

Ultralight dark sectors in experiment



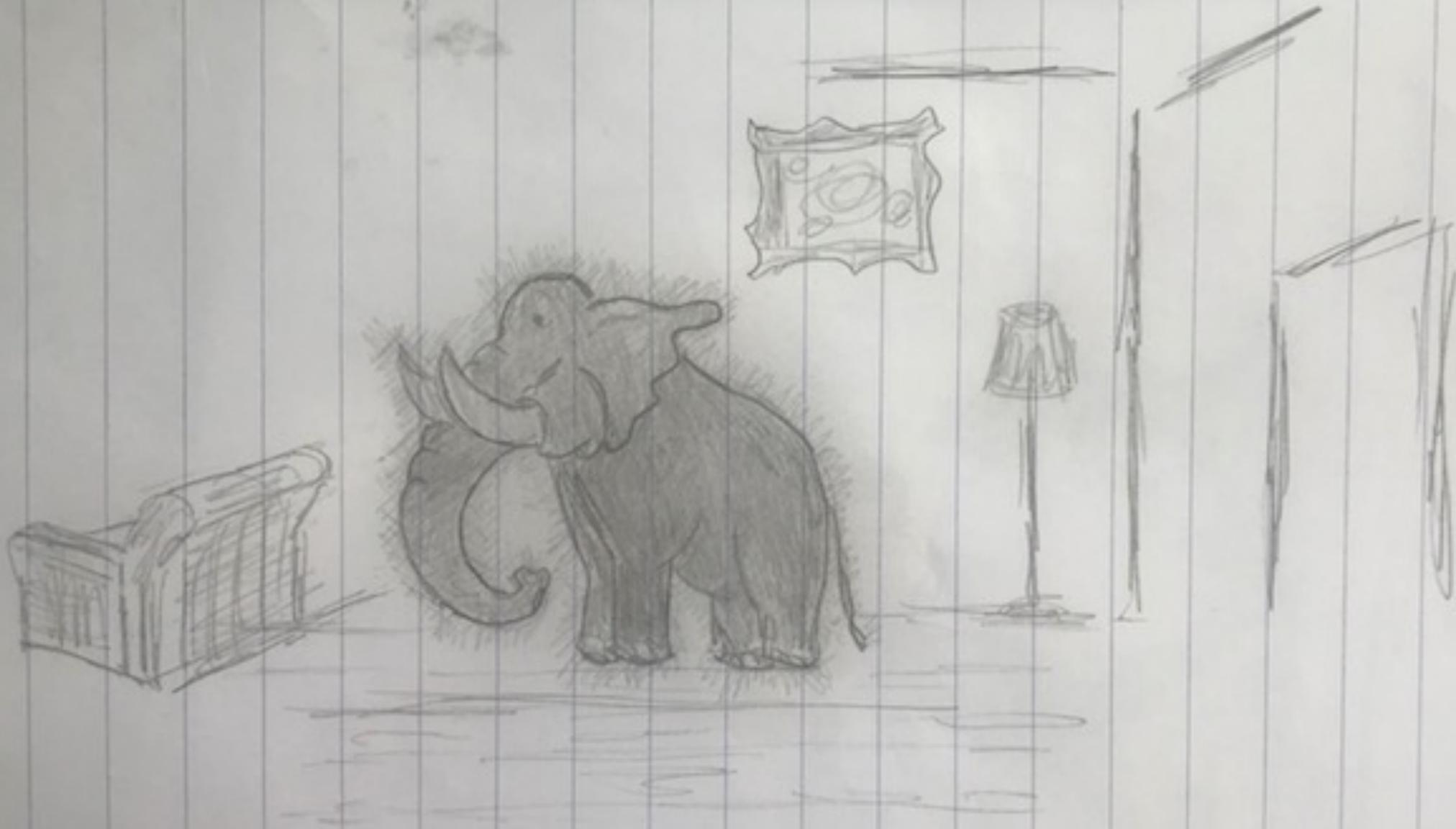
Axel Lindner
Bela Majorowits

Ariel Zhitnitsky

Marie Curie
Pierre Sikivie

Dmitry Budker

Helmholtz Institute, JGU Mainz & Department of Physics, UC Berkeley
ISAPP ESI, Vienna, July 2021



Vasiliki Demas

DARK MATTER "THE ELEPHANT IN THE ROOM"

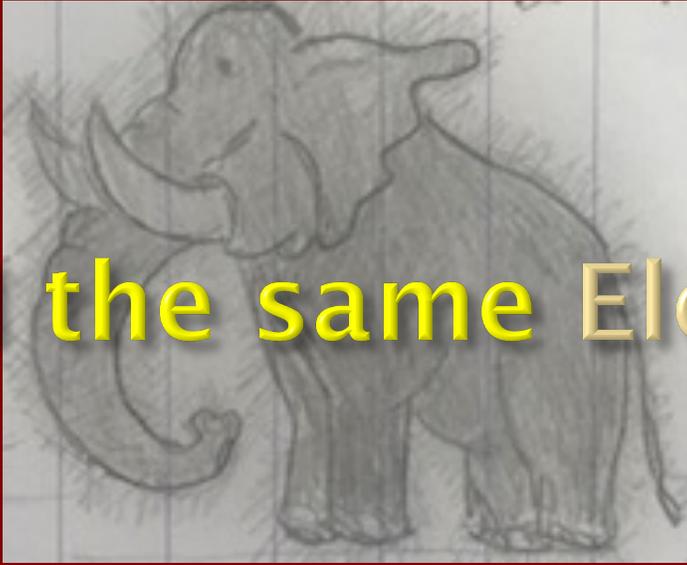
More Elephants!



Matter-antimatter Asymmetry

Similar amount of matter and DM

Dark Energy



One and the same Elephant ?

Strong-CP problem

Hierarchy problem

So what is DM or what mimics it ?

- ▣ A gross misunderstanding of gravity (MOND, ...) ???
- ▣ Proca MHD (finite photon mass) ☹️?
- ▣ Black holes, dark planets, interstellar gas, ... ☹️?
- ▣ WIMPS 😊
- ▣ Ultralight bosonic particles
 - Axions (pseudoscalar) 😊
 - ALPs (pseudoscalar) 😊
 - Dilatons (scalar) 😊
 - Vector particles 😊
 - Tensor particles ???
- ▣ Antiquark Nuggets (AQN) !!!😊!!!

“Most Wanted” file on DM

What do we know?

- ▣ Galactic DM density: $\sim 0.4 \text{ GeV/cm}^3$ (10 GeV/cm³ d.g.)
- ▣ Has to be nonrelativistic: $v/c \sim 10^{-3}$ (cold DM)
- ▣ Has to be **bosonic** if $m < \sim 20 \text{ eV}$ (1 keV dwarf galaxies)
- ▣ “Bosonic Oscillator” with $Q \sim (v/c)^{-2} \sim 10^6$
- ▣ Cannot be lighter than $\sim 10^{-22} \text{ eV}$
- ▣ ... (e.g., BEC ?)

Ultralight Bosonic DM

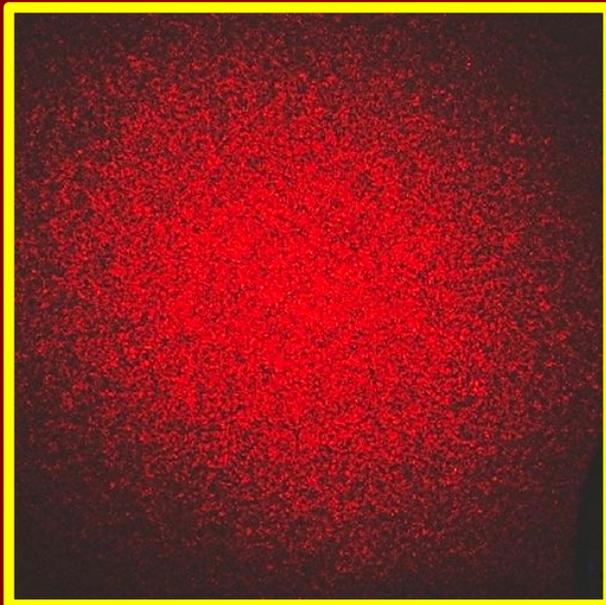
Density

(a) The number density of the dark-matter particles, under the assumption that all dark matter consists of particles of the same mass m is

$$n_{dm} \approx \frac{0.4 \text{ GeV/cm}^3}{mc^2}. \quad (3.13)$$

The 1 MHz frequency corresponds to about $4 \cdot 10^{-9}$ eV, so the density of such particles should be $\approx 10^{17} \text{ cm}^{-3}$. One cubic centimeter of air contains about $3 \cdot 10^{19}$ molecules, the majority of which are nitrogen with 28 nucleons each, which comes to about 10^{21} nucleons per cm^3 , several orders of magnitude more than the above estimate for 1 MHz dark-matter particles.

Spatial pattern = speckle



Coherence time and length

(b) The total energy of a nonrelativistic dark-matter particle is dominated by the rest energy mc^2 with an additional correction on the order of mv^2 , which comes to about 10^{-6} of the rest energy for $v \approx 10^{-3}c$. This means that the de Broglie waves dephase during roughly 10^6 periods of the oscillation whose frequency corresponds to the energy of mc^2 , so that the *coherence time* is

$$\tau_c \approx 10^6 \cdot \left(\frac{mc^2}{2\pi\hbar} \right)^{-1}. \quad (3.14)$$

For 1 MHz dark-matter particles, this comes to $\tau_c \approx 1 \text{ s}$.

Coherence length L_c can be estimated as a product of τ_c and the particle velocity v (we invite the reader to derive this result using the concepts of *phase velocity* and *group velocity* of the de Broglie waves), so that

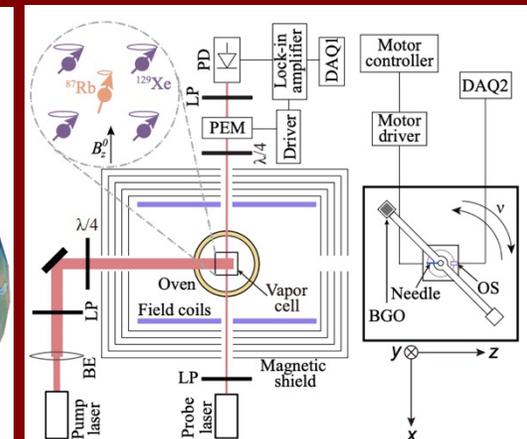
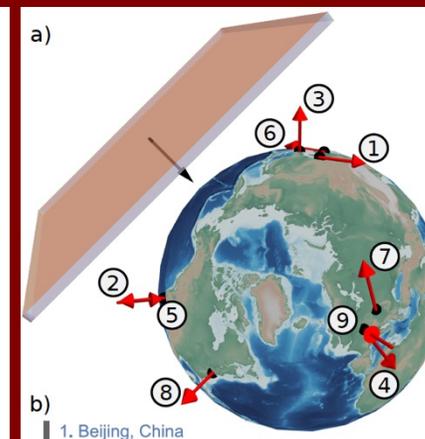
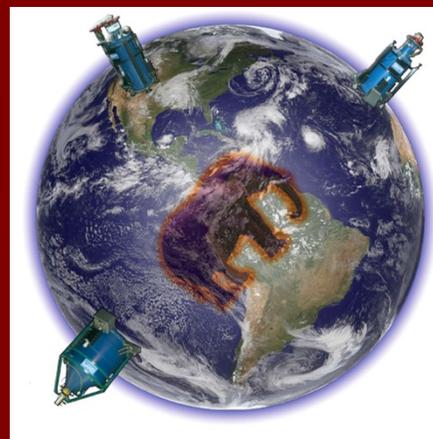
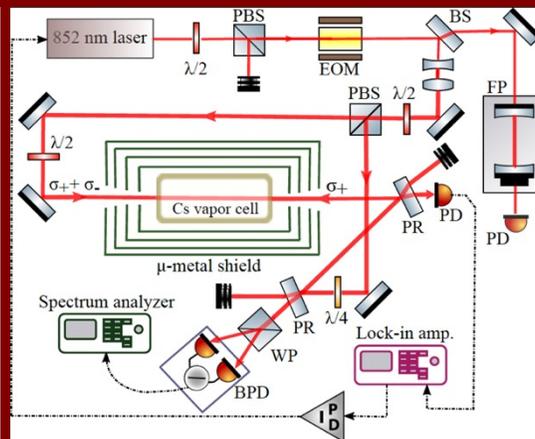
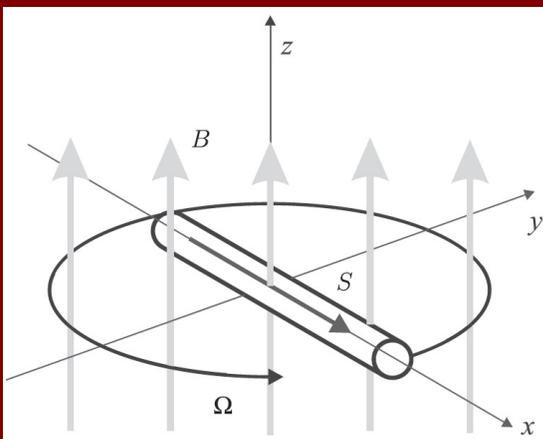
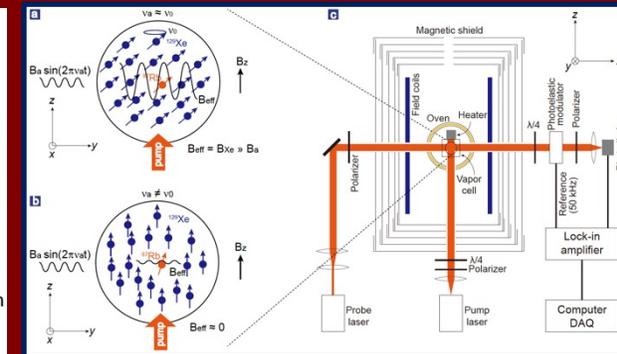
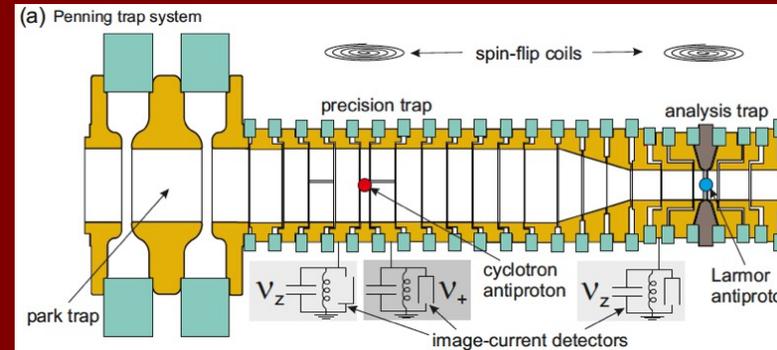
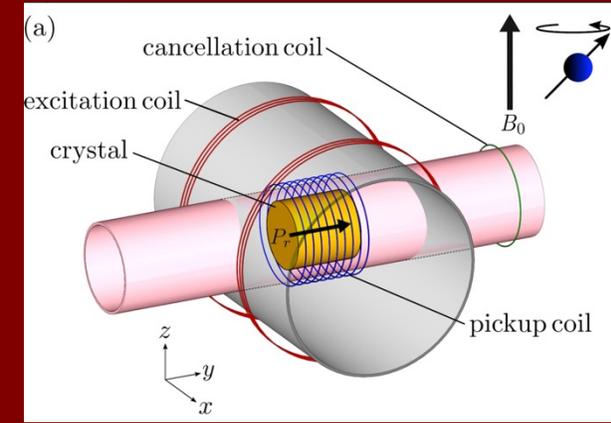
$$L_c \approx 10^3 \cdot \left(\frac{mc^2}{2\pi\hbar c} \right)^{-1}. \quad (3.15)$$

For 1 MHz dark-matter particles, this comes to $L_c \approx 300 \text{ km}$.

From: D. Budker and A. Sushkov *Physics on Your Feet*, 2nd Edition, OUP (forthcoming)

Searching for **Ultralight Bosonic** (and other) **DM**

- **NMR** (CASPEr)
- Spin-based sensors for **DM**: **masers**, **spin amplifiers**
- Spin-based sensors for fifth-force searches (single **NV**, cells)
- **GNOME**, clock networks, hybrid networks
- **Gravimeters**
- **Atomic spectroscopy**
- **Antimatter**
- **Levitated magnets**



Network searches for topological **DM**



The GNOME Experiment

Collaboration website

Global Network of Optical Magnetometers for Exotic searches

Current date: 2017/09/28 21:54:36 GPS

[Show Map Legend](#)

Idea and proof-of-concept:
Annalen der Physik **525**(8-9), 659–70 (2013);
[Phys. Rev. Lett.](#) **110**, 021803 (2013)
System description: [Physics of the Dark Universe](#) **22**, 162-180 (2018), [arXiv:1807.09391](#)



- Network of shielded, GPS-synchronized magnetometers + clocks, interferometers,...
- Sensitive to **topological Dark Matter**: domain walls, axion (ALP) stars, [Phys. Rev. D](#) (2018)
- Multi-messenger astronomy (e.g., look for ALPs from sources of gravitational waves)
- Sensor-correlation techniques resembling those of LIGO/Virgo
- Status: Science Run 2 complete, results to be announced; Run 3: ongoing

GNOME NETWORK 2017



Animation: Arne Wickenbrock

Analysis method for detecting topological defect dark matter with a global magnetometer network

[Physics of the Dark Universe](#) **28**, May 2020, 100494; [arXiv:1912.08727](#)

Hector Masia-Roig^{a,*}, Joseph A. Smiga^{a,*}, Dmitry Budker^{a,b,c}, Vincent Dumont^b, Zoran Grujic^d, Dongok Kim^{e,f}, Derek F. Jackson Kimball^g, Victor Lebedev^d, Madeline Monroy^g, Szymon Pustelny^h, Theo Scholtes^{d,i,1}, Perrin C. Segura^j, Yannis K. Semertzidis^{e,f}, Yun Chang Shin^e, Jason E. Stalnaker^j, Ibrahim Sulai^k, Antoine Weis^d, Arne Wickenbrock^a

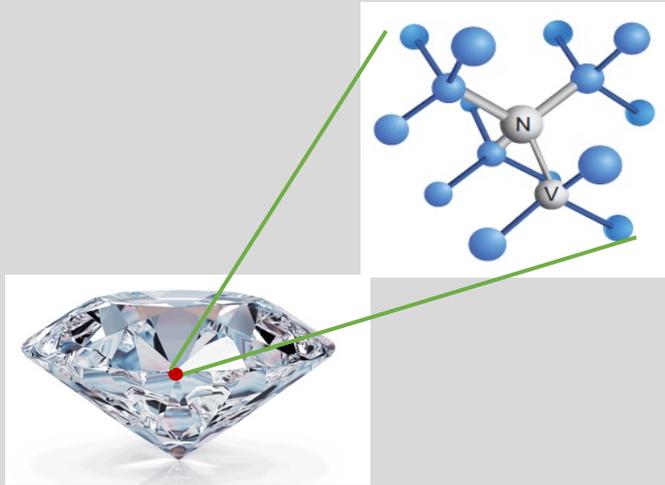
Search for topological defect dark matter using the global network of optical magnetometers for exotic physics searches (GNOME)

[arXiv:2102.13379](#)

Samer Afach,^{1,2} Ben C. Buchler,³ Dmitry Budker,^{1,2,4} Conner Dailey,^{5,*} Andrei Derevianko,⁵ Vincent Dumont,⁶ Nataniel L. Figueroa,^{1,2} Ilja Gerhardt,⁷ Zoran D. Grujić,^{8,9} Hong Guo,¹⁰ Chuanpeng Hao,¹¹ Paul S. Hamilton,¹² Morgan Hedges,³ Derek F. Jackson Kimball,¹³ Dongok Kim,^{14,15} Sami Khamis,¹² Thomas Kornack,¹⁶ Victor Lebedev,⁹ Zheng-Tian Lu,¹⁷ Hector Masia-Roig,^{1,2,†} Madeline Monroy,^{4,13} Mikhail Padniuk,¹⁸ Christopher A. Palm,¹³ Sun Yool Park,^{19,‡} Karun V. Paul,³ Alexander Penafior,¹³ Xiang Peng,¹⁰ Maxim Pospelov,^{20,21} Rayshaun Preston,¹³ Szymon Pustelny,¹⁸ Theo Scholtes,^{9,22} Perrin C. Segura,^{19,§} Yannis K. Semertzidis,^{14,15} Dong Sheng,¹¹ Yun Chang Shin,¹⁴ Joseph A. Smiga,^{1,2,¶} Jason E. Stalnaker,¹⁹ Ibrahim Sulai,²³ Dhruv Tandon,¹⁹ Tao Wang,²⁴ Antoine Weis,⁹ Arne Wickenbrock,^{1,2} Tatum Wilson,¹³ Teng Wu,¹⁰ David Wurm,²⁵ Wei Xiao,¹⁰ Yucheng Yang,¹⁰ Dongrui Yu,¹⁰ and Jianwei Zhang¹⁰

THE SMALLEST “TABLETOP”
ATOMIC SCALE

Utilizing single-spin sensor to search for exotic interactions



NV centers in diamond: single-spin sensors

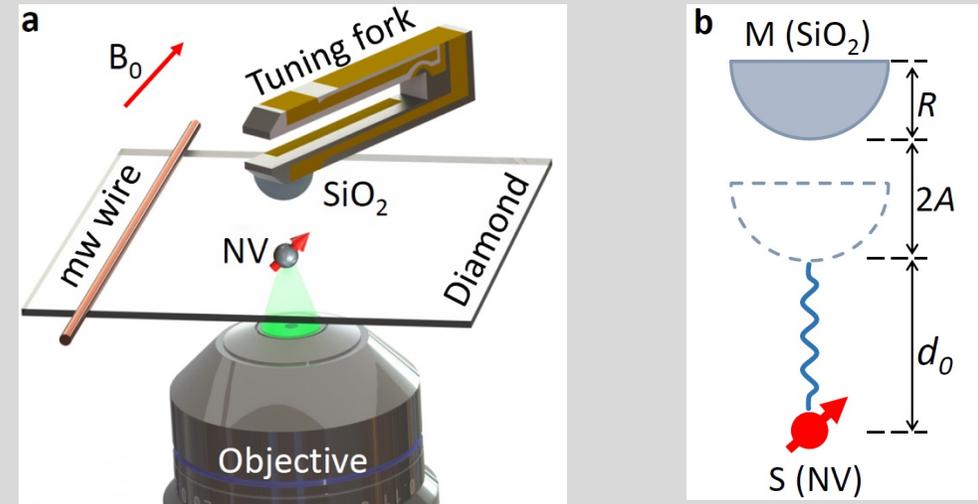


Diagram of the setup: NV sensor + AFM

Features

- ✓ Atomic scale
- ✓ Near surface
- ✓ Precise quantum control
- ✓ NV + AFM

Shorter force range

Good sensitivity

Cancel unwanted signals

The latest catalog of *EXOTIC* potentials

PHYSICAL REVIEW A **99**, 022113 (2019)

Revisiting spin-dependent forces mediated by new bosons: Potentials in the coordinate-space representation for macroscopic- and atomic-scale experiments

Pavel Fadeev,¹ Yevgeny V. Stadnik,¹ Filip Ficek,² Mikhail G. Kozlov,^{3,4} Victor V. Flambaum,^{1,5} and Dmitry Budker^{1,6,7}



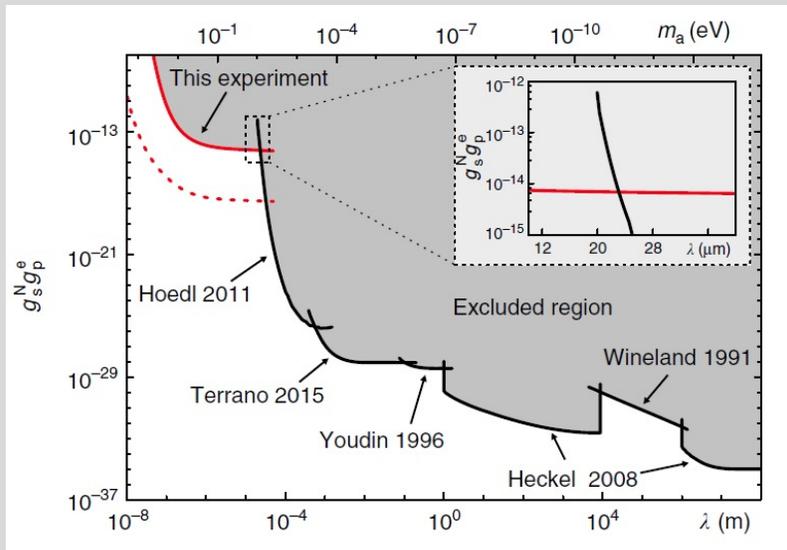
Previous catalogs:

- J. E. Moody and F. Wilczek, Phys. Rev. D 30, 130 (1984)
- B. A. Dobrescu and I. Mocioiu, J. High Energy Phys. 11 (2006)

Search results with NV sensors

Monopole-dipole interaction

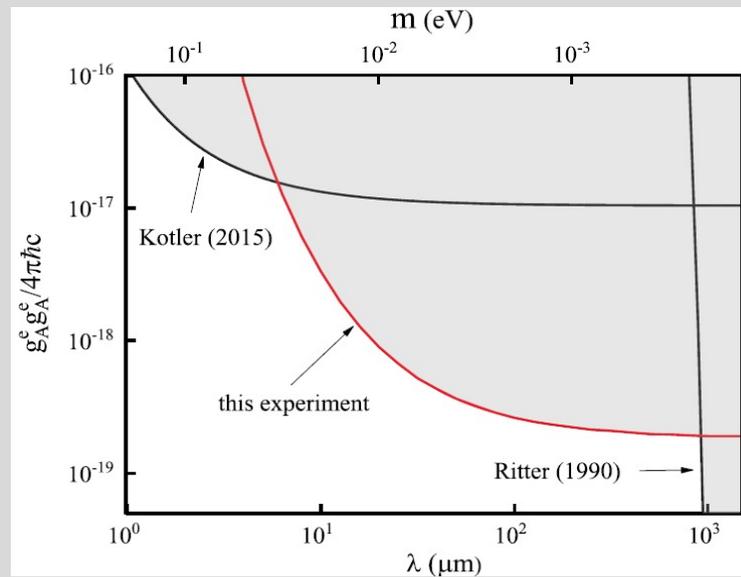
$$V_{\text{sp}}(\mathbf{r}) = \frac{\hbar^2 g_s^N g_p^e}{8\pi m} \left(\frac{1}{\lambda r} + \frac{1}{r^2} \right) e^{-\frac{r}{\lambda}} \boldsymbol{\sigma} \cdot \mathbf{e}_r,$$



Nature Communications 9,739 (2018)

Dipole-dipole interaction

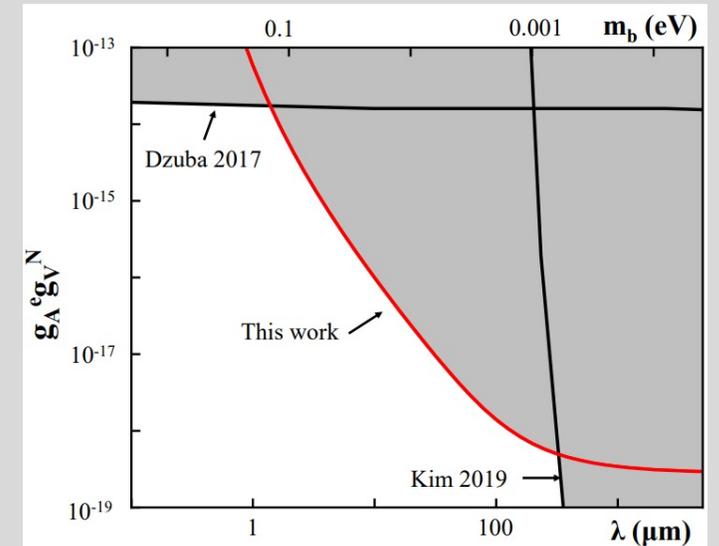
$$H_2 = \frac{g_A^e g_A^e \hbar c}{4\pi \hbar c r} (\boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2) e^{-\frac{r}{\lambda}},$$



Phys. Rev. Lett. 121, 080402 (2018)

Velocity-dependent monopole-dipole interaction

$$V = g_A^e g_V^N \frac{\hbar}{4\pi} (\boldsymbol{\sigma} \cdot \mathbf{v}) \left(\frac{e^{-\frac{r}{\lambda}}}{r} \right),$$

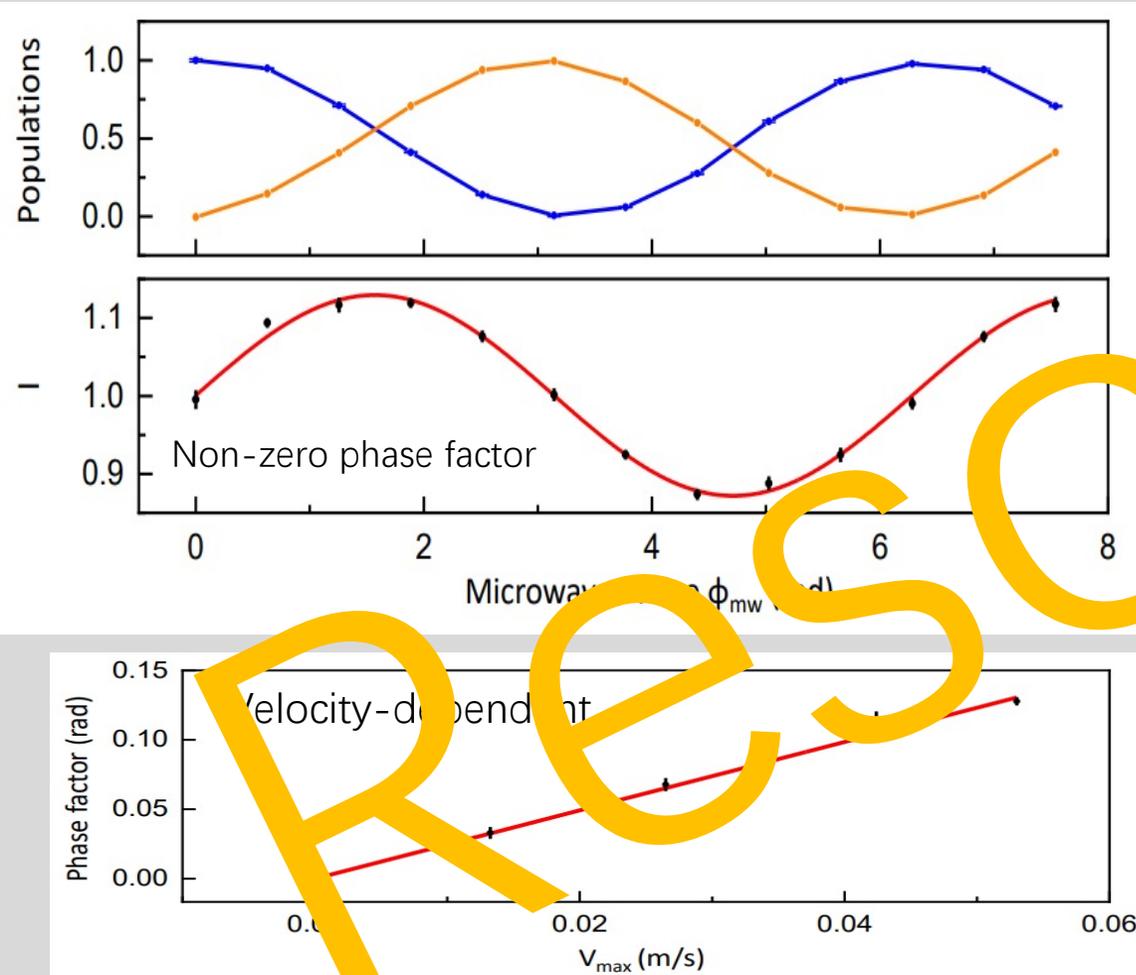


Phys. Rev. Lett. 127, 010501 (2021)

One search yields nonzero signal !

