Gravitational Dynamics Of Scalar Field Dark Matter

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1. Introduction - Scalar Field Dark Matter (SFDM)

Many inconsistencies face the Cold Dark Matter model and WIMPs at astrophysical scales, to name only some of them: the core-cusp problem, the missing satellite problem, the "too big to fail". Alternative models have been created to try to find a solution to all these problems, one of them is what we call the Scalar Field Dark Matter models (SFDM). In these models, dark matter is composed of bosons with masses ranging from 10^{-25} to 1 eV. Some of the advantages of such models are:

1. That they recovers the successes of Λ-CDM at large scales.
2. The appearance of a form of stable equilibrium configuration at is core, the so-called soliton.
3. A smooth density profile at the origin.

The action used in these models is $S = \int d^4x\sqrt{-g}\left[g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right]$ where we will use a parabolic potential $V(\phi) = \frac{1}{2}m^2\phi^2$ (free case).

2. Field To Fluid Picture

In the non-relativistic limit ($\dot{\psi} << m\psi$), we can express $\phi = \frac{\hbar}{\sqrt{2m}}\left(\psi e^{-imt} + \psi^* e^{imt}\right)$. At small-scales, we have the Schrodinger-Poisson system:

$$i\dot{\psi} = -\frac{1}{2m}\nabla^2\psi + m\Phi\psi\,,$$

$$\nabla^2\Phi = 4\pi G m|\psi|^2\,.$$

Using the Madelung transformation (low velocity required), which is $\psi(t,\vec{x}) = \varphi(t,\vec{x})e^{is(t,\vec{x})}$, $\varphi(t,\vec{x}) = \sqrt{\rho(t,\vec{x})/m}$ and $\vec{v}(t,\vec{x}) = \vec{\nabla}s(t,\vec{x})/m$, we can obtain a new system of equations, called the Quantum Euler-Poisson system:

$$\partial_t\rho + \vec{\nabla}\cdot(\rho\vec{v}) = 0\,,$$

$$\partial_t\vec{v} + (\vec{v}\cdot\vec{\nabla})\vec{v} = -\vec{\nabla}\Phi + \frac{1}{2m^2}\vec{\nabla}\cdot\left(\frac{\nabla^2\sqrt{\rho}}{\sqrt{\rho}}\right)\,,$$

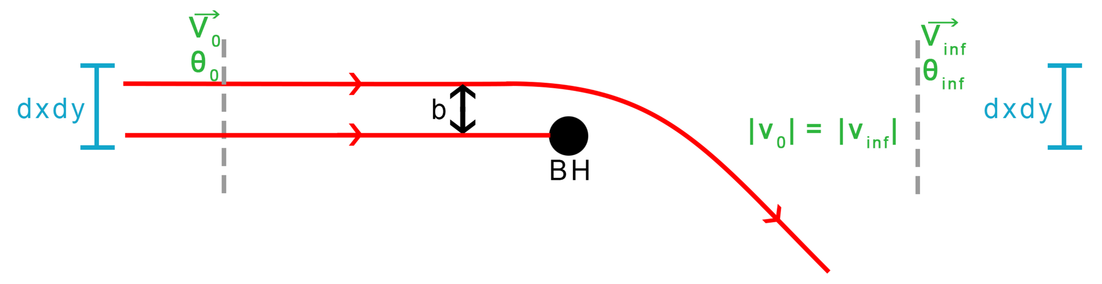
$$\nabla^2\Phi = 4\pi G\rho\,.$$

However, the change can only be done in one way, from field to fluid, since by doing so we lose the wave-like behaviors.

3. Studied System & Goal

We studied a system composed of a Schwarzschild black hole (BH) in motion in a SFDM sea (implying a symmetry over one angle in spherical coordinates). The Schwarzschild metric used being centered on the BH, we will consider that it is the DM particles that move.

We then seek to calculate the dynamical friction, which is define as the loss of momentum of moving objects through gravitational interactions, of such a system, which can help solve some cosmological problems (as globular clusters timing problem) while constraining SFDM mass.



This is a sketch of two particles coming from infinity with a velocity $\vec{v_0}$: One being deviated by a BH and the other being absorbed.

7. References

[1] H. et al., "Ultralight scalars as cosmological dark matter," *Phys. Rev. D*, vol. 95, no. 4, p. 043541, 2017.

[2] B. et al., "Fate of scalar dark matter solitons around supermassive galactic black holes," *Phys. Rev. D*, vol. 101, no. 2, p. 023521, 2020.

