NNPDF2.1
Unbiased PDFs with heavy quark mass effects

Juan Rojo

The **NNPDF Collaboration**: R. D. Ball, V. Bertone,
F. Cerutti, L. Del Debbio, S. Forte,
A. Guffanti, J. I. Latorre, J. R., M. Ubiali

INFN, Sezione di Milano

PDF4LHC, CERN 04/07/2010

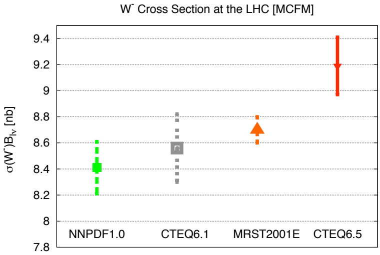
Outline

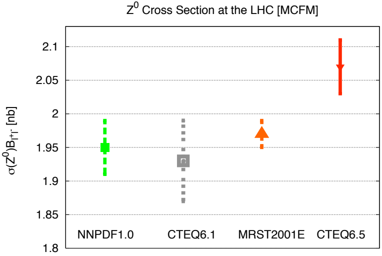
- NNPDF2.1: unbiased PDFs with heavy quark mass effects
- Implications at the PDF - PDF4LHC benchmark study
- LO/LO* NNPDFs and the momentum of quarks and gluons

NNPDF2.1: UNBIASED PDFS WITH HQ MASS EFFECTS

Impact of HQ effects

Heavy quark mass effects were a ∼ 3–sigma effect for CTEQ (2006)
Huge effect - major impact on LHC physics

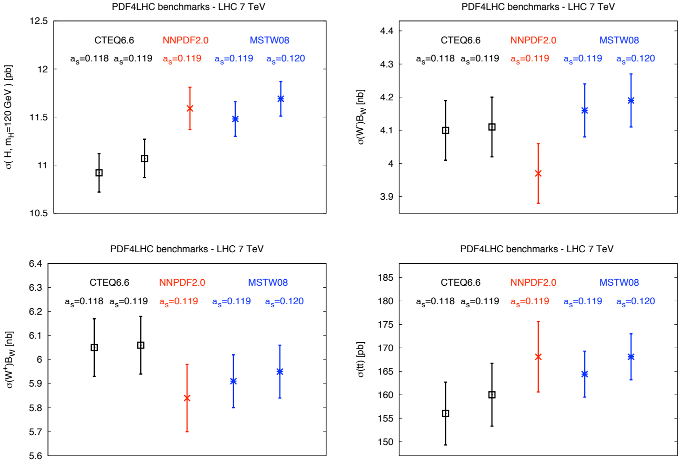




Note though that MTST2001E included heavy quark effects (TR) ....
Are we sure HQ have a so dramatic impact?

Impact of HQ effects

… or effect not so huge? Compare PDF4LHC benchmark observables …

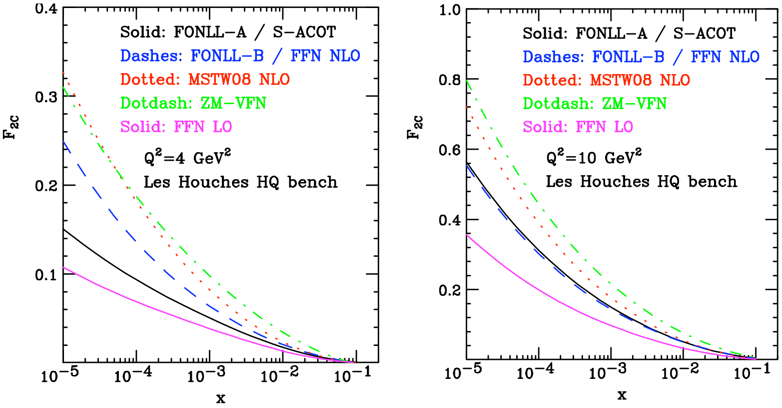


The FONLL GM scheme in NNPDF

- The FONLL General-Mass scheme (arXiv:1001.2312) has several advantages over other GM schemes, such as S-ACOT or TR/TR'
  1. It can applied also in general hadronic processes
  2. It can be formulated at any perturbative order
  3. It allows the combination of different perturbative orders in the massless and massive computations
  4. It can be combined with various prescriptions for the treatment of subleading mass-suppressed terms near threshold, such as χ–scaling or damping factors.

  (see January PDF4LHC meeting, DIS talk)
- FONLL has now been implemented in the `FastKernel` framework, used in the NNPDF analysis
- This has required the analytic computation of the Mellin space HQ $\mathcal{O}(\alpha_s)$ NC and CC coefficient functions
- Accuracy benchmarked againts the Les Houches HQ tables (arXiv:1003.1241)

Les Houches benchmarks: F_2^c NLO schemes summary



- FONLL-A formally identical to S-ACOT if the same threshold prescription adopted
- Difference between FONLL-A and TR' without threshold prescriptions → Frozen, Q^2−independent $\mathcal{O}\left(\alpha_s^2\right)$ term in the latter

The Mellin space $\mathcal{O}(\alpha_s)$ NC HQ coefficient

The $\mathcal{O}(\alpha_s)$ NC massive coefficient function is

$$C_{g,h}^{(n_l),1}(z,\epsilon) = \theta\left(W^2-4m^2\right)\times T_R[(z^2+(1-z)^2+4\epsilon z(1-3z)-8\epsilon^2 z^2)\log\frac{1+v}{1-v} + (8z(1-z)-1-4\epsilon z(1-z))v\,]$$

$$\epsilon = m^2/Q^2\ , \quad v\equiv\sqrt{1-4m^2/W^2}\ , \quad a=(1+4\epsilon)^{-1}$$

has the following Mellin space transform

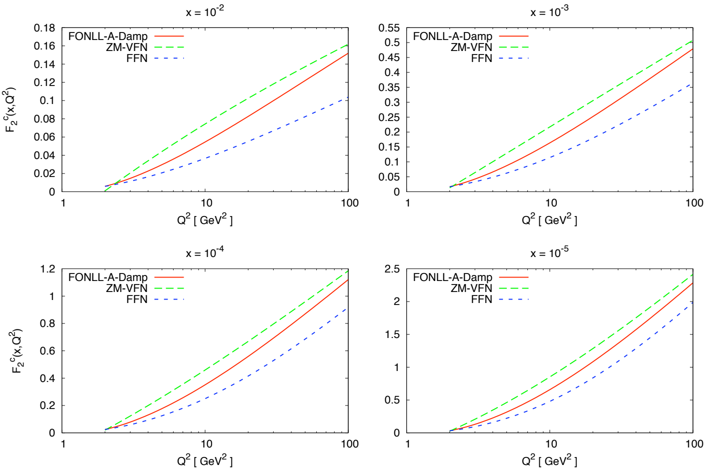
$$C_{g}^{(n_l),1}(N,\epsilon) = T_R a^N\Big\{(\tfrac{1}{N}-1)I(N,a)+(\tfrac{1-3a}{N+1}+9a)I(N+1,a) -(\tfrac{1}{2}\tfrac{1+4a-9a^2}{N+2}-(1+a)(1-9a))I(N+2,a)-a(1-9a)I(N+3,a)\Big\}$$

$$I(N,a)=\frac{\Gamma(N)\Gamma(\frac{1}{2})}{\Gamma(N+\frac{1}{2})}F(\frac{1}{2},N,N+\frac{1}{2};a)$$

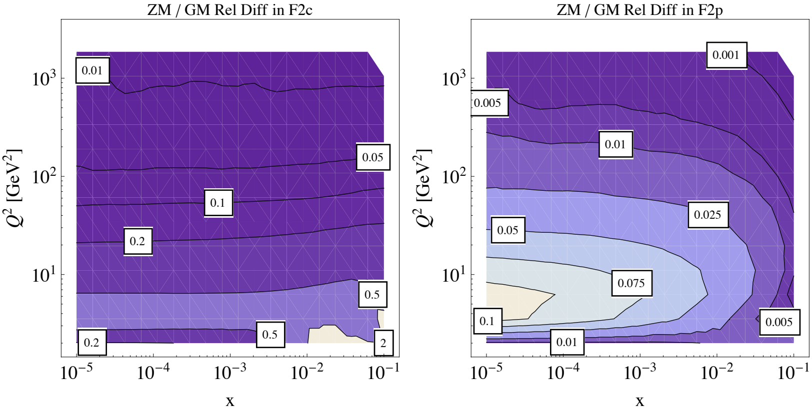
Similar (though longer) expressions for the CC case

FONLL-A GM-VFN vs. ZM-VFN

Smooth interpolation between massive FFN and ZM-VFN schemes



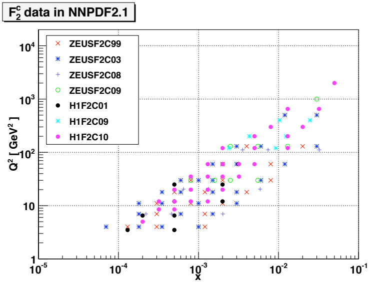
FONLL-A GM-VFN vs. ZM-VFN



- Effect of HQs → At most ∼ 10% in HERA region on F_2^p
- Non-negligible effects also in Fixed Target region

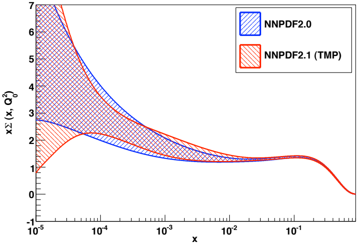
The NNPDF2.1 analysis

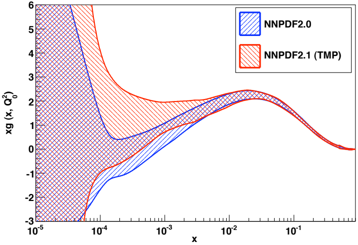
- FONLL-A-Damp as a General Mass scheme for NC and CC DIS observables
- Same dataset as NNPDF2.0 (arXiv:1002.4407), supplemented with HERA F_2^c data
- All results shown still preliminary!



