Preliminary study of the electron beam impedance

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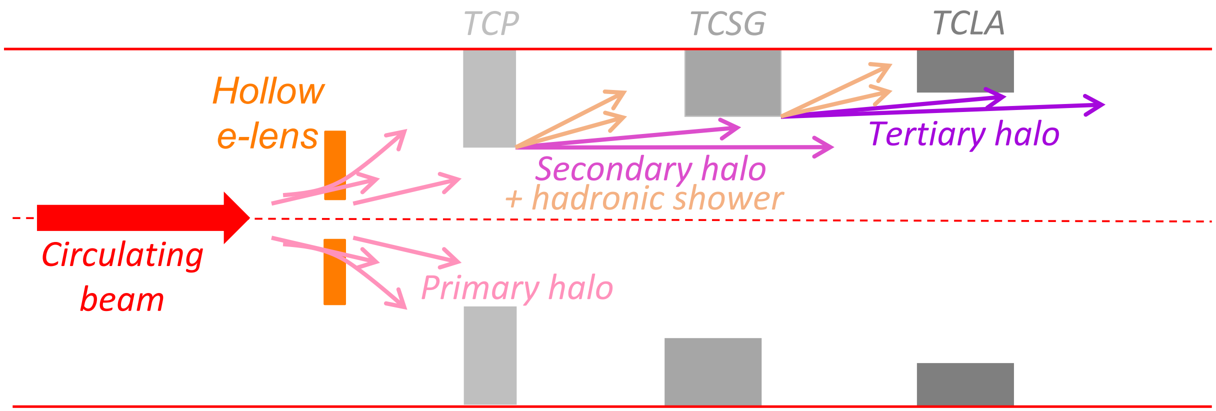
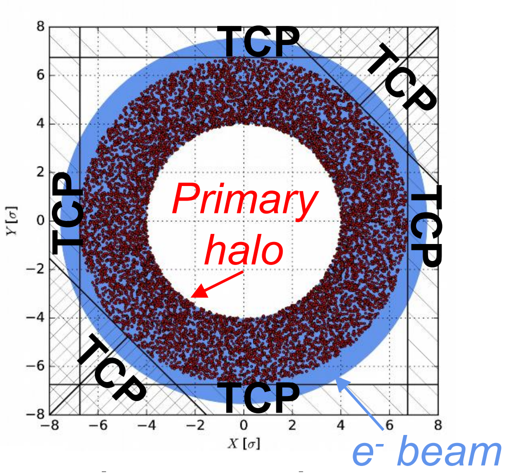
Acknowledgements: D. Mirarchi, C. Zannini.

The hollow electron lens

- Since 2020, the hollow electron lens is in the HL-LHC baseline:





From ***D. Mirarchi***, WP2 15/09/2020

- Regarding impedance & stability, several potential impacts:
  - Depletion of transverse distribution tails, reducing Landau damping ⟶ taken into account in all stability predictions (tails cut at **3.2σ**)
  - Impedance of the physical device ⟶ see previous talk by ***C. Zannini*** and ***B. Salvant***
  - But what about the **impedance of the electron beam** itself?

Previous studies

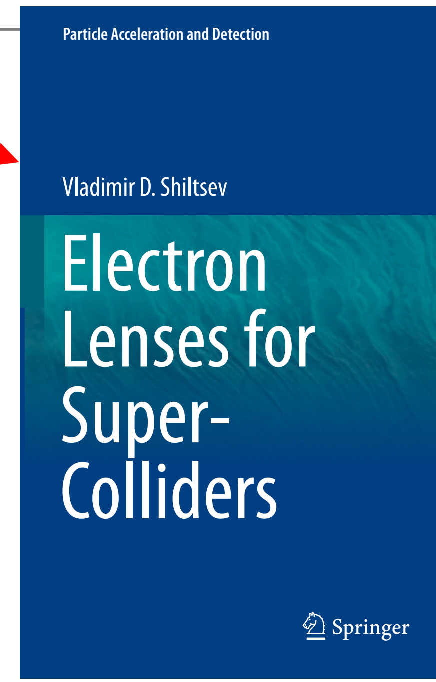
- The issue was studied by **A. Burov** et al in 1999:

PHYSICAL REVIEW E VOLUME 59, NUMBER 3 MARCH 1999

Transverse beam stability with an ''electron lens''

A. Burov,* V. Danilov,† and V. Shiltsev
Fermi National Accelerator Laboratory, Batavia, Illinois 60510
(Received 29 July 1998)