Velocity-dependent monopole-dipole interaction

$$V = f^\perp \frac{\hbar^2}{4\pi m_e c} \boldsymbol{\sigma} \cdot \mathbf{v} \times \hat{r} \left(\frac{1}{\lambda r} - \frac{1}{r^2} \right) e^{-\frac{r}{\lambda}},$$



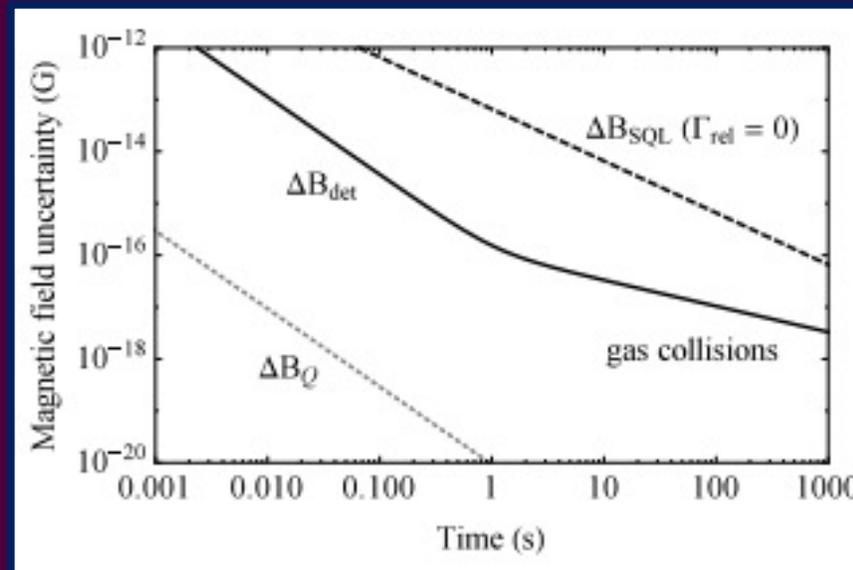
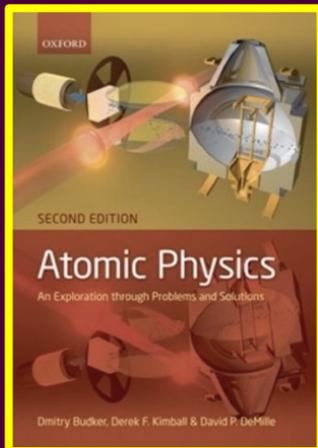
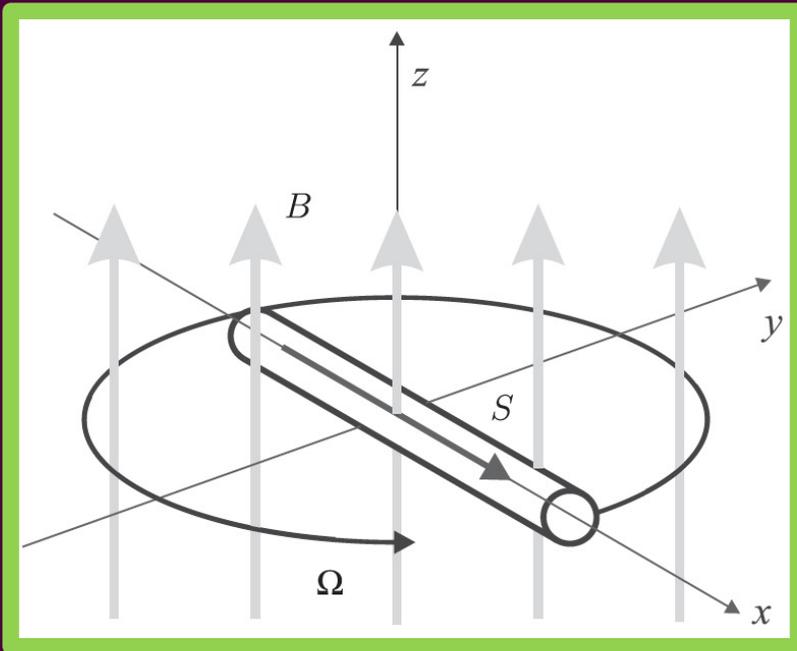
We analyzed several possible sources of the observed signal and they cannot explain the nonzero signal. Further experiments are being carried out to figure out the possible source of this signal.

Possible sources	Contribution to the phase factor (rad)
Tuning fork	$< 10^{-3}$
Charges on the mass	$< 10^{-4}$
Casimir Force	$< 10^{-5}$
Diamagnetism of the mass	$< 10^{-10}$
Effect due to the moving dielectric	$< 10^{-15}$
Nuclear spin in the mass	$< 10^{-15}$



Precessing Ferromagnetic Needle Magnetometer

Derek F. Jackson Kimball,¹ Alexander O. Sushkov,² and Dmitry Budker^{3,4,5}



Levitated Ferromagnets as novel sensors

PHYSICAL REVIEW APPLIED **11**, 044041 (2019)

Dynamics of a Ferromagnetic Particle Levitated over a Superconductor

Tao Wang,^{1,*} Sean Lourette,¹ Sean R. O’Kelley,¹ Metin Kayci,^{1,2} Y.B. Band,³
Derek F. Jackson Kimball,⁴ Alexander O. Sushkov,⁵ and Dmitry Budker^{1,6,7}

Ferromagnetic gyroscopes for tests of fundamental physics

Pavel Fadeev^{1,2,*}, Chris Timberlake³, Tao Wang⁴, Andrea Vinante^{3,5}, Y B Band⁶, Dmitry Budker^{1,2,7} , Alexander O Sushkov⁸, Hendrik Ulbricht³  and Derek F Jackson Kimball⁹ 
Quantum Sci. Technol. **6** (2021) 024006

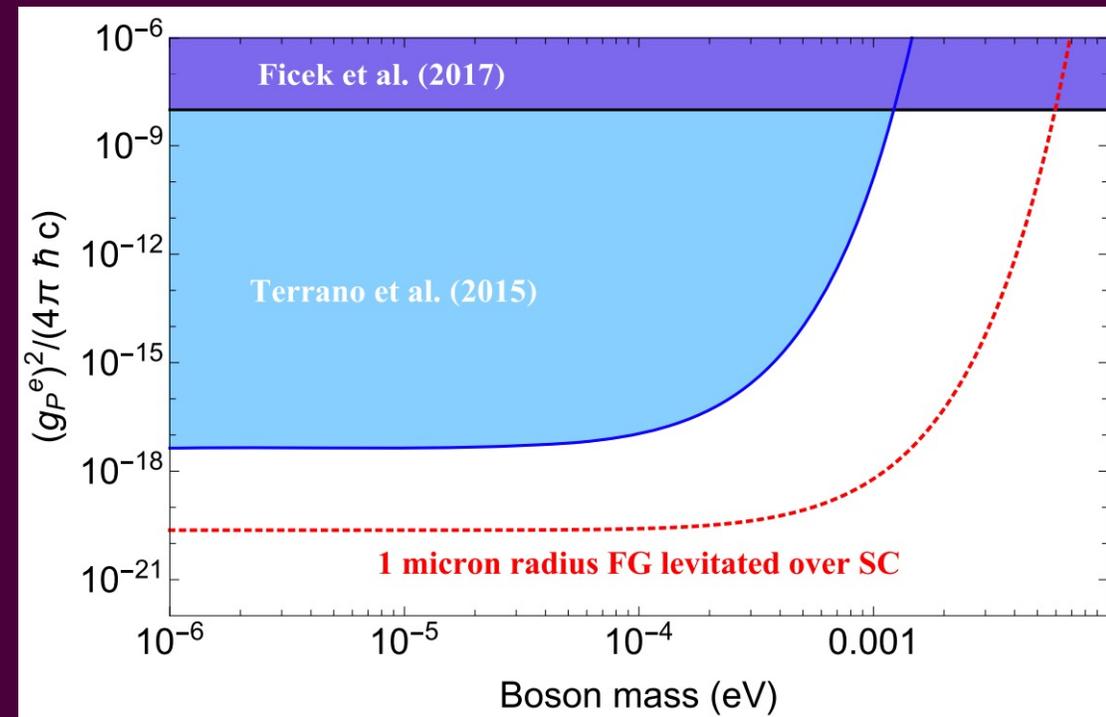
“Exotic” potentials:

(P. Fadeev *et al* 2019 *Phys. Rev. A* **99** 022113)

$$\mathcal{V}_{PP}(\mathcal{R}) = \frac{(g_P^e)^2}{4\pi\hbar c} \frac{\hbar^3}{4m_e^2 c} \left[\mathbf{s}_1 \cdot \mathbf{s}_2 \left(\frac{m_b c}{\hbar \mathcal{R}^2} + \frac{1}{\mathcal{R}^3} + \frac{4\pi}{3} \delta^3(\mathcal{R}) \right) - (\mathbf{s}_1 \cdot \hat{\mathcal{R}}) (\mathbf{s}_2 \cdot \hat{\mathcal{R}}) \left(\frac{m_b^2 c^2}{\hbar^2 \mathcal{R}} + \frac{3m_b c}{\hbar \mathcal{R}^2} + \frac{3}{\mathcal{R}^3} \right) \right] e^{-m_b c \mathcal{R} / \hbar}$$

Experiments @ SOTON, Trento, Paris, Harvard ...

Promising projections!



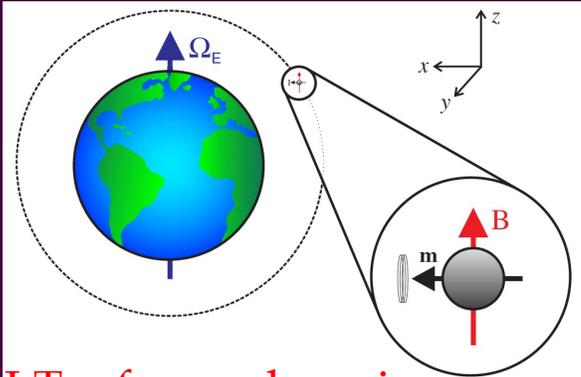
Spin source: 1 mm radius SmCo sphere; 1 mm from FG; integration time $t = 10^6$ s 18

Levitated Ferromagnets as novel sensors

PHYSICAL REVIEW D **103**, 044056 (2021)

Gravity Probe Spin: Prospects for measuring general-relativistic precession of intrinsic spin using a ferromagnetic gyroscope

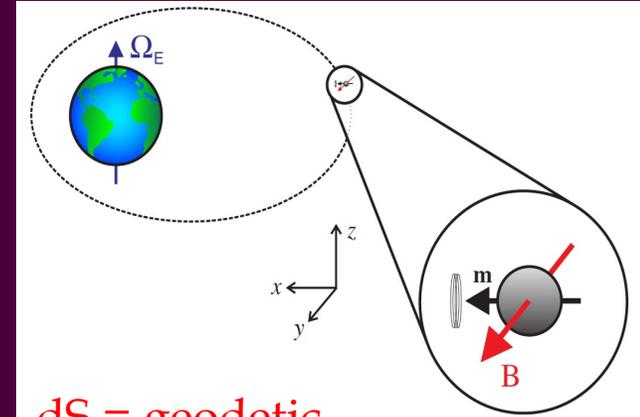
Pavel Fadeev^{1,*}, Tao Wang², Y. B. Band³, Dmitry Budker^{1,4}, Peter W. Graham⁵, Alexander O. Sushkov⁶, and Derek F. Jackson Kimball^{7,†}



LT = frame dragging

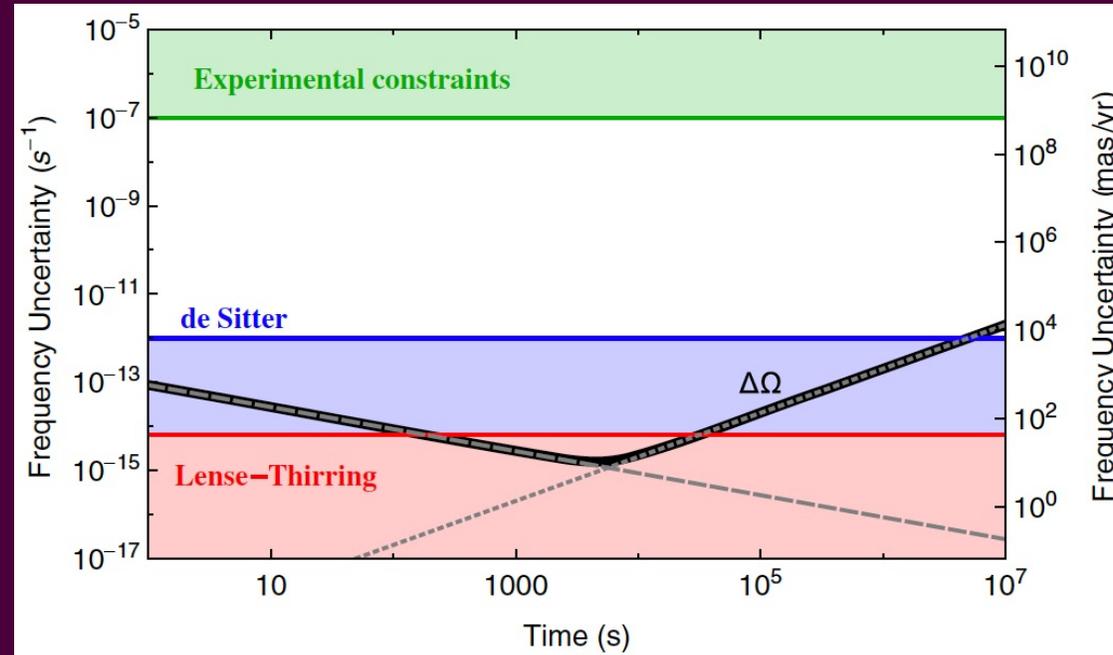
FIG. 2. Conceptual schematic diagram of a “Gravity Probe Spin” experiment. A freely floating spherical FG located within a superconducting shield is in a circular polar orbit. The magnetic field \mathbf{B} (from the frozen flux in the superconducting shields) is oriented parallel to the direction of Earth’s rotation axis Ω_E , both designated to point along z . The insert shows the initial orientation of the FG’s magnetic moment and spin m along the x axis. The pick-up coil measures the FG’s magnetization along x . This geometry is designed for the detection of the Lense-Thirring effect.

$$\Omega_{\text{LT}} \approx g \frac{2 GM}{5 c^2 R} [3(\Omega_E \cdot \hat{R})\hat{R} - \Omega_E]$$



dS = geodetic

FIG. 6. Conceptual schematic diagram of a “Gravity Probe Spin” experiment similar to that shown in Fig. 2 except that the orbit is elliptical and the magnetic field \mathbf{B} is directed along the y -axis, perpendicular to the orbital plane. This geometry is designed for the detection of the de Sitter effect.



$$\Omega_{\text{dS}} \approx g \frac{3 GM}{2 c^2 R^2} (\hat{R} \times \mathbf{v})$$

Another (sneaky) way to search for axions and other exotic particles

Constraints on exotic spin-dependent interactions between electrons from helium fine-structure spectroscopy



Filip Ficek^{1,*}, Derek F. Jackson Kimball², Mikhail Kozlov^{3,4},
Nathan Leefer⁵, Szymon Pustelny¹, and Dmitry Budker^{5,6,7}

¹ *Institute of Physics, Jagiellonian University, Łojasiewicza 11, 30-348 Kraków, Poland*
² *Department of Physics, California State University - East Bay, Hayward, California 94542-3080*

³ *Petersburg Nuclear Physics Institute, Gatchina 188300, Russia*

⁴ *St. Petersburg Electrotechnical University LETI, Prof. Popov Str. 5, 197376 St. Petersburg*

⁵ *Helmholtz Institute Mainz, Johannes Gutenberg University, 55099 Mainz, Germany*

⁶ *Department of Physics, University of California at Berkeley, Berkeley, California 94720-7300, USA*

⁷ *Nuclear Science Division, Lawrence Berkeley National Laboratory, Berkeley, California 94720, USA*

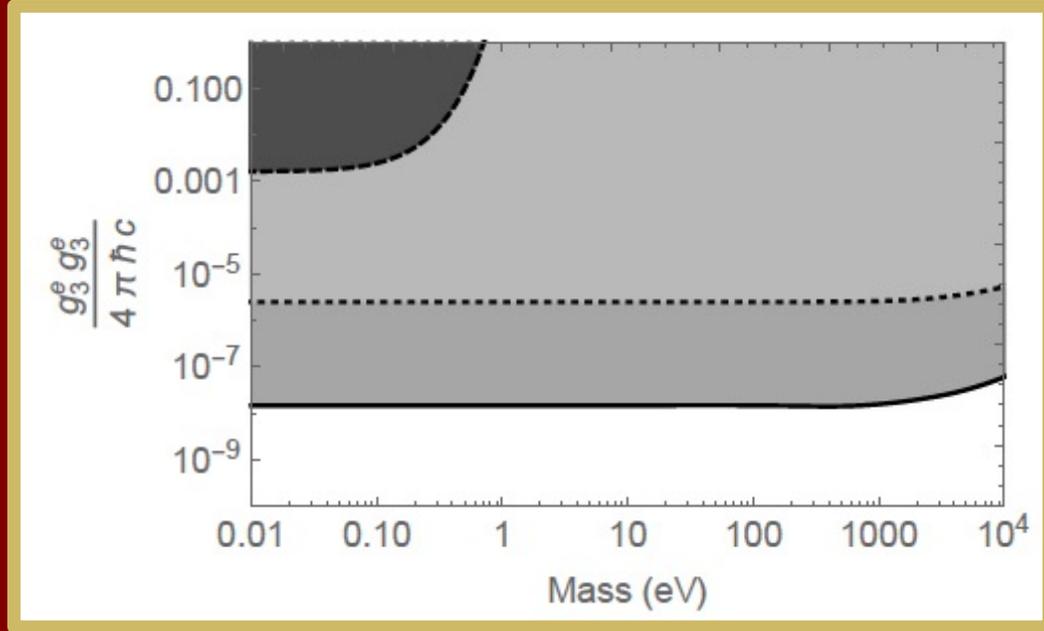
(Dated: August 23, 2016)



TABLE I: Comparison of theoretical (QED-based) and experimental transition energies values between various helium states.

	Theoretical		Experimental		Difference	ΔE
$2^3P_1 - 2^3P_2$	2 291 178.9(1.7) kHz	[35]	2 291 177.69(36) kHz	[30]	1.2(1.7) kHz	4.6 kHz
$2^3P_0 - 2^3P_2$	31 908 131.2(1.8) kHz	[35]	31 908 131.25(30) kHz	[32]	0.1(1.8) kHz	3.2 kHz
$2^3P_0 - 2^3P_1$	29 616 952.3(1.7) kHz	[35]	29 616 951.66(70) kHz	[33]	0.6(1.8) kHz	3.7 kHz
$2^3P_0 - 2^3S_1$	276 764 094.7(3.0) MHz	[36]	276 764 094.7073(21) MHz	[31]	0.0(3.0) MHz	
$2^3P_1 - 2^3S_1$	276 734 477.7(3.0) MHz	[36]	276 734 477.7525(20) MHz	[31]	0.1(3.0) MHz	
$2^3P_2 - 2^3S_1$	276 732 186.1(2.9) MHz	[36]	276 732 186.621(15) MHz	[31]	0.5(2.9) MHz	

A sneaky way to look for ALPs



$$V_2 = \frac{g_2^e g_2^e}{4\pi\hbar c} \hbar c (\mathbf{s}_1 \cdot \mathbf{s}_2) \frac{e^{-r_{12}/\lambda}}{r_{12}},$$

$$V_3 = \frac{g_3^e g_3^e}{4\pi\hbar c} \frac{\hbar^3}{4m_e^2 c} \left[\mathbf{s}_1 \cdot \mathbf{s}_2 \left(\frac{1}{\lambda r_{12}^2} + \frac{1}{r_{12}^3} \right) - (\mathbf{s}_1 \cdot \mathbf{e}_{12})(\mathbf{s}_2 \cdot \mathbf{e}_{12}) \left(\frac{1}{\lambda^2 r_{12}} + \frac{3}{\lambda r_{12}^2} + \frac{3}{r_{12}^3} \right) \right] e^{-r_{12}/\lambda},$$

$$V_4 = \frac{g_4^e g_4^e}{4\pi\hbar c} \frac{i\hbar^3}{4m_e^2 c} (\mathbf{s}_1 + \mathbf{s}_2) \cdot \left[(\nabla_1 - \nabla_2) \times \mathbf{r}_{12}, \left(\frac{1}{r_{12}^3} + \frac{1}{\lambda r_{12}^2} \right) e^{-r_{12}/\lambda} \right]_+,$$

$$V_8 = \frac{g_8^e g_8^e}{4\pi\hbar c} \frac{\hbar^3}{4m_e^2 c} \left[\mathbf{s}_1 \cdot (\nabla_1 - \nabla_2), \left[\mathbf{s}_2 \cdot (\nabla_1 - \nabla_2), \frac{e^{-r_{12}/\lambda}}{r_{12}} \right]_+ \right]_+,$$

FIG. 1: Constraints (at the 90% confidence level) on the dimensionless coupling constant $g_3^e g_3^e / (4\pi\hbar c)$ as a function of the boson mass. The dashed line and dark gray fill shows the constraint for electrons from Ref. [35]. The dotted line and light gray fill show the constraint derived from analysis of positronium, also discussed in [35]. The solid line and medium gray fill shows the constraint from a comparison between theory and experiment for the $2^3P_2 - 2^3P_1$ transition frequency in He.

Also: antiprotonic He

$(\bar{p}, \text{He}^{++}, e^-)$

PHYSICAL REVIEW LETTERS **120**, 183002 (2018)

Constraints on Exotic Spin-Dependent Interactions Between Matter and Antimatter from Antiprotonic Helium Spectroscopy

Filip Ficek,^{1,*} Pavel Fadeev,^{2,3} Victor V. Flambaum,^{2,4} Derek F. Jackson Kimball,⁵ Mikhail G. Kozlov,^{6,7} Yevgeny V. Stadnik,² and Dmitry Budker^{2,8,9}

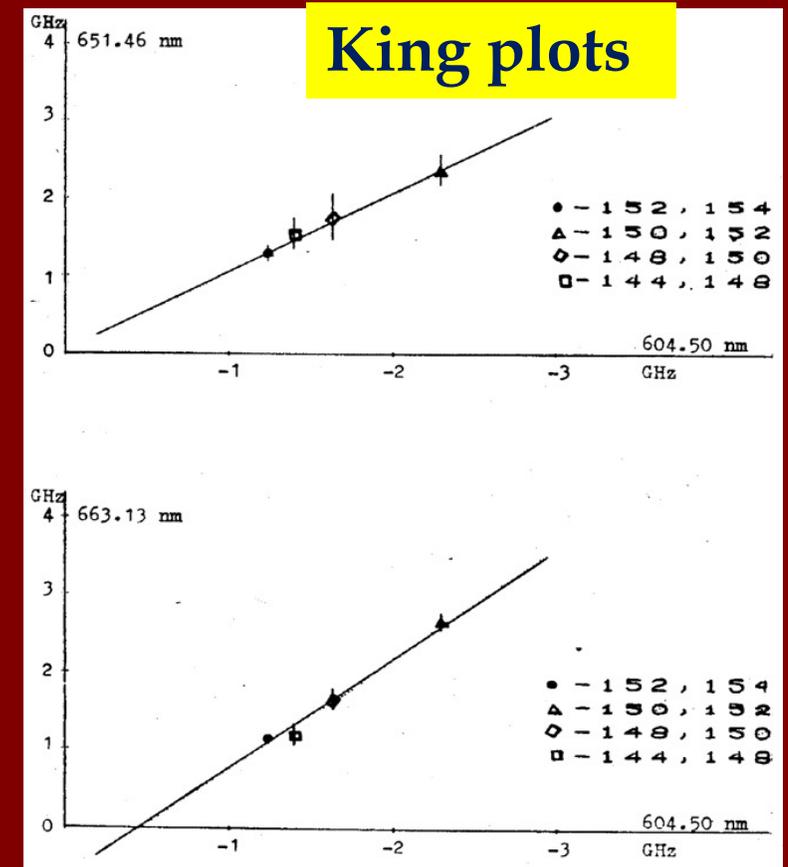
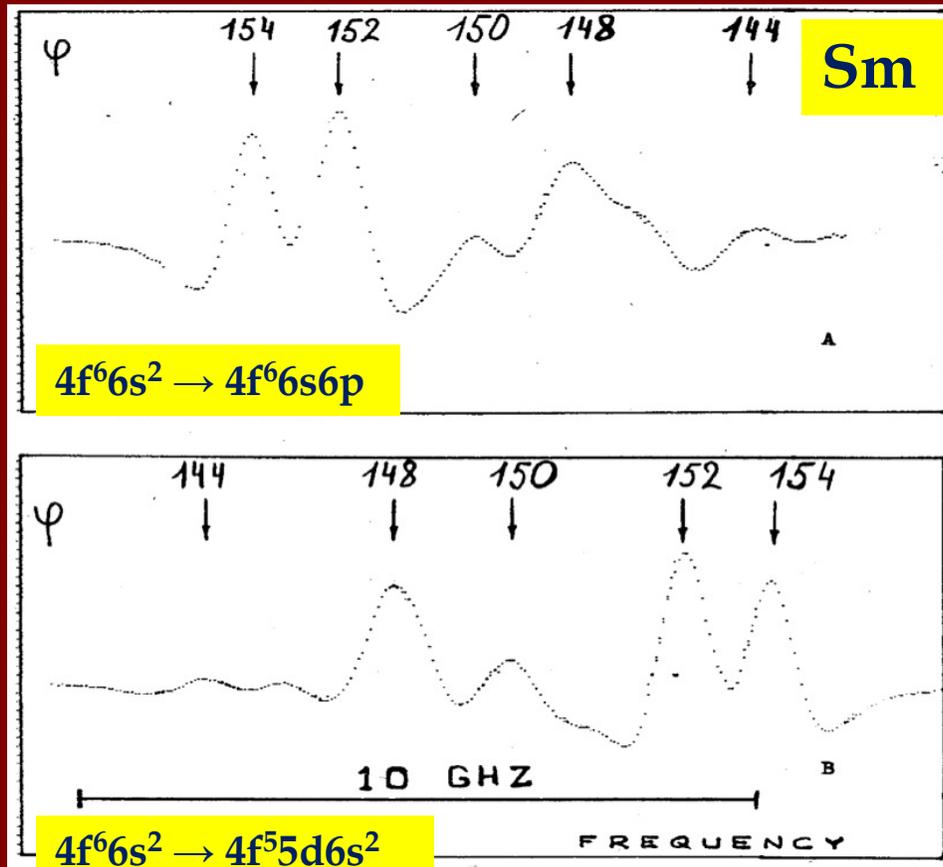
Another sneaky way to look for new physics

PHYSICAL REVIEW LETTERS **120**, 091801 (2018)

Probing New Long-Range Interactions by **Isotope Shift** Spectroscopy

Julian C. Berengut,^{1,*} Dmitry Budker,^{2,3,4,†} Cédric Delaunay,^{5,‡} Victor V. Flambaum,^{1,§} Claudia Frugiuele,^{6,||}
Elina Fuchs,^{6,¶} Christophe Grojean,^{7,8,**} Roni Harnik,^{9,††} Roei Ozeri,^{10,‡‡} Gilad Perez,^{6,§§} and Yotam Soreq^{11,|||}