Experiment	Set	Ref.	Points	$x_{\rm min}$	$x_{\rm max}$	$Q^2_{\rm min}$	$Q^2_{\rm max}$
ZEUSF2C			69 (62)				
	ZEUSF2C99	[4]	21 (18)	$5\,10^{-5}$ $(1.3\,10^{-4})$	0.02	1.8 (4)	130
	ZEUSF2C03	[5]	31 (27)	$3\,10^{-5}$ $(7\,10^{-5})$	0.03	2.0 (4.0)	500
	ZEUSF2C08	[6]	9	$2.2\,10^{-4}$	0.032	7.0	112
	ZEUSF2C09	[7]	8	$8\,10^{-4}$	0.03	30	1000
H1F2C			47 (45)				
	H1F2C01	[8]	12 (10)	$5\,10^{-4}$	$3.2\,10^{-3}$	1.5	60
	H1F2C09	[9]	6	$2.4\,10^{-4}$	0.025	120	400
	H1F2C10	[10]	29	$2\,10^{-4}$	0.05	5.0	2000
Total			3554				

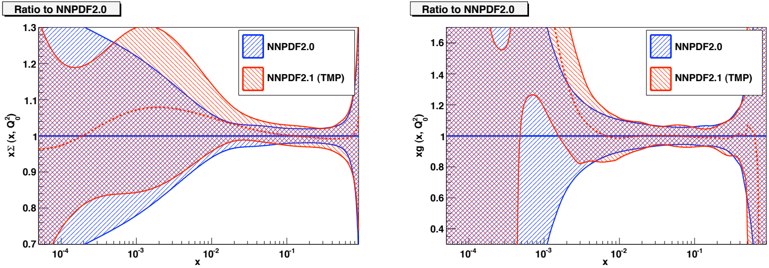
The NNPDF2.1 analysis - PDFs at $Q_0^2 = 2$ GeV^2





- HQ mass effects and F_2^c data enhance the singlet and the gluon PDFs at moderate and small-x
- NNPDF2.1 always within 1–sigma of NNPDF2.0 → HQ effects important though not dramatic
- Harder small-x gluon partly from constraints of $F_2^c(x, Q^2)$ data

The NNPDF2.1 analysis - PDFs at $Q_0^2 = 2\ \mathrm{GeV}^2$



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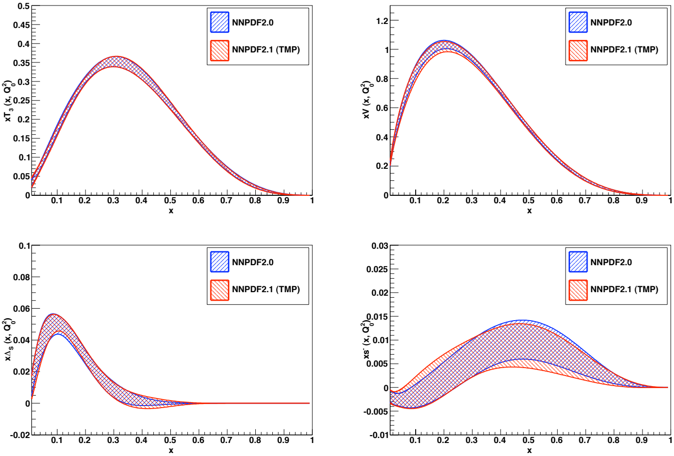
The NNPDF2.1 analysis - PDFs at $Q_0^2 = 2\ \mathrm{GeV}^2$

Large-x valence PDFs consistently unaffected by HQ effects



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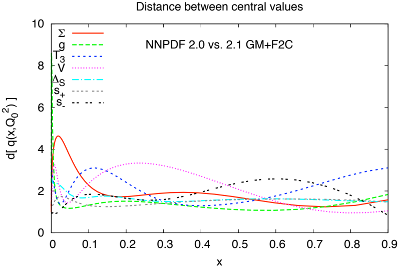
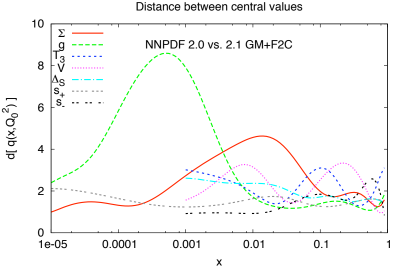




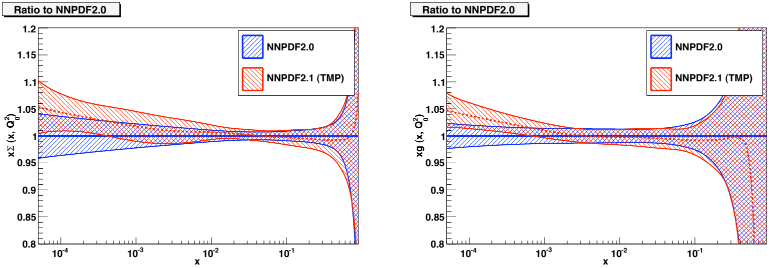
- Compute distances between PDF sets to quantify HQ impact

$$d^2\left(\langle q^{(1)}\rangle, \langle q^{(2)}\rangle\right) = \frac{\left(\langle q^{(1)}\rangle_{(1)} - \langle q^{(2)}\rangle_{(2)}\right)^2}{\sigma^2_{(1)}[\langle q^{(1)}\rangle] + \sigma^2_{(2)}[\langle q^{(2)}\rangle]} \, , \quad \sigma^2_{(i)}[\langle q^{(i)}\rangle] = \frac{1}{N_{\mathrm{rep}}^{(i)}}\sigma^2_{(i)}[q^{(i)}] \tag{1}$$

- $d \sim 5$ for the singlet at $x \sim 10^{-2}$ at $Q_0^2 = 2$ GeV^2
- $d \sim 8$ for the gluon at $x \sim 10^{-3}$ at $Q_0^2 = 2$ GeV^2

The NNPDF2.1 analysis - PDFs at $Q_0^2 = 10^4\ \mathrm{GeV}^2$

Compare PDFs at the LHC scale → Assess effects of quark-gluon mixing in DGLAP evolution



Note greatly reduced small-x PDF uncertainties

NNPDF2.0 and 2.1 always consistent within uncertainties

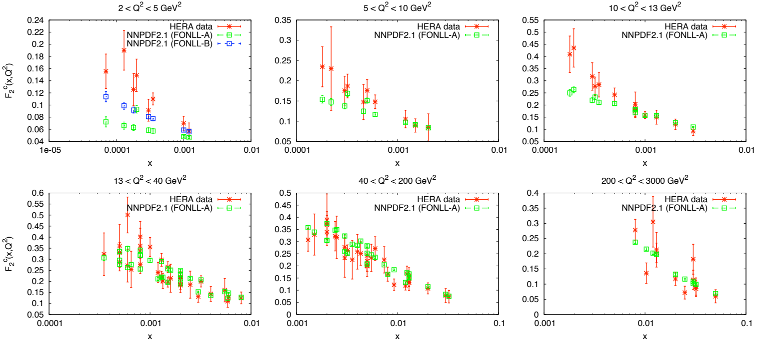
The NNPDF2.1 analysis - Dataset description (Prel.)

