Preliminary study of the electron beam impedance

N. Mounet

Acknowledgements: D. Mirarchi, C. Zannini.
The hollow electron lens

Since 2020, the hollow electron lens is in the HL-LHC baseline:

- Hollow e-lens
- Circulating beam
- TCP
- TCSG
- TCLA
- Secondary halo + hadronic shower
- Tertiary halo
- Primary halo

From D. Mirarchi, WP2 15/09/2020

Regarding impedance & stability, several potential impacts:

- Depletion of transverse distribution tails, reducing Landau damping
  → taken into account in all stability predictions (tails cut at 3.2σ)
- Impedance of the physical device
  → see previous talk by C. Zannini and B. Salvant
- But what about the impedance of the electron beam itself?
Previous studies

- The issue was studied by A. Burov et al in 1999:

**Transverse beam stability with an “electron lens”**

A. Burov,* V. Danilov,† and V. Shiltsev

*Fermi National Accelerator Laboratory, Batavia, Illinois 60510*

(Received 29 July 1998)

- This study is partly reproduced in V. Shiltsev’s book
- … and used as reference in the conceptual design of hollow lens, by G. Stancari et al (CERN-ACC-2014-0248):

> “An early concern on the use of electron lenses […] was the stability of the beams. […] In particular, a displaced head of the circulating bunch could distort the electron beam, whose electromagnetic fields could in turn act back on the bunch tail, causing oscillations in the electron trajectory and a fast transverse mode coupling instability […]. The electron beam is made stiff by increasing the axial solenoidal field, reducing its effective impedance […].“
If a proton passes by with an offset, it kicks the electrons which are at the same longitudinal position.

The electrons start spiraling under the action of the solenoid field.
If a proton passes by with an offset, it **kicks the electrons** which are at the same longitudinal position.

The electrons start **spiraling** under the action of the **solenoid** field.

At a later time, a “test” proton behind the initial one, will in turn see offset electrons and receive a **kick** (in both the x and y directions → coupling).
A simple model for the wake

- Assuming both proton and electron beams are uniform and of same radius $a_e$, and looking only at the first order perturbation of the beams space-charge fields (i.e. keeping everything linear),
- electrons are kicked by a displaced proton slice, then move transversely under the sole effect of the solenoid field (Larmor oscillations), and finally kick the protons behind, at a later time.

$\Rightarrow$ we get wake functions: for a source proton displaced by $\Delta x$ and $\Delta y$

$$W_x = W \sin(kz) \Delta x - W (1 - \cos(kz)) \Delta y$$
$$W_y = W (1 - \cos(kz)) \Delta x + W \sin(kz) \Delta y$$

with

$$W = -\left(\frac{1}{4\pi\varepsilon_0}\right)^2 \frac{4L_e I_e}{a_e^4 \beta_e e^2 B} \frac{(1 + \beta_p \beta_e)^2}{\beta_p + \beta_e}$$

$$k = \frac{\omega_L}{(\beta_p + \beta_e)c}, \quad \omega_L = \frac{eB}{m_e \gamma_e}$$

See A. Burov et al, PRE 59, 3 (1999) (converted to SI units and using PyHEADTAIL sign convention)

- $L_e = 3 \text{ m}$,
- $I_e = 5 \text{ A}$,
- $\gamma_e = (1 - \beta_e^2)^{-1} = 1.029$ (15 keV e$^-$),
- $\beta_p \approx 1$ (top energy)
- $B = 5 \text{ T}$
- $a_e = 2.9 \text{ mm}$ (9.4 $\sigma$ with 2.5 $\mu$m emittance).

(Parameters from D. Mirarchi)
A simple model for the TMCI

- In absence of chromaticity and of any other kind of impedance, a simple formula can be found for the transverse mode coupling instability threshold, in terms of $B$ field:

$$B_{thr} \approx 39 \frac{eN_p \sqrt{\xi_x \xi_y}}{a_e^2 \sqrt{Q_s |Q_x - Q_y|}},$$

with the figures of merit of the e-lens defined as

$$\xi_x = \frac{\beta_x L_e r_p L_e (1+\beta_e)}{2\pi a_e^2 \gamma_p e \beta_e c}, \quad \xi_y = \frac{\beta_y L_e r_p L_e (1+\beta_e)}{2\pi a_e^2 \gamma_p e \beta_e c}.$$

- In HL, one gets stability as soon as

$$B > B_{thr} \approx 0.07 \text{ T}$$

Even multiplying by the extra factor found $(1 + \beta_p \beta_e)^2$ (see previous slide), one gets $B_{thr} \approx 0.1 \text{ T}$.

⇒ It seems very far from the nominal $B = 5 \text{ T}$. 

From A. Burov et al, PRE 59, 3 (1999) (converted to SI units)

$$\beta_x = \beta_y = 280 \text{ m},$$

$$Q_s = 2.1 \times 10^{-3},$$

$$|Q_x - Q_y| = 0.01,$$

$$N_p = 2.3 \times 10^{11} \text{ p+/bunch},$$

$$r_p = 1.535 \times 10^{-18} \text{ m},$$

$$\gamma_p = (1 - \beta_p^2)^{-\frac{1}{2}} = 7460.52,$$

$$\xi_x = \xi_y = 1.8 \times 10^{-3}. $$
Checking the TMCI threshold with PyHEADTAIL:

One gets $B_{\text{thr}} \approx 0.077$ T.

Many thanks to Carlo Zannini for providing an initial HL-LHC PyHEADTAIL script.
Still several shortcomings

- Some assumptions of the model are not true in the case of the HL-LHC electron lens: the electron beam is hollow (so clearly not uniform), and the proton beam is Gaussian and typically much much smaller.

- The wake extends up to $\sim 52 \text{ ns} \rightarrow$ potential multibunch effects.

- Only TMCI was checked, at zero chromaticity.
  $\rightarrow$ potential weak headtail instabilities at higher chromaticities ($Q' \sim 15$),
  $\rightarrow$ more generally, one should add the electron beam impedance to the full model and check the impact.
Comparing the total wake of the HL-LHC model (latest update, retracted collimators, see WP2 28/07/2020) with the electron-beam wake model:

⟹ significantly smaller than the dipolar total wake (x or y) within the bunch,
⟹ but coupled terms much stronger than the rest of the wake.
In a standard operational configuration (Q’=15, 100 turns damper), adding the wake from the electron beam on top of the full HL-LHC model (both dipolar and coupled terms):

\[ \implies \text{no strong impact} \] on the most unstable plane (y), even at a lower B field and at a lower x-y tune difference (rise times stay within \( \sim10\% \)).
The electron beam of an electron lens can trigger coherent instabilities, in particular TMCI-like.

These can be strongly mitigated with the solenoid field. In the case of HL-LHC, as low as $B=0.1$ T should be enough to avoid TMCI coming from the e-beam alone.

The simple wake model from A. Burov et al, shows nevertheless that coupled terms in the e-beam wake are larger than those of the full HL-LHC model.

Still, single-bunch instabilities in a standard operational configuration ($Q'=15$, damper 100 turns) seem not to be strongly affected.

Several shortcomings of the model need to be addressed:
- the respective size and transverse shape of the protons and electron beams have to be modelled better,
- potential multibunch effects to be looked at.
Appendix
Comparing the total wake of the HL-LHC model (latest update, retracted collimators, see WP2 28/07/2020) with the electron-beam wake model:

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