Isotope Shifts: a century of FUN



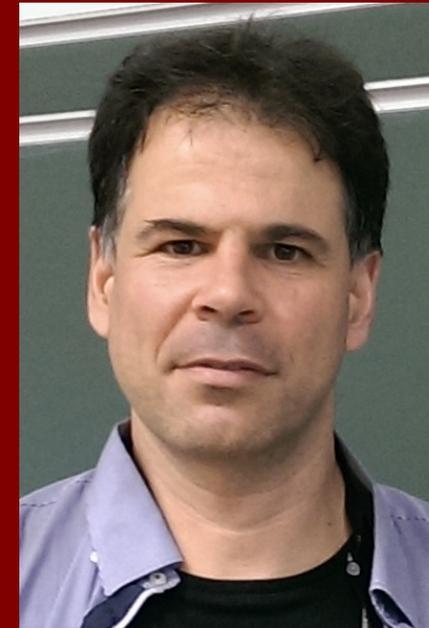
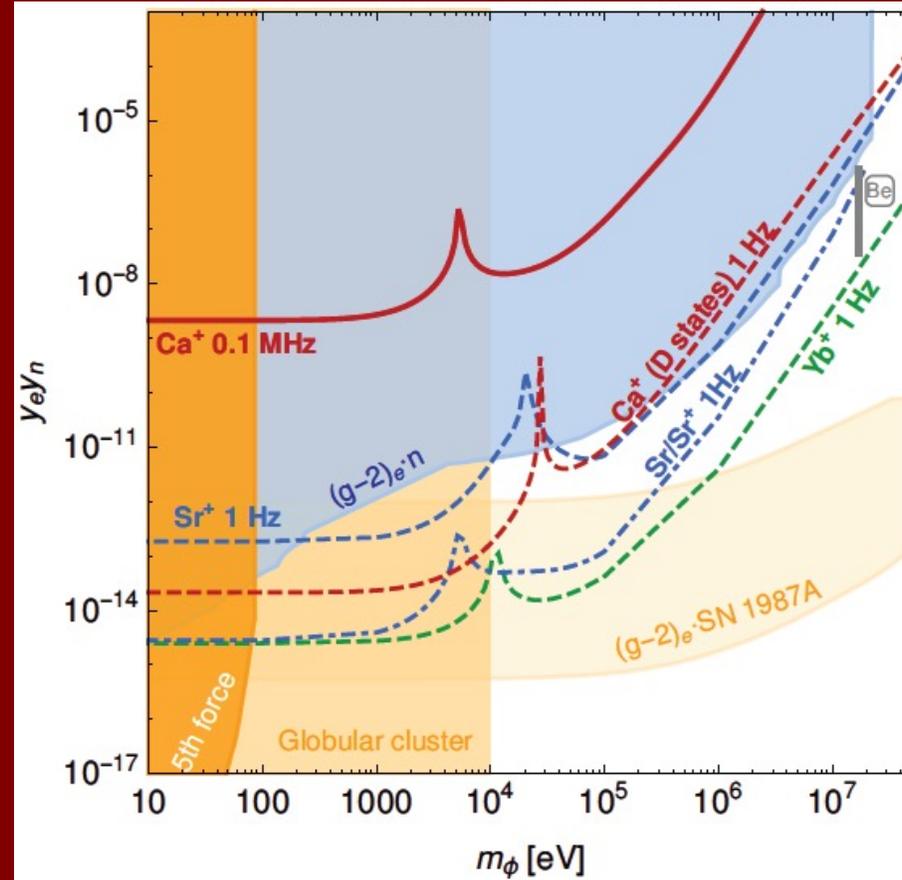
Plots from DB's Diploma Thesis (Novosibirsk, 1985)

Another sneaky way to look for new physics

PHYSICAL REVIEW LETTERS 120, 091801 (2018)

Probing New Long-Range Interactions by **Isotope Shift** Spectroscopy

Julian C. Berengut,^{1,*} Dmitry Budker,^{2,3,4,†} Cédric Delaunay,^{5,‡} Victor V. Flambaum,^{1,§} Claudia Frugiuele,^{6,||} Elina Fuchs,^{6,¶} Christophe Grojean,^{7,8,**} Roni Harnik,^{9,††} Roei Ozeri,^{10,‡‡} Gilad Perez,^{6,§§} and Yotam Soreq^{11,|||}



Precision isotope shifts in ytterbium and implications for new physics

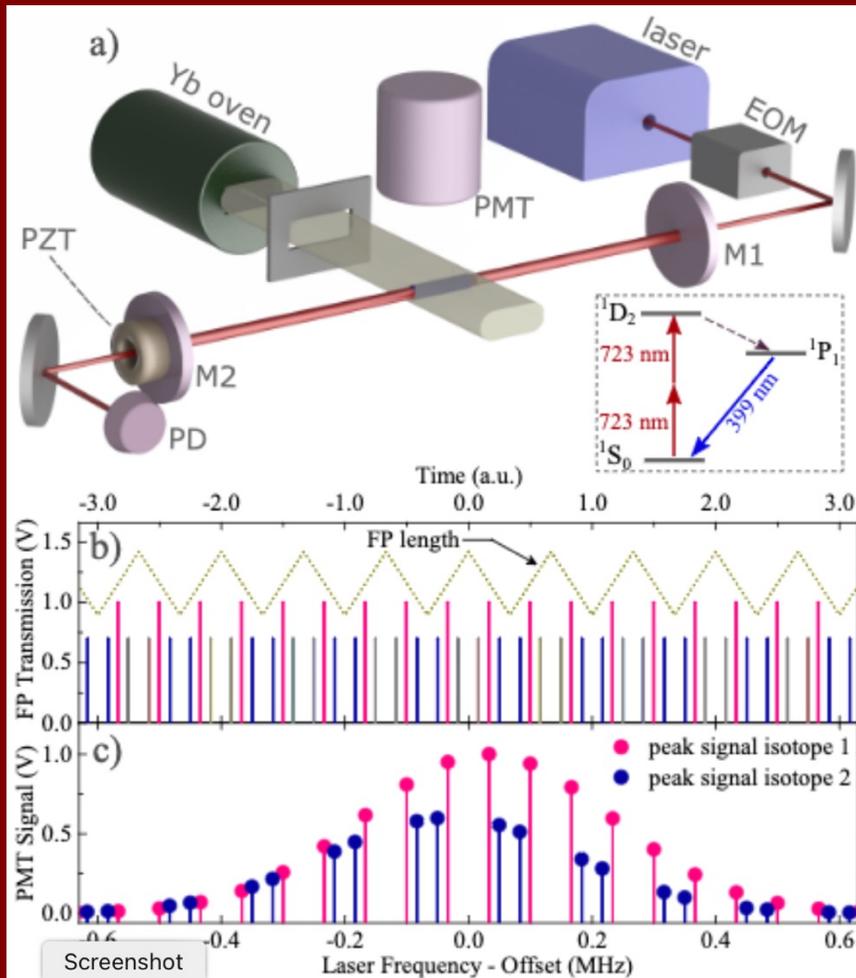
N. L. Figueroa,* D. Budker, and D. Antypas

Johannes Gutenberg-Universität Mainz, Helmholtz-Institut Mainz, Mainz 55128, Germany

J. C. Berengut, V. A. Dzuba, and V. V. Flambaum

School of Physics, University of New South Wales, Sydney, New South Wales 2052, Australia

(Dated: June 8, 2021)



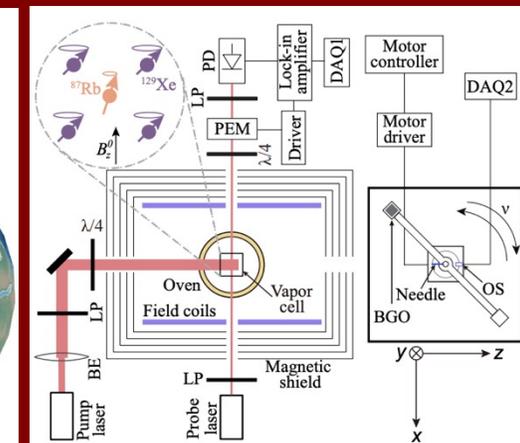
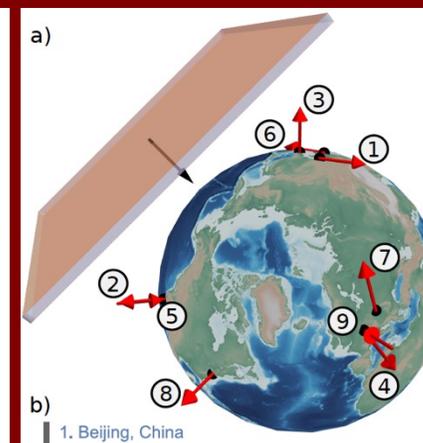
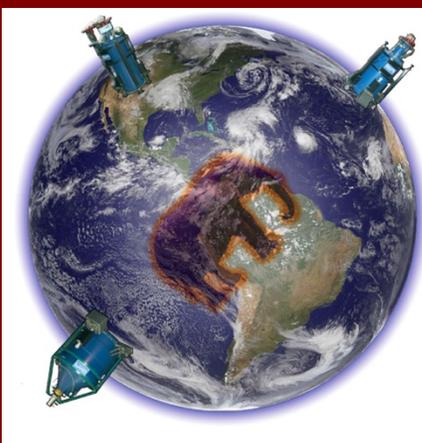
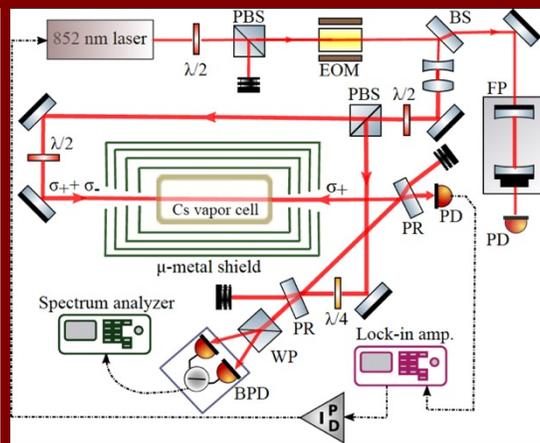
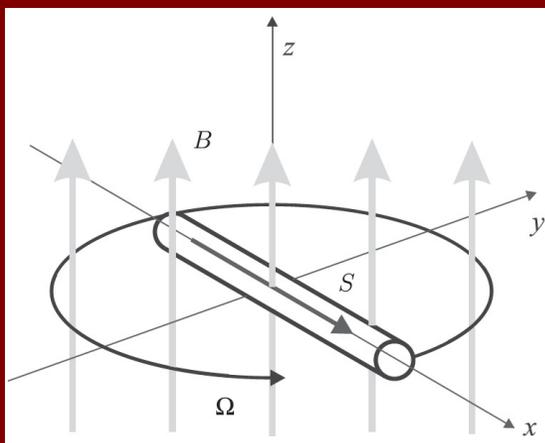
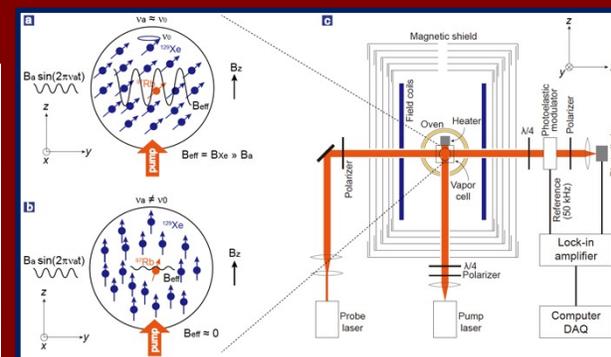
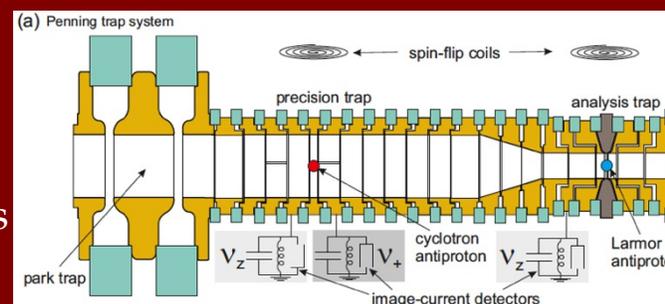
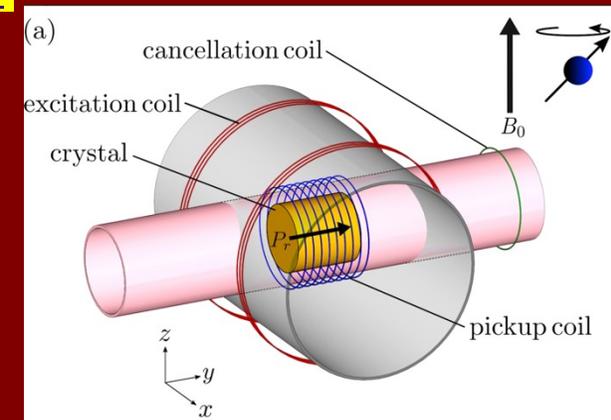
King Plot

Coming here soon

~300 Hz per isotope pair

In search...

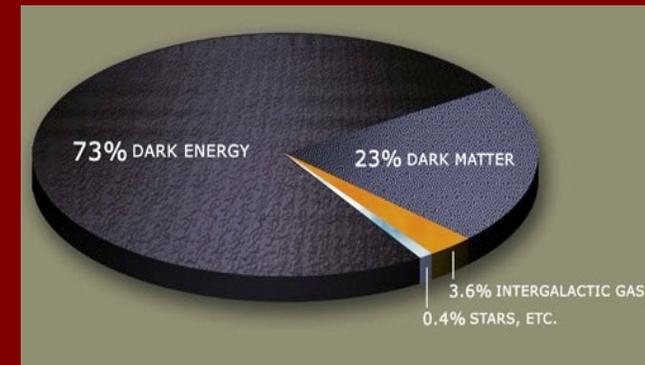
- D. Aybas, et al, Search for axion-like dark matter using solid-state nuclear magnetic resonance, [Phys. Rev. Lett.](#) **126**, 141802 (2021)
- M. Jiang, H. Su, A. Garcon, X. Peng, and D. Budker, Search for axion-like dark matter with spin-based amplifiers, [arXiv:2102.01448](#) (2021)
- H. Su, et al, Search for exotic spin-dependent interactions with a spin-based amplifier, [arXiv:2103.15282](#) (2021)
- S. Afach, et al, Search for topological defect dark matter using the global network of optical magnetometers for exotic physics searches (GNOME); [arXiv:2102.13379](#) (2021)
- N. L. Figueroa, et al, Dark matter searches using accelerometer-based networks, [Quantum Sci. Technol.](#) **6** 034004 (2021)
- D. Antypas, et al, Fast apparent oscillations of fundamental constants, [ANNALEN DER PHYSIK](#) **1900566** (2020)
- P. Fadeev, et al, Ferromagnetic Gyroscopes for Tests of Fundamental Physics, [Quant. Sci. Tech.](#) **6**(2) 024006 (2021)
- C. Smorra, et al, Direct limits on the interaction of antiprotons with axion-like dark matter. [Nature](#) **575**, 310-314 (2019)



Part II : let us dig in!

Why Axions (ALPs) ?

- Big clean-up ?
 - Strong CP problem
 - Dark Matter
 - Dark Energy
 - Baryon asymmetry of the Universe
 - Hierarchy?
 - ...



<http://earthsky.org/space/>

How to search for Axions (ALPs) ?

Axion (ALP) Interactions

Gravity

+

Gauge Fields

$$\frac{a}{f_a} F_{\mu\nu} \tilde{F}^{\mu\nu}$$

$$\frac{a}{f_a} G_{\mu\nu} \tilde{G}^{\mu\nu}$$

Fermions

$$\frac{\partial_\mu a}{f_a} \bar{\Psi}_f \gamma^\mu \gamma_5 \Psi_f$$

Most Searches,
DM radio

(nEDM, CASPER-**E**)

(CASPER-**Gradient**, **GNOME**, QUAX)

Dark Matter search with **NMR**

Key Ideas:

- **Dark Matter** could be a “classical” field
- Not screened by shielding
- Oscillating at frequency: mc^2/h
- Relatively narrow line: $\Delta\nu/\nu \sim 10^{-6}$



→ Cosmic **Axion Spin-Precession Experiment(s)**

CASPEr



CASPEr searches for **DM** via:

$$\frac{a}{f_a} G_{\mu\nu} \tilde{G}^{\mu\nu}$$

↑
⋮

coupling to gluons

→ creates oscillating nucleon electric dipole moment (EDM)
this is why axions were invented

→ spin σ to axion coupling:

$$H_e \propto a \sigma \cdot \mathbf{E}^*$$

CASPEr-electric

$$\frac{\partial_\mu a}{f_a} \bar{\Psi}_f \gamma^\mu \gamma_5 \Psi_f$$

↑
⋮

coupling to fermions

→ via axion field gradient

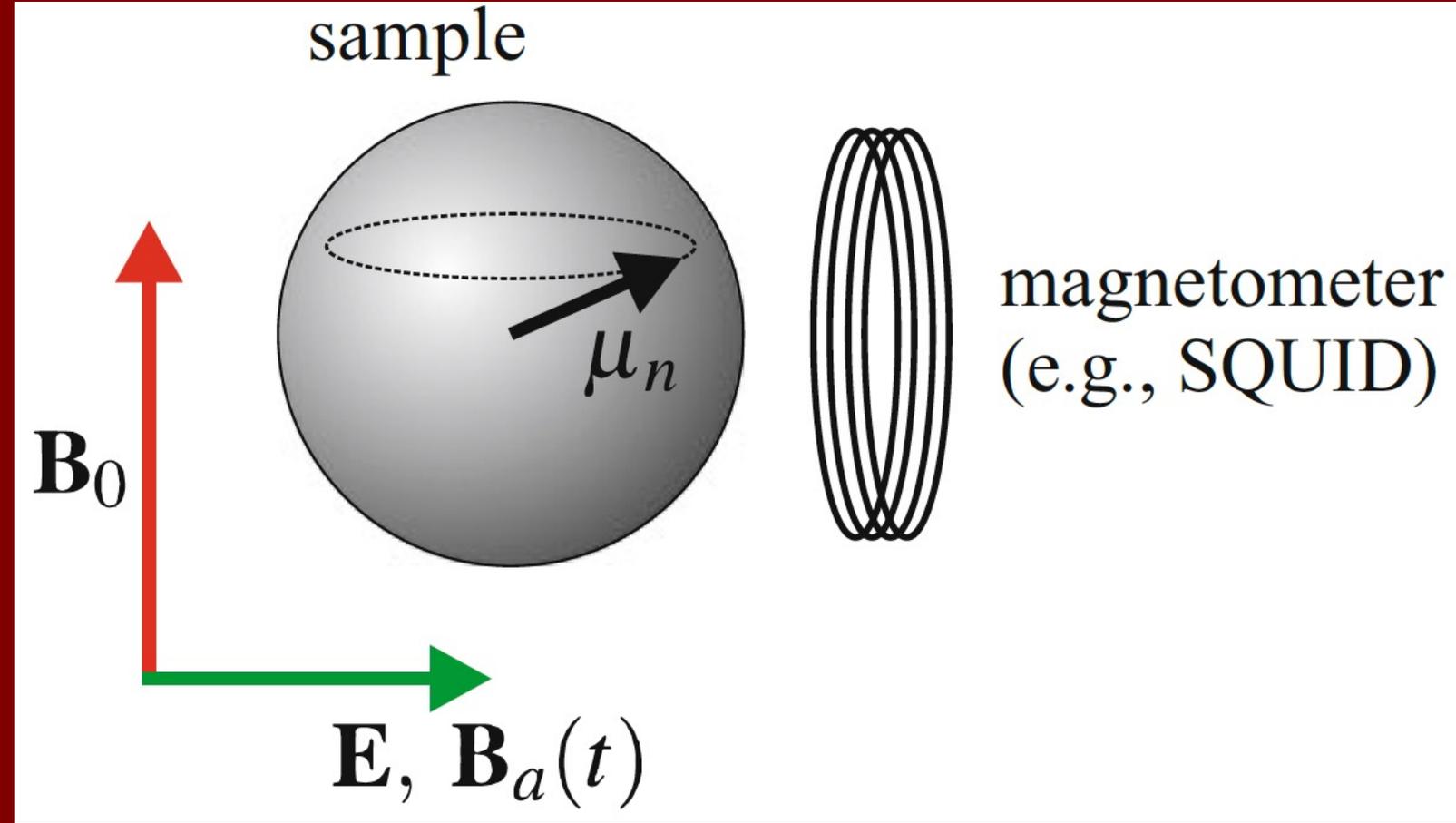
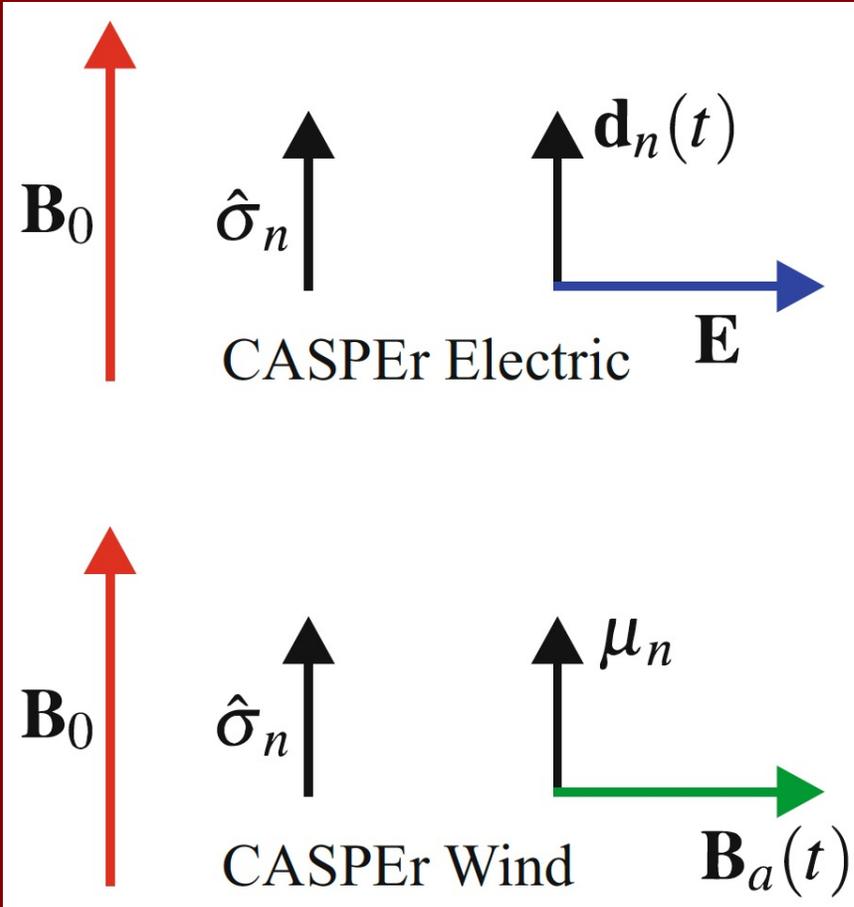
→ spin σ to axion gradient coupling:

$$H_g \propto \sigma \cdot \nabla a$$

CASPEr-gradient

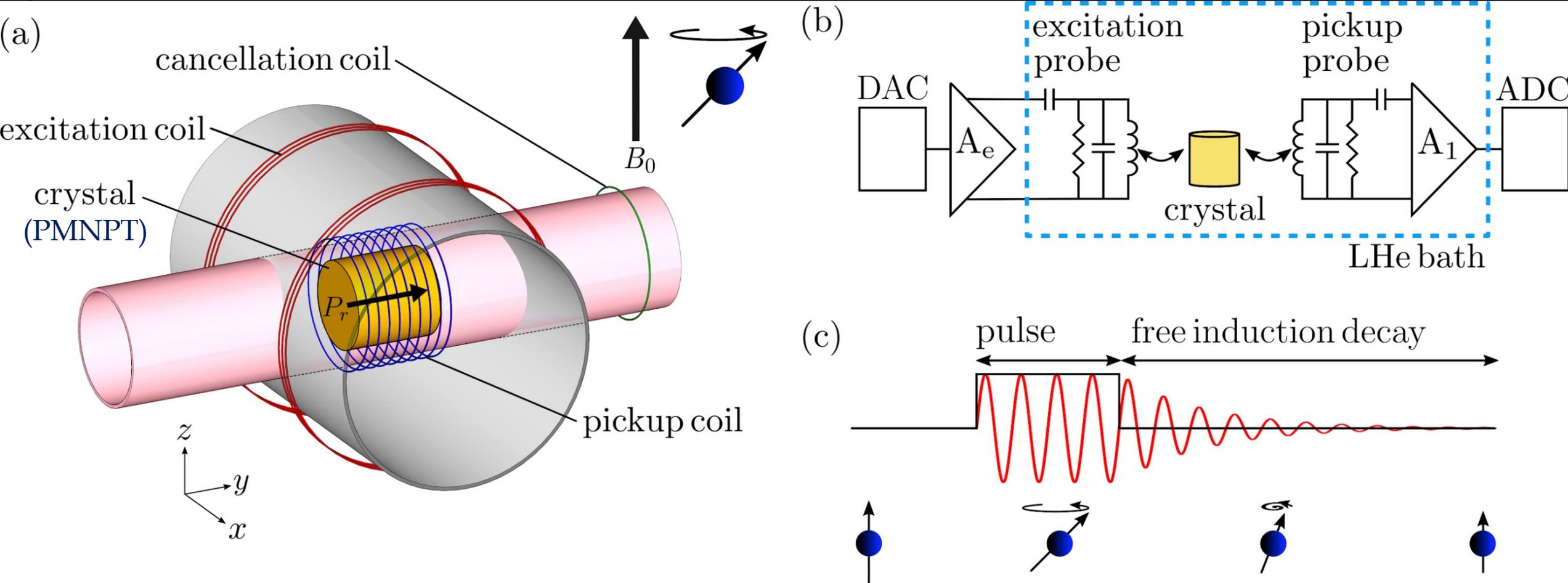
CASPEr (Cosmic Axion Spin Precession Experiments) searches for experimental signatures of these couplings

DM search with NMR (CASPER)



D. F. Jackson Kimball *et. al.* in G. Carosi, G. Rybka (eds.), Microwave Cavities and Detectors for Axion Research, Springer Proceedings in Physics 245, https://doi.org/10.1007/978-3-030-43761-9_13

CASPER-Boston

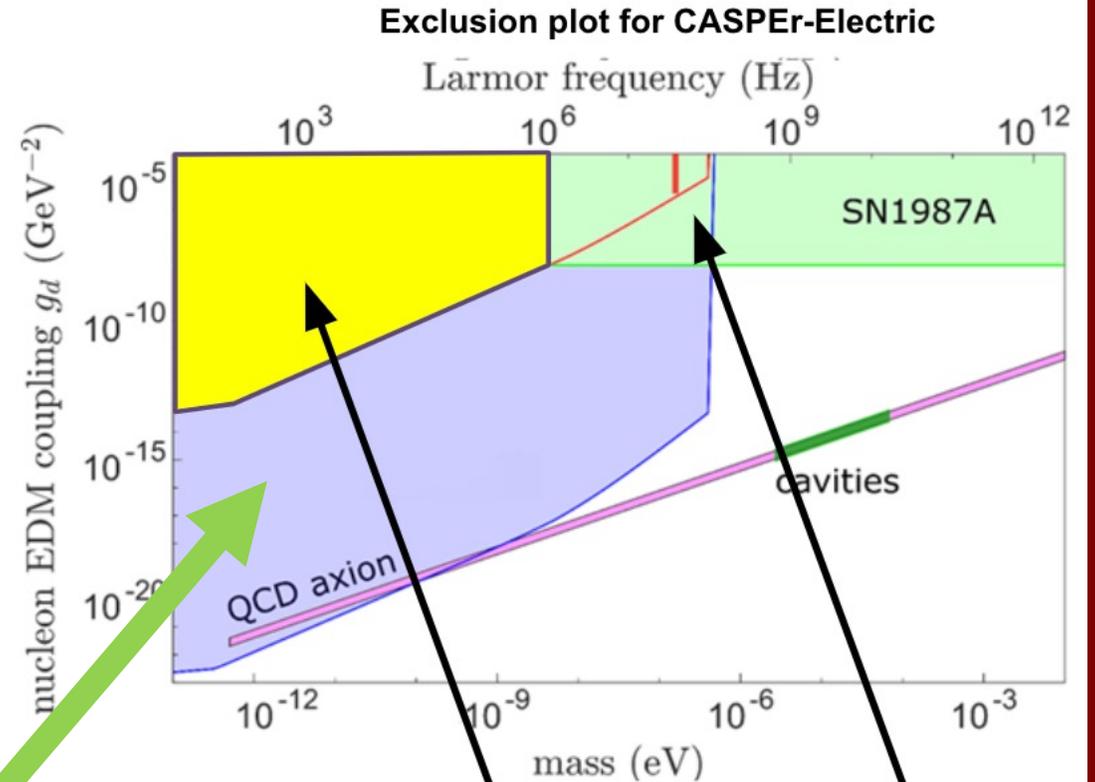


CASPER-Boston



CASPER-Electric Generation 2 - Low Field

- Successful Gen 1 – High field
- **Generation 2 – Low field**
- **Frequency:** up to 1 MHz
- Goes below astronomical limits
- **Differences:**
 - Pickup: SQUID
 - SC wires