Experiment/Set	2.0	2.0 + GM	2.1
Total	**1.20**	**1.23**	**1.21**
NMC-pd	1.04	*1.03*	0.96
NMC	1.69	*1.51*	1.61
SLAC	1.30	*1.31*	1.31
BCDMS	1.30	*1.19*	1.21
HERA1-av	1.13	*1.28*	1.11
HERA1-NCep	**1.32**	*1.56*	**1.29**
HERA1-NCem	0.85	*0.88*	0.82
HERA1-CCep	0.97	*0.97*	0.96
HERA1-CCem	0.57	*0.57*	0.57
ZEUS-H2	1.24	*1.27*	1.23
ZEUSF2C	*1.80*	2.14	1.89
H1F2C	*1.67*	1.70	1.59
CHORUS	1.20	*1.18*	1.19
NTVDMN	0.70	*0.71*	0.71

- Overall fit quality almost identical between GM and ZM fits
- Quality of FT DIS data (NMC, BCDMS) improves in the GM fit as compared to ZM
- Quality of fit to HERA-I data unaffected
- Heavy quark effects are absorbed into the PDFs in the ZM fit of HERA1-NCep data
- Fit to F_2^c data not completely satisfactory (see after)

Impact of F_2^c data in NNPDF2.1

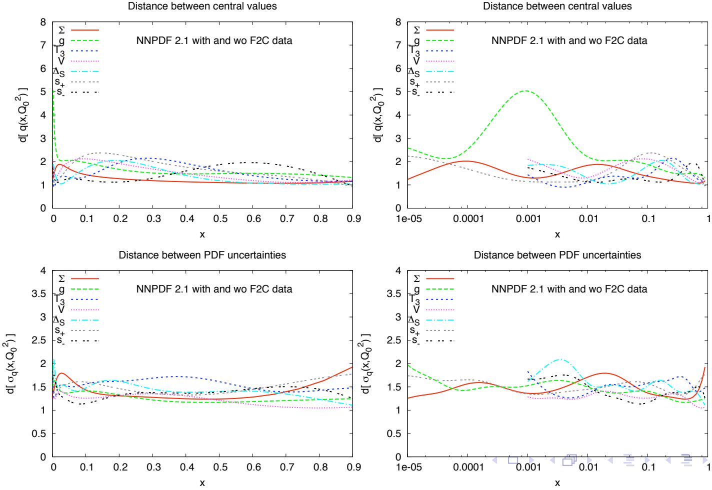
Good description of F_2^c data except at the smallest x and Q^2 bins
FONLL-A does not account for large $\mathcal{O}\left(\alpha_s^2\right)$ corrections to F_2^c in the FFNS



Update analysis with Combined HERA F_2^c dataset and with the FONLL-B GM scheme

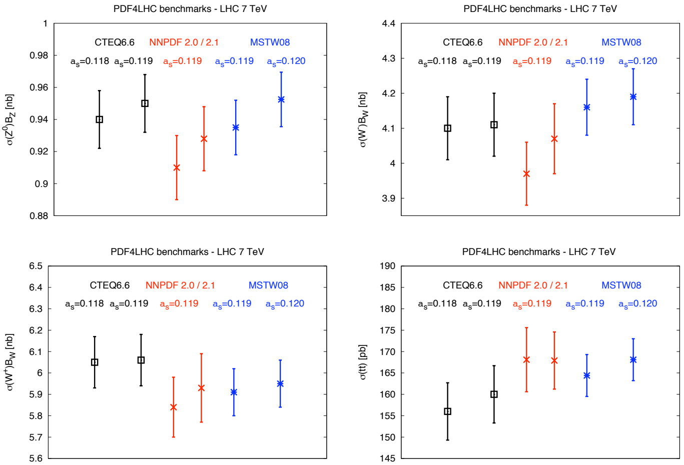
Impact of F_2^c data in NNPDF2.1

F_2^c data lead to an important constraint on the small-x gluon
$\rightarrow \sim 1/2$–sigma shift at $x \sim 10^{-3}$

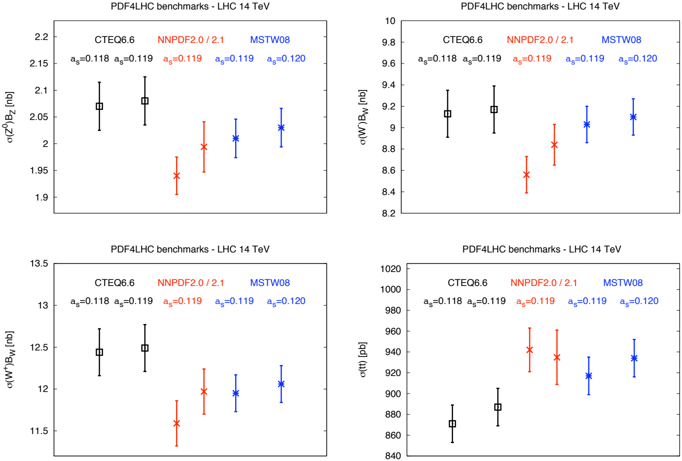


NNPDF2.1: IMPACT AT THE LHC AND THE PDF4LHC BENCHMARKS

PDF4LHC benchmarks revisited - 7 and 14 TeV



PDF4LHC benchmarks revisited - 7 and 14 TeV



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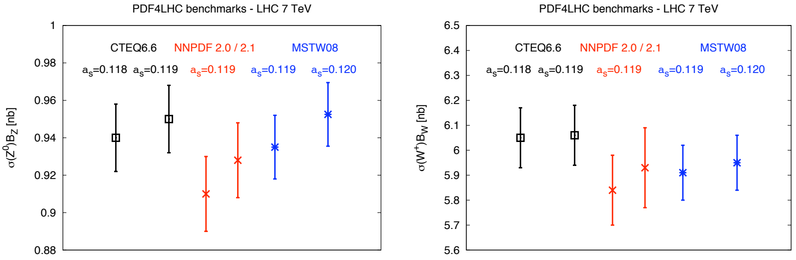


- HQ mass effects and F_2^c data amount to an about $\sim$ 1–sigma shift in LHC observables at 7 TeV and at 14 TeV
- NNPDF2.1 predictions in excellent agreement with MSTW08 for all observables
- Only marginal agreement with CTEQ6.6 for most observables (also Higgs)
- Using common α_s increases the agreement

LO/LO* NNPDFs AND THE MOMENTUM OF QUARKS AND GLUONS

LO/LO* NNPDFs

- LO PDFs are a necessary input for LO event generators
- Existing global LO fits provide a much worse description to data than at NLO

PDF set	Ref	$\chi^2_{\mathrm{LO}}/\chi^2_{\mathrm{NLO}} - 1$ (approx)
TS07	arXiv:0711.2473	$\sim 25\%$
CT09	arXiv:0910.4183	$\sim 30\%$

- Suggestions in the literature to change $\alpha_s\,(M_Z)$ from its LO to its NLO value or remove the momentum sum rule (MSR)
- To examine the situation within NNPDF, we have produced LO, LO* (NLO α_s + No MSR) and NLO* (No MSR) fits
- This analysis is based on the same dataset and settings as **NNPDF2.0** fit

Results (Preliminary)

Experiment	LO	LO*	NLO	NLO*
Total $\chi^2_{\rm tot}/N_{\rm dat}$	**1.29**	**1.32**	**1.21**	**1.22**
NMC-pd	0.88	0.90	1.01	1.01
NMC	1.67	1.68	1.68	1.68
SLAC	2.03	2.17	1.30	1.34
BCDMS	1.28	1.38	1.30	1.30
HERAI-AV	1.39	1.40	1.13	1.19
CHORUS	1.31	1.37	1.20	1.21
NTVDMN	0.81	0.72	0.70	1.21
ZEUS-H2	1.41	1.43	1.24	1.52
DYE605	0.56	0.57	0.84	0.81
DYE866	0.84	0.81	1.25	1.27
CDFWASY	1.35	1.28	1.79	1.94
CDFZRAP	3.44	3.16	1.86	1.88
D0ZRAP	1.41	1.19	0.55	0.56
CDFR2KT	0.88	0.89	0.79	0.87
D0R2CON	1.21	1.23	0.92	0.98
Mom Int	1	1.08 ± 0.05	1	1.01 ± 0.02
$\langle \mathrm{TL} \rangle \cdot 10^{-3}$	**18.9**	**19.7**	**16.3**	**16.6**

Results (Preliminary)

Experiment	LO	LO*	NLO	NLO*
Total $\chi^2_{\rm tot}/N_{\rm dat}$	**1.29**	**1.32**	**1.21**	**1.22**
Mom Int	1	1.08 ± 0.05	1	1.01 ± 0.02
$\langle \rm TL \rangle \cdot 10^{-3}$	**18.9**	**19.7**	**16.3**	**16.6**

- LO fit quality worse than at NLO, but moderate increase $\sim 6\%$
- LO* fit quality not better than standard LO fit
- At LO, total momentum is 1.08 ± 0.05, compatible with 1 at the 90% C.L.
- Training lengths increase at LO $\rightarrow$ The neural nets to accomodate both data and missing NLO corrections

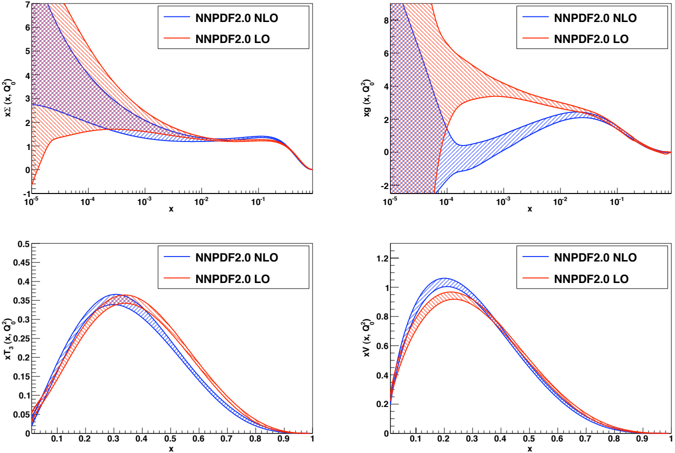
For the NLO* fit, excellent agreement with asymptotic pQCD predictions for quark/gluon momentum fractions

$$\int_0^1 dx\, x\Sigma\left(x, Q^2\right)\Big|_{\rm NLO*} = 0.532 \pm 0.010 \,, \quad \int_0^1 dx\, xg\left(x, Q^2\right)\Big|_{\rm NLO*} = 0.474 \pm 0.015 \,,$$

$$\int_0^1 dx\, x\Sigma\left(x, Q^2\right)\Big|_{\rm pQCD} = \frac{3N_f}{16 + 3N_f} = 0.529 \,, \quad \int_0^1 dx\, xg\left(x, Q^2\right)\Big|_{\rm pQCD} = 0.471 \,,$$

