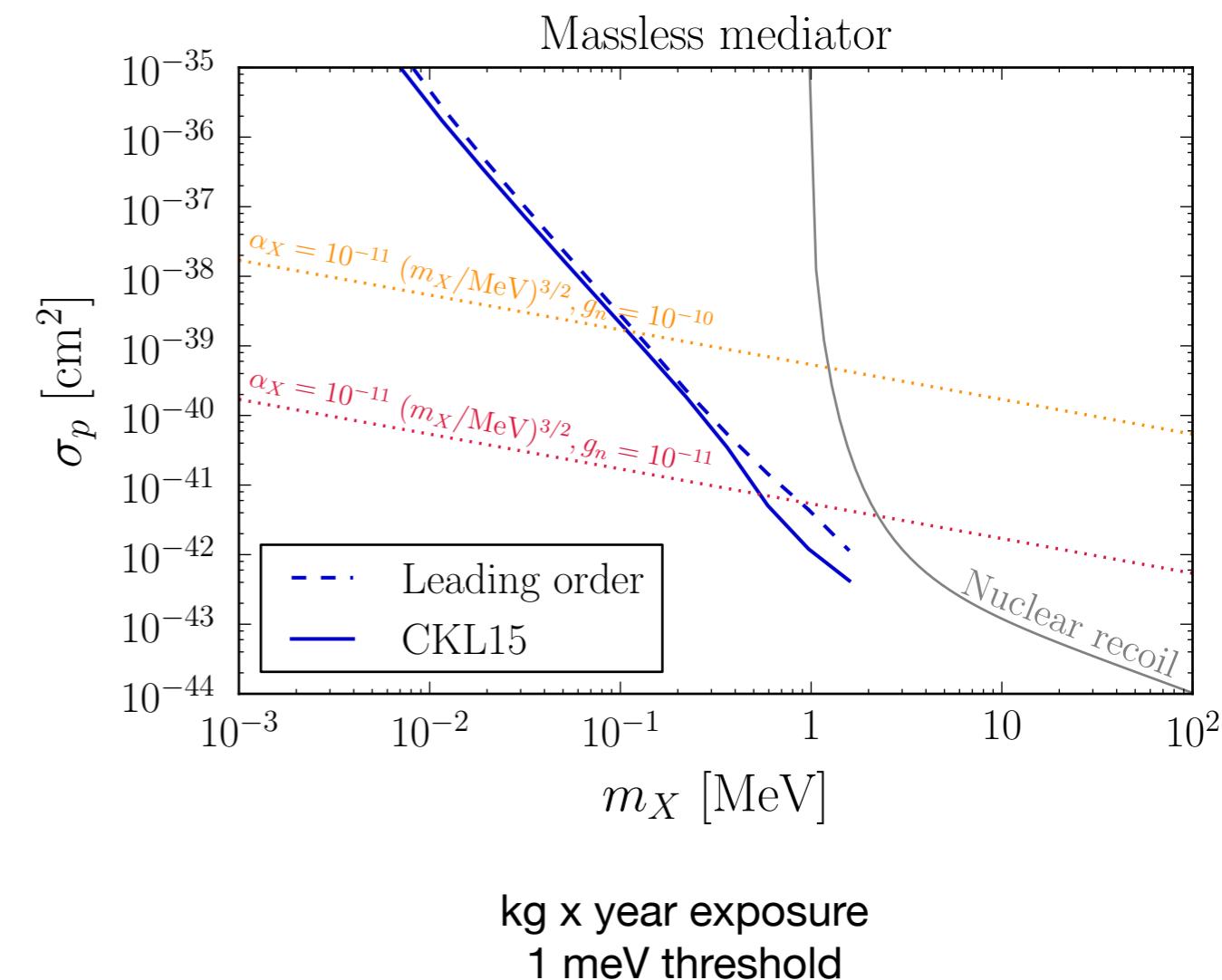


Comparison with simulation

q^4 scaling is reproduced in the simulation data*



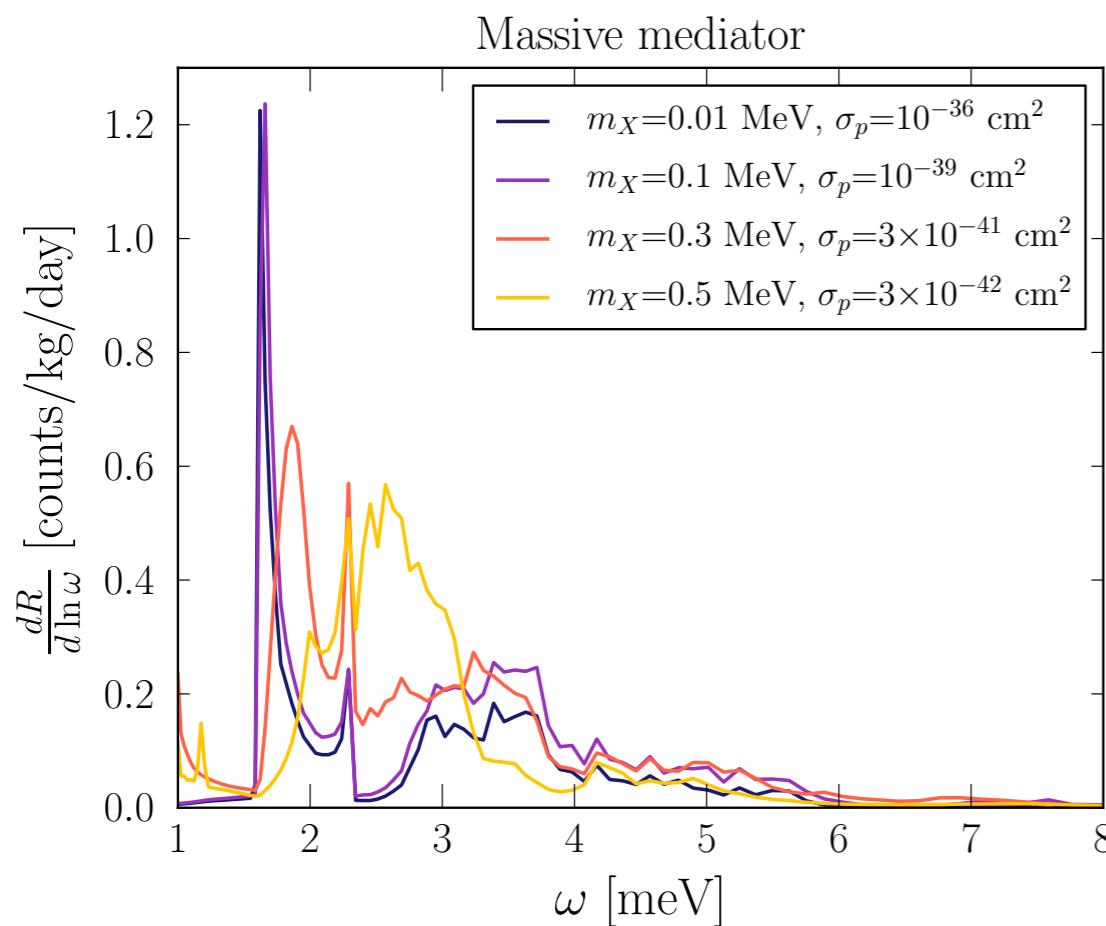
Reach agrees within a factor of ~ 2