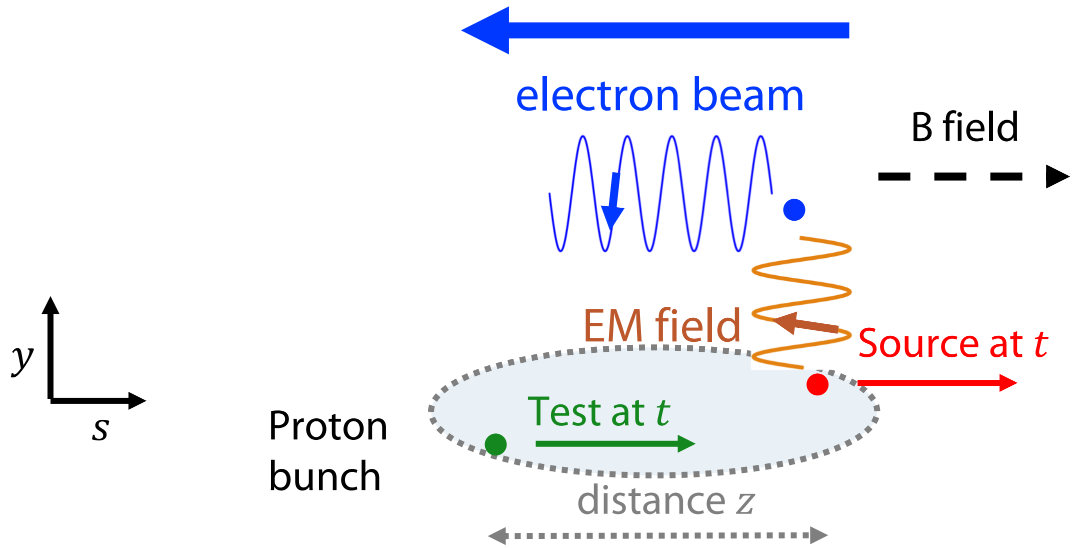
- This study is partly reproduced in **V. Shiltsev**'s book



- … and used as reference in the conceptual design of hollow lens, by **G. Stancari** et al (***CERN-ACC-2014-0248***):

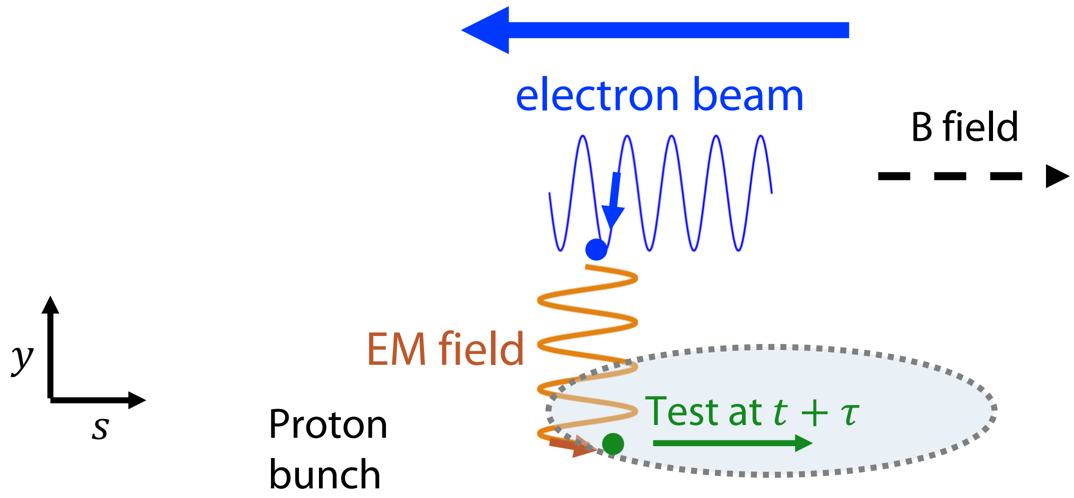
"An early concern on the use of electron lenses […] was the stability of the beams. [..] In particular, a displaced head of the circulating bunch could distort the electron beam, whose electromagnetic fields could in turn act back on the bunch tail, causing oscillations in the electron trajectory and a fast transverse mode coupling instability […].
The electron beam is made stiff by increasing the axial solenoidal field, reducing its effective impedance […]."