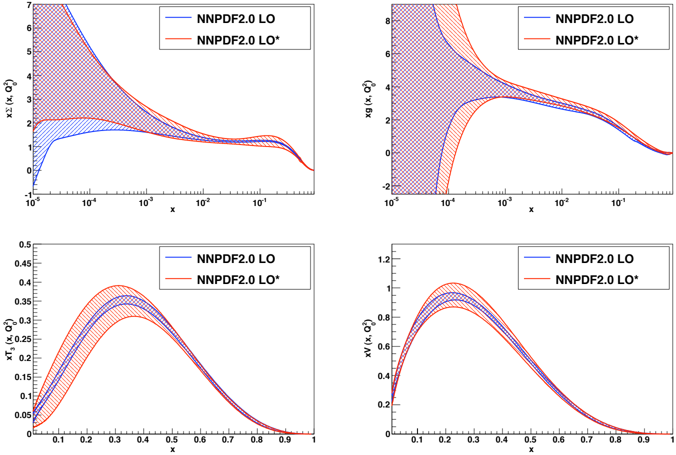
LO/LO* NNPDFs (Prel)

LO vs NLO: Differences for all PDFs, much harder small-x gluon



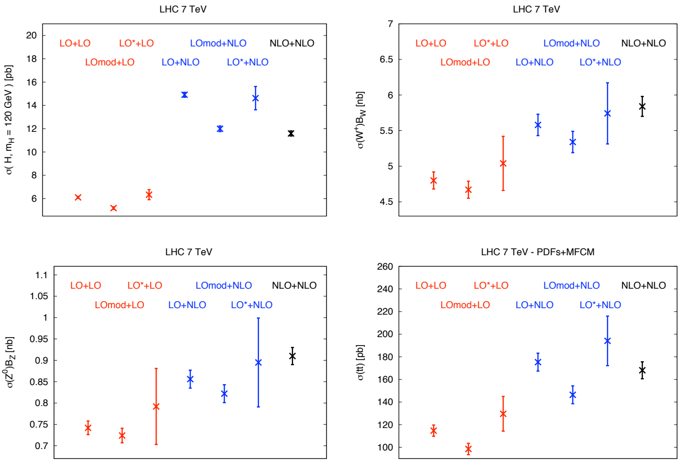
LO/LO* NNPDFs (Prel)

LO vs LO*: PDF uncertainties at LO grow enormously once MSR removed



LHC observables with LO/LO* PDFs

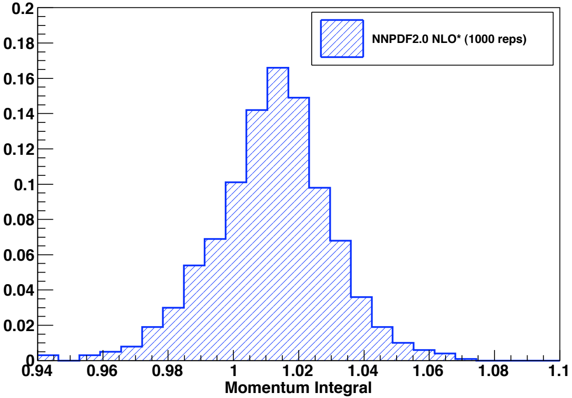
LO*+LO closer to NLO+NLO than other cases
(but case-by-case statement - cannot be taken as general rule)



The momentum integral at NLO

$$\int_0^1 dxx\,[\Sigma(x)+g(x)] = 1.01 \pm 0.02 \qquad (\mathrm{NNPDF20\,NLO*})$$

Impressive agreement with theoretical expectations with very small uncertainties
Most precise determination of the momentum of quarks and gluons in the proton



Summary

NNPDF2.1 improves on all existing NLO PDF sets:

- All relevant hard scattering data - including the HERA-I combined dataset
- Exact NLO theory for all hadronic processes - No K–factor approximations
- Up-to-date theory for heavy quark effects: the FONLL General-Mass scheme
- Extremely flexible input parametrizations - No bias from simple functional forms
- Statistically faithful PDF uncertainties - No arbitrary tolerances, no gaussian/linear assumptions, unbiased normalization treatment
- Exact propagation of PDF uncertainties to arbitrary observables, exact treatment of the PDF-α_s correlation

Thanks for your attention!

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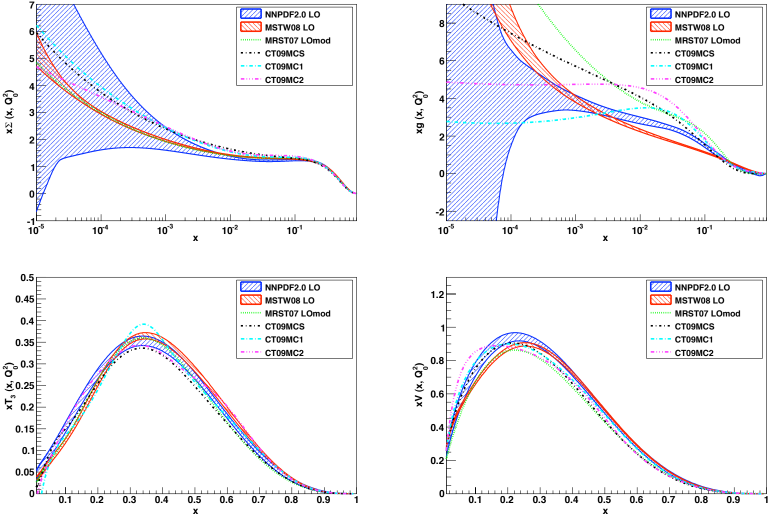
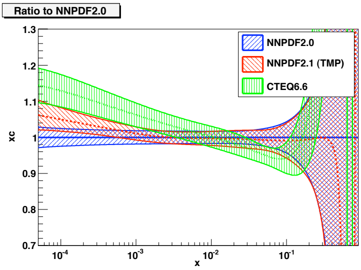
EXTRA MATERIAL

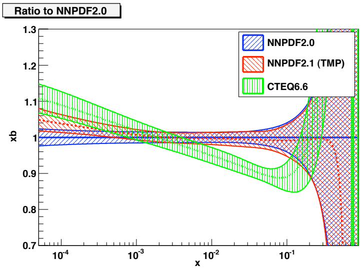
NNPDF LO/LO* vs. TS07/CT09



Heavy quark PDFs and luminosities

Ratio to NNPDF2.0 at $Q^2 = 10^4$ GeV^2

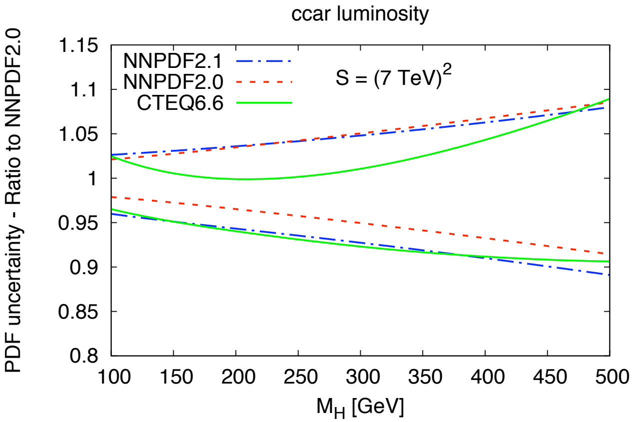




- Same pattern for $c(x, Q^2)$ and $b(x, Q^2)$ (Common evolution from singlet and gluon)
- Systematic discrepancy in b PDF for $x \in [0.01, 0.1]$ unrelated to HQs