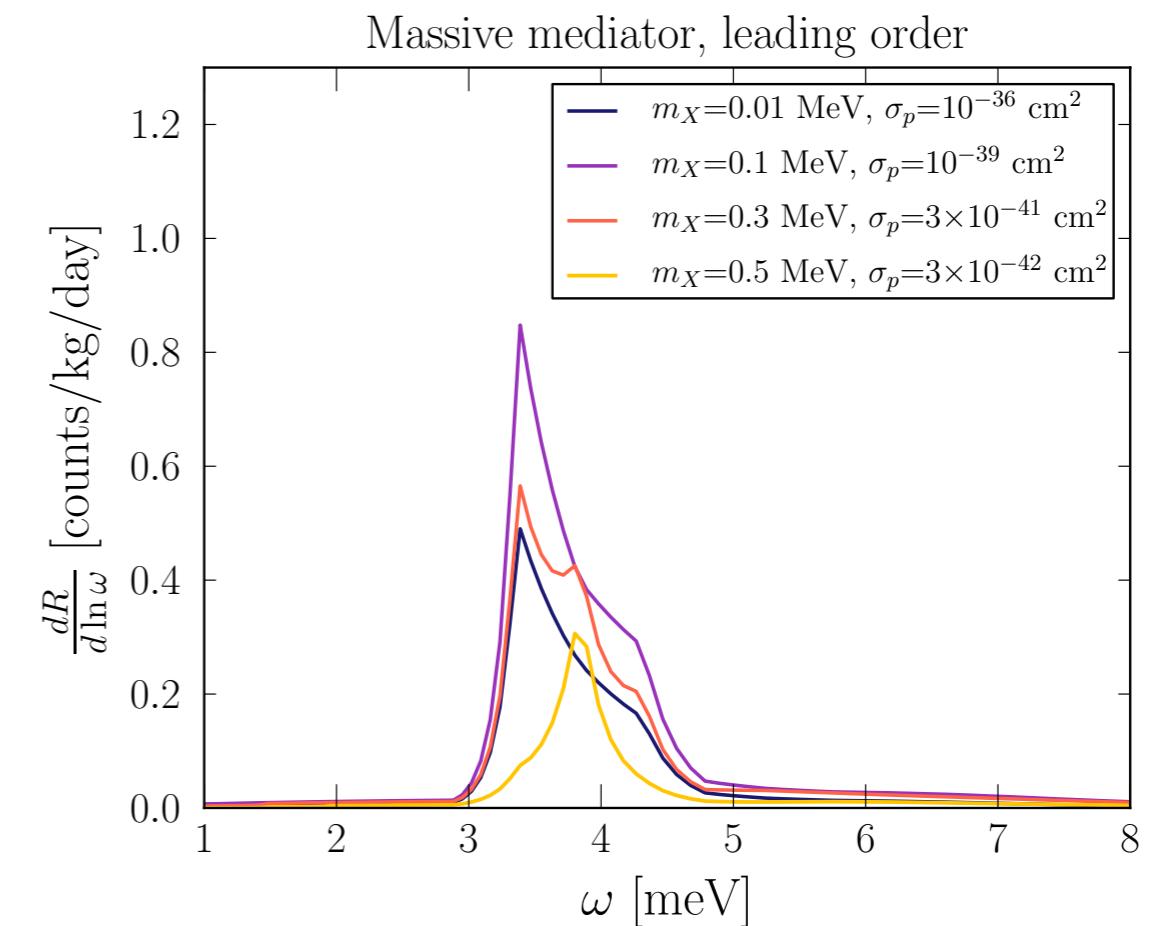
* Campbell, Krotscheck and Lichtenegger (2015)

Spectrum

Simulation data



Our computation



Only qualitative agreement on the differential rate, but integrated rate roughly right