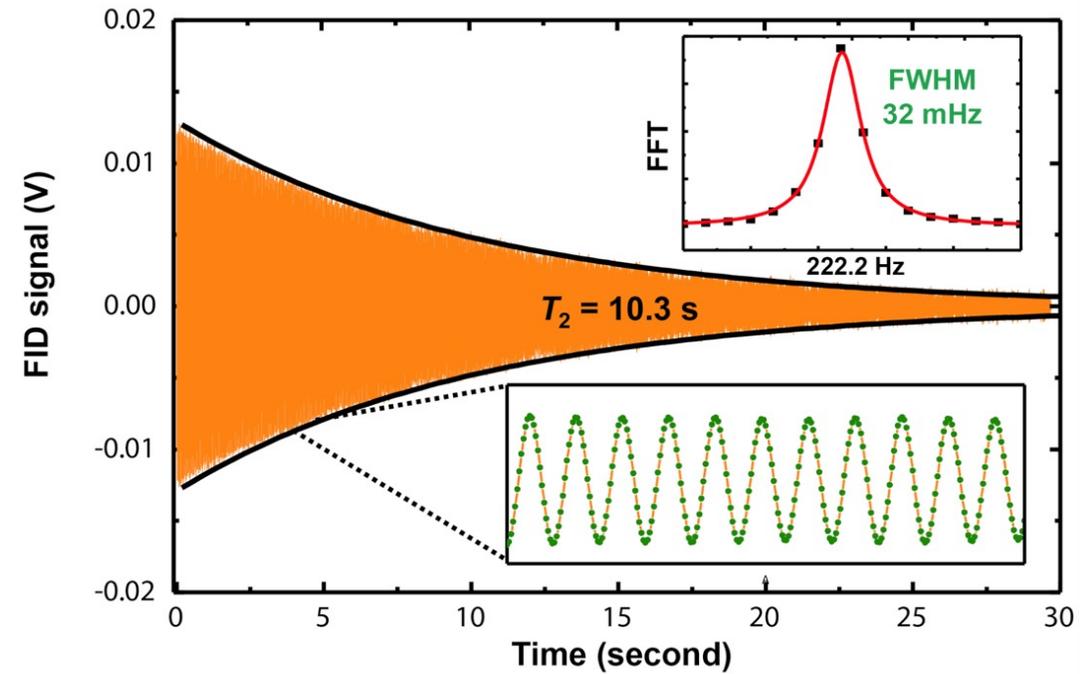
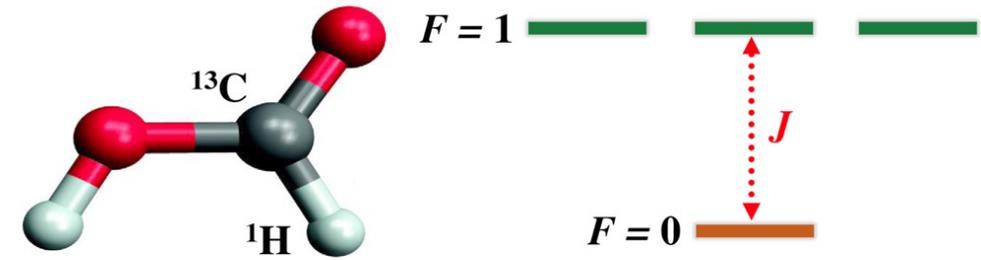
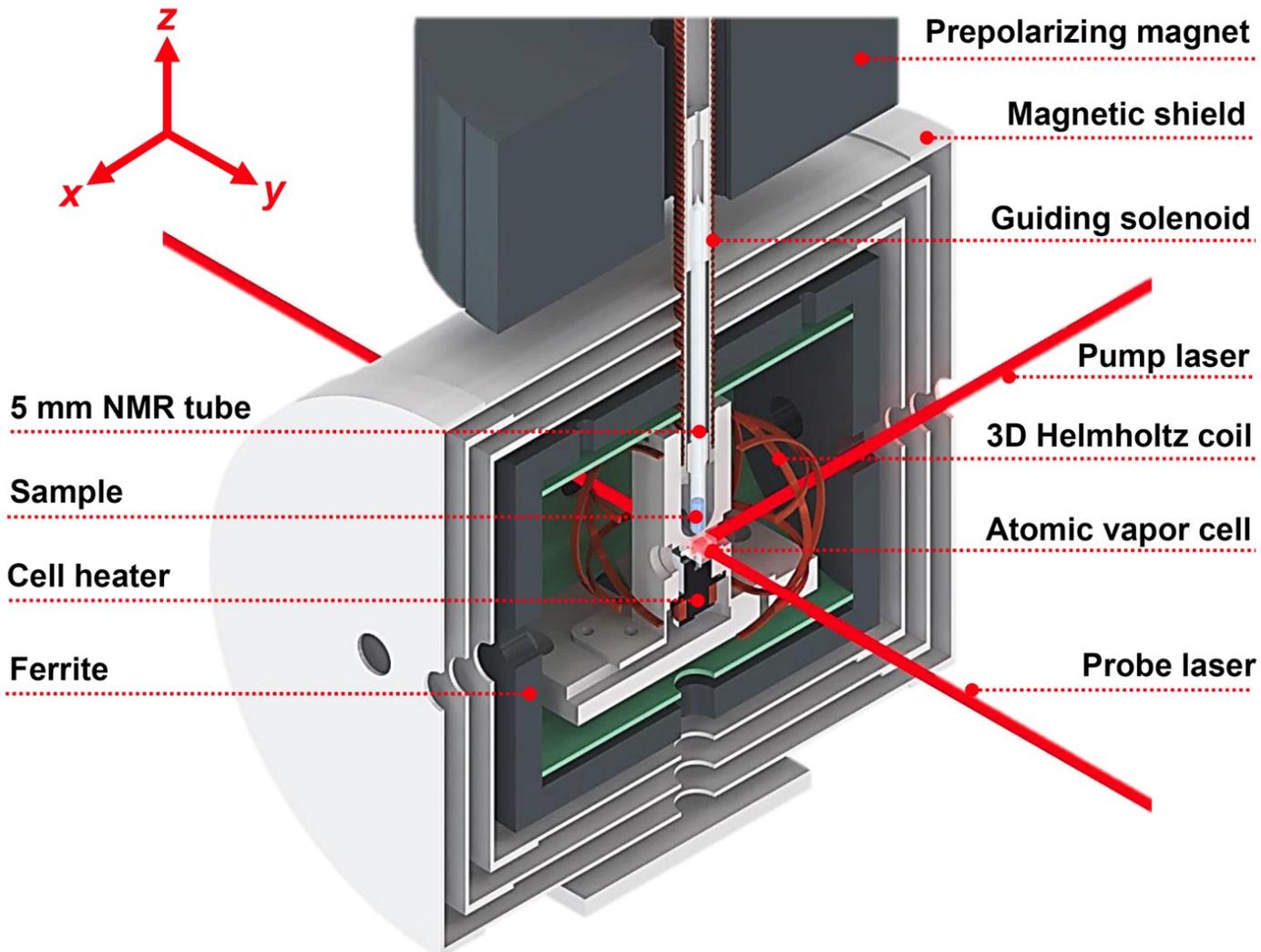


Gen 2 is looking for ALPs below 1 MHz with SQUIDs

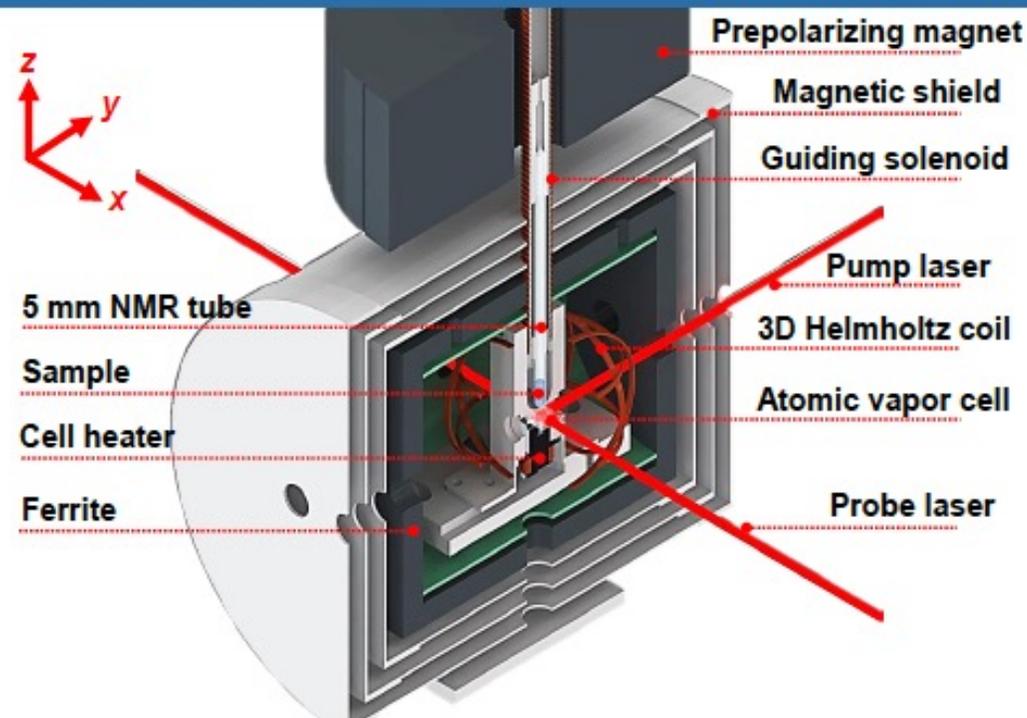
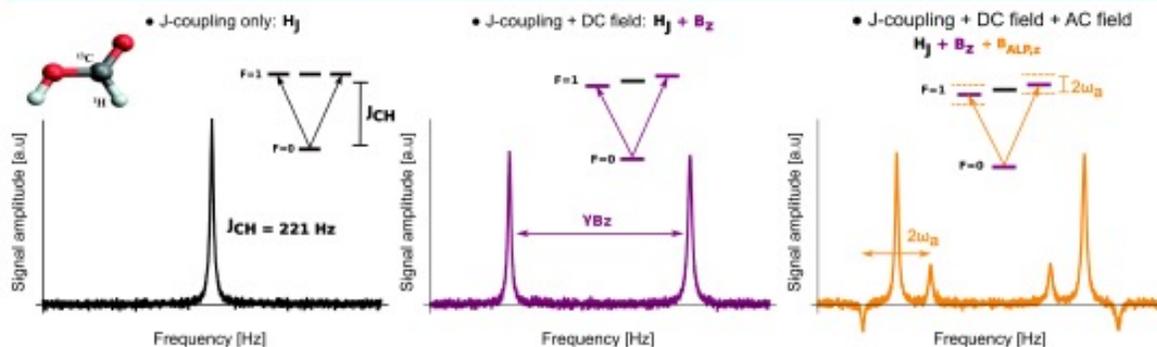
CASPER Gen 1
CASPER Gen 2

Gen 3 : Big Sample + Hyperpolarization !

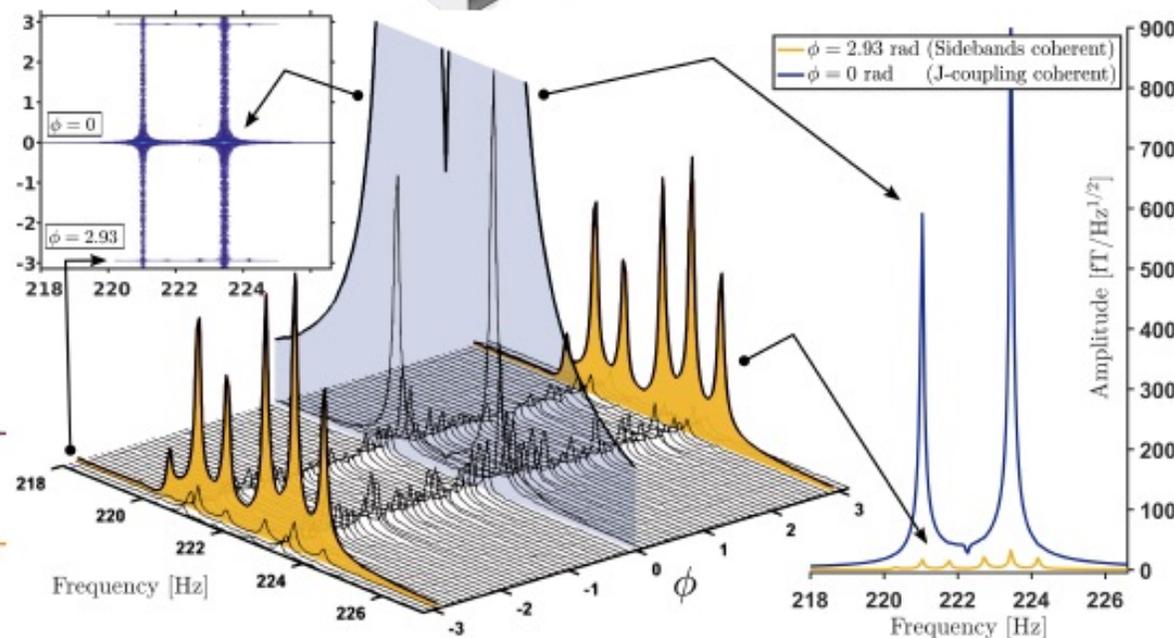
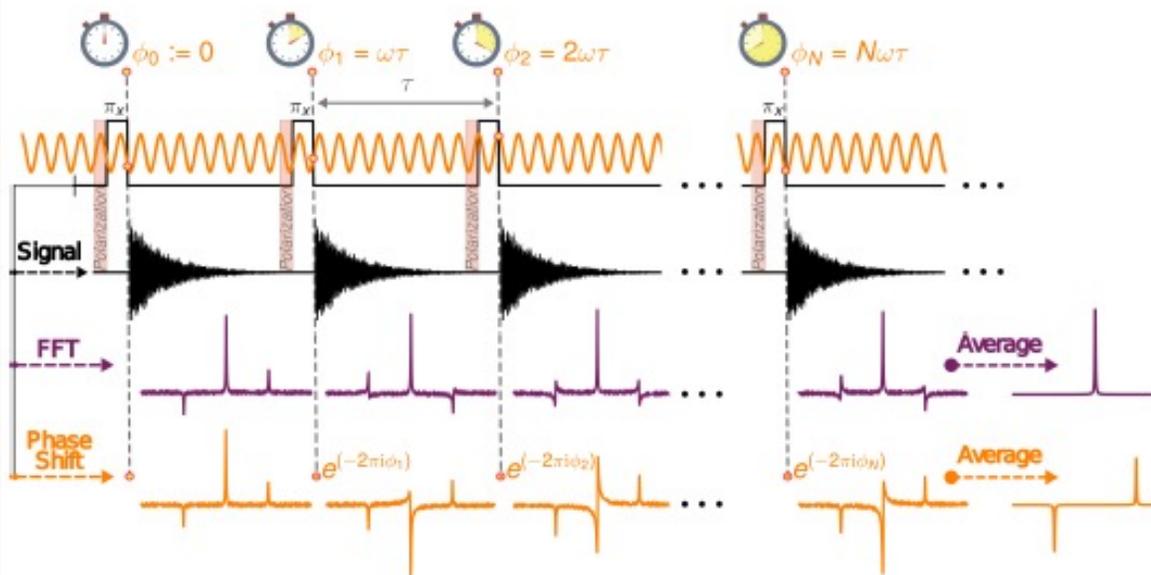
Zero-field NMR



CASPER With Zero- to Ultralow-Field NMR



- ▶ Search for dark-matter-induced sidebands
- ▶ Coherent averaging of arbitrary frequency via post-processing phase cycling
- ▶ Upcoming sensitivity improvements with PHIP

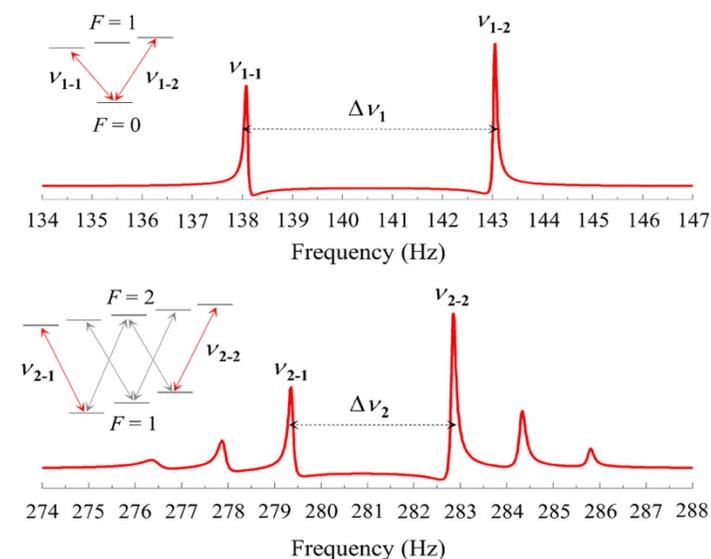
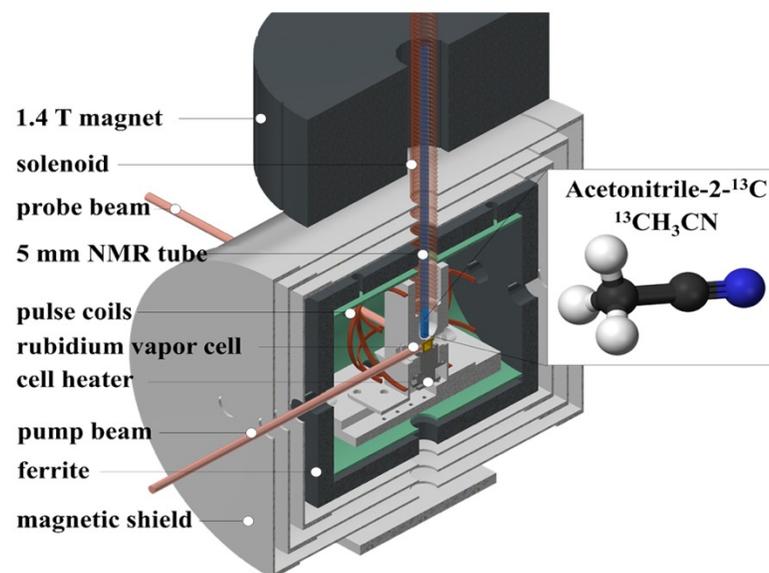
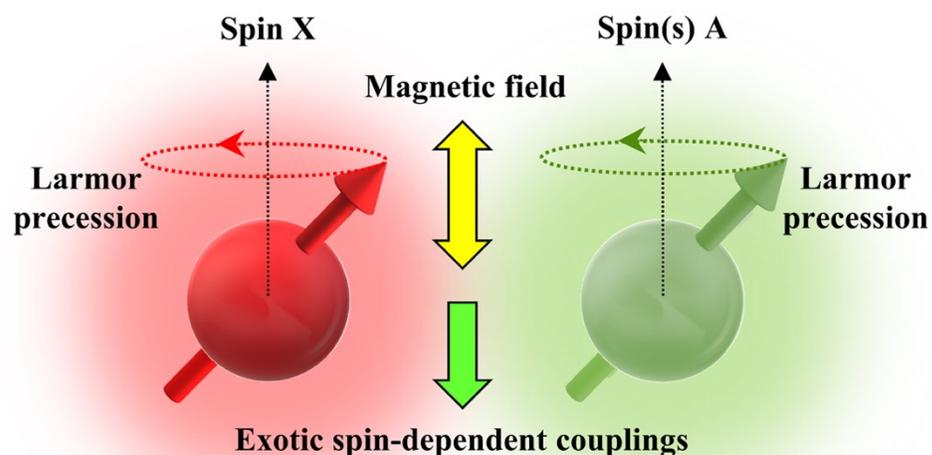


Nuclear-spin comagnetometer based on a liquid of identical molecules

Phys. Rev. Lett.

Teng Wu, John W. Blanchard, Derek F. Jackson Kimball, Min Jiang, and Dmitry Budker

Accepted 15 June 2018



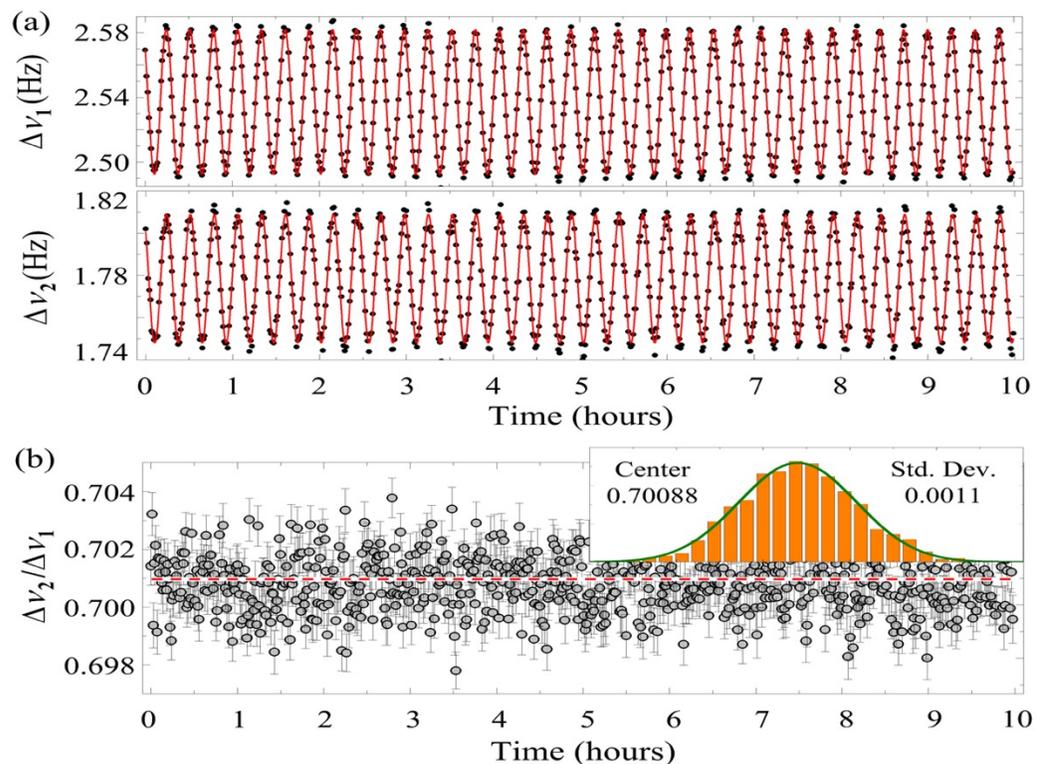
Nuclear-spin comagnetometer based on a liquid of identical molecules

Phys. Rev. Lett.

Teng Wu, John W. Blanchard, Derek F. Jackson Kimball, Min Jiang, and Dmitry Budker

Accepted 15 June 2018

✓ Suppression of Magnetic Field Fluctuations

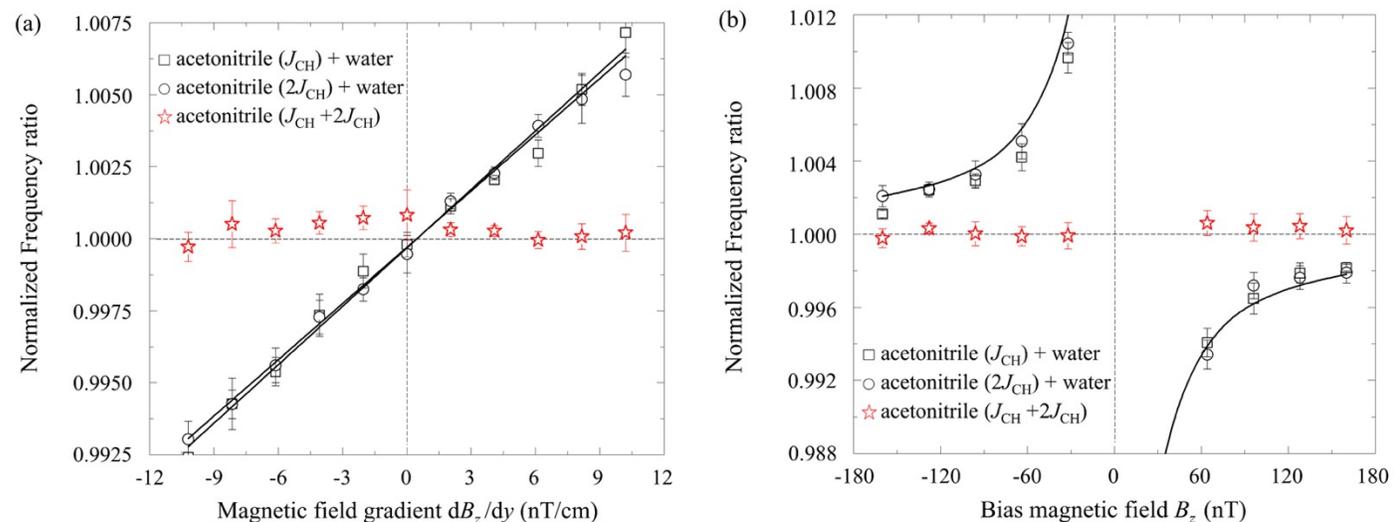


Sample: 2- ^{13}C -acetonitrile ($^{13}\text{CH}_3\text{CN}$), $\sim 100 \mu\text{L}$, 5 mm NMR tube

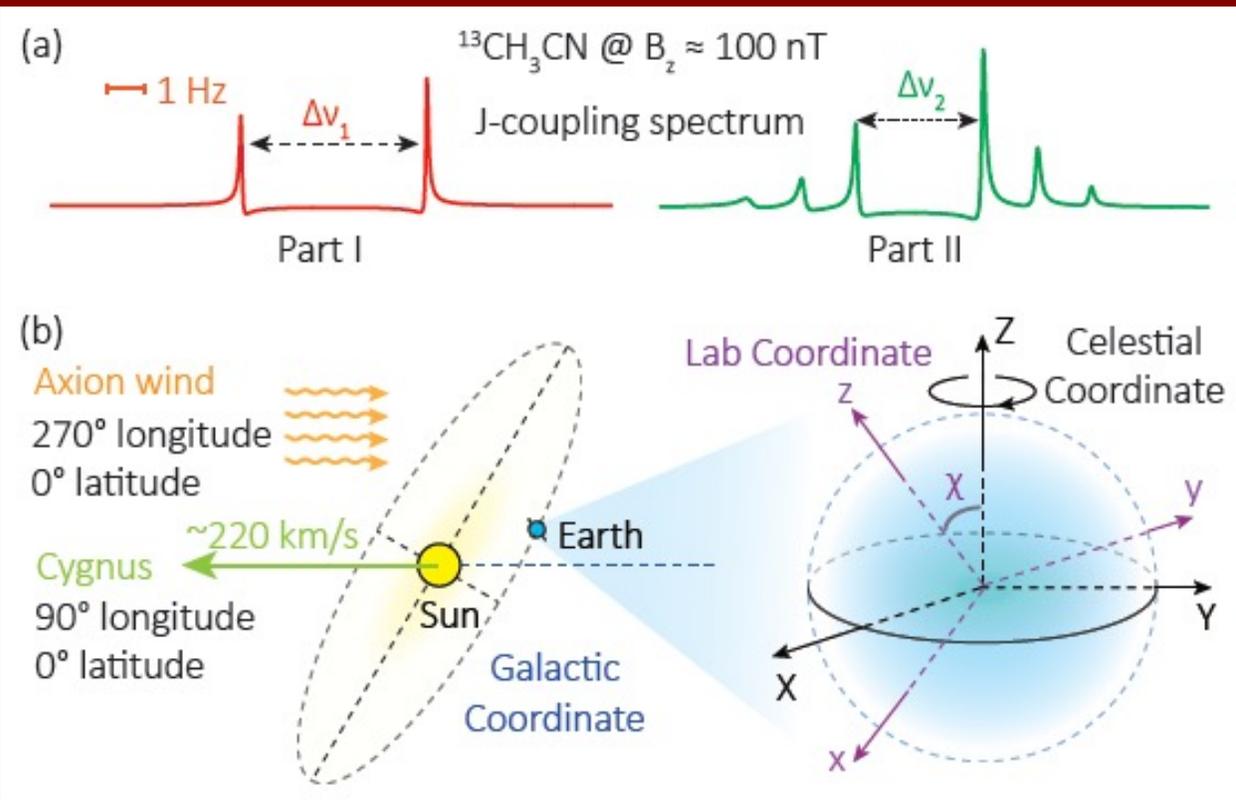
Polarization method: 1.4 Tesla Halbach magnet

Detection method: Rubidium atomic magnetometer ($10 \text{ fT/Hz}^{1/2}$)

✓ Suppression of Magnetic Field Gradient Systematic Effects

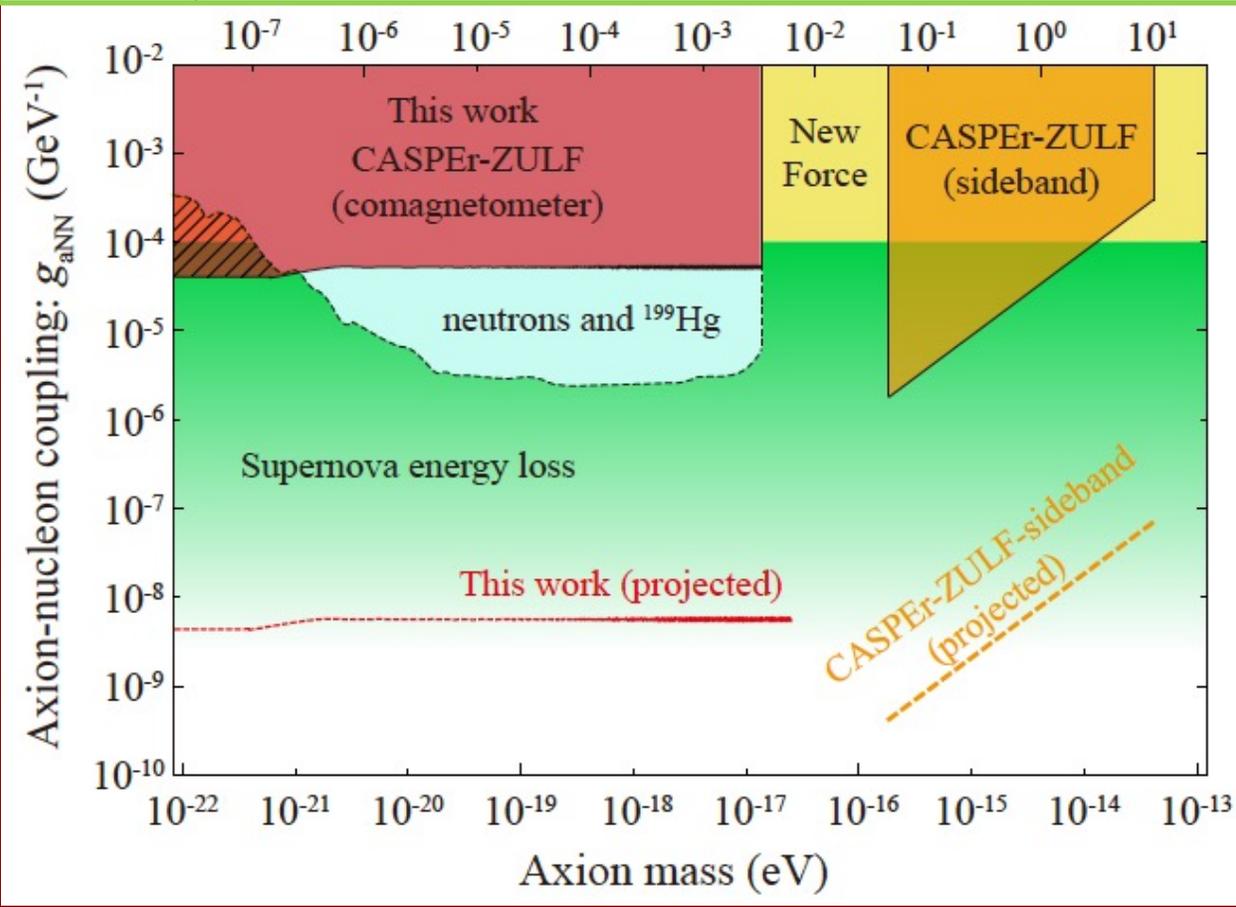


ZULF-comagnetometer



PHYSICAL REVIEW LETTERS **122**, 191302 (2019)