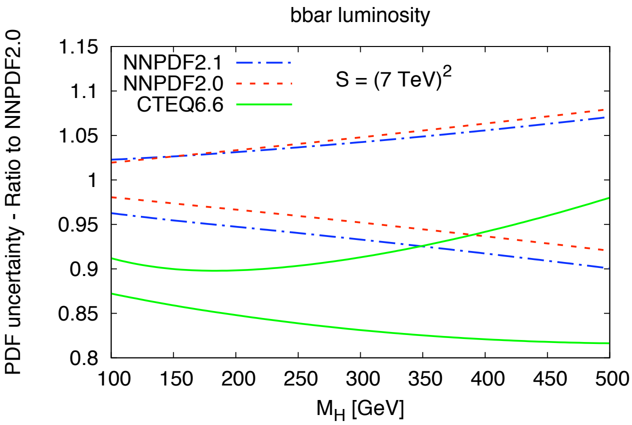
Heavy quark PDFs and luminosities

Luminosity $c\bar{c}$ at 7 TeV

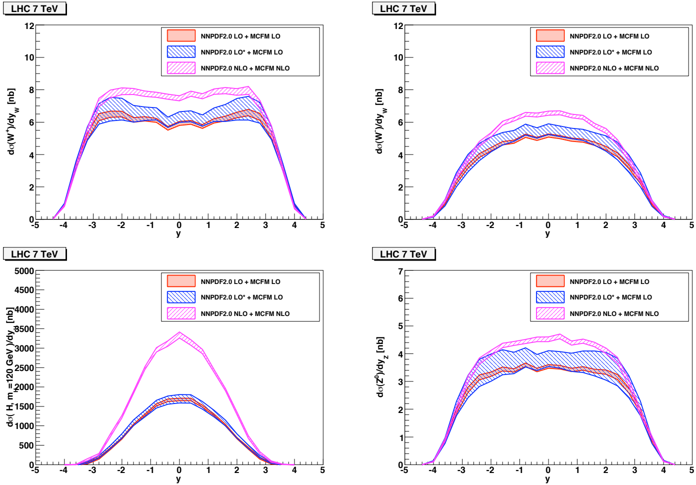


Heavy quark PDFs and luminosities

Luminosity $b\bar{b}$ at 7 TeV



LHC observables with LO/LO* PDFs



FONLL in a nutshell

- Express the massive result $F^{(n_l)}$ in terms of the massless PDFs and α_s (non trivial from $\mathcal{O}\left(\alpha_s^2\right)$)

$$F^{(n_l)}(x,Q^2) = x\int_x^1 \frac{dy}{y}\sum_{i=q,\bar{q},g} B_i\left(\frac{x}{y},\frac{Q^2}{m^2},\alpha_s^{(n_l+1)}(Q^2)\right) f_i^{(n_l+1)}(y,Q^2),$$

- Define massless limit of the massive computation as

$$F^{(n_l,0)}(x,Q^2) \equiv x\int_x^1 \frac{dy}{y}\sum_{i=q,\bar{q},g} B_i^{(0)}\left(\frac{x}{y},\frac{Q^2}{m^2},\alpha_s^{(n_l+1)}(Q^2)\right) f_i^{(n_l+1)}(y,Q^2),$$

$$\lim_{m\to 0}\left[B_i\left(x,\frac{Q^2}{m^2}\right) - B_i^{(0)}\left(x,\frac{Q^2}{m^2}\right)\right] = 0$$

- The FONLL approximation is then

$$F^{\mathrm{FONLL}}(x,Q^2) \equiv F^{(d)}(x,Q^2) + F^{(n_l)}(x,Q^2),$$
$$F^{(d)}(x,Q^2) \equiv \left[F^{(n_l+1)}(x,Q^2) - F^{(n_l,0)}(x,Q^2)\right]$$

Important technical advantage: PDFs and α_s expressed always in the (n_l+1) scheme

FONLL in a nutshell

- Far from threshold, $Q^2 \gg m^2$ $F^{(n_l,0)}(x,Q^2) \sim F^{(n_l)}(x,Q^2) \to$ the massless computation recovered

$$F^{\rm FONLL}(x,Q^2) \sim F^{(n_l+1)}(x,Q^2)$$

- Near threshold the "difference term" is formally higher order but unreliable, so one can correct it by mass suppressed terms, using for example a damping factor (FONLL default)

$$F^{(d,\,th)}(x,Q^2) \equiv f_{\rm thr}(x,Q^2)F^{(d)}(x,Q^2), \quad f_{\rm thr}(x,Q^2) = \Theta(Q^2-m^2)\left(1-\frac{Q^2}{m^2}\right)^2,$$

or some form of $\chi-$scaling,

$$F^{(d,\,\chi)}(x,Q^2) \equiv F^{(d)}(x,Q^2) = x\int_{\chi(x,Q^2)}\frac{dy}{y}C\left(\frac{\chi(x,Q^2)}{y},\alpha(Q^2)\right)f(y,Q^2),$$

$$F^{(d,\,\chi,v2)}(x,Q^2) \equiv F^{(d)}(\chi(x,Q^2),Q^2), \quad \chi = x\left(1+\frac{4m^2}{Q^2}\right).$$

The choice of threshold prescription represent an intrinsic ambiguity of the matching procedure. Can this ambiguity be minimized?

Perturbative ordering in FONLL

Three FONLL schemes for different ordering of the perturbative expansion can be defined:

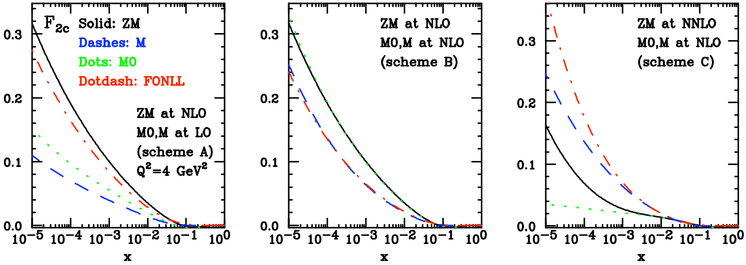
1. Scheme A → $\mathcal{O}(\alpha_s)$ in massless and in massive
2. Scheme B → $\mathcal{O}(\alpha_s)$ in massless and $\mathcal{O}(\alpha_s^2)$ in massive
3. Scheme C → $\mathcal{O}(\alpha_s^2)$ in massless and in massive

In any of the three schemes, any threshold prescription can be implemented
These schemes can be related to existing approaches

1. Scheme A is identical to S-ACOT
2. Scheme B was formulated with similar scope as TR (use the information from the $\mathcal{O}(\alpha_s^2)$ massive computation in a NLO GM-VFN scheme), but they turn to be different
3. Scheme C should be S-ACOT at NNLO?

$F_{2c}(x, Q^2)$ in FONLL

The different contributions to FONLL for $F_{2c}(x, Q^2)$:



In FONLL scheme B ZM$\sim$M0 even at $Q^2 \sim 20$ GeV^2 , so FONLL$\sim$Massive
Greatly reduced sensitivity to choice of (arbitrary) threshold prescription present in scheme A

In all schemes mass-suppressed corrections are important even at moderate Q^2

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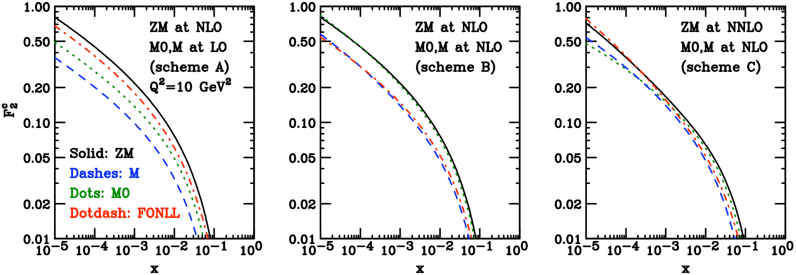

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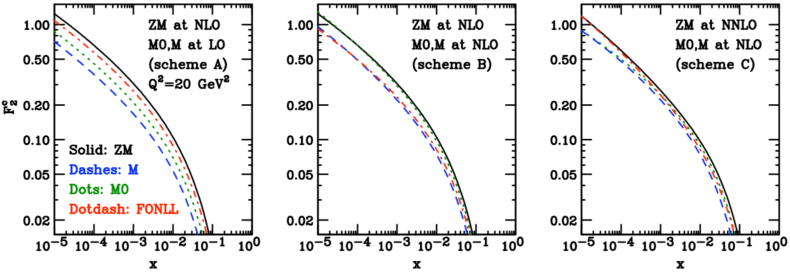

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In all schemes mass-suppressed corrections are important even at moderate Q^2

Results: F_{2c} in FONLL vs. S-ACOT

The $\chi-$scaling threshold prescription used in S-ACOT-χ can be implemented in two alternative ways (with the difference being subleading)

- $x \to \chi$ replacement only inside convolutions

$$F^{(\chi)}(x,Q^2) = x\int_{\chi(x,Q^2)} \frac{dy}{y} C\left(\frac{\chi(x,Q^2)}{y},\alpha(Q^2)\right) f(y,Q^2),$$

- $x \to \chi$ replacement in the structure function argument

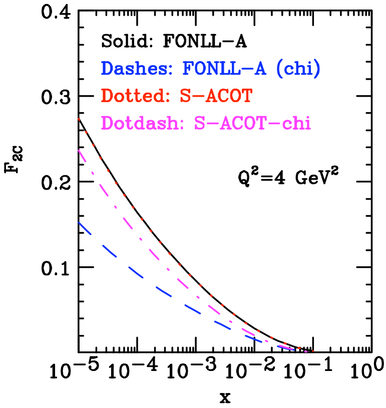
$$F^{(\chi,v2)}(x,Q^2) = \chi(x,Q^2)\int_{\chi(x,Q^2)} \frac{dy}{y} C\left(\frac{\chi(x,Q^2)}{y},\alpha(Q^2)\right) f(y,Q^2),$$

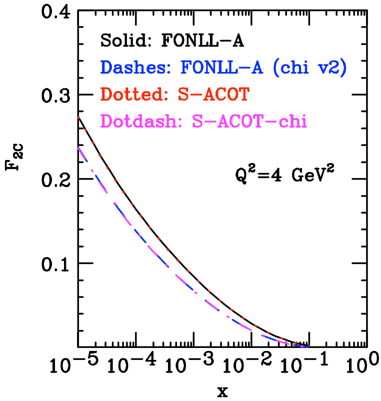
$$\chi(x,Q^2) = x\left(1+\frac{4m^2}{Q^2}\right).$$