4. Free Scalar Field Dark Matter

We choose the initial velocity to be $\vec{v}_0 = v_{inf}\vec{e_z}$. The energy-momentum tensor is:

$$T_{\mu\nu} = -g_{\mu\nu}\left(\frac{1}{2}\partial_\alpha\phi\partial^\alpha\phi + V(\phi)\right) + \partial_\mu\phi\partial_\nu\phi\,.$$

From the dynamical friction formula, which is $\vec{F} = -\oint dS_j T_{jz}\vec{e_z}$ and using the fluid picture, we obtain $\vec{F} \approx -\oint dxdy\rho v_z^2$. After some calculations to obtain ρ and v_z expressions at infinity, the dynamical friction of our system is then

$$\boxed{F = 2\pi\rho_0 v_{inf}^2\left[\frac{G^2M^2h(v_{inf})}{v_{inf}^4}ln\left(\frac{(b^+)^2v_{inf}^4 + G^2M^2h(v_{inf})}{(b^-)^2v_{inf}^4 + G^2M^2h(v_{inf})}\right) + \frac{(b^-)^2}{2}\right]},$$

with $h(v_{\rm inf}) = 1 - 4v_{inf}^2 + 4v_{inf}^4$. The first term corresponds to the difference between particles outgoing and incoming, the second one to particles absorbed by the BH. The first term correspond exactly to what we can obtain with a standard astrophysical object instead of a BH [1].

5. Self-Interacting Scalar Field Dark Matter

We add a new potential term $V_I(\phi) = \lambda\phi^4/4$ corresponding to self-interactions.
From the Klein-Gordon equation

$$\frac{\partial^2\phi}{\partial t^2} - \sqrt{\frac{f}{h^3}}\vec{\nabla}\cdot\left(\sqrt{fh}\vec{\nabla}\phi\right) + f\frac{\partial V}{\partial\phi} = 0\,,$$

with $f(r), h(r)$ the isotropic metric functions, we can express

$$\boxed{\phi(r,\theta) = \phi_0(r,\theta)cn[\omega(r,\theta)t - \mathbf{K}(k)\beta(r,\theta), k(r,\theta)]}.$$

[2] where ϕ_0 is the amplitude, $cn[u,k]$ is the Jacobi elliptic function, $\omega = 2\mathbf{K}\omega_0/\pi$ is the angular frequency and $\mathbf{K}$ is the complete elliptic integral of the first kind.

Supposing $k^2 << 1$ and of the form $k^2 = k_0^2 + C/r$ as a first approximation, the only unknown function is β that we will find using the conservation equation $\nabla_\mu T_0^\mu = 0$ and the steady state property of the system $<\dot{\rho}>= 0$, which will give us $\vec{\nabla}\cdot\left(\rho_{eff}^{(0)}(k)\vec{\nabla}\beta\right) = 0$. The boundary conditions (at small radius $\beta = C_0 ln(1/r + C_1)$, at large radius $\beta = v_0 r cos(\theta)$) allow us to obtain an expression

$$\boxed{\beta(r,\theta) = A_0 ln\left(\frac{1}{r} + \gamma\right) + A_1 v_0 r^{\sqrt{2}}{}_2F_1\left(a,b,c;-\gamma r\right)cos(\theta)},$$

where A_0, A_1, γ, a, b, c are constants. Then including a correction on the effective density so $\rho_{eff} = \rho_{eff}^{(0)} + \rho_{eff}^{(1)}$, we have $\vec{\nabla}\cdot\left[\left(\rho_{eff}^{(0)} + \rho_{eff}^{(1)}\right)\vec{\nabla}\beta\right] = \vec{\nabla}\cdot\left[\rho_{eff}^{(0)}\vec{\nabla}\beta\right] + S^{(0)} = 0$.
Numerically, we will try to obtain results using an iteration based on the assumption that the corrections are small: $\beta^{(i)}$, $\rho^{(i)} \rightarrow \rho^{(i+1)} \rightarrow S^{(i+1)} \rightarrow \beta^{(i+1)} \rightarrow \rho^{(i+2)} \rightarrow \ldots$
Then, we will use the dynamical friction as for a free SFDM.

6. Conclusions & Prospects

Considering the system of a Schwarzschild black hole moving through a dark matter sea:

1. The results obtained for the dynamical friction with a free scalar field are consistent with what is currently known + correction due to the black hole absorbing particles.
2. For future work, we hope that we will obtain good results with a self-interacting scalar field, which will mean that the induced correction on β is of the order of a perturbation.

We will extend to the case of a Kerr black hole, which will induce a loss of angular symmetry.