Electron beam impedance



- If a proton passes by with an offset, it kicks the electrons which are at the same longitudinal position.
- The electrons start spiraling under the action of the solenoid field.

Electron beam impedance



- If a proton passes by with an offset, it kicks the electrons which are at the same longitudinal position.
- The electrons start spiraling under the action of the solenoid field.
- At a later time, a "test" proton behind the initial one, will in turn see offset electrons and receive a kick (in both the x and y directions ⟶ coupling).

A simple model for the wake

- Assuming both proton and electron beams are uniform and of same radius a_e, and looking only at the first order perturbation of the beams space-charge fields (i.e. keeping everything linear),
- electrons are kicked by a displaced proton slice, then move transversely under the sole effect of the solenoid field (Larmor oscillations), and finally kick the protons behind, at a later time.

$\Longrightarrow$ we get **wake functions**: for a source proton displaced by Δx and Δy

$$W_x = W sin(kz)\, \Delta x - W(1-\cos(kz))\, \Delta y$$

$$W_y = W(1-cos(kz))\, \Delta x + W sin(kz)\, \Delta y$$

Coupled terms

See ***A. Burov*** et al, PRE 59, 3 (1999) (converted to SI units and using PyHEADTAIL sign convention)

with $W = -\left(\frac{1}{4\pi\varepsilon_0}\right)^2 \frac{4L_e I_e}{a_e^4 \beta_e c^2 B} \frac{(1+\beta_p\beta_e)^2}{\beta_p+\beta_e}$,

$k = \frac{\omega_L}{(\beta_p+\beta_e)c}$, $\omega_L = \frac{eB}{m_e\gamma_e}$

Absent from original formula

$L_e = 3$ m,
$I_e = 5$ A,
$\gamma_e = (1-\beta_e^2)^{\frac{-1}{2}} = 1.029$ (15 keV e-),
$\beta_p \approx 1$ (top energy)
$B = 5$ T
$a_e = 2.9$ mm (9.4 σ with 2.5 µm emittance).

(Parameters from ***D. Mirarchi***)

A simple model for the TMCI

- In absence of chromaticity and of any other kind of impedance, a simple formula can be found for the transverse mode coupling instability threshold, in terms of B field:

$$B_{thr} \approx 39 \frac{eN_p\sqrt{\xi_x\xi_y}}{a_e^2\sqrt{Q_s|Q_x - Q_y|}},$$

From ***A. Burov*** et al, PRE 59, 3 (1999) (converted to SI units)

with the figures of merit of the e-lens defined as

$$\xi_x = \frac{\beta_x L_e r_p I_e (1+\beta_e)}{2\pi a_e^2 \gamma_p e \beta_e c}, \quad \xi_y = \frac{\beta_y L_e r_p I_e (1+\beta_e)}{2\pi a_e^2 \gamma_p e \beta_e c}$$

$$\beta_x = \beta_y = 280 \text{ m},$$
$$Q_s = 2.1\times10^{-3},$$
$$|Q_x - Q_y| = 0.01,$$
$$N_p = 2.3\times10^{11} \text{ p+/bunch},$$
$$r_p = 1.535\times10^{-18} \text{ m},$$
$$\gamma_p = \left(1 - \beta_p^2\right)^{\frac{-1}{2}} = 7460.52,$$
$$\xi_x = \xi_y = 1.8\times10^{-3}.$$