$F^{(\chi)}(x,Q^2)$ used in CTEQ6.6, while $F^{(\chi,v2)}(x,Q^2)$ implemented in MSTW2008

Results: F_{2c} in FONLL vs. S-ACOT

S-ACOT is identical to FONLL scheme A
S-ACOT-χ is identical to FONLL scheme A with χ scaling (v2)





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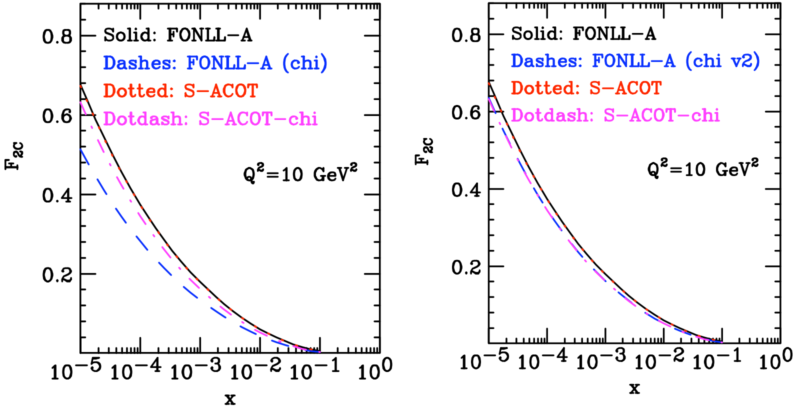
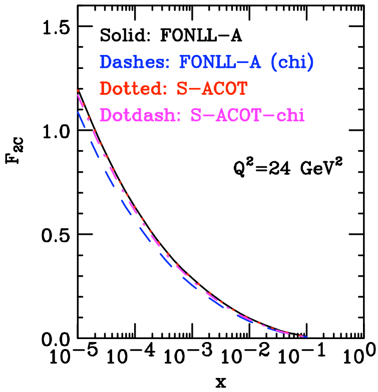
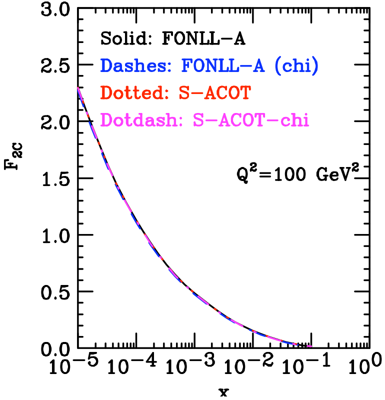


Results: F_{2c} in FONLL vs. S-ACOT

As Q^2 increases all schemes are identical (threshold effects negligible)



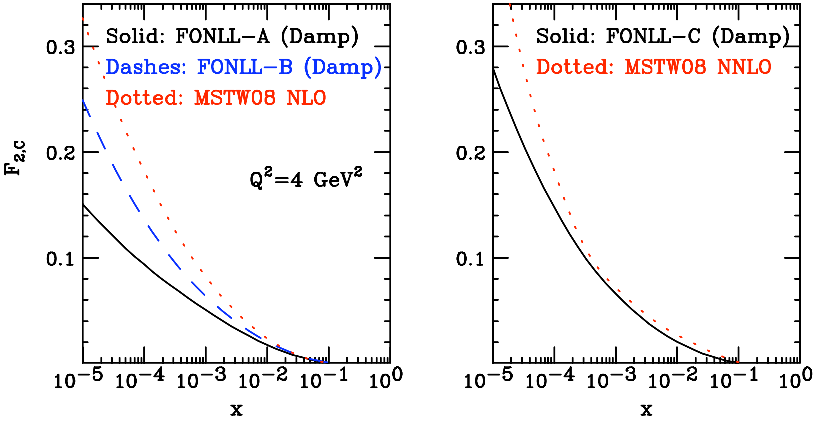


Results: F_{2c} in FONLL vs. S-ACOT

- FONLL-A (plain) is identical to S-ACOT (both for F_{2c} and for F_{Lc})
- FONLL-A is identical to S-ACOT-χ once the proper threshold prescription is adopted
- The S-ACOT-χ numbers provided by F. Olness use a different $\chi-$scaling than the ones used in the CTEQ6.6 fit (P. Nadolsky)
- It is crucial to carefully state the threshold prescription used in each case $\rightarrow$ In FONLL scheme A (and in S-ACOT) the effect of the threshold prescription can be as large as the resummation itself
- The default threshold prescription used in FONLL (damping factor) falls between the two implementations of $\chi-$scaling

Results: F_{2c} in FONLL vs. MSTW08

With default threshold prescriptions:



Results: F_{2c} in FONLL vs. MSTW08

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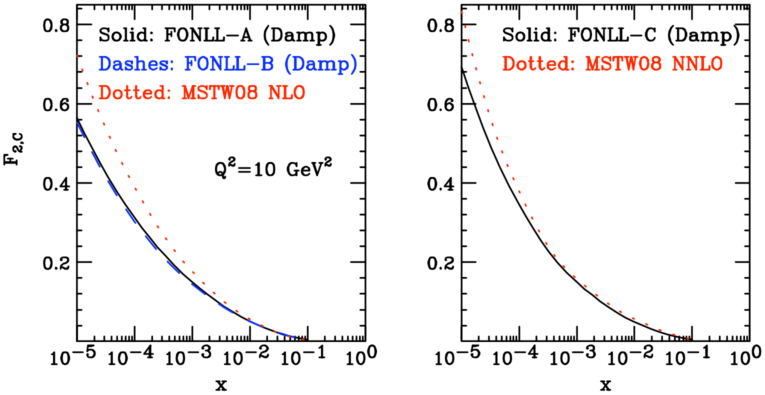
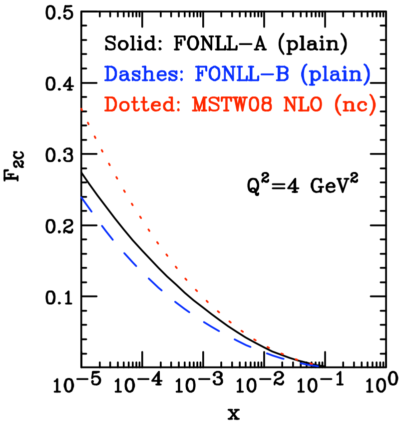


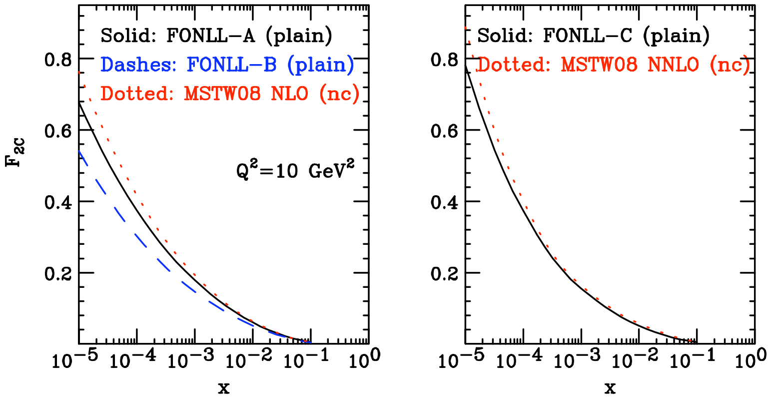
Results: F_{2c} in FONLL vs. MSTW08

With threshold prescriptions switched off:



Results: F_{2c} in FONLL vs. MSTW08

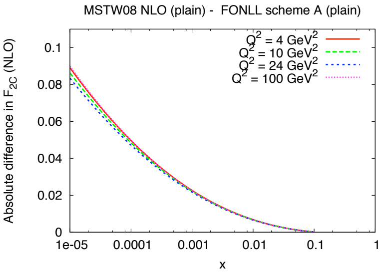
With threshold prescriptions switched off:

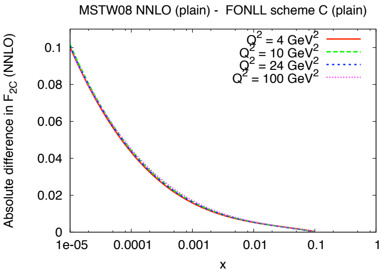


Results: F_{2c} in FONLL vs. MSTW08

The only difference for $F_{2c}(x, Q^2)$ between FONLL scheme A (and scheme C) and MSTW08 NLO (and NNLO) (without threshold prescriptions) is a Q^2-independent matching term f in MSTW08:

$$\text{FONLL-A - MSTW08-NLO} = f\left(x, \alpha_s\left(m_c^2\right)\right)$$





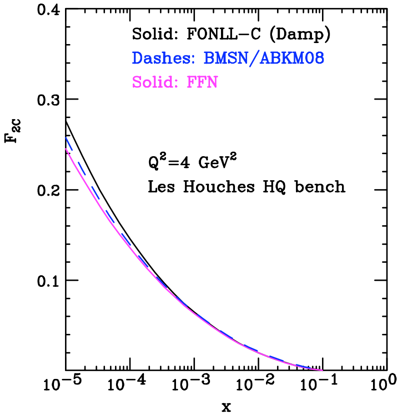
The same conclusions holds for S-ACOT vs. MSTW08 NLO