Teng Wu,¹ John W. Blanchard,¹ Gary P. Centers,¹ Nataniel L. Figueroa,¹ Antoine Garcon,¹ Peter W. Graham,² Derek F. Jackson Kimball,³ Surjeet Rajendran,⁴ Yevgeny V. Stadnik,¹ Alexander O. Sushkov,⁵ Arne Wickenbrock,¹ and Dmitry Budker^{1,4,6}



Recent **strong** limits (not CASPER):

- I. M. Bloch, Y. Hochberg *et al* (2020+2021) **comagnetometer**
- Jiang Min *et al* (2021) **spin amplifier + maser**

CASPER: NMR based ALP-search program

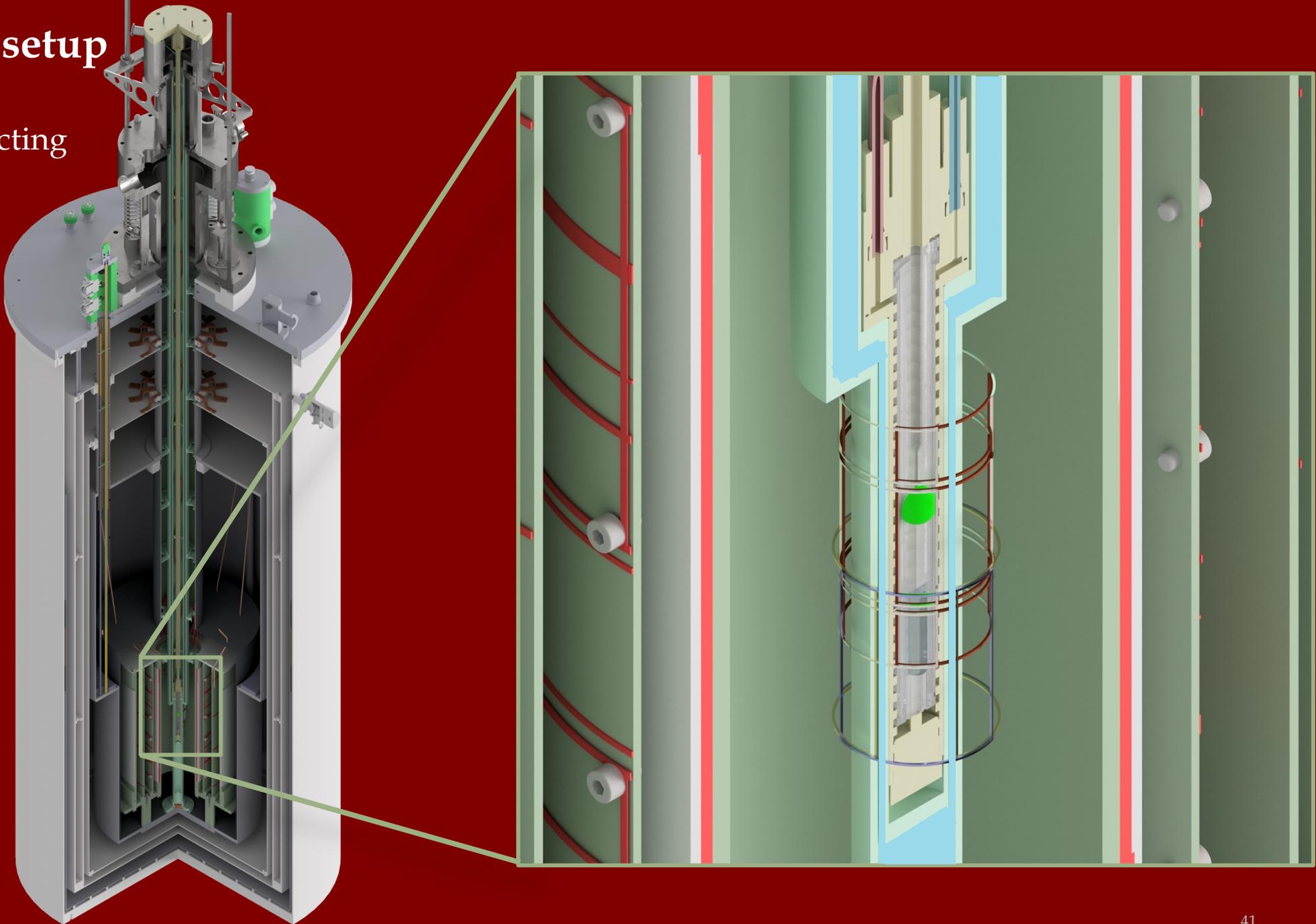
- Higher frequencies: data runs start in 2021 (**virus** permitting)!



Cryogenics magnet; $B < 0.15$ T (< 1.6 MHz for ^{129}Xe)

CASPER-gradient setup

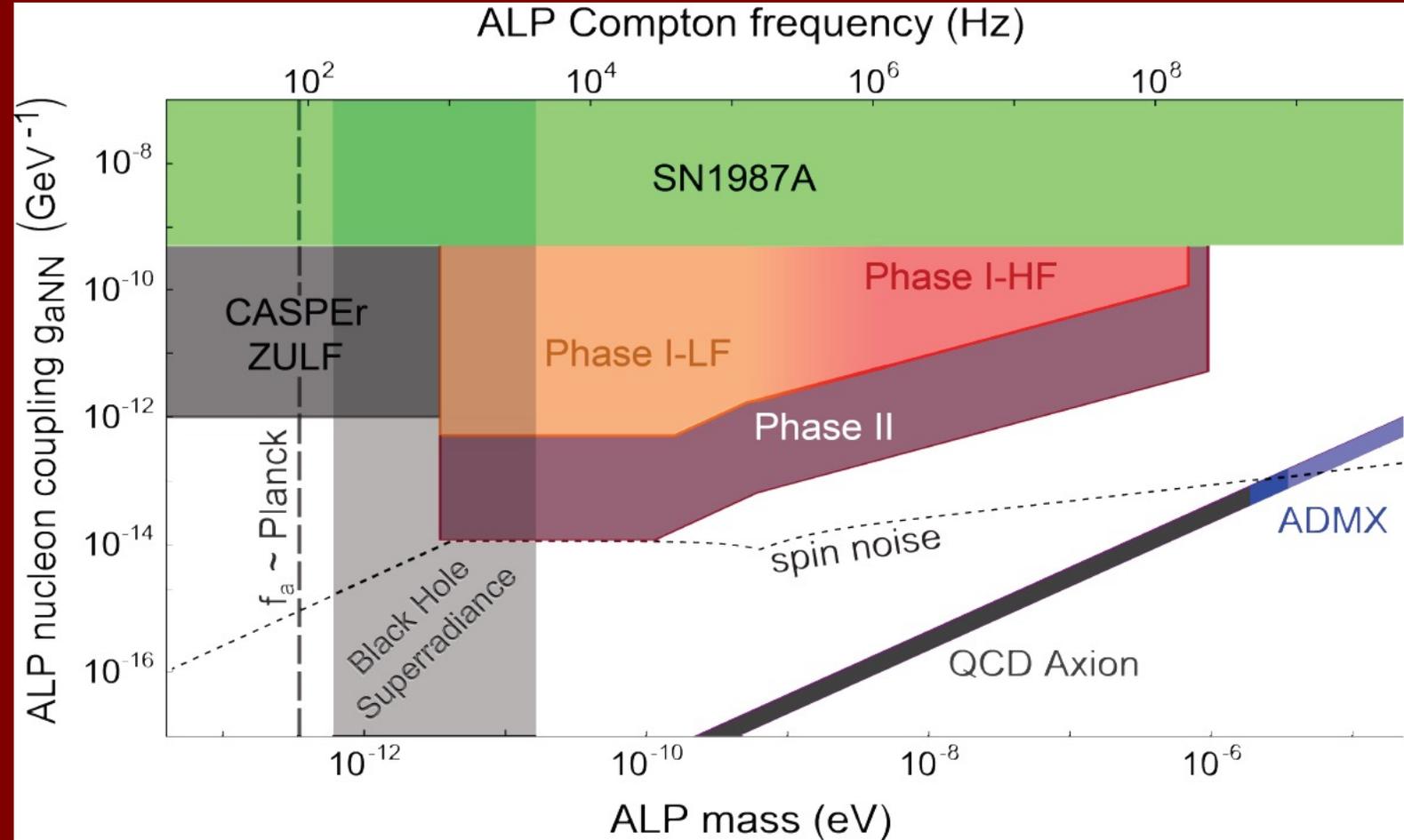
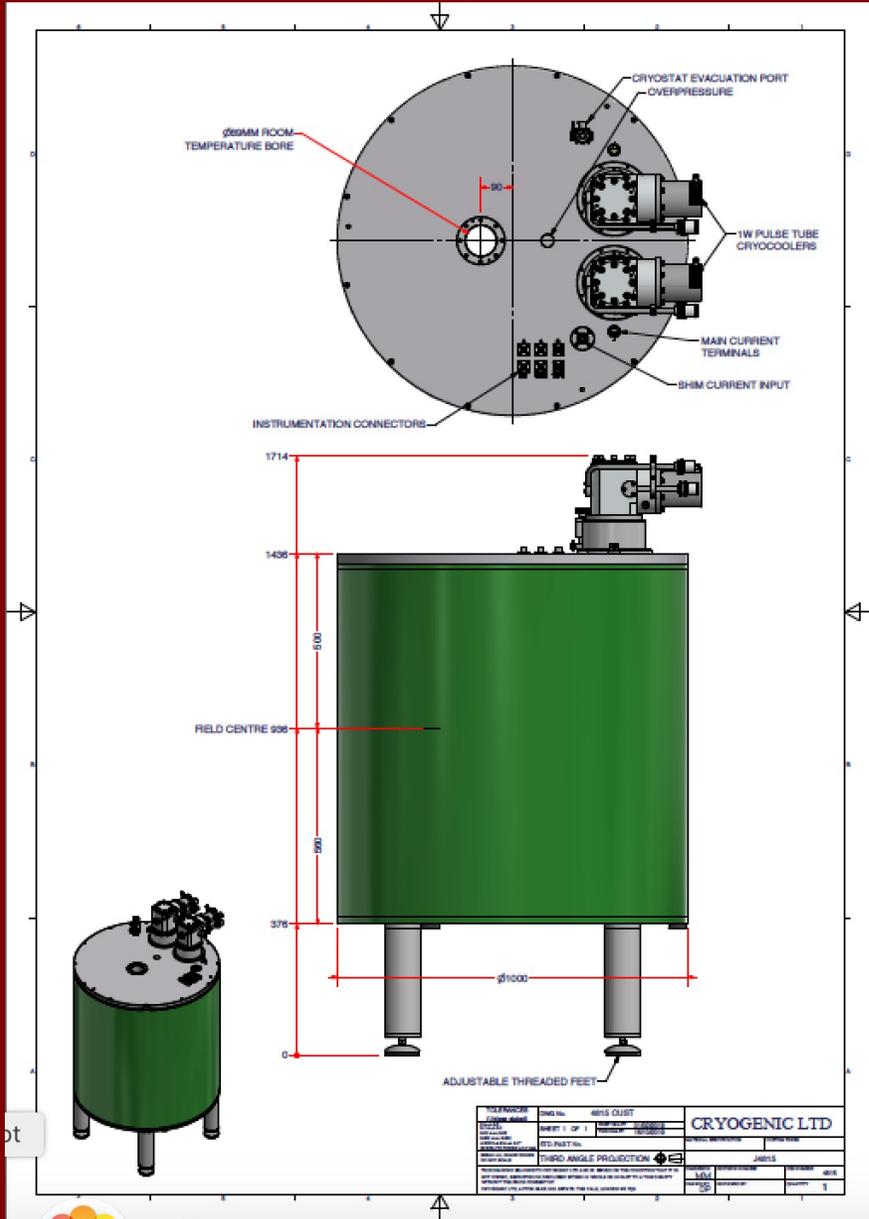
Cryogenics Lt.
0.1 T (1 kG) superconducting
Cold, wet bore
Superconducting shield



H. Bekker

CASPER: NMR based ALP-search program

- Even higher frequencies are in the plan (~2 years):



Cryogenics magnet; $B < 14.1$ T (166 MHz for ^{129}Xe)

Search for axion-like dark matter with spin-based amplifiers

Min Jiang,^{1,2,3, a)} Haowen Su,^{1,2,3, a)} Antoine Garcon,^{4,5} Xinhua Peng,^{1,2,3, b)} and Dmitry Budker^{4,5,6}

¹⁾Hefei National Laboratory for Physical Sciences at the Microscale and Department of Modern Physics, University of Science and Technology of China, Hefei 230026, China

²⁾CAS Key Laboratory of Microscale Magnetic Resonance, University of Science and Technology of China, Hefei 230026, China

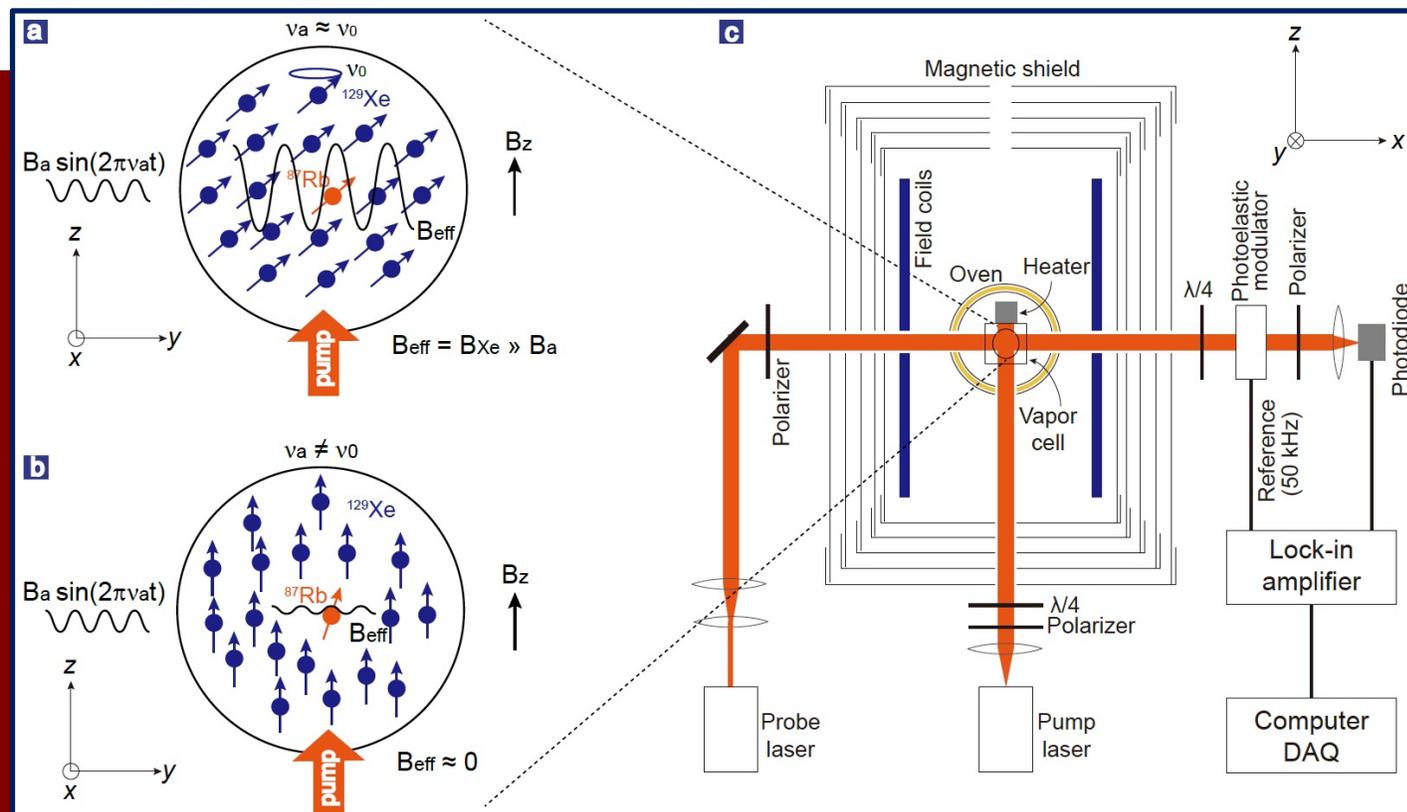
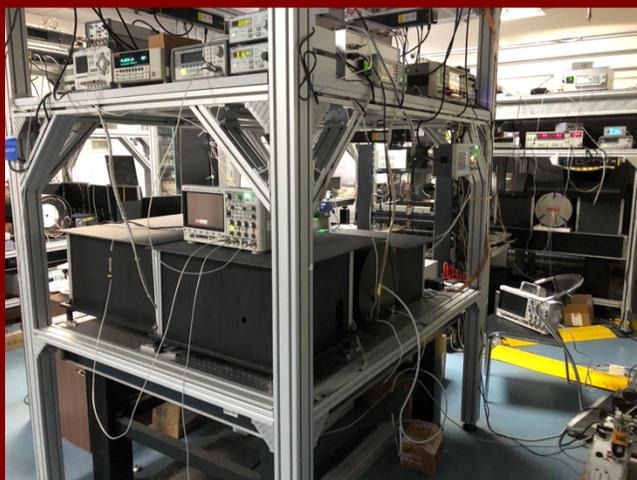
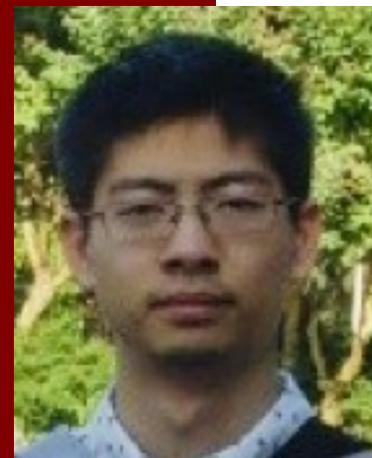
³⁾Synergetic Innovation Center of Quantum Information and Quantum Physics, University of Science and Technology of China, Hefei 230026, China

⁴⁾Helmholtz-Institut, GSI Helmholtzzentrum für Schwerionenforschung, Mainz 55128, Germany

⁵⁾Johannes Gutenberg University, Mainz 55128, Germany

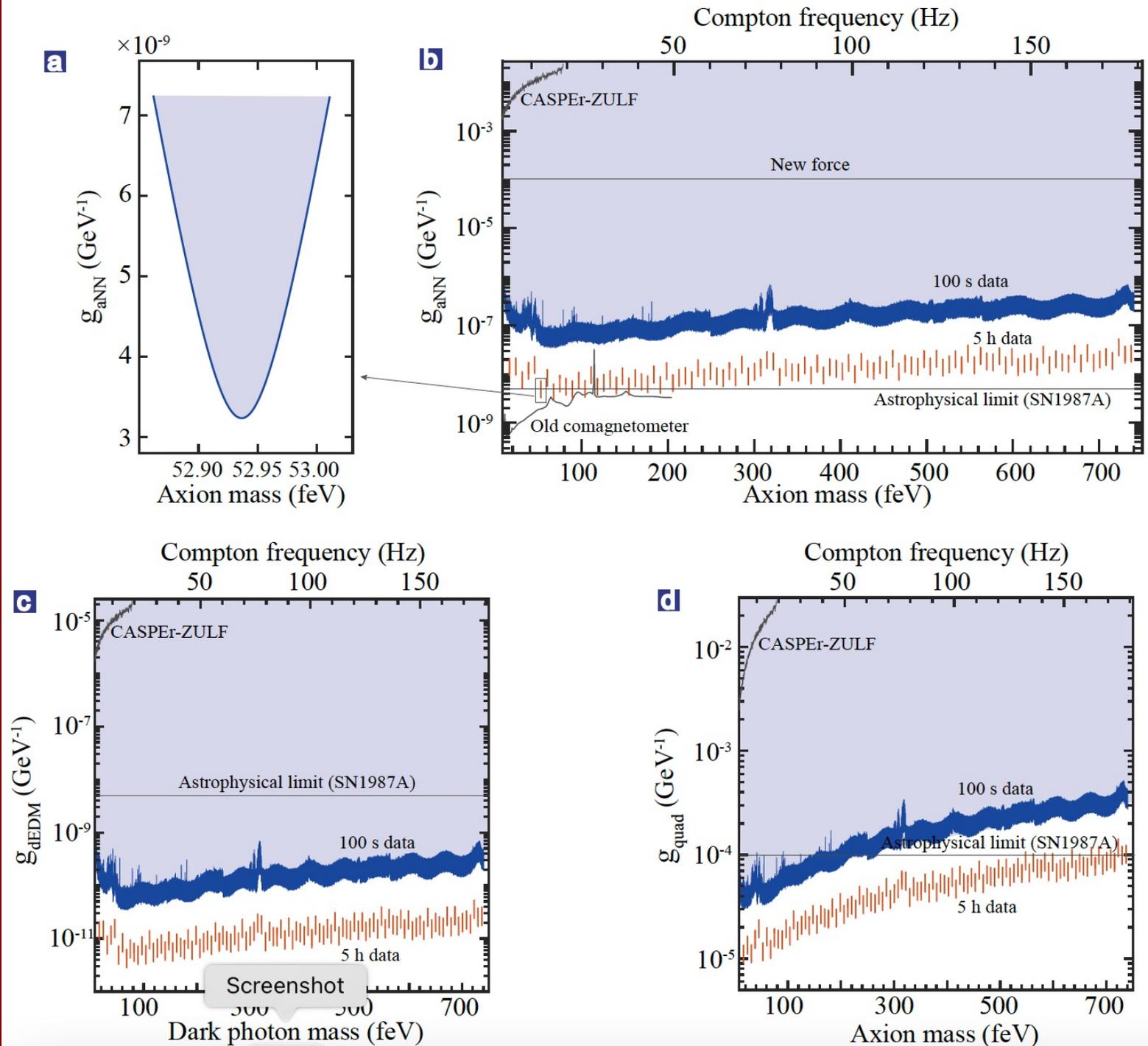
⁶⁾Department of Physics, University of California, Berkeley, CA 94720-7300, USA

(Dated: 3 February 2021)

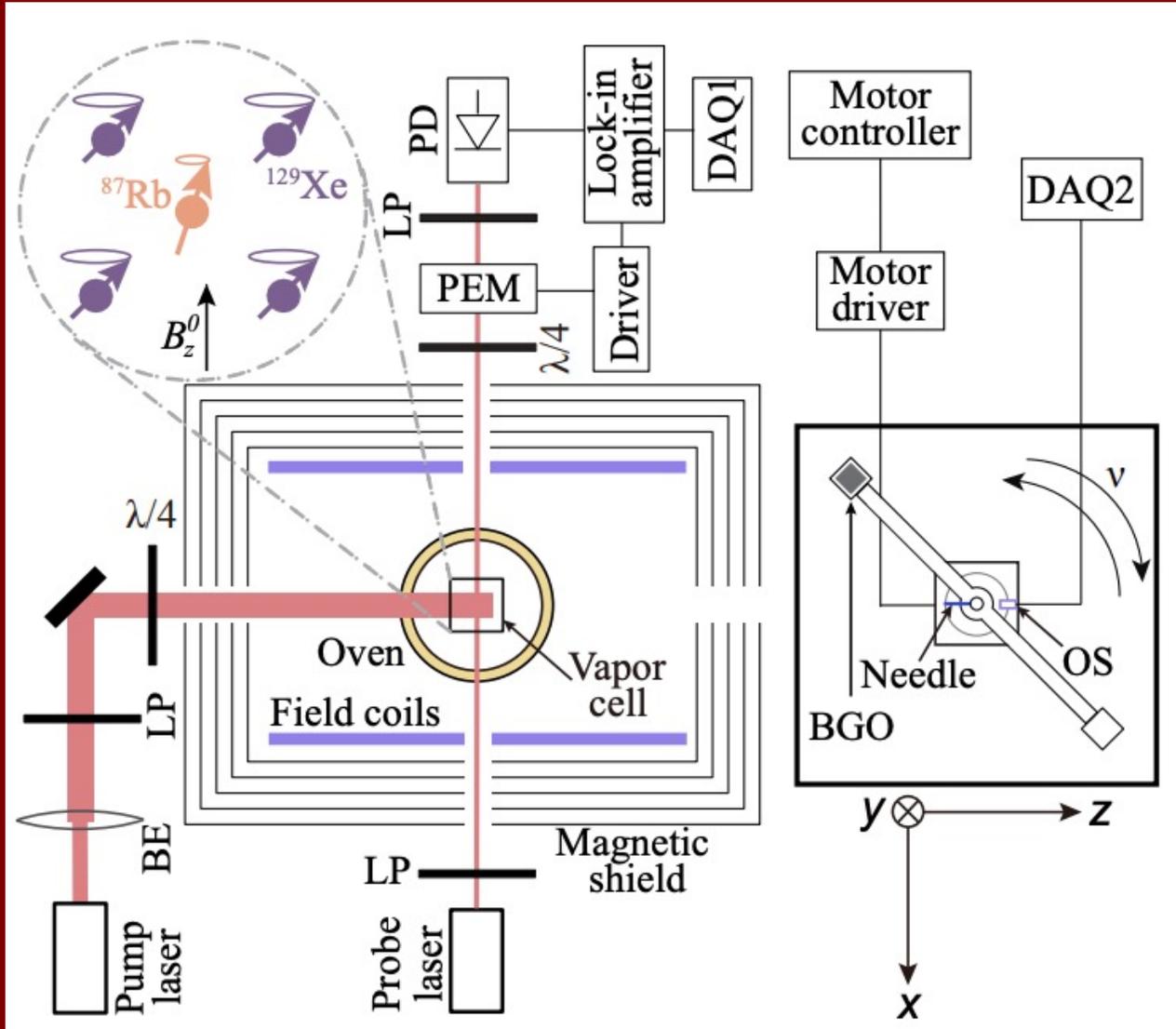


[arXiv:2102.01448](https://arxiv.org/abs/2102.01448)

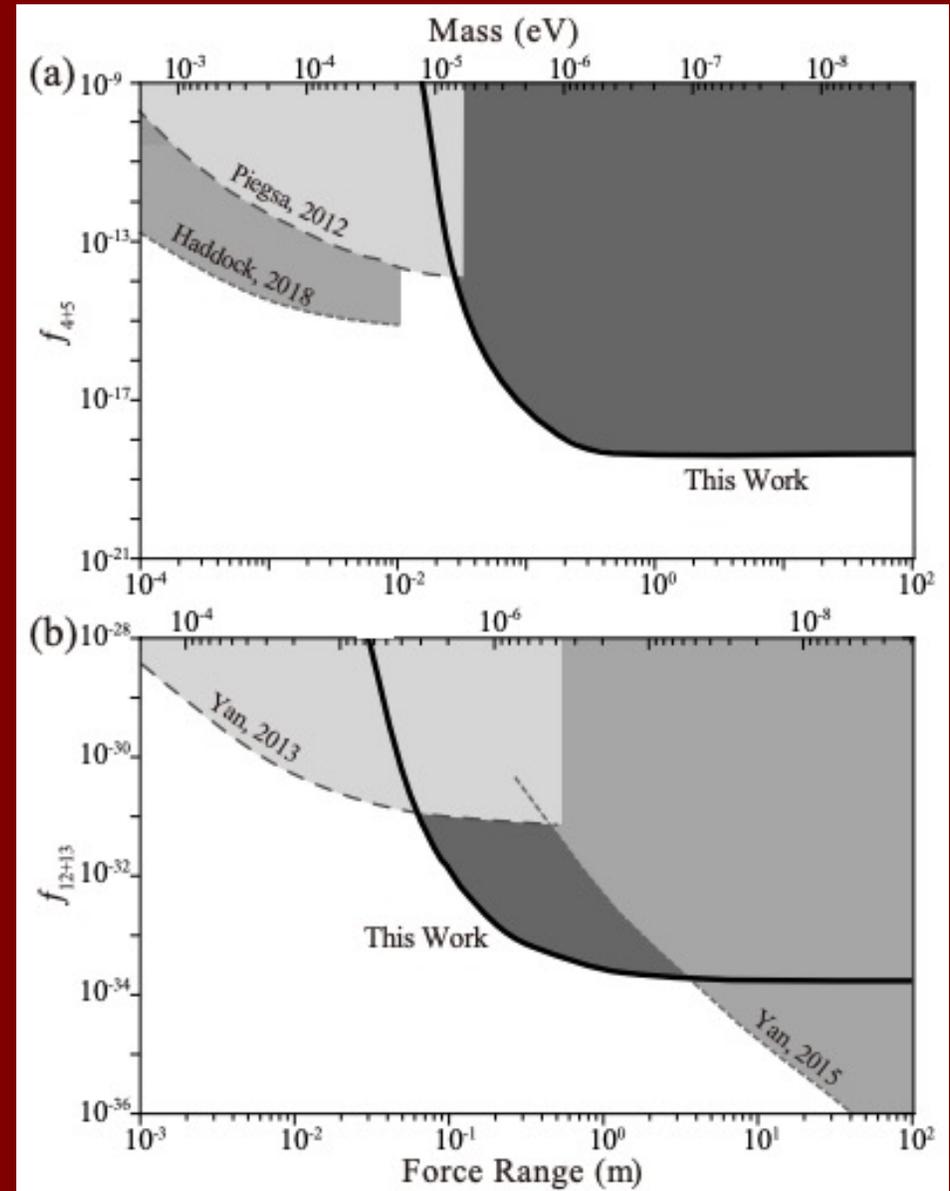
[arXiv:2102.01448](https://arxiv.org/abs/2102.01448)



Spin-Amplifier search for “fifth forces” (USTC)



Haowen Su, et al, arXiv:2103.15282



Floquet maser

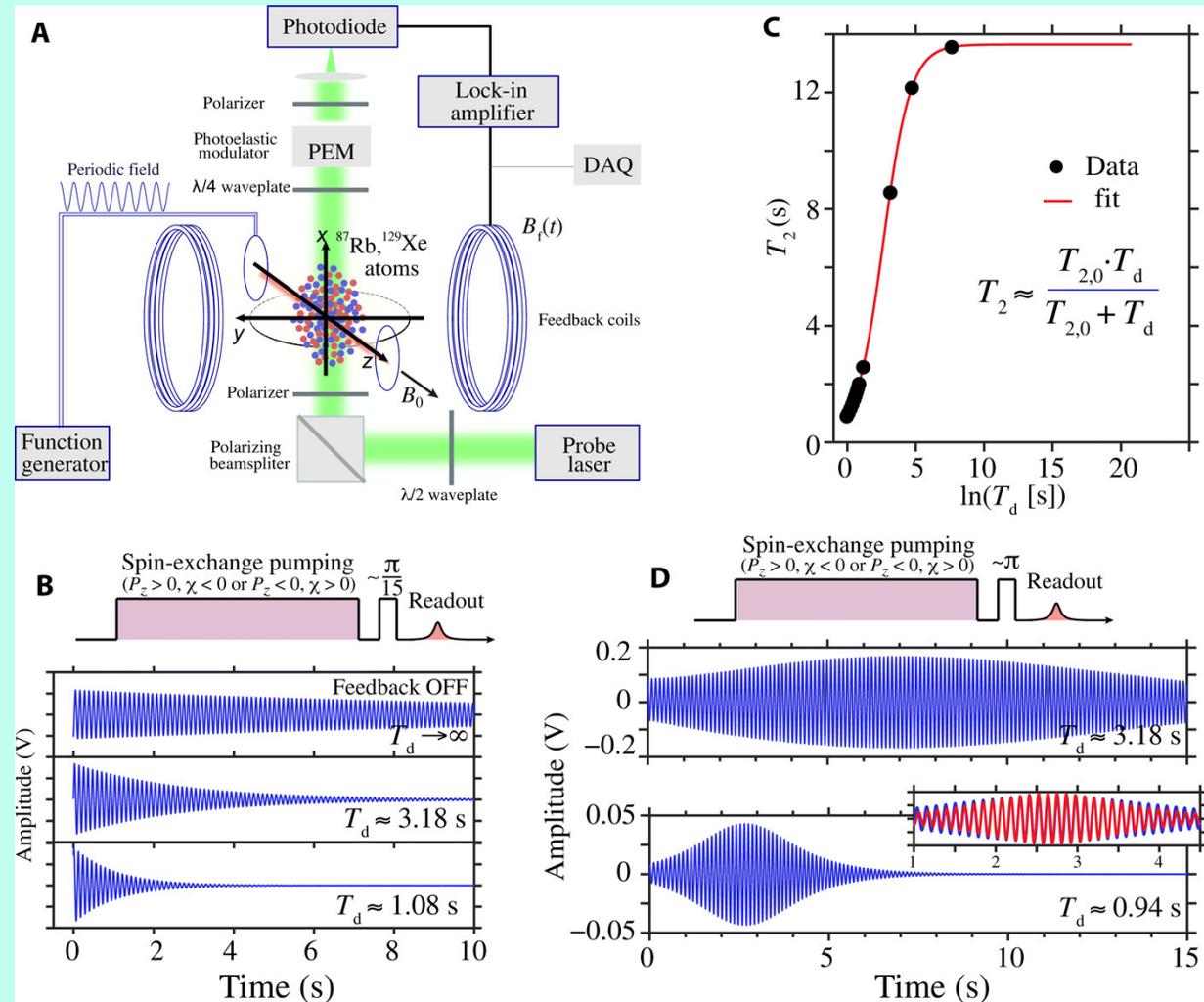
Science Advances 17 Feb 2021:
Vol. 7, no. 8, eabe0719
DOI: 10.1126/sciadv.abe0719

Min Jiang^{1,2,3}, Haowen Su^{1,2,3}, Ze Wu^{1,2,3}, Xinhua Peng^{1,2,3,*} and Dmitry Budker^{4,5,6}

+ See all authors and affiliations

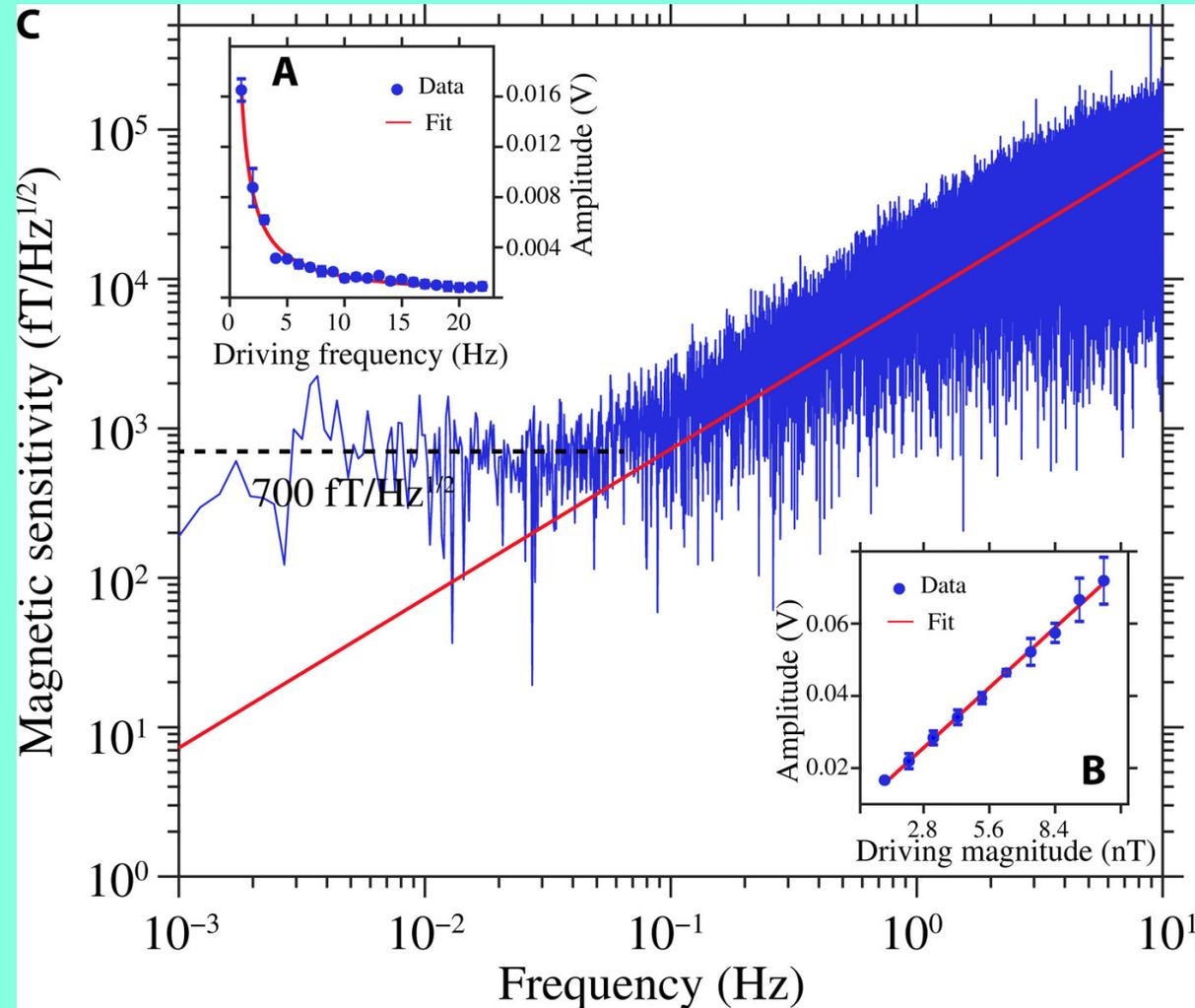


Schematic of experimental setup and damping feedback mechanism



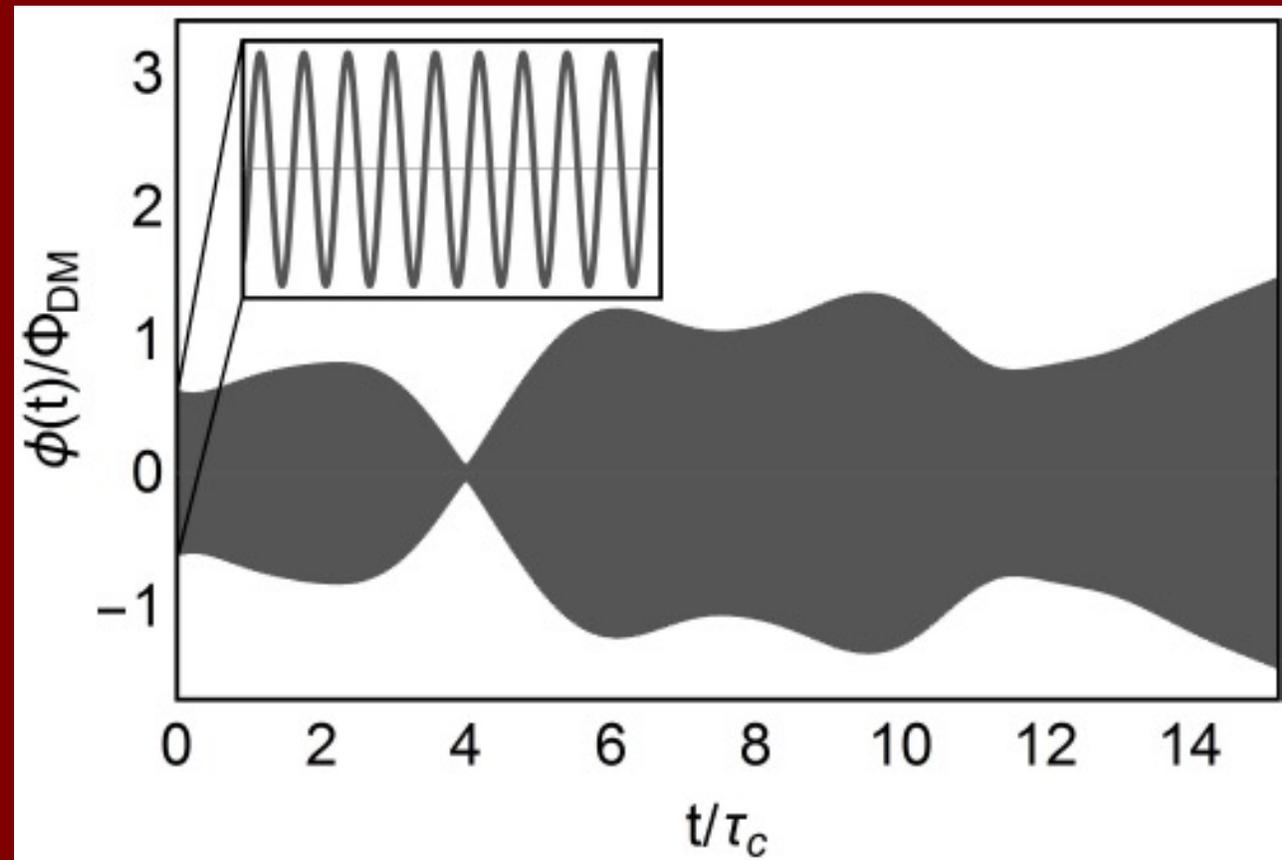
Min Jiang et al. Sci Adv 2021;7:eabe0719

Maser-based magnetometry on the first-order Floquet sideband of ^{129}Xe



Min Jiang et al. *Sci Adv* 2021;7:eabe0719

Stochastic nature of bosonic DM important for measurements not longer than coherence time

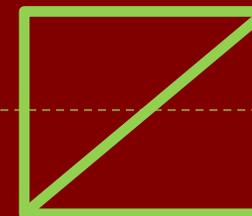


Gary P. Centers et al , [arXiv:1905.13650](https://arxiv.org/abs/1905.13650) (2019-2021)

Analogy with chaotic light



Thermal light source



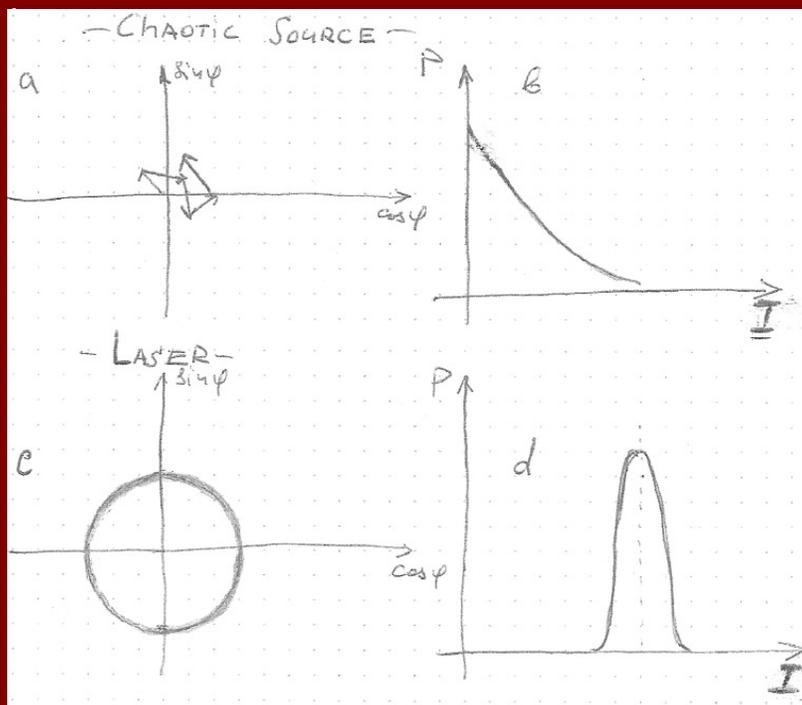
Polarizer



Detector

➤ Q: What is the most probable **instantaneous intensity** ?

➤ A: **zero**



$$p(E_s) \propto \exp \left\{ -\frac{E_s^2}{2NE_0^2} \right\} dE_s. \quad (5.26)$$

The combined probability distribution for the independent cosine and sine amplitudes is then

$$p(E_c, E_s) \propto \exp \left\{ -\frac{E_s^2 + E_c^2}{2NE_0^2} \right\} dE_s dE_c = \exp \left\{ -\frac{I}{\bar{I}} \right\} d\varphi E dE, \quad (5.27)$$

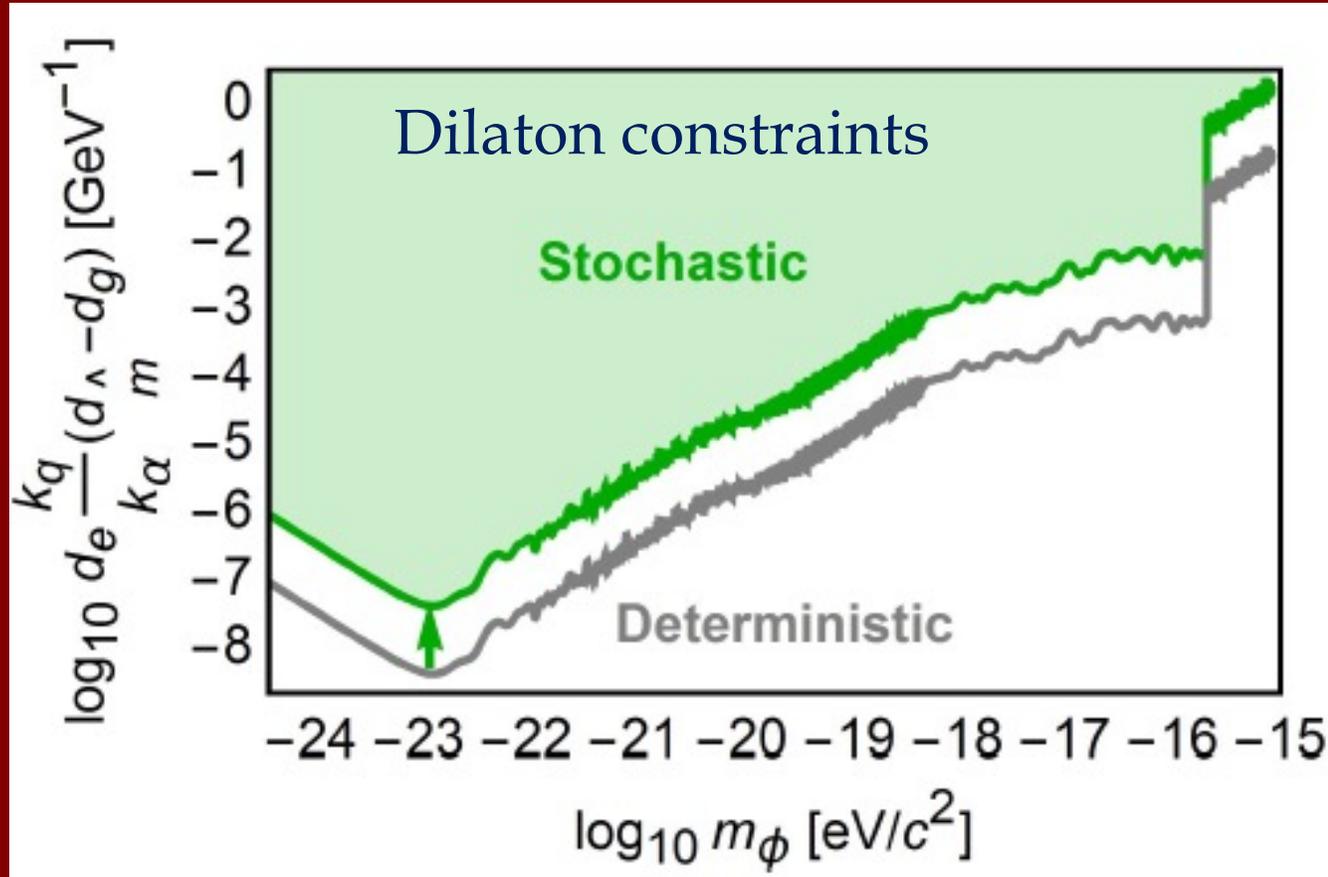
where we have introduced the instantaneous cycle-averaged intensity I , its mean value \bar{I} , the *phase angle* φ , and the total field amplitude E , which is non-negative.

We can now convert this into a distribution of cycle-averaged instantaneous intensity by integrating over φ and using the fact that $I \propto E^2$, so, correspondingly, $dI \propto 2EdE$. With this, Eq. (5.27) becomes

$$p(I) \propto \exp \left\{ -\frac{I}{\bar{I}} \right\} dI, \quad (5.28)$$

From: D. Budker and A. Sushkov *Physics on Your Feet*, OUP 2015

Significant effect on low-frequency DM searches



- Also **velocity** (magnitude and direction)

Gary P. Centers, John W. Blanchard, Jan Conrad, Nataniel L. Figueroa, Antoine Garcon, Alexander V. Gramolin, Derek F. Jackson Kimball, Matthew Lawson, Bart Pelssers, Joeseoph A. Smiga, Yevgeny Stadnik, Alexander O. Sushkov, Arne Wickenbrock, Dmitry Budker, and Andrei Derevianko, *Stochastic fluctuations of bosonic dark matter*, [arXiv:1905.13650](https://arxiv.org/abs/1905.13650) (2019)

Atomic clock searches for scalar **DM**

DILATON DM ?



Searching for dilaton dark matter with atomic clocks

Asimina Arvanitaki*

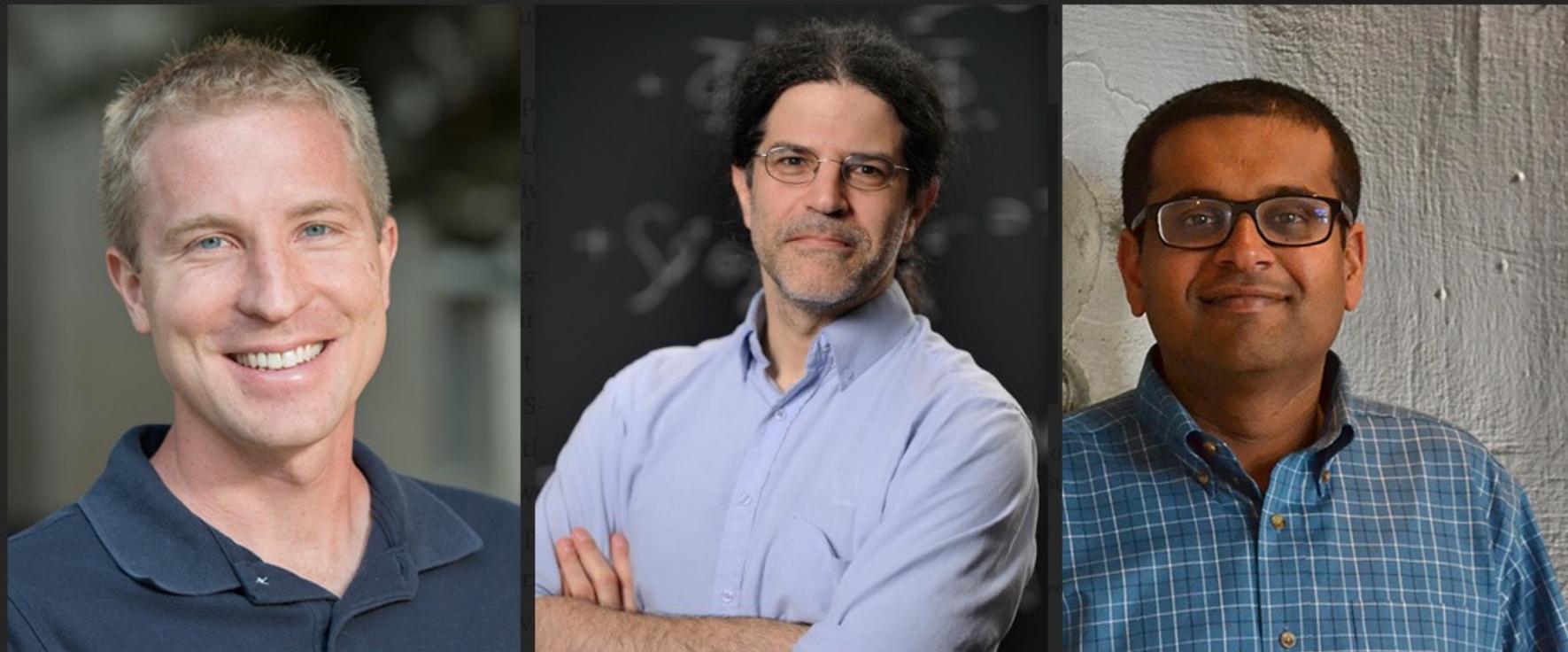
Perimeter Institute for Theoretical Physics, Waterloo, Ontario, N2L 2Y5, Canada

Junwu Huang[†] and Ken Van Tilburg[‡]

*Stanford Institute for Theoretical Physics, Department of Physics,
Stanford University, Stanford, CA 94305, USA*

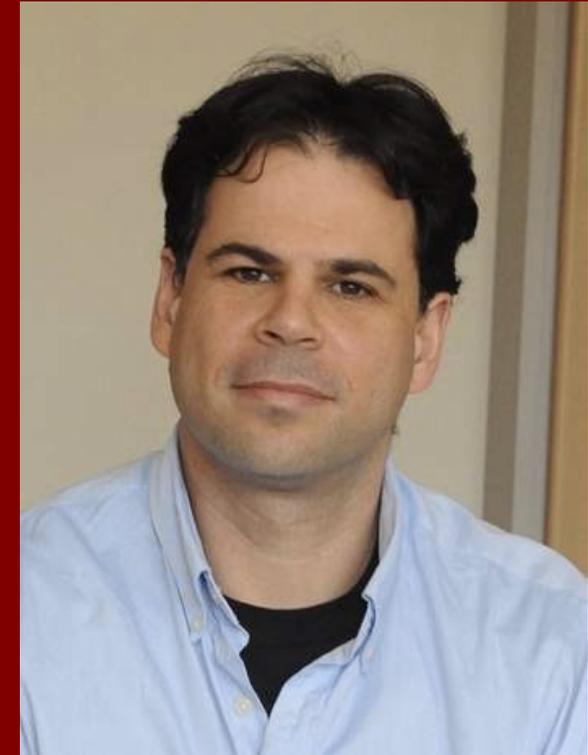
(Dated: May 14, 2014)

Relaxions in the dark



cosmological constant — a mystifyingly tiny number that defines the
From left: Peter Graham of Stanford University, David Kaplan of Johns Hopkins
University and Surjeet Rajendran of the University of California, Berkeley.
drives the accelerating expansion of the universe.

—
Linda A. Cicero/Stanford News Service, Will Kirk/Johns Hopkins University, Sarah Wittmer



Gilad Perez
Weizmann Institute

RELAXION



- ◆ Relaxion => solves **hierarchy** and **strong CP** problems

Graham, Kaplan & Rajendran (15); Hook, Marques-Tavares; Gupta, Komargodski, Perez & Ubaldi (16);
Davidi, Gupta, Perez, Redigolo & Shalit; Gupta; Nelson & Prescod-Weinstein (17)

- ◆ Axion-like particle but mixes with the Higgs => has **scalar interactions**

Flacke, Frugiuele, Fuchs, Gupta & Perez; Choi & Im (16)

- ◆ Minimal model provides viable axion-like dark matter (DM);

for: $10^{-10} \text{eV} \lesssim m_{\phi \equiv \text{relax}} \lesssim 10^{-3} \text{eV}$

Banerjee, Kim & Perez (18)

- ◆ DM can form **stars & halos around Earth** \ w large over densities



Relaxion stars and their detection via atomic physics

Abhishek Banerjee, Dmitry Budker, Joshua Eby, Hyungjin Kim & Gilad Perez

[COMMUNICATIONS PHYSICS](#) (2020) 3:1 [arXiv:1902.08212](#)

Fast changing “constants” ?

- Jun Ye *et al*: cavity-clock comparison
- Even faster: A “weekend” experiment @ Mainz ?