- In HL, one gets stability as soon as

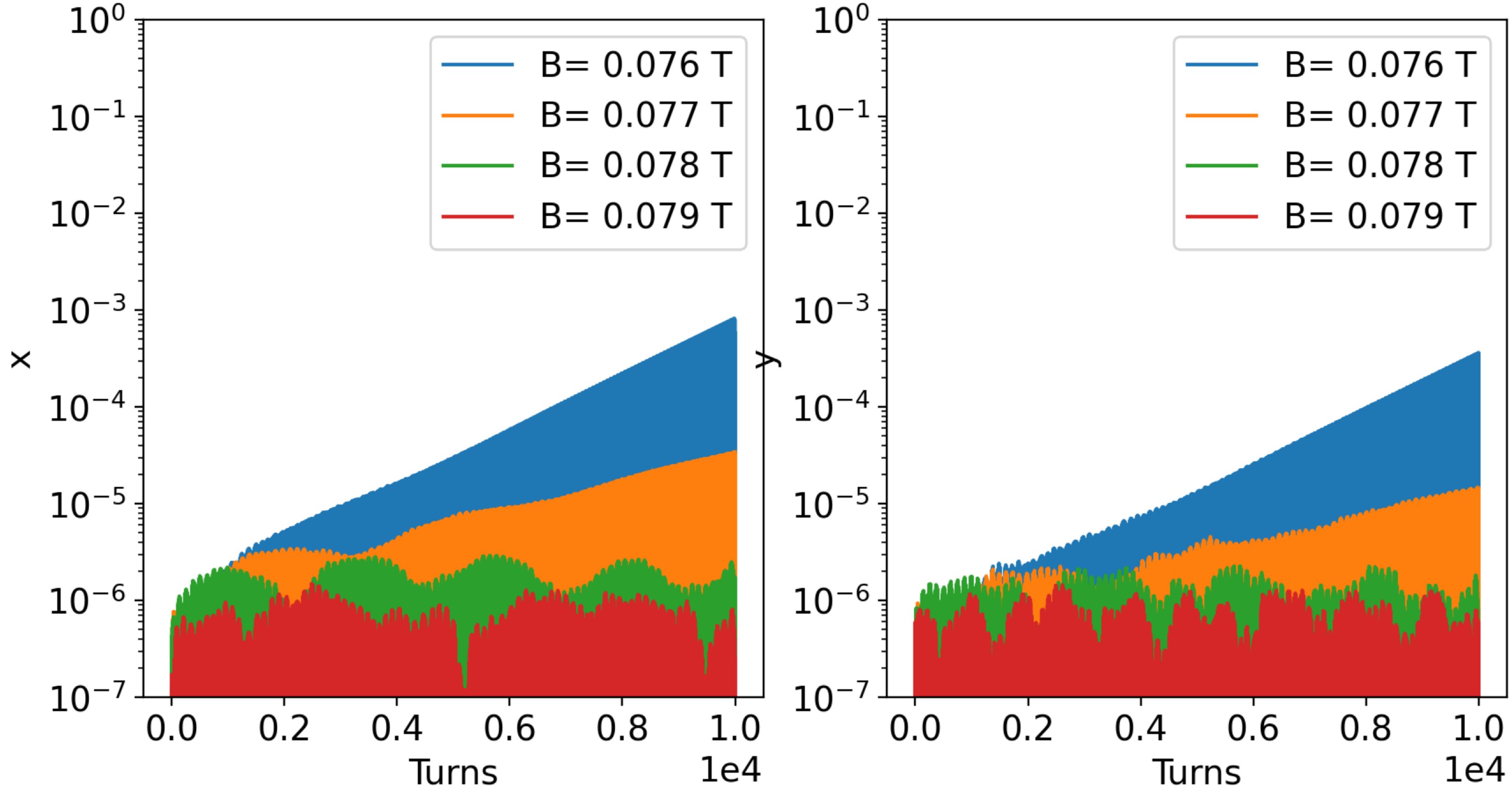
$$\text{B} > B_{thr} \approx 0.07 \text{ T}$$

Even multiplying by the extra factor found $\left(1 + \beta_p\beta_e\right)^2$ (see previous slide), one gets $B_{thr} \approx 0.1$ T.

$\Longrightarrow$ It seems very far from the nominal $B = 5$ T.

E-beam TMCI with PyHEADTAIL

- Checking the TMCI threshold with PyHEADTAIL:



$\Longrightarrow$ one gets $B_{thr} \approx 0.077$ T.

Many thanks to ***Carlo Zannini*** for providing an initial HL-LHC PyHEADTAIL script.