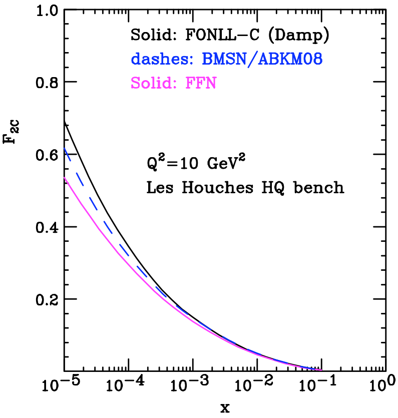
Results: F_{2c} in FONLL vs. MSTW08

Summary of the TR/MSTW08 vs. FONLL comparison

- FONLL scheme B was formulated with a similar motivation than TR → Use all information from the $\mathcal{O}\left(\alpha_s^2\right)$ massive computation in the NLO GM-VFN scheme
- In practice, since TR freeze their $\mathcal{O}\left(\alpha_s^2\right)$ term at $Q^2 = m_c^2$, for F_{2c} TR and FONLL-B turn out to be alternative schemes
- TR NLO is S-ACOT/FONLL-A plus the constant (subleading) term, and shares with these schemes the large dependence on the choice of (arbitrary) threshold prescription (unlike FONLL-B which is unaffected by this choice of prescription)
- Similar conclusions for TR NNLO and FONLL-C: identical up to a Q^2-independent subleading term
- For F_{Lc} instead the TR ordering leads to similar results between FONLL-B and MSTW08.

Results: F_{2c} in FONLL vs. BMSN/ABKM08





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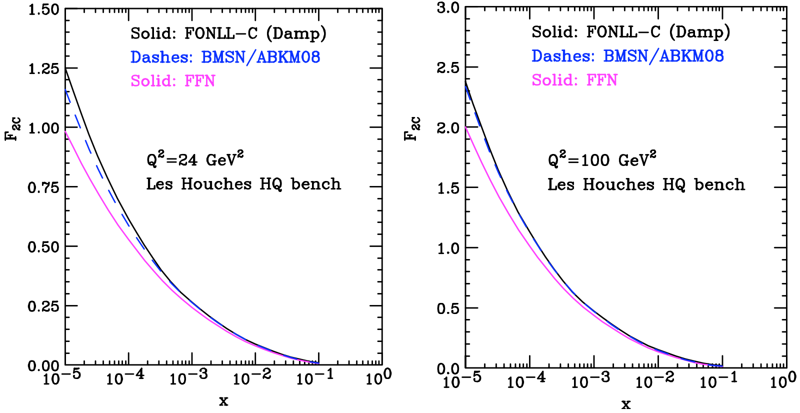
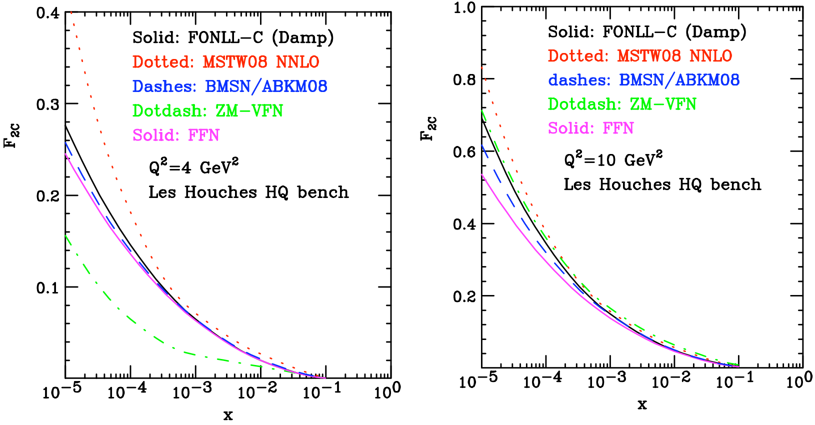
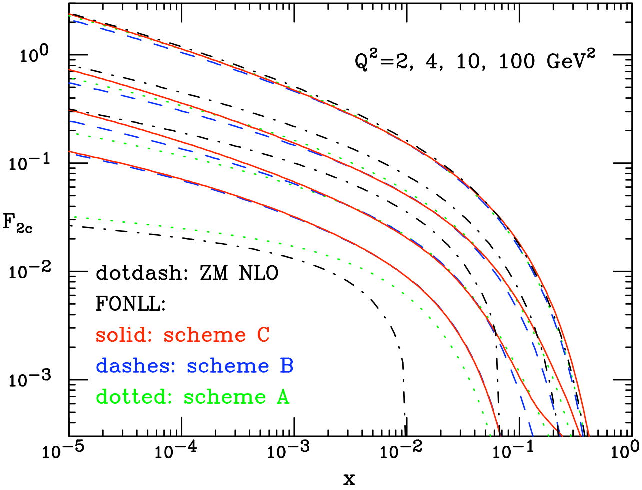


LH HQ benchmarks: F_2^c NNLO schemes summary

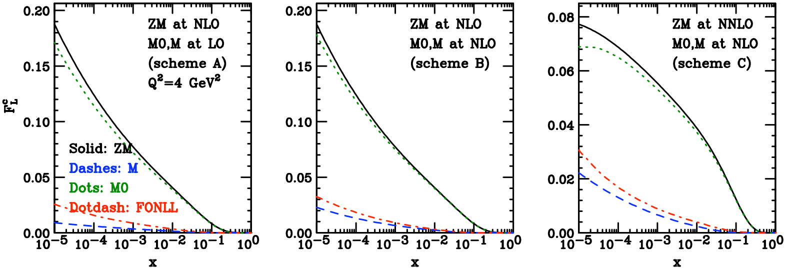


$F_{2c}(x, Q^2)$ in FONLL - Summary



$F_{Lc}(x, Q^2)$ in FONLL

The different contributions to FONLL for $F_{Lc}(x, Q^2)$



In FONLL scheme B ZM∼M0 even at $Q^2 \sim 20$ GeV^2 , so FONLL∼Massive

Reduced sensitivity to choice of (arbitrary) threshold prescription present in scheme A

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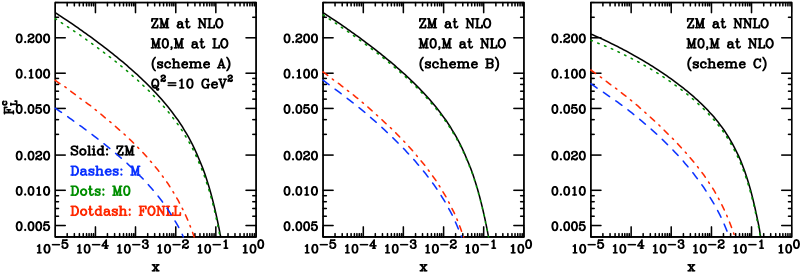

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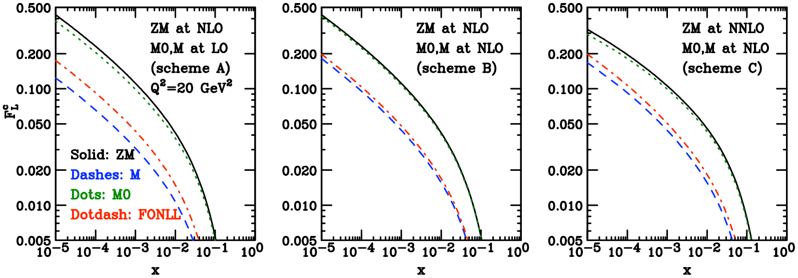
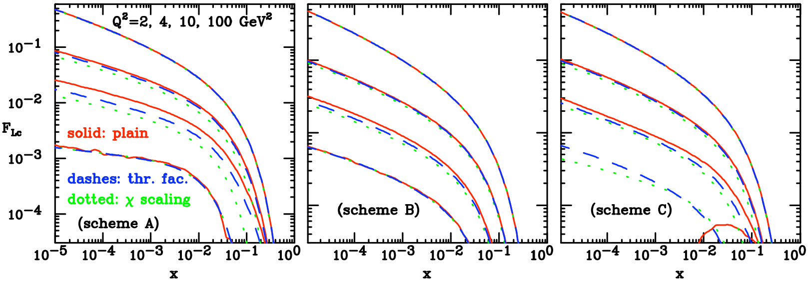


In FONLL scheme B ZM∼M0 even at $Q^2 \sim 20$ GeV^2 , so FONLL∼Massive

Reduced sensitivity to choice of (arbitrary) threshold prescription present in scheme A

$F_{Lc}(x, Q^2)$ in FONLL - threshold prescriptions

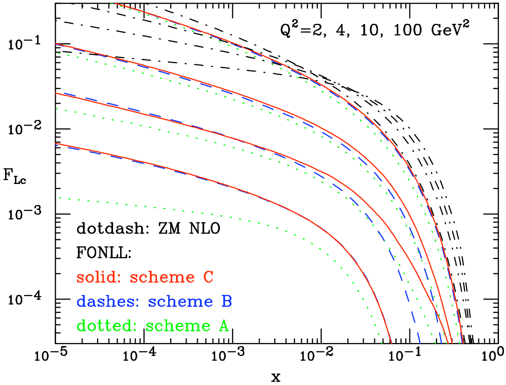
The FONLL result for $F_{Lc}(x, Q^2)$ with different threshold prescriptions