Dr. Dionysis Antypas
Helmholtz Institute, JGU Mainz



Oleg Tretiak
HIM, JGU Mainz



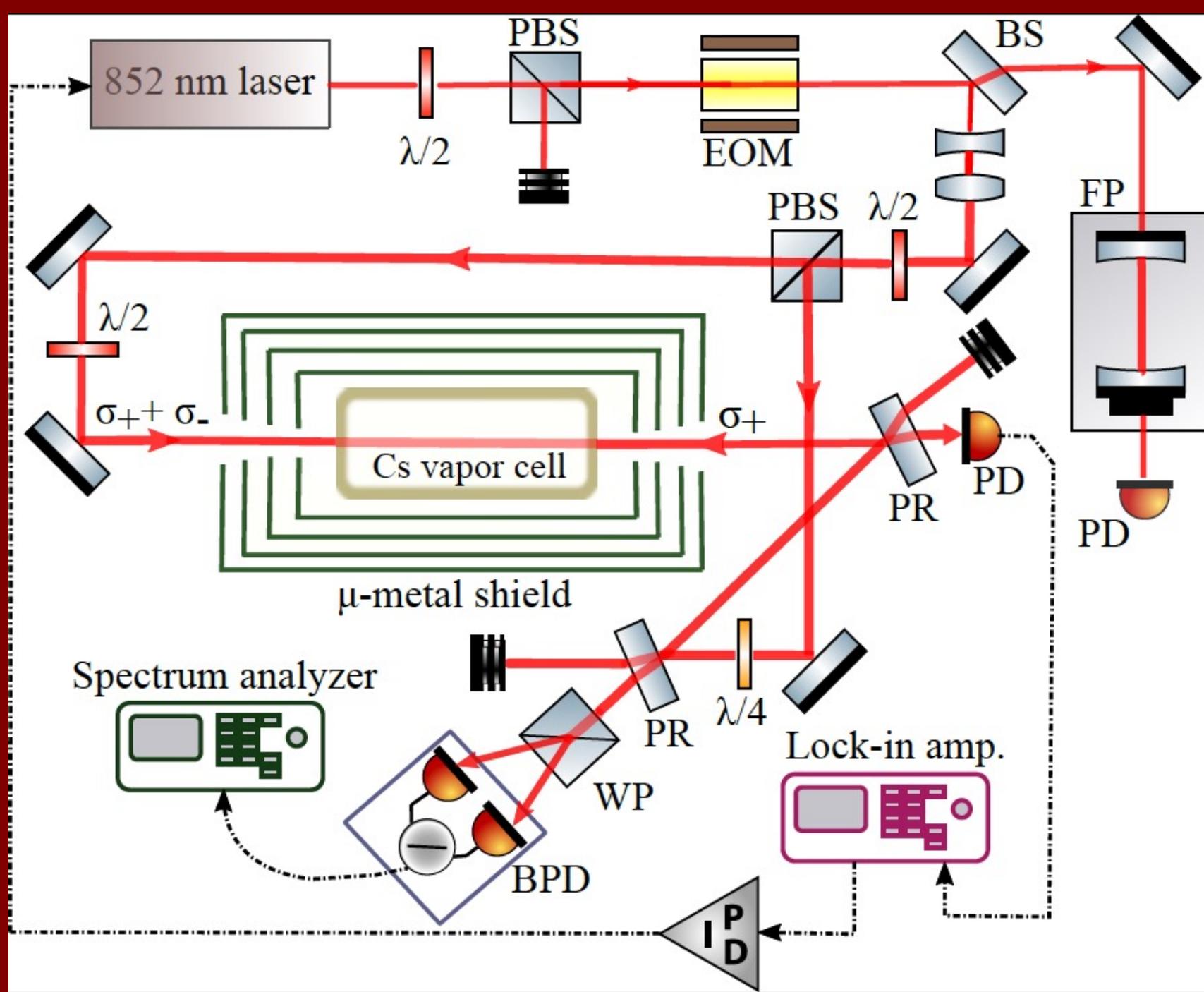
Prof. Roee Ozeri
Weizmann Institute



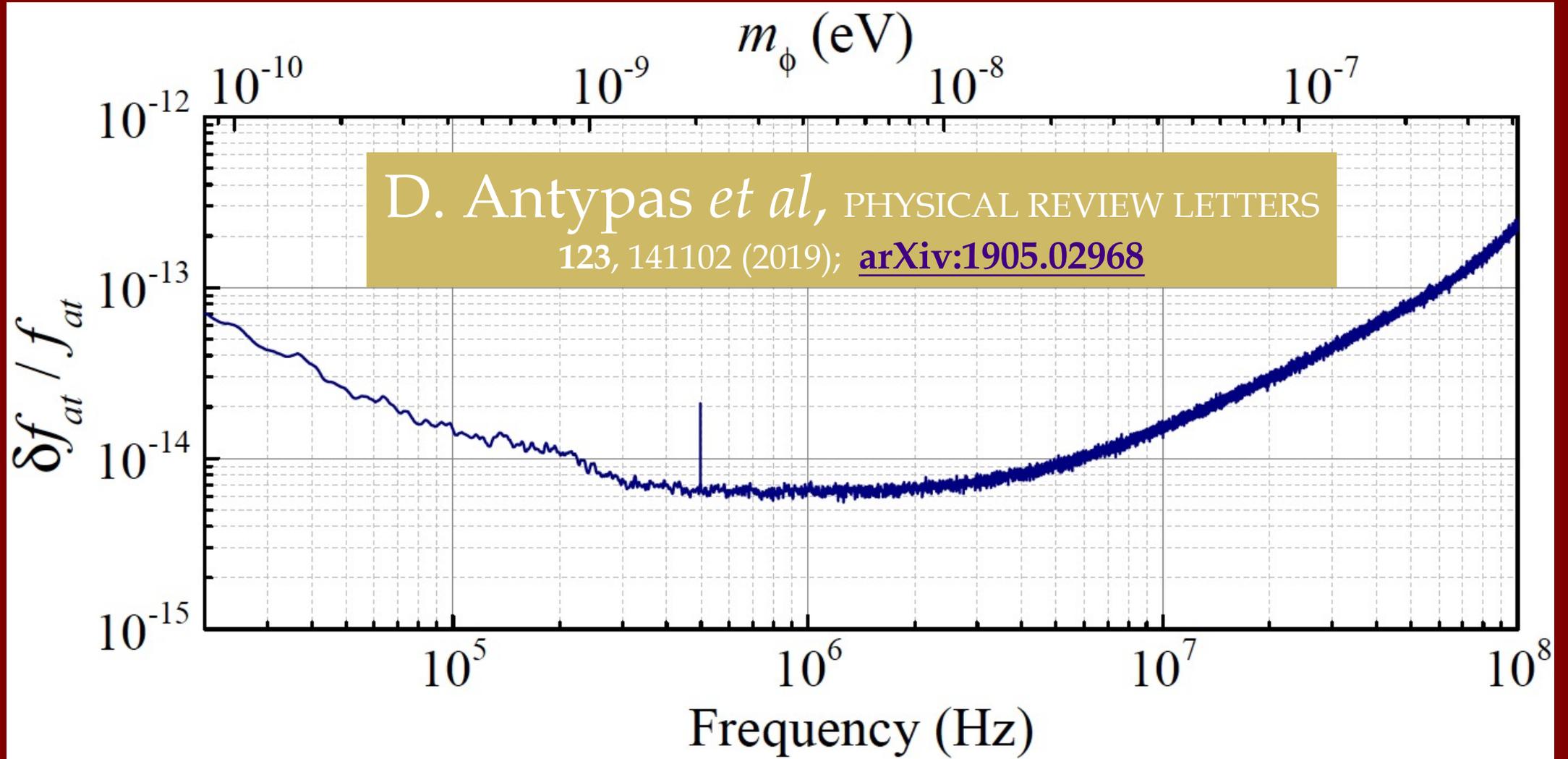
Antoine Garcon
HIM, JGU Mainz

FAST OSCILLATING SCALAR **DM**

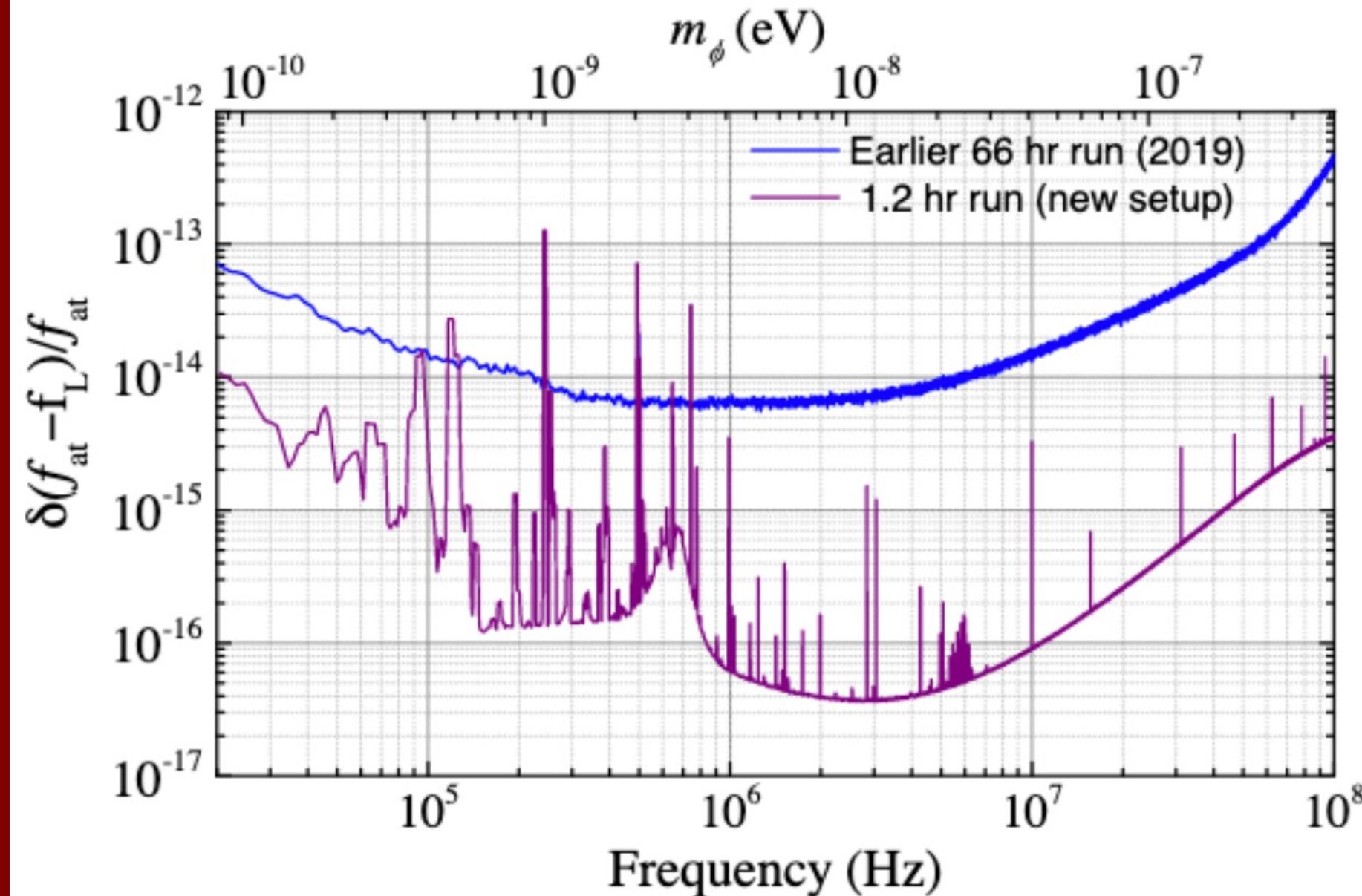
Weekend **R**e_{laxion} **S**earch **L**aboratory



Cs D2 line (852 nm)



* Sensitive to variation of α and m_e $2d_\alpha + d_{m_e}$



- Projection: 10^{-18} (Cs)
- WReSL-molecules with S. Schiller group
- Sensitive to oscill. of nuclear mass

Fast Variation of Fundamental “Constants”

- Dionysios Antypas, Dmitry Budker, Victor V. Flambaum, Mikhail G. Kozlov, Gilad Perez, and Jun Ye,

[ANNALEN DER PHYSIK 2020, 1900566; arXiv:1912.01335](#)

- **Old thinking:** only *dimensionless* constants may vary
- **New thinking:** $m_e / \langle m_e \rangle$; $\alpha / \langle \alpha \rangle$, ... are **OK**
- Origin of apparent variations: **bosonic fields**
- BSM (and BBSM) physics is tricky!

$$\begin{aligned}\mathcal{L}_{\text{free}} = & -\frac{1}{4\alpha} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} (\partial_\mu \varphi \partial^\mu \varphi - m^2 \varphi^2) \\ & + \mathcal{L}_{\text{kin}}^{\text{SM}} - \sqrt{2} \frac{m_e}{v} H \bar{L}_e e_R + h.c. \\ & - \mu^2 H^\dagger H + \lambda (H^\dagger H)^2 + \mu_{\phi h} \phi H^\dagger H\end{aligned}$$

Hypothetical Internal Object search with superconducting gravimeters

A network of superconducting gravimeters as a detector of matter with feeble nongravitational

Eur. Phys. J. D **74**, 115 (2020) coupling [arXiv:1912.01900](https://arxiv.org/abs/1912.01900)

Wenxiang Hu¹, Matthew Lawson^{2,3}, Dmitry Budker^{3,4}, Nataniel L. Figueroa³, Derek F. Jackson Kimball⁵, Allen P. Mills Jr.⁶, and Christian Voigt⁷



- Tunnel-through-the-Earth problem ($T \approx 80$ min)
- Generalization to Hypothetical Internally Orbiting matter (HIO)
- Earth is NOT of uniform density
- Small amplitude, near center:

$$T = \frac{2\pi}{\omega_h} = \frac{2\pi}{\sqrt{\frac{4\pi}{3} G \rho_0}} \approx 55 \text{ min}$$

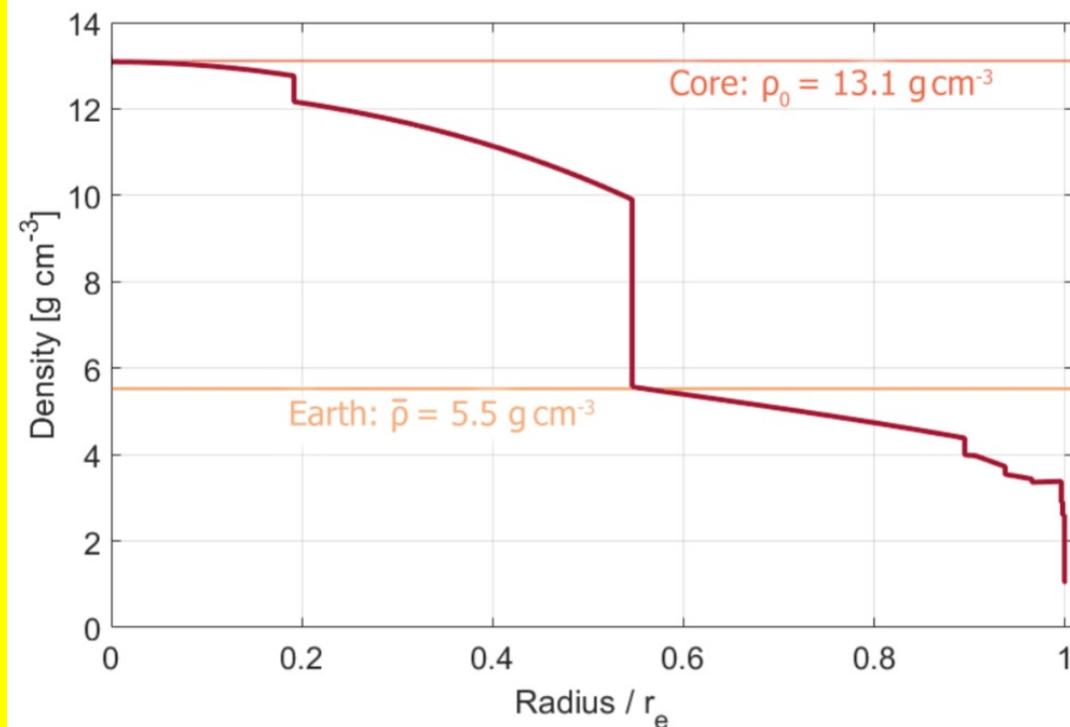


Figure 1: The density profile of the Earth based on the Preliminary Reference Earth model (PREM) [5]. $r_e = 6371$ km is the mean radius of the Earth, $\bar{\rho} = 5.51$ g cm⁻³ is the average density of the Earth, and $\rho_0 = 13.1$ g cm⁻³ is the density at the Earth's center.

Gravimeter Network

4132 Rev. Sci. Instrum., Vol. 70, No. 11, November 1999

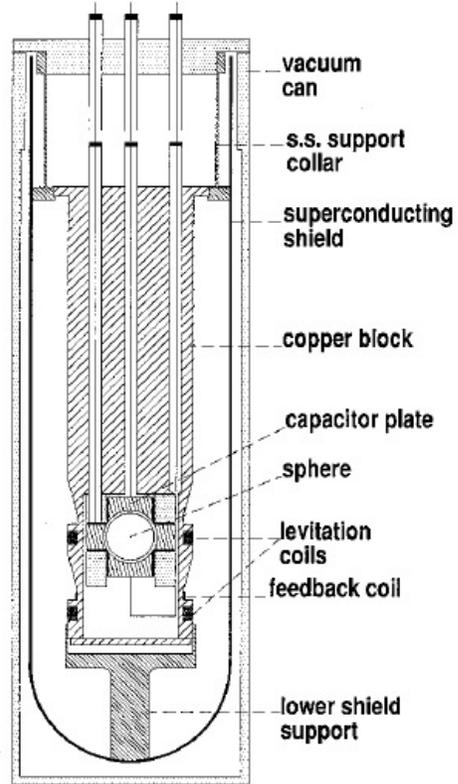


FIG. 1. Diagram of the cryogenic portion of the superconducting gravimeter.

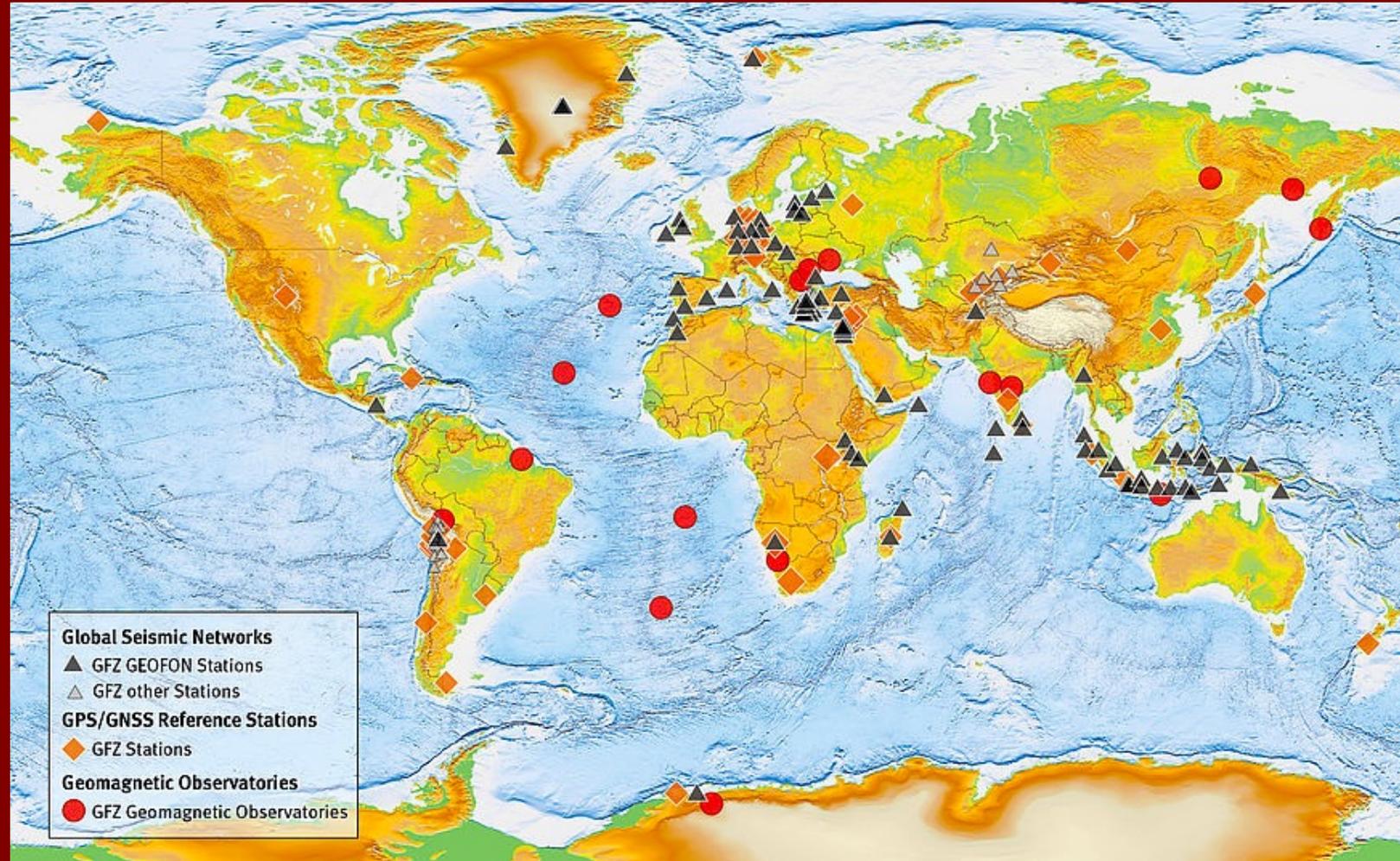


Figure credit: <https://www.gfz-potsdam.de/en/scientific-infrastructure/research-infrastructures/global-observatories/>

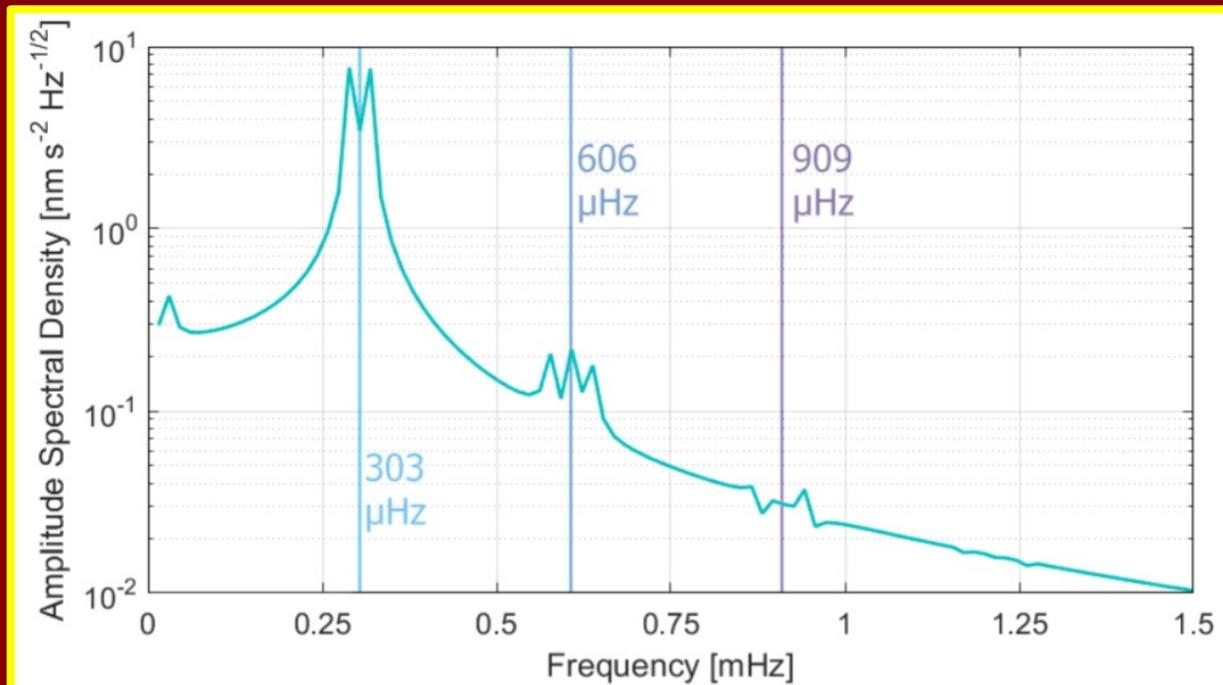


Figure 3: The theoretical spectrum of the HIO orbiting near the center of the Earth, the supposed mass of HIO here is 10^{16} kg, and the amplitude of the oscillation is $0.1 r_e$. In the spectrum, there are signals centered near the first (around $303 \mu\text{Hz}$) and higher (around $606 \mu\text{Hz}$ and $909 \mu\text{Hz}$) harmonics due to the non-linearity of the force. Rotation of the Earth (seen as a small lowest-frequency peak) also leads to splitting of the first- and second-harmonic lines.

See also: C. J. Horowitz and R. Widmer-Schmidrig. Gravimeter search for compact dark matter objects moving in the Earth; arXiv 1912.00940, PRL 2019

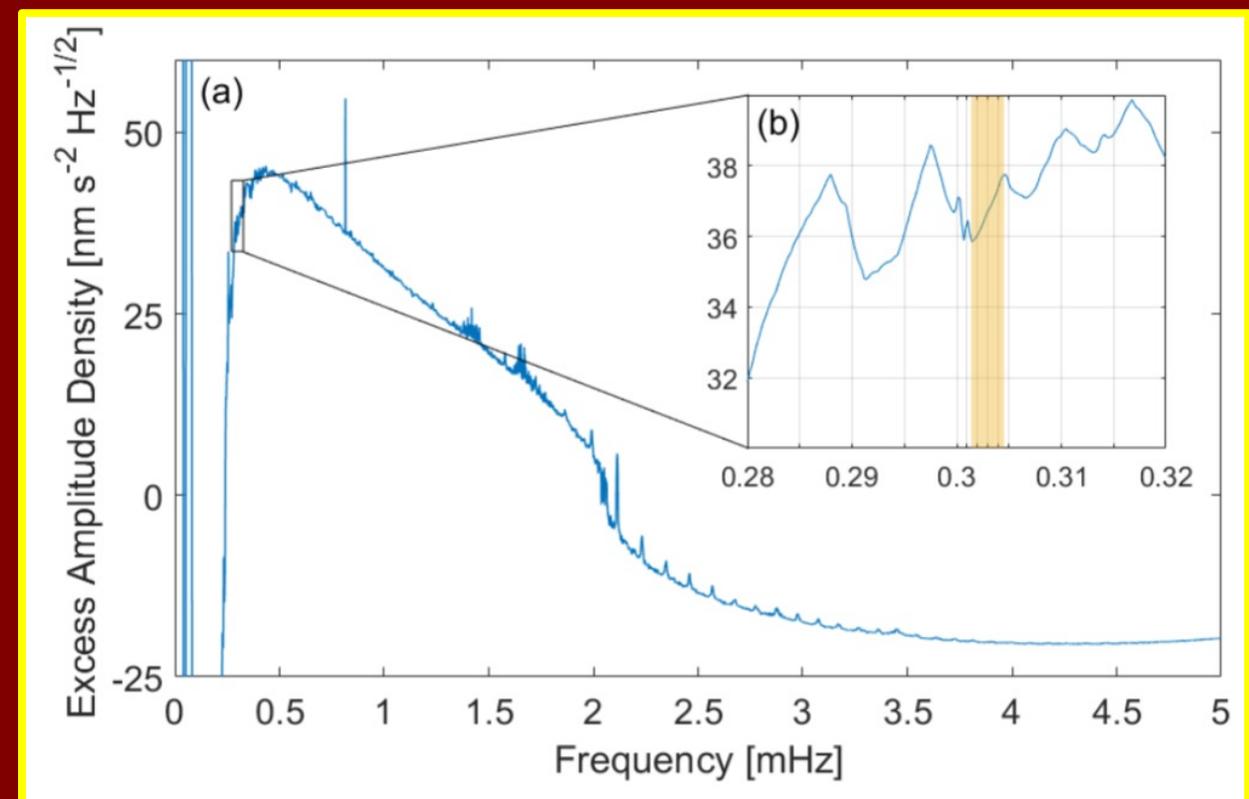


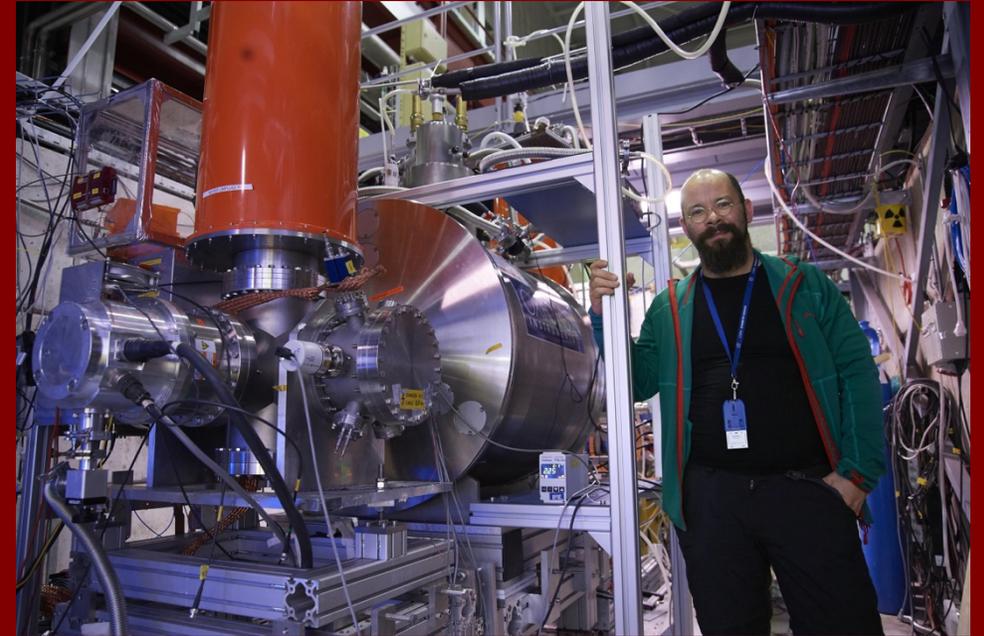
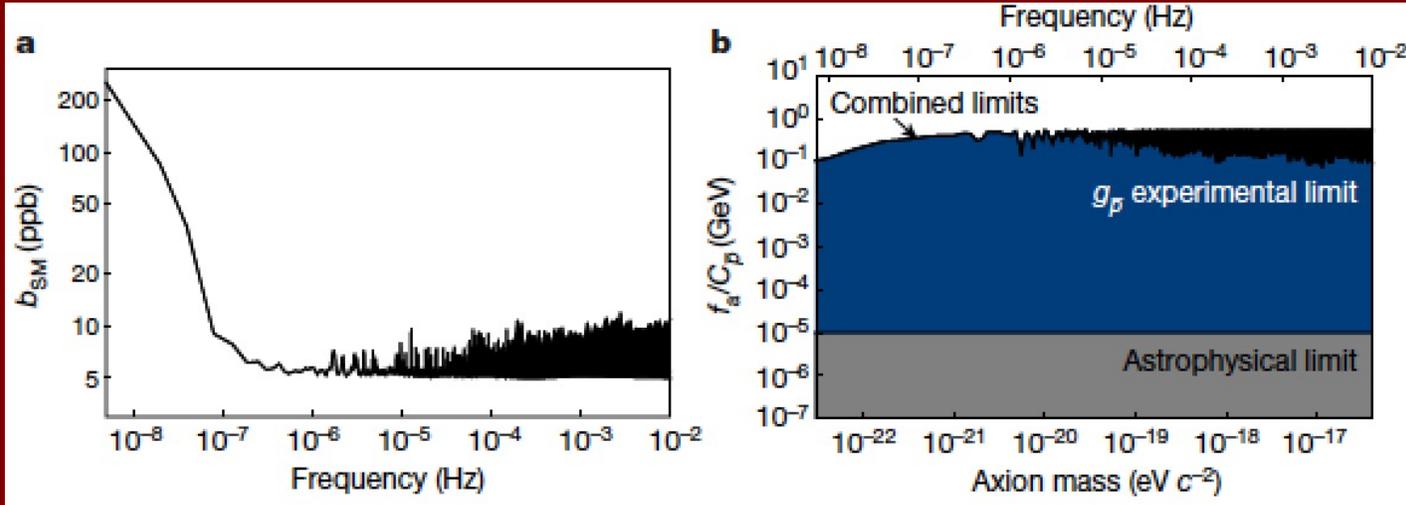
Figure 4: (a) The amplitude spectral density of the IGETS level 3 data sets, with background removal performed prior to averaging. The inset (b) shows details around $303 \mu\text{Hz}$, where the signal from a HIO orbiting near the center of the Earth would lie. The vertical orange strip highlights the range used to look for HIO related oscillations. The large spike around $800 \mu\text{Hz}$ is due to the ${}_0S_0$ “breathing” mode of Earth [23, 5, 24].

Dark Matter search with antimatter

Dark-matter search with antimatter !

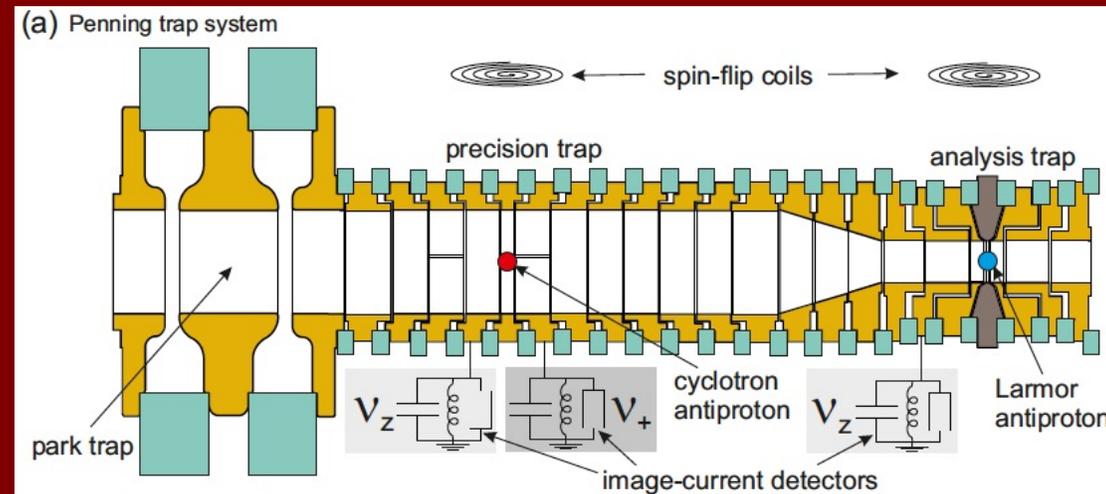
Direct limits on the interaction of antiprotons with axion-like dark matter

310 | Nature | Vol 575 | 14 November 2019



Stefan Ulmer

- ▣ Collaboration with BASE
- ▣ Search for ALP-induced antiproton spin precession



Ch. Smorra



Y. Stadnik⁶⁶

Axion Quark Nuggets

Axion quark nuggets and how a global network can discover them

Dmitry Budker*

*Johannes Gutenberg-Universität Mainz (JGU)—Helmholtz-Institut, 55128 Mainz, Germany
and Department of Physics, University of California, Berkeley, California 94720-7300, USA*

Victor V. Flambaum[†]

*School of Physics, University of New South Wales, Sydney 2052, Australia
and Johannes Gutenberg-Universität Mainz (JGU)—Helmholtz-Institut, 55128 Mainz, Germany*

Xunyu Liang[‡] and Ariel Zhitnitsky[§]

Department of Physics and Astronomy, University of British Columbia, Vancouver V6T1Z1, Canada

<https://arxiv.org/abs/2003.07363>

Axion Quark Nuggets. SkyQuakes and Other Mysterious Explosions

Dmitry Budker*

*Johannes Gutenberg-Universität Mainz - Helmholtz-Institut,
GSI Helmholtzzentrum für Schwerionenforschung, 55128 Mainz, Germany
Department of Physics, University of California, Berkeley, CA, 94720-7300, USA*

Victor V. Flambaum[†]

*School of Physics, University of New South Wales, Sydney 2052, Australia
Johannes Gutenberg-Universität Mainz (JGU) - Helmholtz-Institut,
GSI Helmholtzzentrum für Schwerionenforschung 55128 Mainz, Germany*

Ariel Zhitnitsky[‡]

Department of Physics and Astronomy, University of British Columbia, Vancouver, Canada

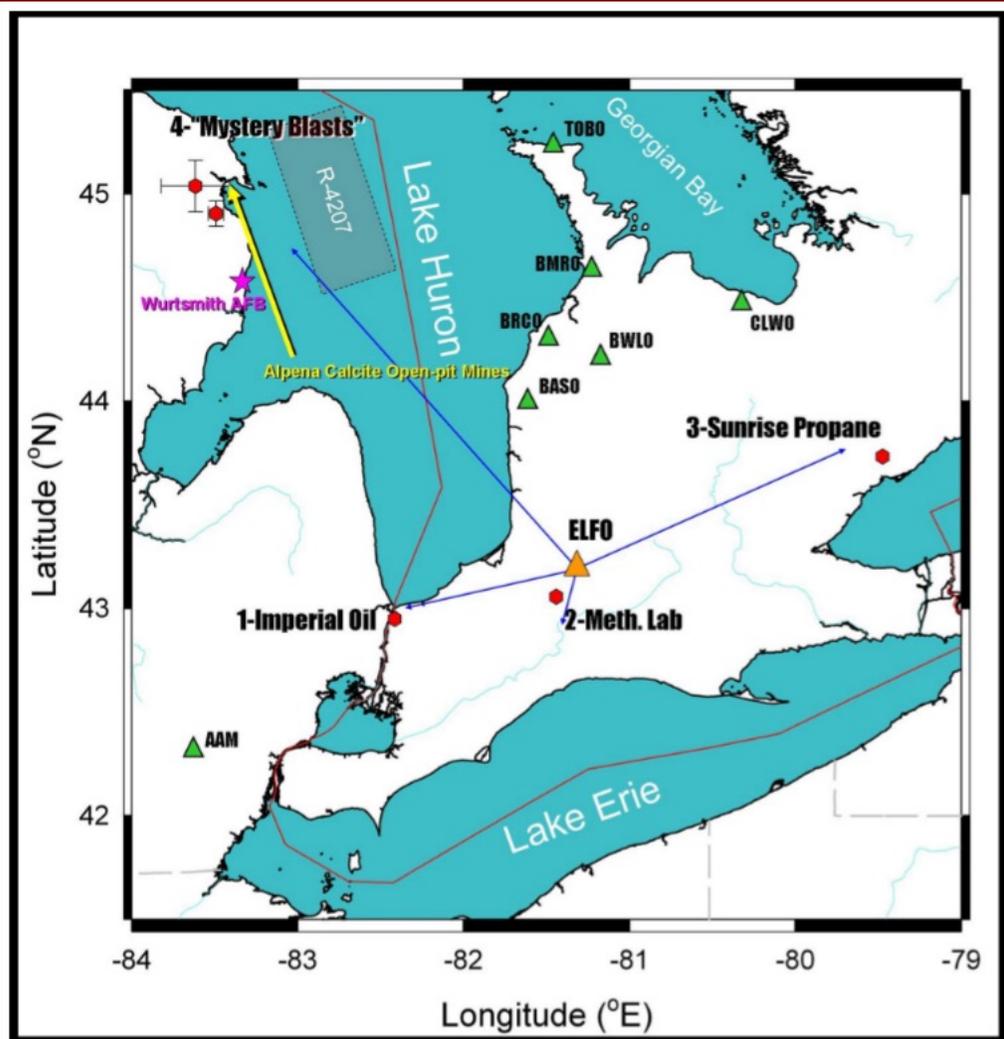


FIG. 1. Location of ELFO and seismic stations in the area, adopted from [4]. One degree along the latitude corresponds to 112 km. i.e. $1^\circ \approx 112$ km, while along the longitude $1^\circ \approx 82$ km. It explains our benchmark 300 km in eqs. (29) and (30) which covers the relevant area shown on the map.

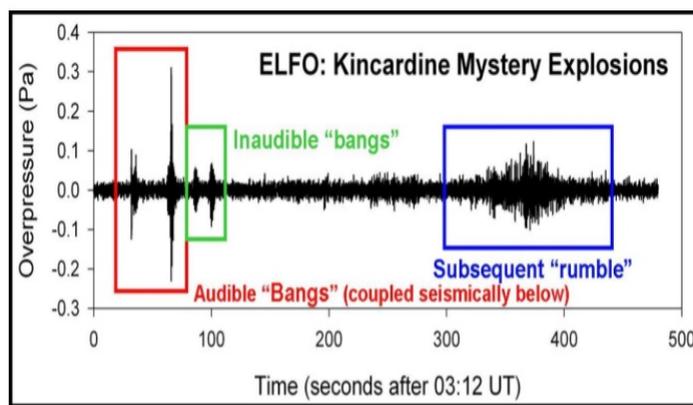


Figure 4a (observed blast)

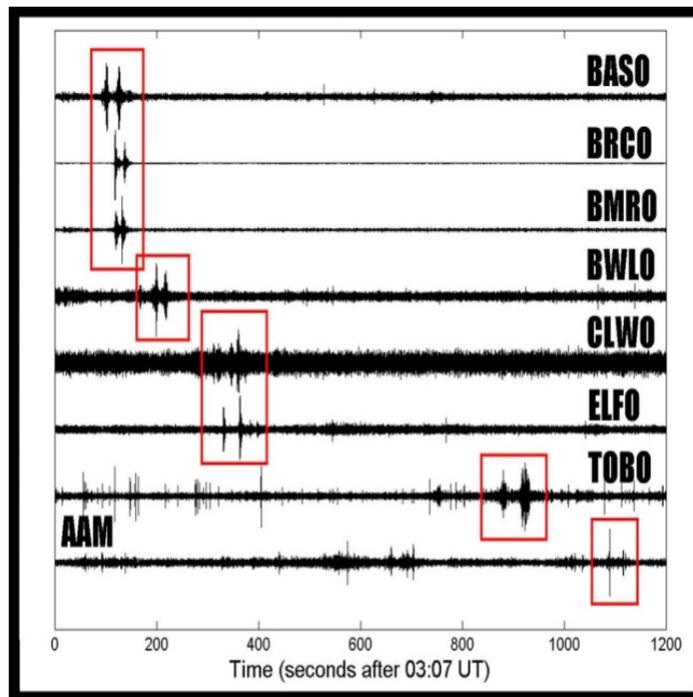


Figure 4b

**AQN
detected?**

← Acoustic

← Seismic

DARK MATTER has nowhere to hide!

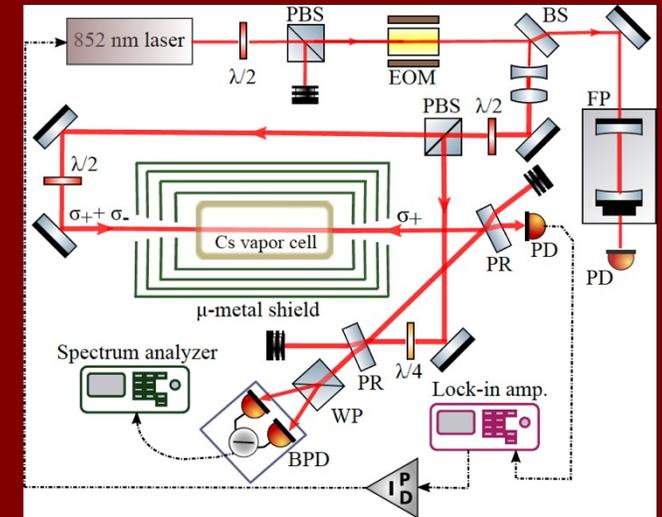
Ultralight bosonic DM searches with networks



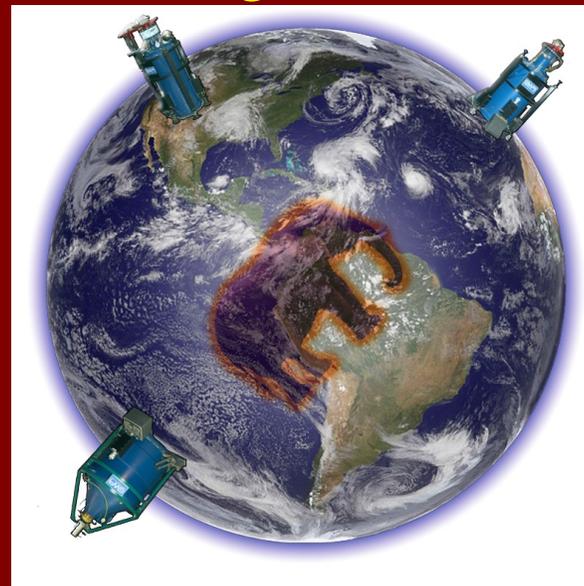
Elephants...



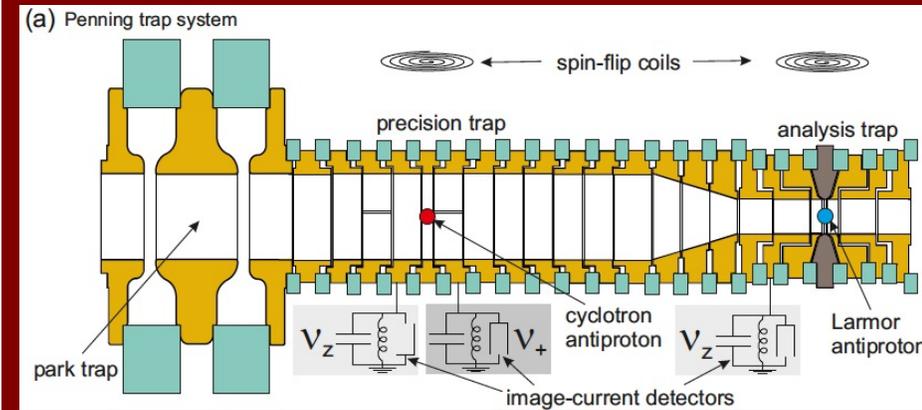
... with "clocks"



...with gravimeters

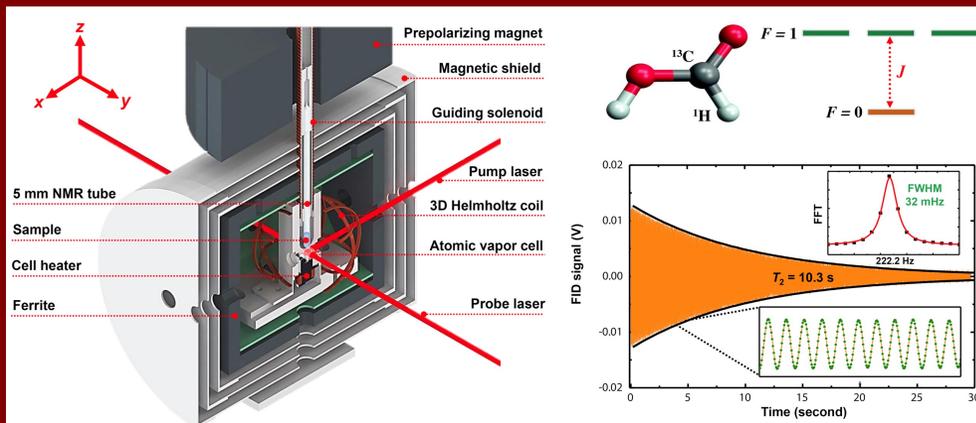


...with antimatter



...with seismometers

...with NMR



Exercises



FIG. 1 How can many musicians collaborate to make beautiful music? Drawing courtesy of Olga Budker.

? Problem 1.1: Galactic rotation curves

Consider a star in a circular orbit at the periphery of a galaxy of mass M , such that most of the galaxy's mass is contained within the star's orbital radius R . How does the star's orbital velocity scale with R under these assumptions? Given that we observe flat galactic rotation curves, what can we assume is the radial dependence of the dark matter density?

From: *Ultralight Bosonic Dark Matter*,
D. F. Jackson Kimball and Karl van Bibber, Springer 2021

Solution

Problem 1.1: Galactic rotation curves

Since most of the galaxy's mass M is within the radius R of the star's circular orbit, the star's centripetal acceleration (v^2/R , where v is the star's rotational velocity) is equal to the gravitational pull of the galaxy divided by the star's mass:

$$\frac{v^2}{R} \approx \frac{G_N M}{R^2}, \quad (11.1)$$

and so

$$v \approx \sqrt{\frac{G_N M}{R}}. \quad (11.2)$$

Thus $v \propto 1/\sqrt{R}$. If we assume that, in fact, the galaxy's mass is dominated by a spherical distribution of dark matter so that

$$M(R) \approx \int_0^R 4\pi \rho_{\text{dm}}(r) r^2 dr, \quad (11.3)$$

where $\rho_{\text{dm}}(r)$ is the dark matter density, to obtain the observed flat rotation curve we demand that $\rho_{\text{dm}}(r) \propto 1/r^2$. This yields $M(R) \propto R$, and thus based on Eq. (11.2), v is independent of R .

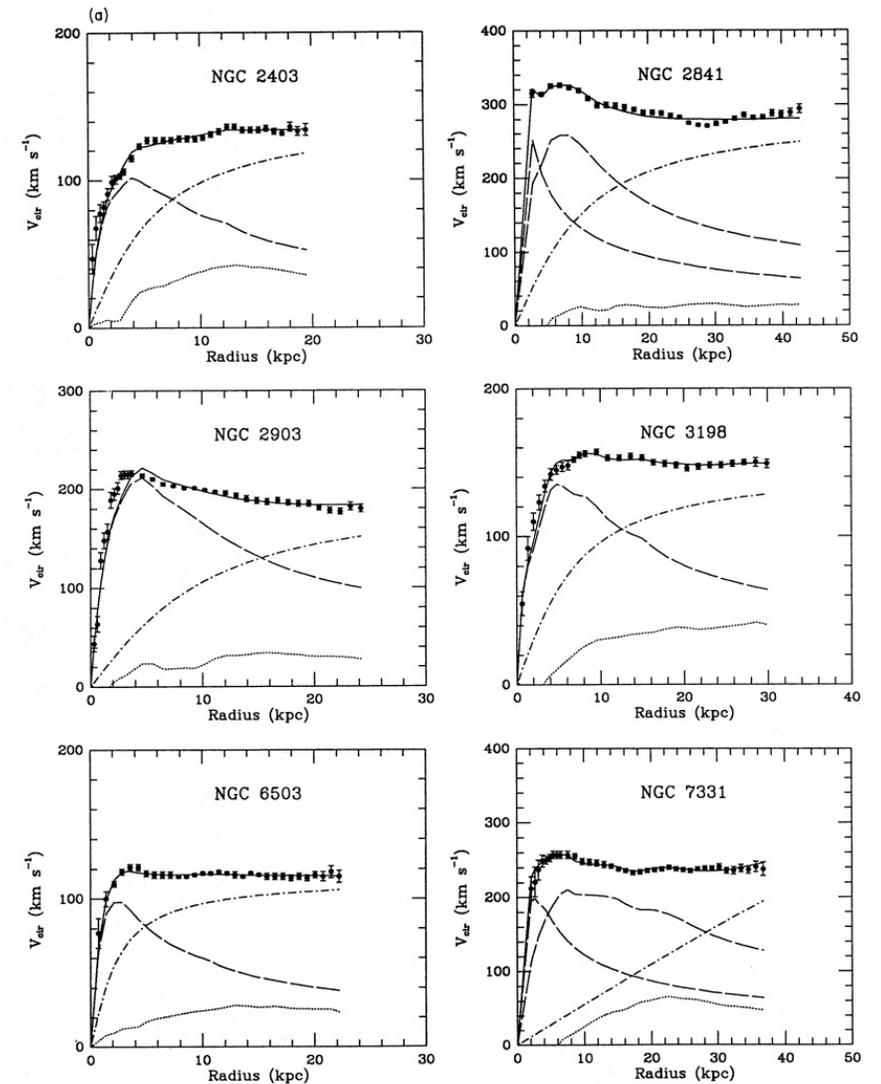


Figure 1. Three-parameter dark-halo fits (solid curves) to the rotation curves of sample galaxies. The rotation curves of the individual components are also shown: the dashed curves are for the visible components, the dotted curves for the gas, and the dash-dot curves for the dark halo. The fitting parameters are the mass-to-light ratio of the disc (M/L), the halo core radius (r_c), and the halo asymptotic circular velocity (V_h). The galaxies from the sample of Begeman are shown in (a) and the lower luminosity galaxies in (b). Best-fit values for the free parameters are given in columns 2, 3 and 4 of Table 2.

Mon. Not. R. astr. Soc. (1991) 249, 523-537

Extended rotation curves of spiral galaxies: dark haloes and modified dynamics

K. G. Begeman, A. H. Broeils and R. H. Sanders
Kapteyn Astronomical Institute, University of Groningen, 9700 AV Groningen, The Netherlands

? Problem 1.2: Minimum mass of fermionic dark matter

Derive a lower limit on the mass of a spin-1/2 fermionic dark matter candidate based on the facts that (a) the average mass density of dark matter in the Milky Way is $\rho_{\text{dm}} \approx 0.4 \text{ GeV/cm}^3$ [73] and (b) the escape velocity of the Milky Way galaxy is $v_{\text{esc}} \approx 2 \times 10^{-3} c$ [74].

From: *Ultralight Bosonic Dark Matter*,

D. F. Jackson Kimball and Karl van Bibber, Springer 2021

Solution

Problem 1.2: Minimum mass of fermionic dark matter

The spin-statistics theorem demands that only a single fermion can occupy a given quantum state, and so there is an upper bound on the possible fermion density in the dark matter halo. There is also an upper bound on the speed of the dark matter particles: to remain trapped within the gravitational potential of the galaxy, they cannot exceed the escape velocity v_{esc} . These two bounds conspire to set a lower limit m^* on the fermionic dark matter mass m .

The existence of a lower bound on m can be understood qualitatively in the following way. The maximum number density of fermions is capped at $\sim 1/\lambda_{\text{dB}}^3$, where λ_{dB} is the de Broglie wavelength of the fermions; this is the case when there is about one fermion per quantum state. This caps the mass density at about $\rho_{\text{max}} \sim m/\lambda_{\text{dB}}^3$. Since $\lambda_{\text{dB}} \propto 1/m$, $\rho_{\text{max}} \propto m^4$. Thus if m is too small, ρ_{max} is smaller than ρ_{dm} and the fermions cannot obtain the observed density of dark matter.

To derive a numerical value for m^* , we begin by considering the number of quantum states dN in a differential volume of phase space, which is given by dividing the phase space volume by h^3 :

$$dN = 2 \frac{d^3\mathbf{r}d^3\mathbf{p}}{h^3}, \quad (11.4)$$

where we have included an additional factor of 2 to account for the spin degree of freedom for the spin-1/2 fermions. To find the maximum possible density, we assume that every possible quantum state is occupied, starting from the state with the smallest possible momentum up to the Fermi momentum p_F (the case of a zero-temperature Fermi gas). The density of quantum states $n_Q = dN/dV$, where $dV = d^3\mathbf{r}$ is the volume element, and so the maximum number density $n_{\text{max}} = n_Q$ is found by integrating over the possible momenta in spherical coordinates

$$n_{\text{max}} = \frac{2}{h^3} \int_0^{p_F} 4\pi p^2 dp = \frac{8\pi p_F^3}{3h^3}. \quad (11.5)$$

Requiring that $p_F \leq mv_{\text{esc}}$ and also $\rho_{\text{max}} = mn_{\text{max}} \geq \rho_{\text{dm}}$, we obtain the relation

$$\frac{8\pi m^4 v_{\text{esc}}^3}{3h^3} \geq \rho_{\text{dm}}, \quad (11.6)$$

and find that the minimum mass of a fermionic dark matter particle is

$$m^* = \sqrt[4]{\frac{3h^3 \rho_{\text{dm}}}{8\pi v_{\text{esc}}^3}}. \quad (11.7)$$

Numerically, given that $\rho_{\text{dm}} \approx 0.4 \text{ GeV/cm}^3$ and $v_{\text{esc}} \approx 2 \times 10^{-3}c$, we find

$$m^* \approx 10 \text{ eV}. \quad (11.8)$$

? Problem 1.3: Ultralight bosonic dark matter waves

Suppose that dark matter consists mostly of bosons with mass $m_b c^2 = 10^{-6}$ eV. What are the Compton frequency and Compton wavelength of such bosons? Recalling that the virialized velocity of dark matter in the Milky Way is $\approx 10^{-3}c$, what is the de Broglie wavelength λ_{dB} of such bosons? Given that the local dark matter density is $\rho_{\text{dm}} \approx 0.4 \text{ GeV}/\text{cm}^3$, estimate how many bosons occupy a volume corresponding to λ_{dB}^3 ? Repeat these estimates for dark matter bosons with mass $m_b c^2 = 10^{-12}$ eV.

? Problem 1.4: Coherence of ultralight bosonic dark matter fields

Given that the characteristic width of the UBDM velocity distribution in the Milky Way is $\Delta v \approx 10^{-3}c$, derive τ_{coh} and L_{coh} for the UBDM field. Carry out numerical estimates of τ_{coh} and L_{coh} for the boson masses considered in Problem 1.3 ($m_b c^2 = 10^{-6}$ eV and $m_b c^2 = 10^{-12}$ eV). What would be the corresponding Q -factor for the UBDM in the Milky Way, $Q = \omega/\Delta\omega$?

Problem 1.3: Ultralight bosonic dark matter waves

The Compton frequency is given by

$$\omega_c = \frac{mc^2}{\hbar} \quad (11.9)$$

and the Compton wavelength is given by

$$\lambda_c = \frac{2\pi c}{\omega_c} = \frac{2\pi\hbar}{mc}. \quad (11.10)$$

A useful numerical quantity to recall for such “back-of-the-envelope” estimates is $\hbar c \approx 200 \text{ eV} \cdot \text{nm}$. For $m_b c^2 = 10^{-6} \text{ eV}$, we find

$$\omega_c \approx \frac{(10^{-6} \text{ eV}) \times (3 \times 10^{10} \text{ cm/s})}{200 \times 10^{-7} \text{ eV} \cdot \text{cm}} \approx 2\pi \times 240 \text{ MHz}, \quad (11.11)$$

and

$$\lambda_c \approx \frac{2\pi \times 3 \times 10^{10} \text{ cm/s}}{1.5 \times 10^9 \text{ s}^{-1}} \approx 130 \text{ cm}. \quad (11.12)$$

The de Broglie wavelength is given by

$$\lambda_{\text{dB}} = \frac{2\pi\hbar}{mv} = \frac{\lambda_c c}{v}, \quad (11.13)$$

and so for virialized dark matter with $v \approx 10^{-3}c$ and $m_b c^2 = 10^{-6} \text{ eV}$, $\lambda_{\text{dB}} \approx 10^5 \text{ cm} \approx 1 \text{ km}$. Since the axions are nonrelativistic, their kinetic energy is small compared to their rest energy, and we can estimate that each axion carries about $m_a c^2$ of energy. Based on the the dark matter density ρ_{dm} , the average number of bosons $\langle N_b \rangle$ occupying a “quantum volume” λ_{dB}^3 can be estimated to be

$$\langle N_b \rangle \approx \frac{\rho_{\text{dm}} \lambda_{\text{dB}}^3}{m_b c^2}. \quad (11.14)$$

For $m_b c^2 = 10^{-6} \text{ eV}$, $\langle N_b \rangle \approx 10^{30}$. Clearly the mode density for such UBDM is quite large!

To see how these estimates change for lighter bosons, we note how each quantity scales with m_b : $\omega_c \propto m_b$, λ_c and λ_{dB} are $\propto 1/m_b$, and $\langle N_b \rangle \propto 1/m_b^4$. Thus for $m_b c^2 = 10^{-12} \text{ eV}$, we find $\omega_c \approx 2\pi \times 240 \text{ Hz}$, $\lambda_c \approx 2 \times 10^7 \text{ cm}$, $\lambda_{\text{dB}} \approx 10^{11} \text{ cm}$, and $\langle N_b \rangle \approx 10^{54}$.

Problem 1.4: Coherence of ultralight bosonic dark matter fields

Since UBDM is cold (i.e., nonrelativistic), the energy of an UBDM particle is the sum of its rest energy and its kinetic energy:

$$E \approx m_b c^2 + \frac{1}{2} m_b v^2, \quad (11.15)$$

and the spread in observed energies due to the virialized velocity distribution is

$$\Delta E \approx \frac{1}{2} m_b \Delta v^2. \quad (11.16)$$

Thus, there is a spread of observed frequencies $\Delta\omega$ for UBDM:

$$\frac{\Delta\omega}{\omega} = \frac{\Delta E}{E} \approx \frac{\Delta v^2}{2c^2}. \quad (11.17)$$

Since $\Delta v^2 \approx 10^{-6}c^2$, the UBDM waves dephase after $\approx 10^6$ oscillations: this is when the accumulated phase differences become ~ 1 . The Q -factor for UBDM in the Milky Way is thus $Q \approx 10^6$. This gives a coherence time of

$$\tau_{\text{coh}} \approx 10^6 \frac{2\pi\hbar}{m_b c^2}. \quad (11.18)$$

The coherence length is given by the product of τ_{coh} and the average boson velocity, $v \approx 10^{-3}c$, which is the de Broglie wavelength:

$$L_{\text{coh}} = v\tau_{\text{coh}} = \lambda_{\text{dB}} \approx 10^3 \frac{2\pi\hbar}{m_b c}. \quad (11.19)$$

For $m_b c^2 = 10^{-6} \text{ eV}$, $\tau_{\text{coh}} \approx 4 \text{ ms}$ and $L_{\text{coh}} \approx 10^5 \text{ cm} \approx 1 \text{ km}$. For $m_b c^2 = 10^{-12} \text{ eV}$, $\tau_{\text{coh}} \approx 4000 \text{ s} \approx 1 \text{ hour}$ and $L_{\text{coh}} \approx 10^{11} \text{ cm}$.