Still several shortcomings

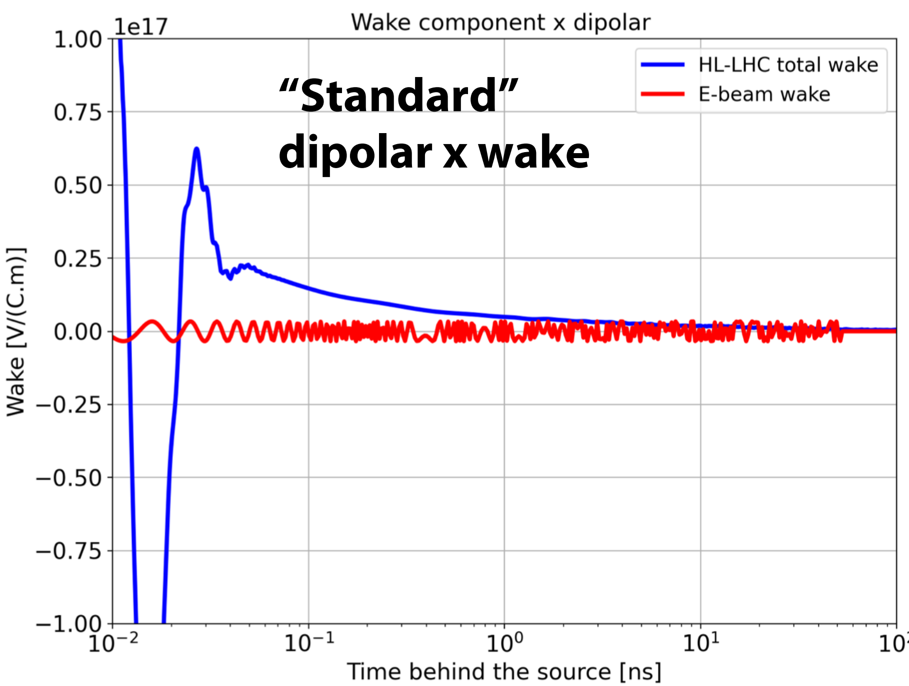
- Some assumptions of the model are not true in the case of the HL-LHC electron lens: the electron beam is hollow (so clearly not uniform), and the proton beam is Gaussian and typically much smaller.

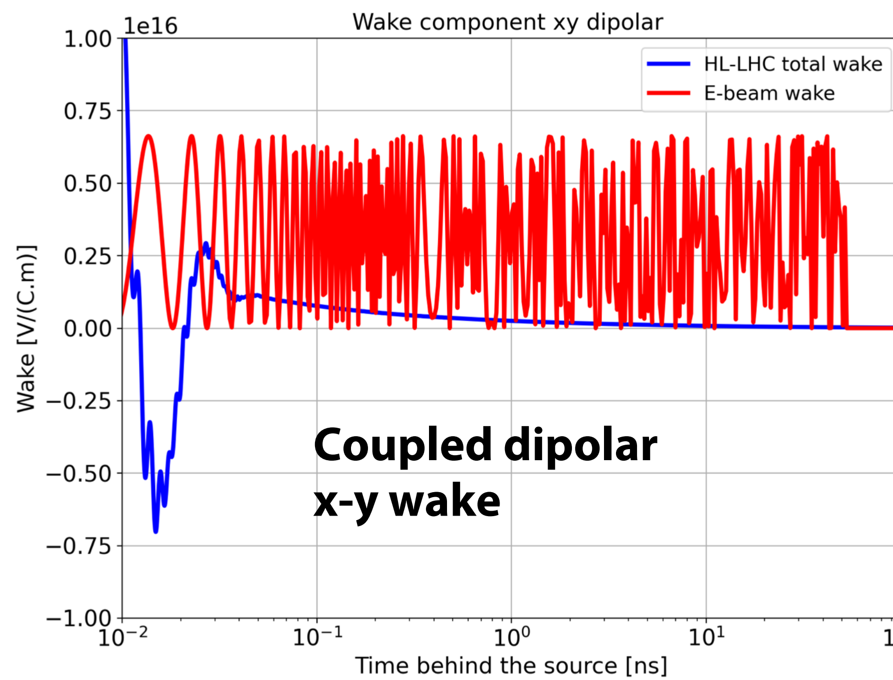
- The wake extends up to ~52 ns ⟶ potential **multibunch** effects.

- Only TMCI was checked, at zero chromaticity.
⟶ potential weak headtail instabilities at higher chromaticities (Q'~15),
⟶ more generally, one should add the electron beam impedance to the full model and check the impact.

Electron beam wake vs. total budget

- Comparing the total wake of the HL-LHC model (latest update, retracted collimators, see WP2 28/07/2020) with the electron-beam wake model:



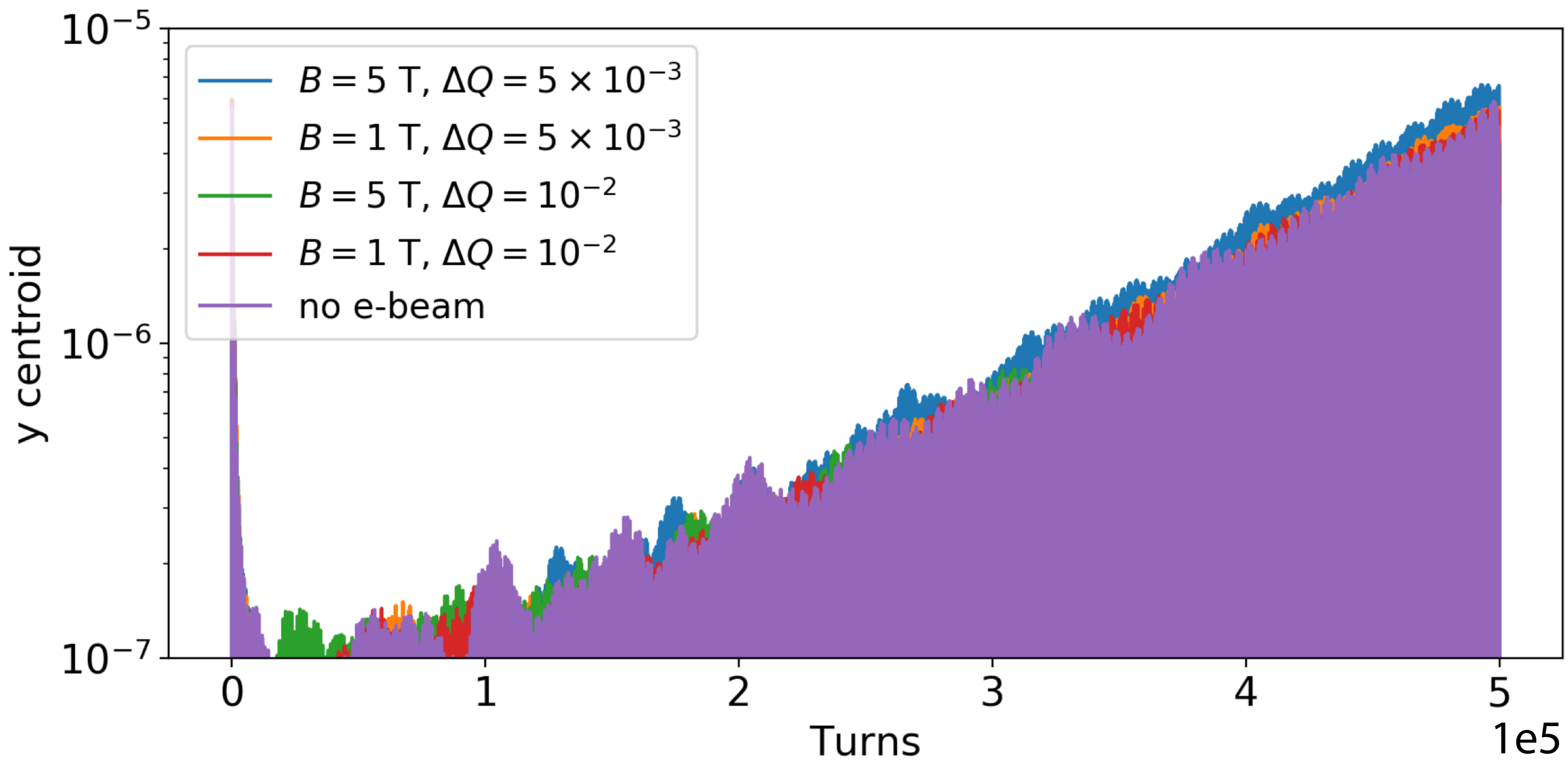


$\Longrightarrow$ significantly smaller than the dipolar total wake (x or y) within the bunch,

$\Longrightarrow$ but coupled terms much stronger than the rest of the wake.

Impact of electron beam on stability

- In a standard operational configuration (Q'=15, 100 turns damper), adding the wake from the electron beam on top of the full HL-LHC model (both dipolar and coupled terms):



$\Longrightarrow$ no strong impact on the most unstable plane (y), even at a lower B field and at a lower x-y tune difference (rise times stay within ~10%).

Conclusion

- The electron beam of an electron lens can trigger coherent instabilities, in particular TMCI-like.
- These can be strongly mitigated with the solenoid field.
  ⟶ In the case of HL-LHC, as low as B=0.1 T should be enough to avoid TMCI coming from the e-beam alone.
- The simple wake model from A. Burov et al, shows nevertheless that coupled terms in the e-beam wake are larger than those of the full HL-LHC model.
- Still, single-bunch instabilities in a standard operational configuration (Q'=15, damper 100 turns) seem not to be strongly affected.
- Several shortcomings of the model need to be addressed:
  - the respective size and transverse shape of the protons and electron beams have to be modelled better,
  - potential multibunch effects to be looked at.

Appendix

Electron beam wake vs. total budget

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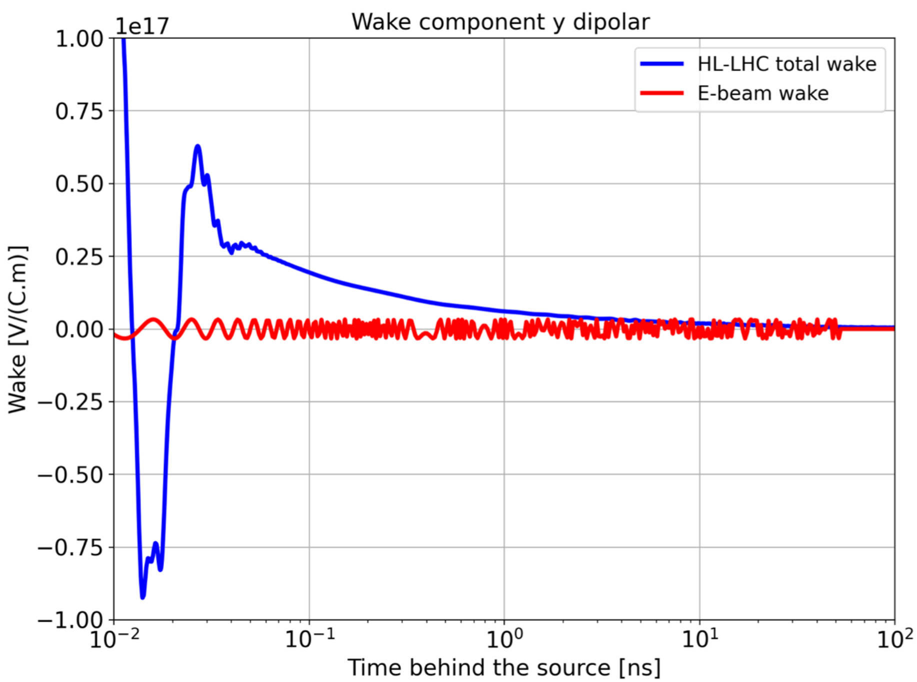

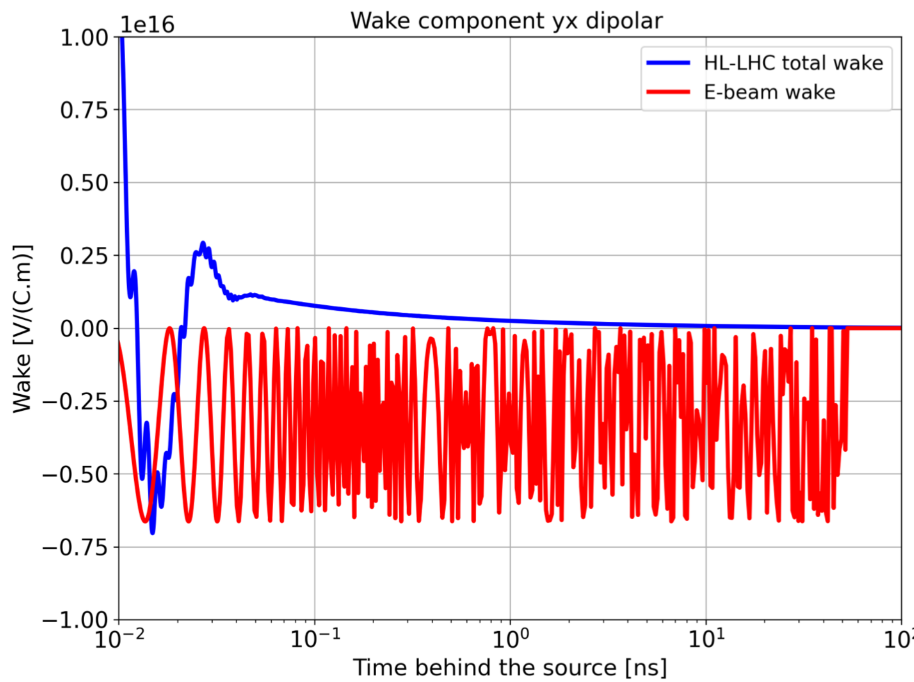

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