In FONLL the ambiguity due to choice of (arbitrary) threshold prescription present in scheme A dissapears in scheme B

This threshold ambiguity can be as large as the resummation itself

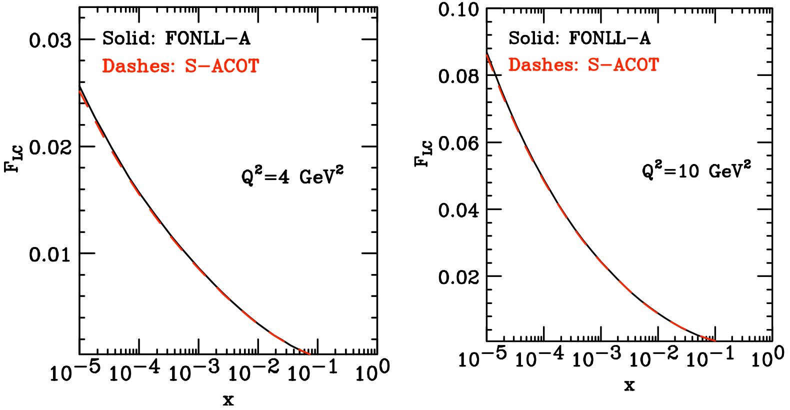
$F_{Lc}(x, Q^2)$ in FONLL - Summary



The massless is very far from FONLL even at large Q^2 for $F_{Lc}(x, Q^2)$

Results: F_{Lc} in FONLL vs. S-ACOT

S-ACOT is identical to FONLL scheme A also for F_{Lc}



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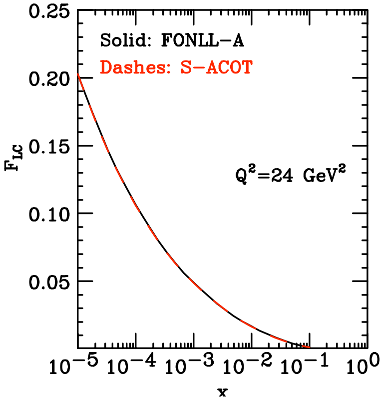
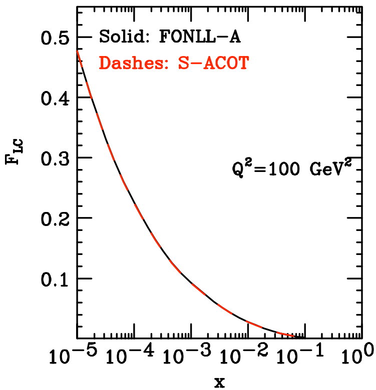
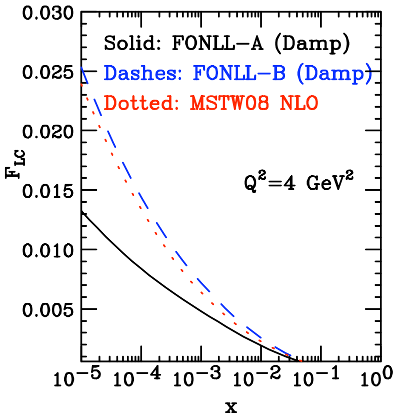




Results: F_{Lc} in FONLL vs. MSTW08

With default threshold prescriptions:



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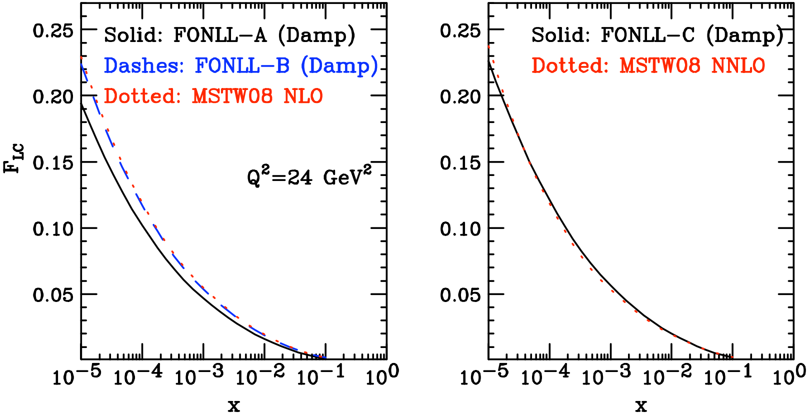

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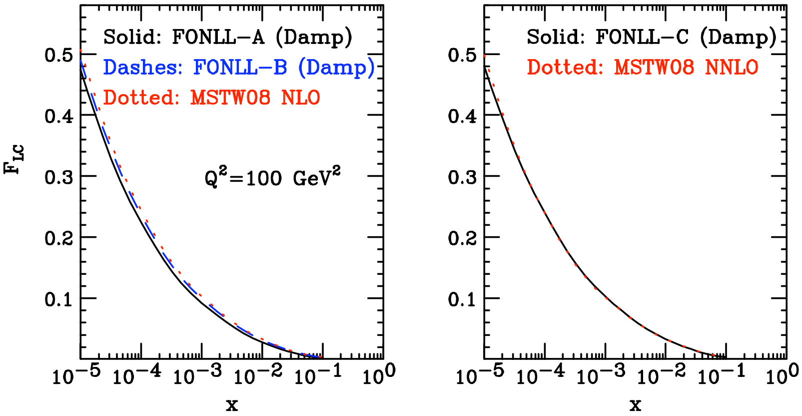
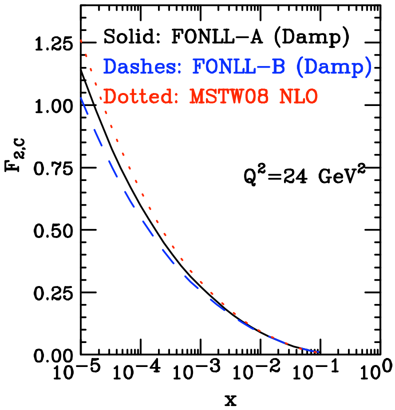
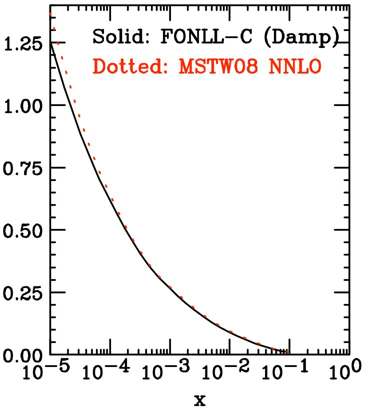


FONLL vs. MSTW08

With default threshold prescriptions:





FONLL vs. MSTW08

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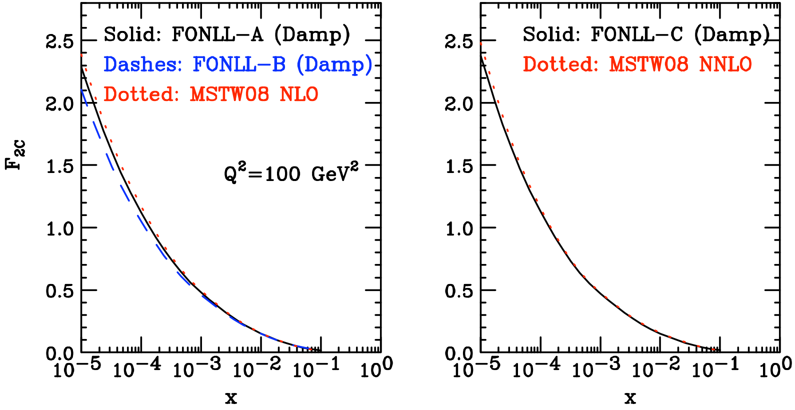
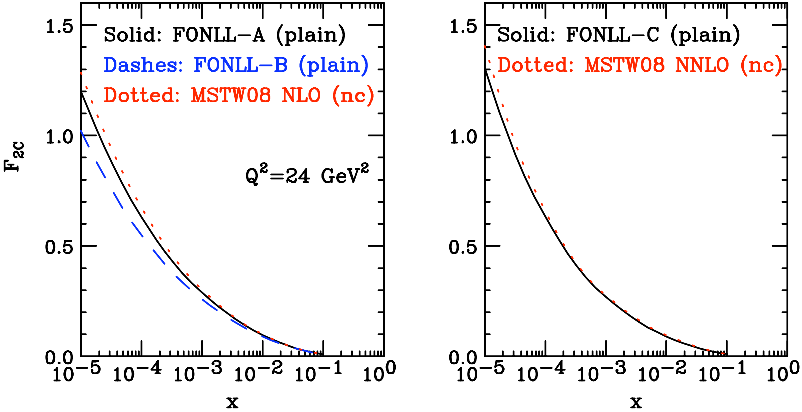


FONLL vs. MSTW08

With threshold prescriptions switched off:



FONLL vs. MSTW08

With threshold prescriptions switched off: