Spin Interference in Jet Substructure from Light-ray OPE

Hua Xing Zhu
Zhejiang University

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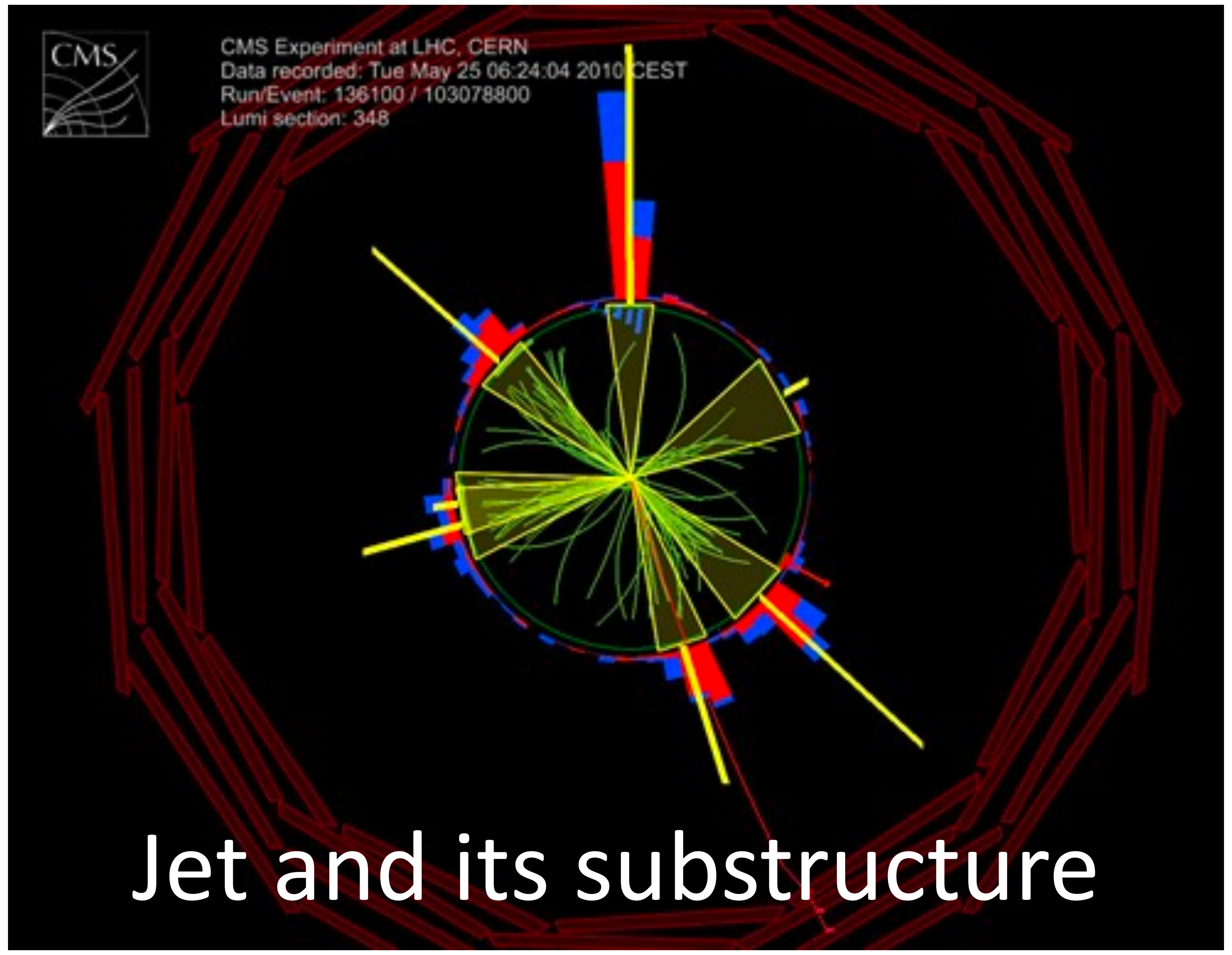
CERN QCD Lunch

1912.11050, with Hao Chen, Ming-xing Luo, Ian Moult, Tong-Zhi Yang, XiaoYuan Zhang
2011.02492, with Hao Chen, Ian Moult + work to appear
with Hao Chen, Ian Moult, Joshua Sanders, work in progress

BSM search



Dynamics of QCD at higher energy:

perturbative calculation

alphaS determination

parton shower

hadronization/power corrections

quantum nature of jet

Laboratory for QFT:

Scattering amplitudes

Feynman integrals

Wilson loops

integrability of twist operators

light-ray operator

- Numerous substructure observable proposed in the last decades. These observables are designed to achieve different goals: maximizing BSM signal, reducing soft hadron contamination, ……

- For deciphering the dynamics of QCD, guiding principles are (subject to theorists own tastes):

  - better perturbative behavior

  - nice factorization properties

  - simple operator definition

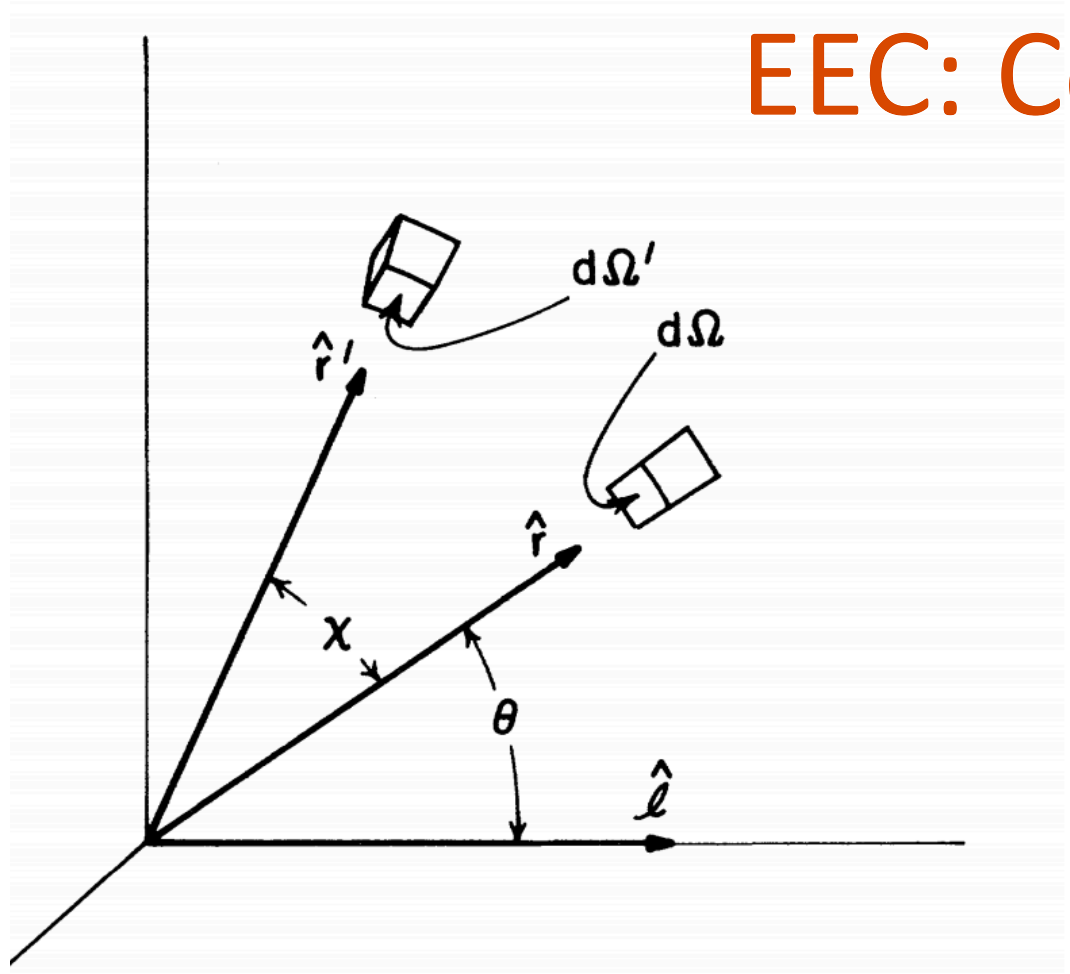
Energy Correlations in Electron-Positron Annihilation: Testing Quantum Chromodynamics

C. Louis Basham, Lowell S. Brown, Stephen D. Ellis, and Sherwin T. Love
Department of Physics, University of Washington, Seattle, Washington 98195
(Received 21 August 1978)

An experimental measure is presented for a precise test of quantum chromodynamics. This measure involves the asymmetry in the energy-weighted opening angles of the jets of hadrons produced in the process $e^+e^- \to$ hadrons at energy W. It is special for several reasons: It is reliably calculable in asymptotically free perturbation theory; it has rapidly vanishing (order $1/W^2$) corrections due to nonperturbative confinement effects; and it is straightforward to determine experimentally.

Energy Correlations

Basham, Brown, Ellis, Love, 1978



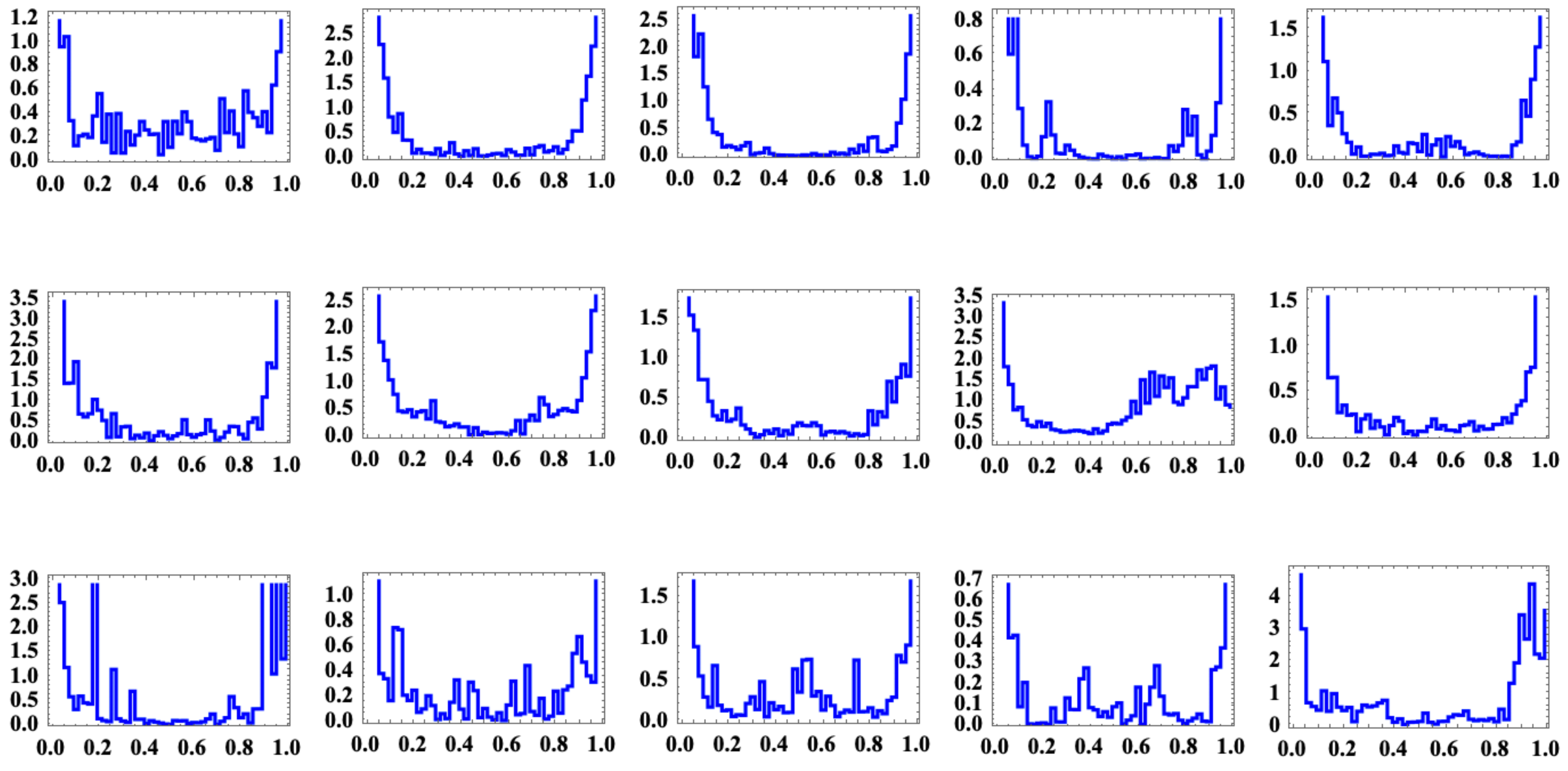
EEC: Correlation of energy deposition between two detector at an angle χ

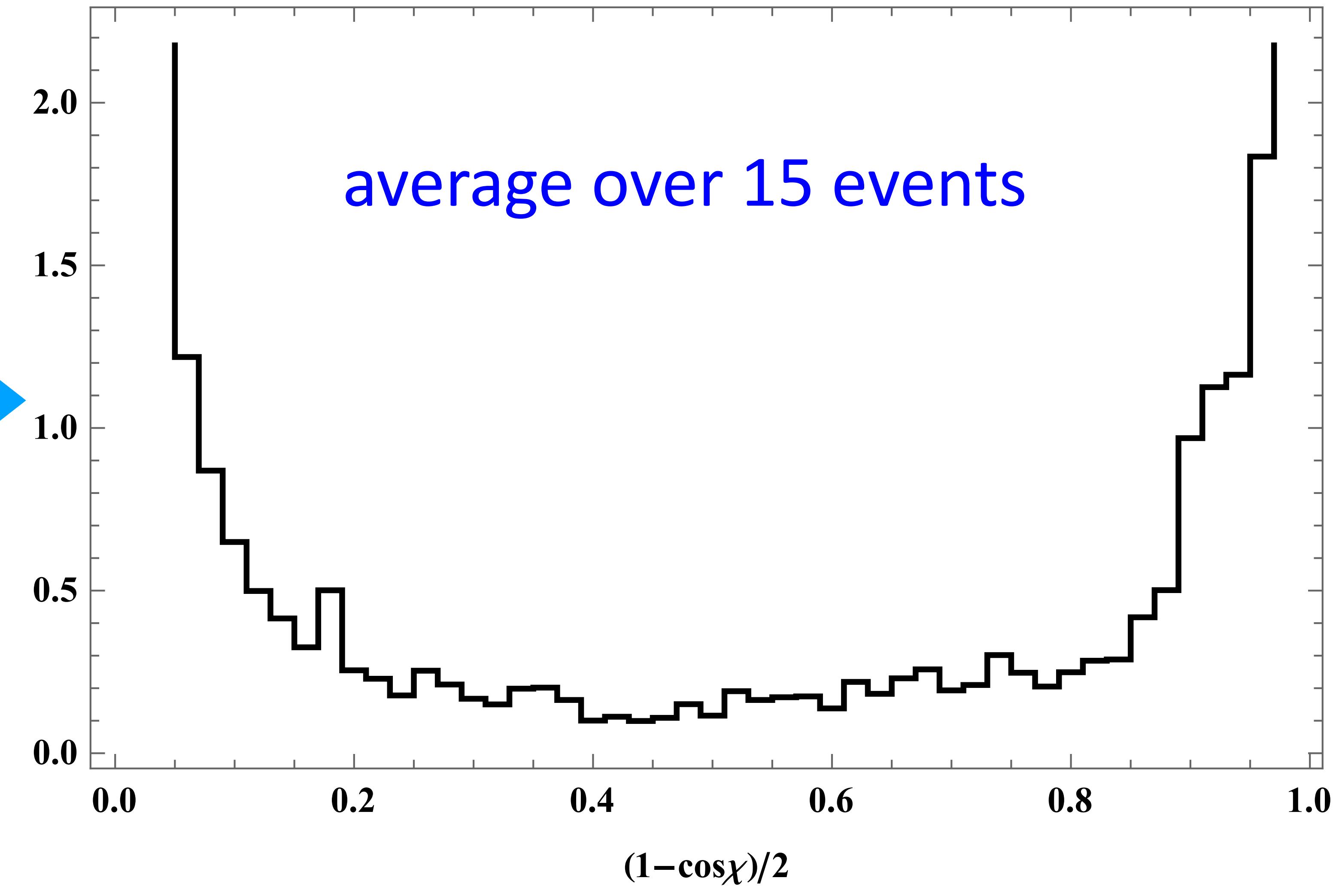
$$\mathrm{EEC}(\chi) = \frac{1}{N}\sum_{\mathrm{events}}^{N}\sum_{i,j}^{N_{\mathrm{particles}}} \frac{E_i E_j}{E_{\mathrm{tot}}^2}\left(\frac{1}{\Delta\chi}\int_{\chi-\Delta\frac{\chi}{2}}^{\chi+\Delta\frac{\chi}{2}} \delta(\chi'-\chi_{ij})\,d\chi'\right)$$

$$\mathrm{EEC}(\chi) = \frac{1}{\sigma_{\mathrm{tot}}}\sum_{i,j}\int d\sigma_{e^+e^-\to i,j+X}\frac{E_i E_j}{E_{\mathrm{tot}}^2}\delta(\chi-\chi_{ij})$$

Weighted cross section

Measurement on a single event gives a function





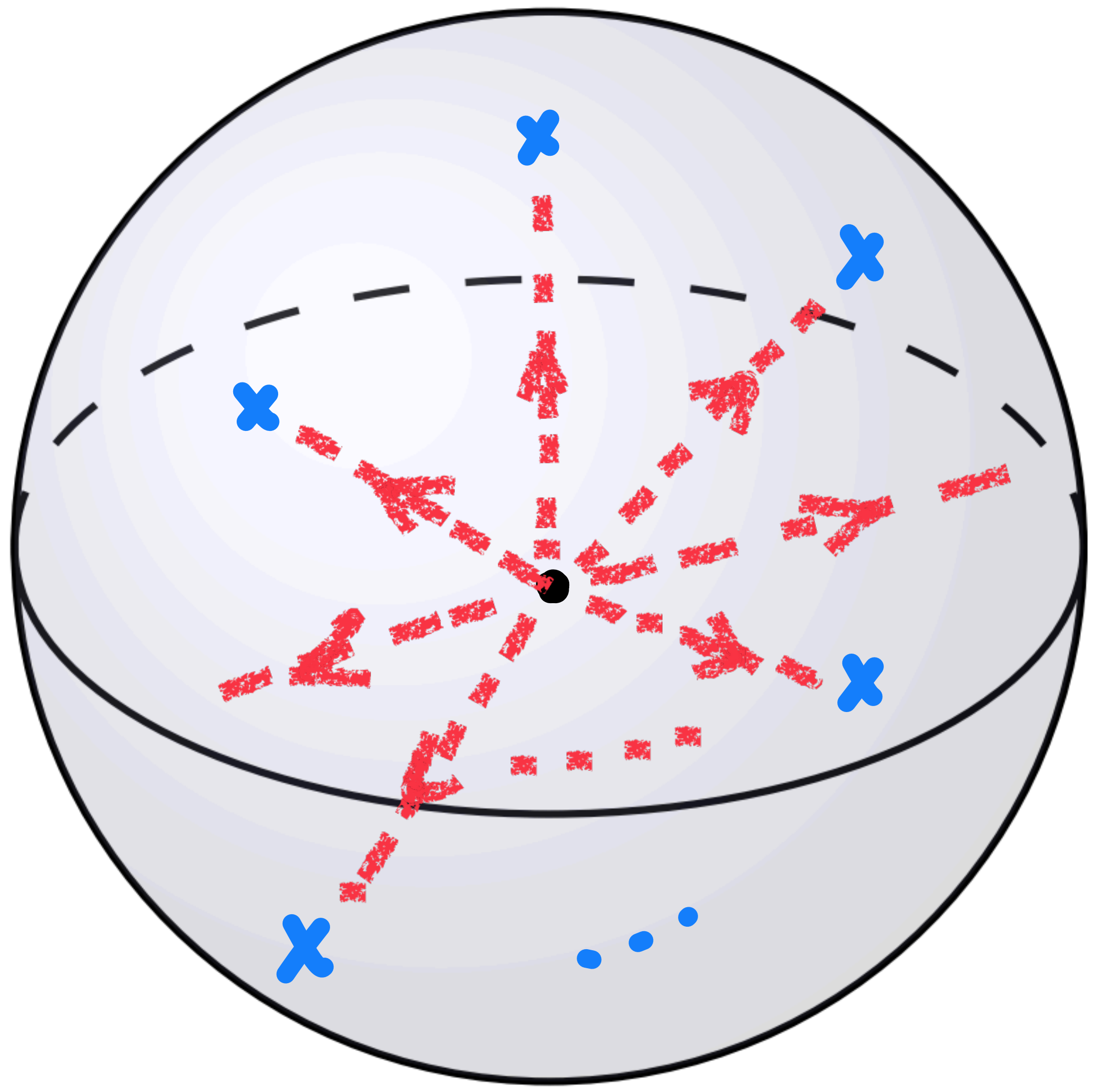
N-point energy correlator and celestial sphere

$$\mathrm{EEC}(\{\chi\}) = \frac{1}{\sigma_{\mathrm{tot}}} \sum_{i_1, i_2, \ldots, i_N} \int d\sigma_{e^+e^- \to i_1, \ldots, i_N + X} \frac{E_{i_1} E_{i_2} \cdots E_{i_N}}{E_{\mathrm{tot}}^N} \delta(\chi_{12} - \chi_{i_1 i_2}) \cdots \delta(\chi_{N-1,N} - \chi_{i_{N-1} i_N})$$

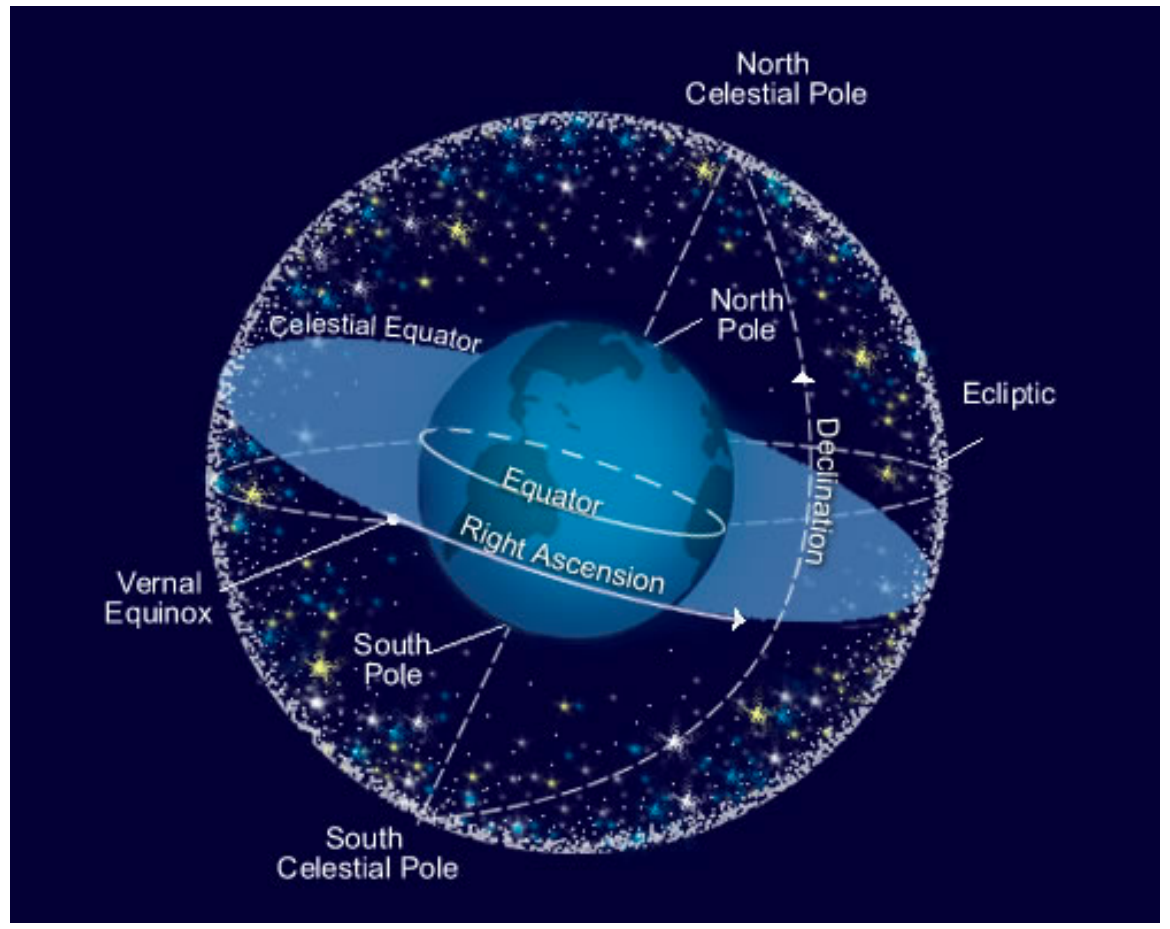
Place N energy detector at N marked point on the celestial sphere

Weighted cross section parameterized by N(N-1)/2 angles on the celestial sphere

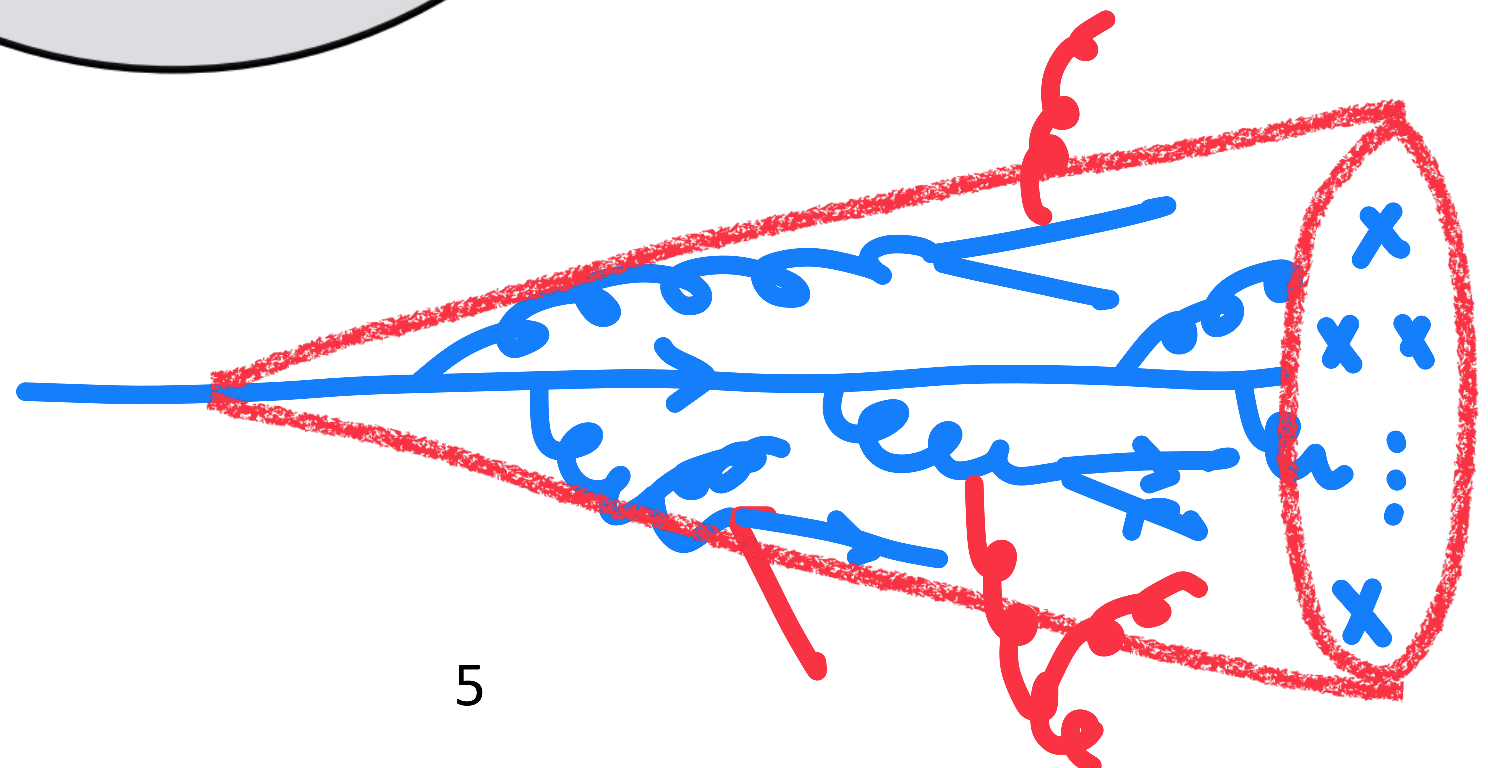
collider celestial sphere



astronomy celestial sphere



collinear limit
jet substructure



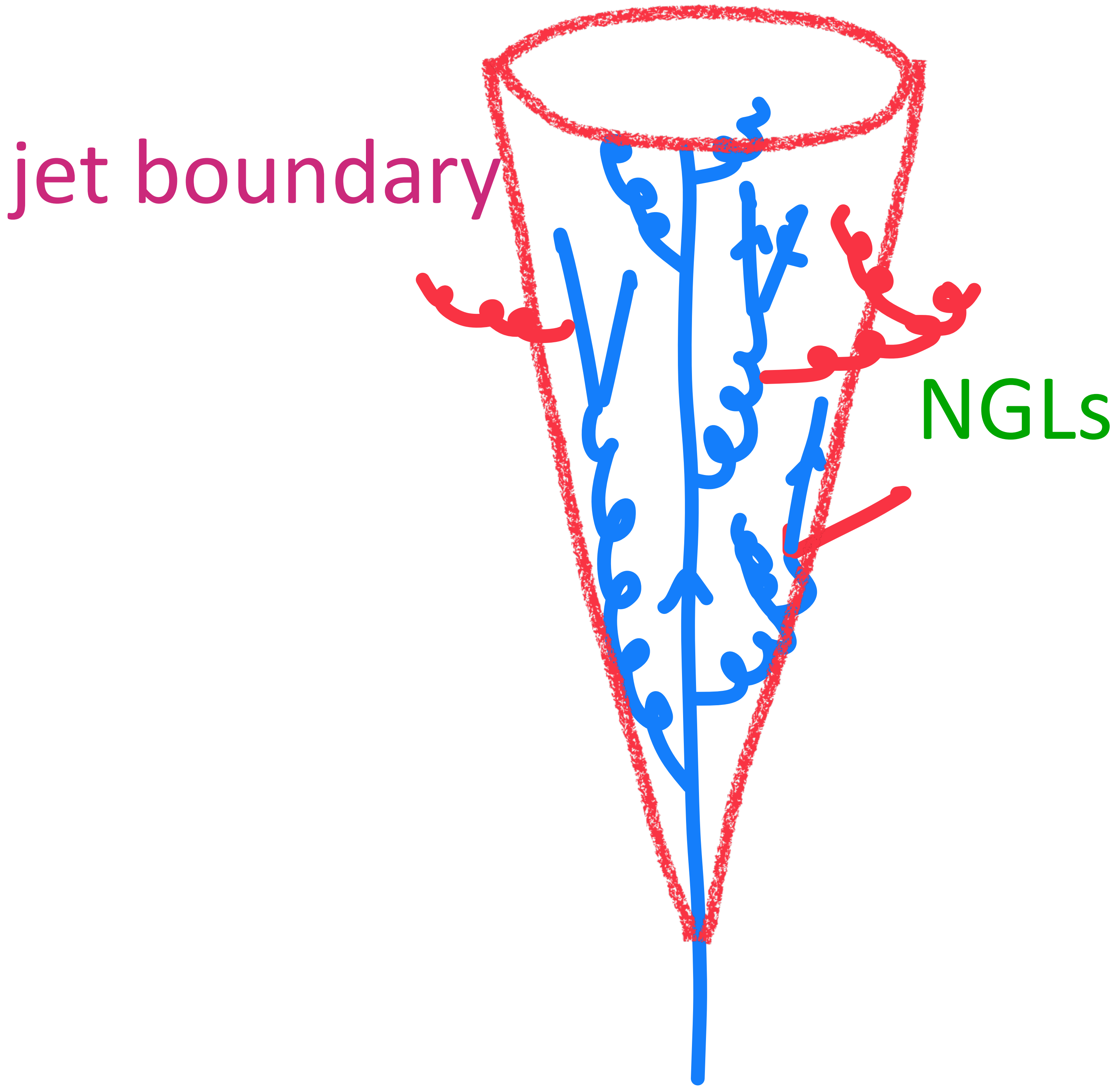
$\chi_{ab} \ll 1$

Simplicity of perturbative energy correlators

$$J \sim \int d\Phi_{\text{coll}} P_{1\to n}(\{\xi_i\},\{s_{ij}\})\delta[O_{\text{jet}} - f(\{\xi_i\},\{s_{ij}\};\text{algorithm})]$$

$$\text{jet substructure} = H \otimes J \otimes S$$

$$S \sim \sum_{k_s} \delta[O_{\text{jet}} - f(\{k_s\},\{n_i\};\text{algorithm})]\langle\Omega||W_{n_1}W_{n_2}\cdots W_{n_J}(0)|^2|\Omega\rangle$$



For energy correlators, many of the complications absent:

1. Absent of algorithm
2. No soft contribution at leading power, S=1
3. Simplification in collinear phase space

$$d\Phi_c^{(3)} = ds_{123}ds_{12}ds_{13}ds_{23}\delta(s_{123}-s_{12}-s_{13}-s_{23})d\xi_1 d\xi_2 d\xi_3 \delta(1-\xi_1-\xi_2-\xi_3) \times \frac{4\Theta(-\Delta)(-\Delta)^{-1/2-\epsilon}}{(4\pi)^{5-2\epsilon}\Gamma(1-2\epsilon)},$$

$$s_{ij} = \xi_i \xi_j \frac{\chi_{ij}^2}{4} Q^2$$

$$\Delta = (\xi_3 s_{12} - \xi_1 s_{23} - \xi_2 s_{13})^2 - 4\xi_1\xi_2 s_{13}s_{23}$$

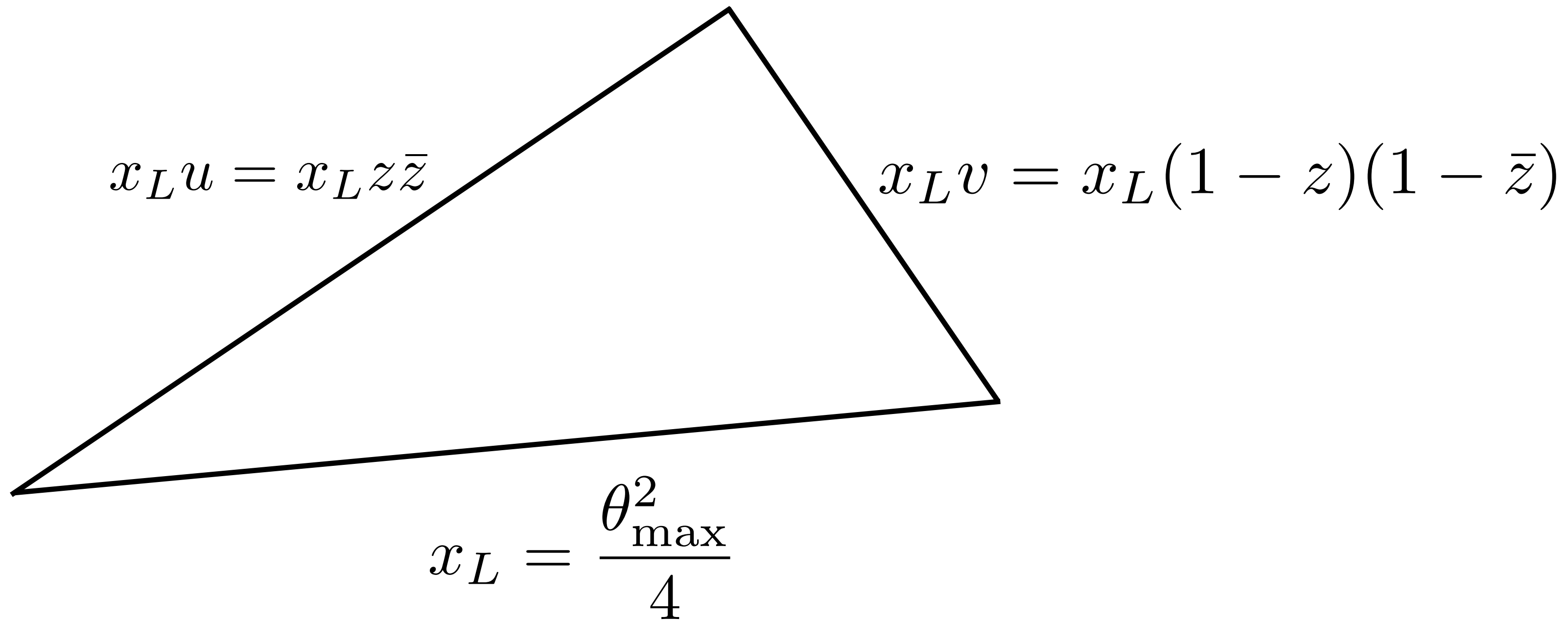
projection onto celestial variable

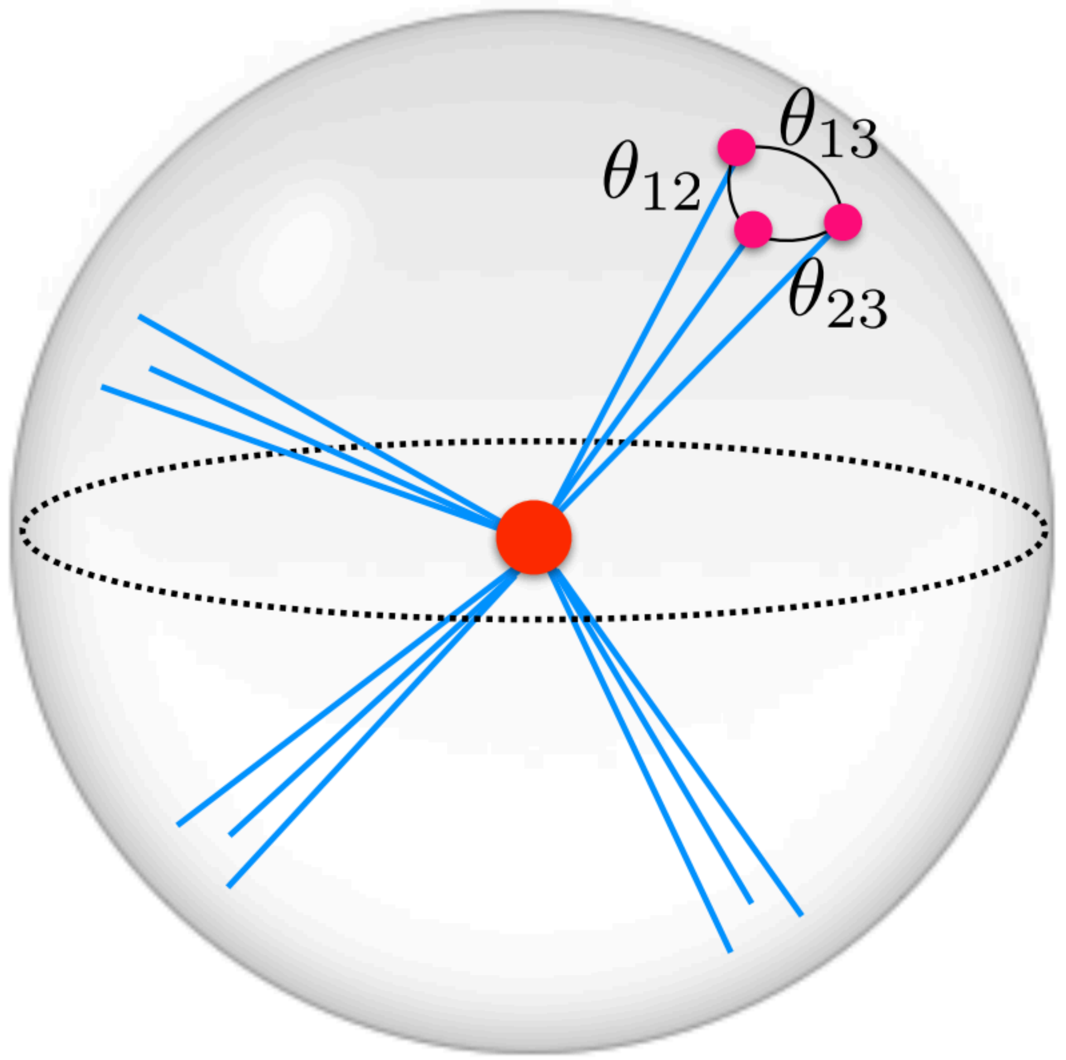
$$\Delta = (\xi_1\xi_2\xi_3)^2 \lambda\left(\frac{\chi_{12}^2}{4},\frac{\chi_{13}^2}{4},\frac{\chi_{23}^2}{4}\right)$$

complete factorization of bulk variables and celestial variables

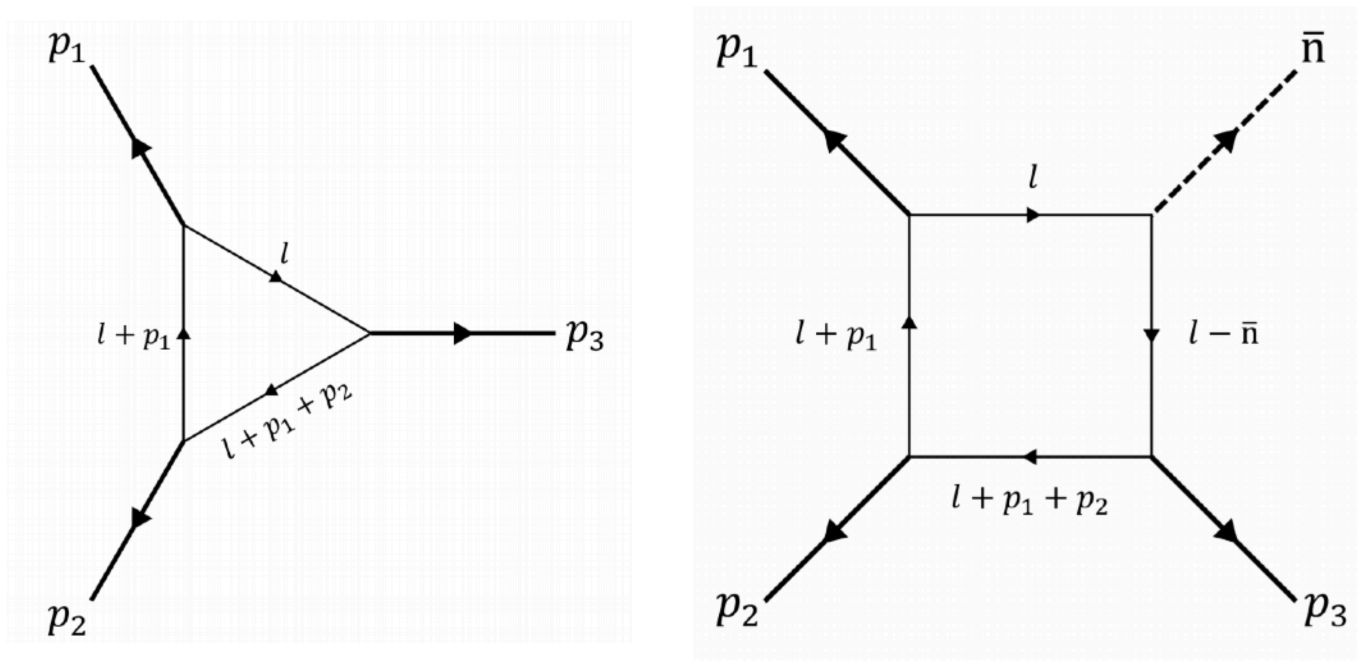
Analytic results for a three-prong substructure

three-point energy correlator





factorization of bulk and celestial variables



“almost” 2D euclidean correlators

N=4 SYM

$$\begin{aligned} G_{\mathcal{N}=4}(z) &= \frac{1+u+v}{2uv}(1+\zeta_2) - \frac{1+v}{2uv}\log(u) - \frac{1+u}{2uv}\log(v) \\ &- (1+u+v)(\partial_u+\partial_v)\Phi(z) + \frac{(1+u^2+v^2)}{2uv}\Phi(z) + \frac{(z-\bar{z})^2(u+v+u^2+v^2+u^2v+uv^2)}{4u^2v^2}\Phi(z) \\ &+ \frac{(u-1)(u+1)}{2u^2v^2}D_2^+(z) + \frac{(v-1)(v+1)}{2u^2v}D_2^+(1-z) + \frac{(u-v)(u+v)}{2uv}D_2^+\left(\frac{z}{z-1}\right), \end{aligned} \tag{2.4}$$

$$\Phi(z) = \mathrm{Li}_2(z) - \mathrm{Li}_2(\bar{z}) + \frac{1}{2}(\ln(1-z) - \ln(1-\bar{z}))\ln(z\bar{z})$$

$$D_2^+(z) = \left(\mathrm{Li}_2(1-|z|^2) + \frac{1}{2}\ln(|1-z|^2)\ln|z|^2\right)$$

Plan for the remaining talk

- Light-ray operator and its OPE in QCD
- Two-point energy correlator (EEC) from Light-ray OPE; reciprocity
- Squeeze limit of three-point energy correlator
- Hidden conformal symmetry on the celestial sphere

Factorization v.s. OPE

conventional observables

$$\mu^2 \frac{dD_{h/i}(z,\mu^2)}{d\mu^2} = \sum_j \int_z^1 \frac{d\xi}{\xi} P^T_{ji}(\xi) D_{h/j}\left(\frac{z}{\xi},\mu^2\right)$$

Weighted cross section

$$D^{(J)}_{h/i}(\mu^2) = \int_0^1 \frac{dz}{z} z^J D_{h/i}(z,\mu^2)$$

$$\mu^2 \frac{d}{d\mu^2} D^{(J)}_{h/i}(\mu^2) = \sum_j \gamma^T_{ji}(J) D^{(J)}_{h/j}(\mu^2)$$

$$\gamma^T_{ji}(J) = \int_0^1 \frac{dz}{z} z^J P^T_{ji}(z)$$

1. Weighted cross section convert factorization convolution to ordinary RG equation
2. Evolution kernels are Mellin moment of time-like splitting kernels. Through (generalized) Gribov-Lipatov reciprocity converts to anomalous dimension of **local** Wilson operators
3. Both properties are realized by energy correlators automatically

Energy flow operator

- Energy flow operator measures the time-integrated energy recorded by a detector placed at a point **n** on celestial sphere.

$$\mathcal{E}(\vec{n}) = \lim_{r\to\infty} r^2 \int_0^\infty dt\, \vec{n}_i T^{0i}(t, r\vec{n})$$

1. Proposed as a operator definition of measurement — Sveshnikov, Tkachov, 95; Tkachov, 95
2. Used to study non-perturbative power corrections — Korchemsky, Oderda, Sterman, 97; Korchemsky, 98; Belitsky, Korchemsky, Sterman, 01; Bauer, Fleming, Lee, Sterman; 08
3. Application in conformal collider — Hofman, Maldacena, 08
4. Calculations with Mellin amplitude technique — Belitsky, Hohenegger, Korchemsky, Sokatchev, Zhiboedov, 13; Henn, Sokatchev, K. Yan, Zhiboedov, 19;
5. Non-perturbative convergent OPE in CFT — Kravchuk, Simmons-Duffin, 18; Kologlu, Kravchuk, Simmons-Duffin, Zhiboedov, 19; C.H. Chang, Kologlu, Kravchuk, Simmons-Duffin, Zhiboedov, 20
6. Perturbative OPE in QCD — H. Chen, Moult, HXZ, 20

Energy flow operator

- Energy flow operator measures the time-integrated energy recorded by a detector placed at a point **n** on celestial sphere.

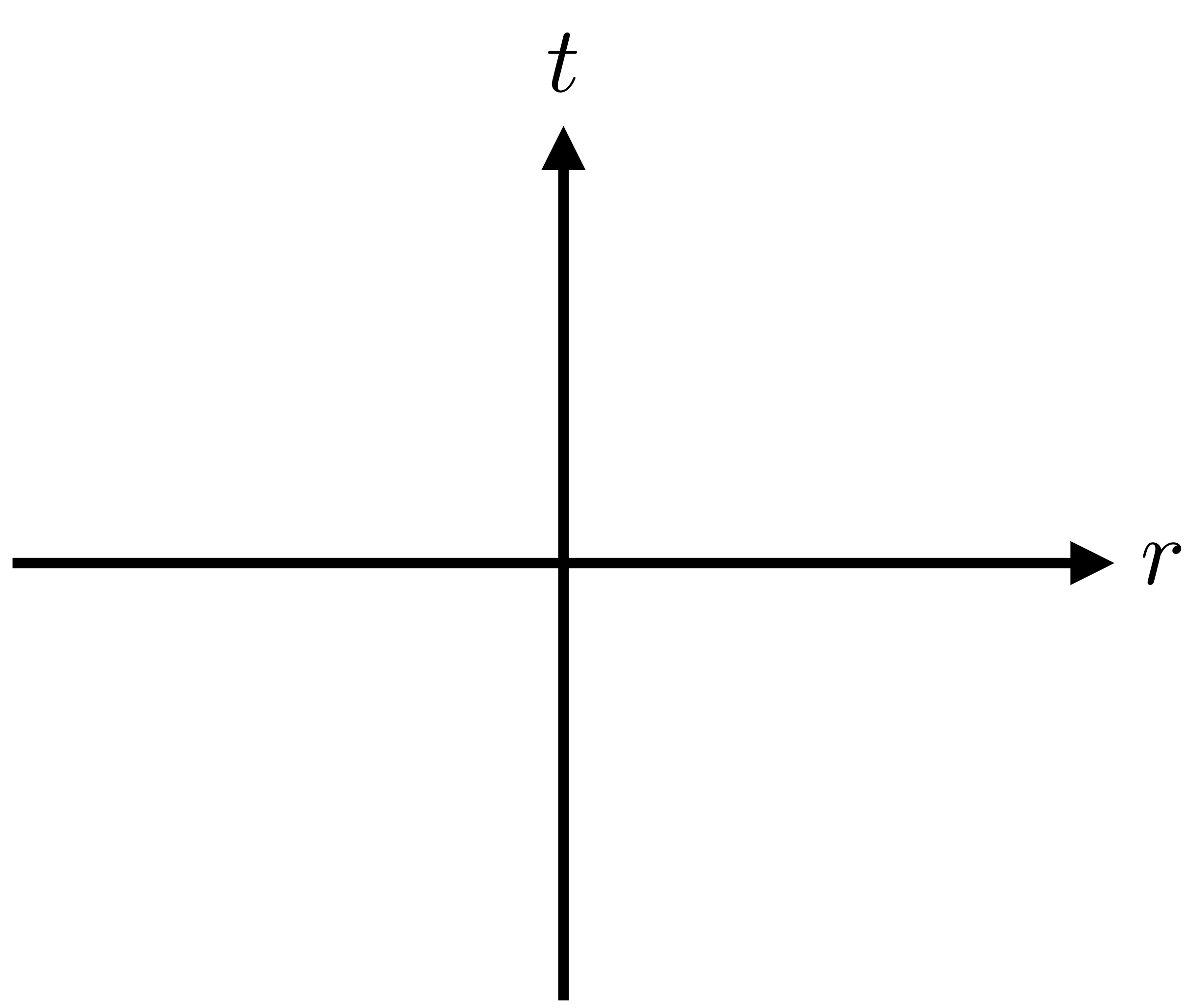
light-transform of local operator

$$\mathcal{E}(\vec{n}) = \lim_{r\to\infty} r^2 \int_0^\infty dt\ \vec{n}_i T^{0i}(t, r\vec{n})$$

string operator

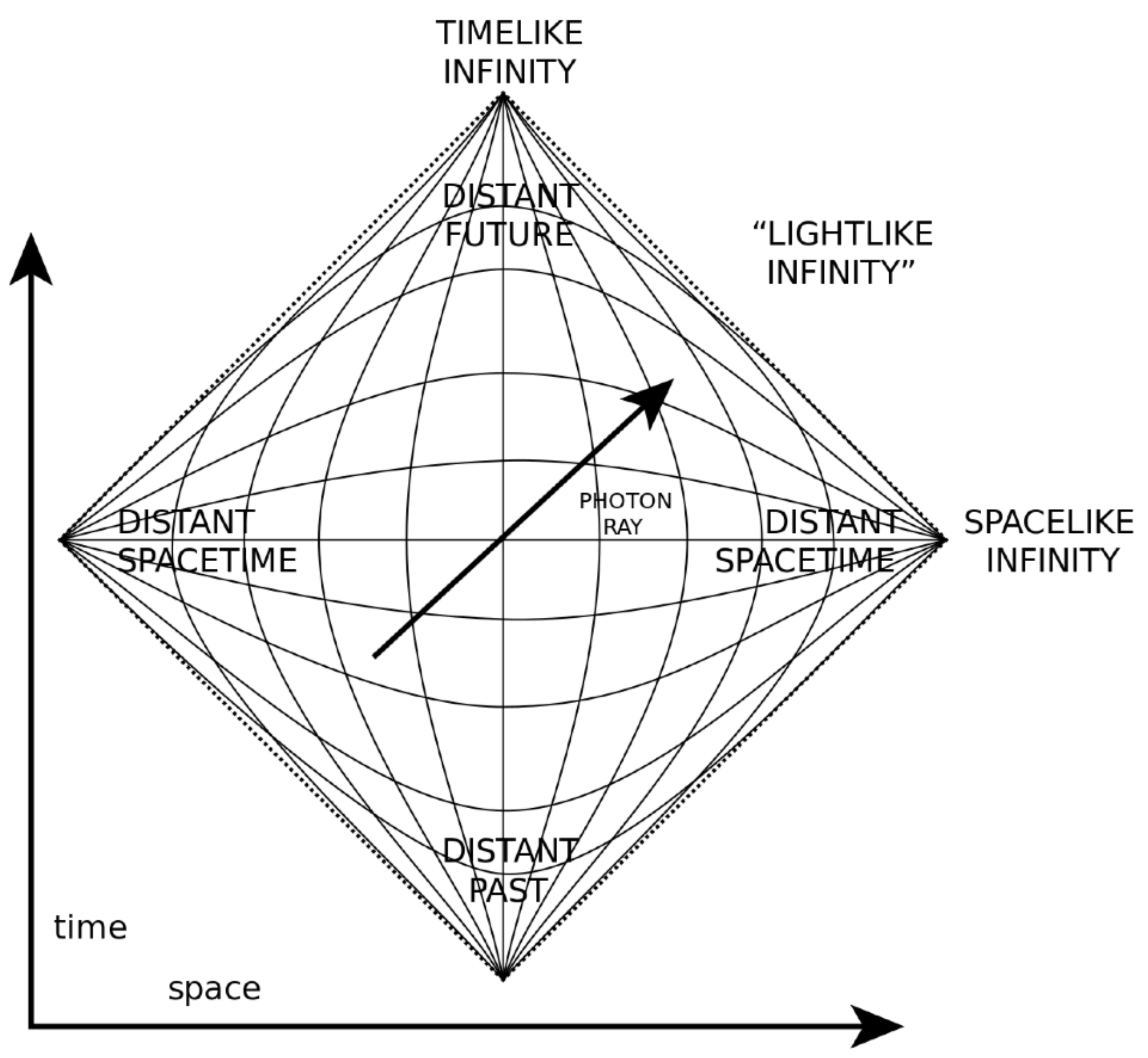
light-ray operator

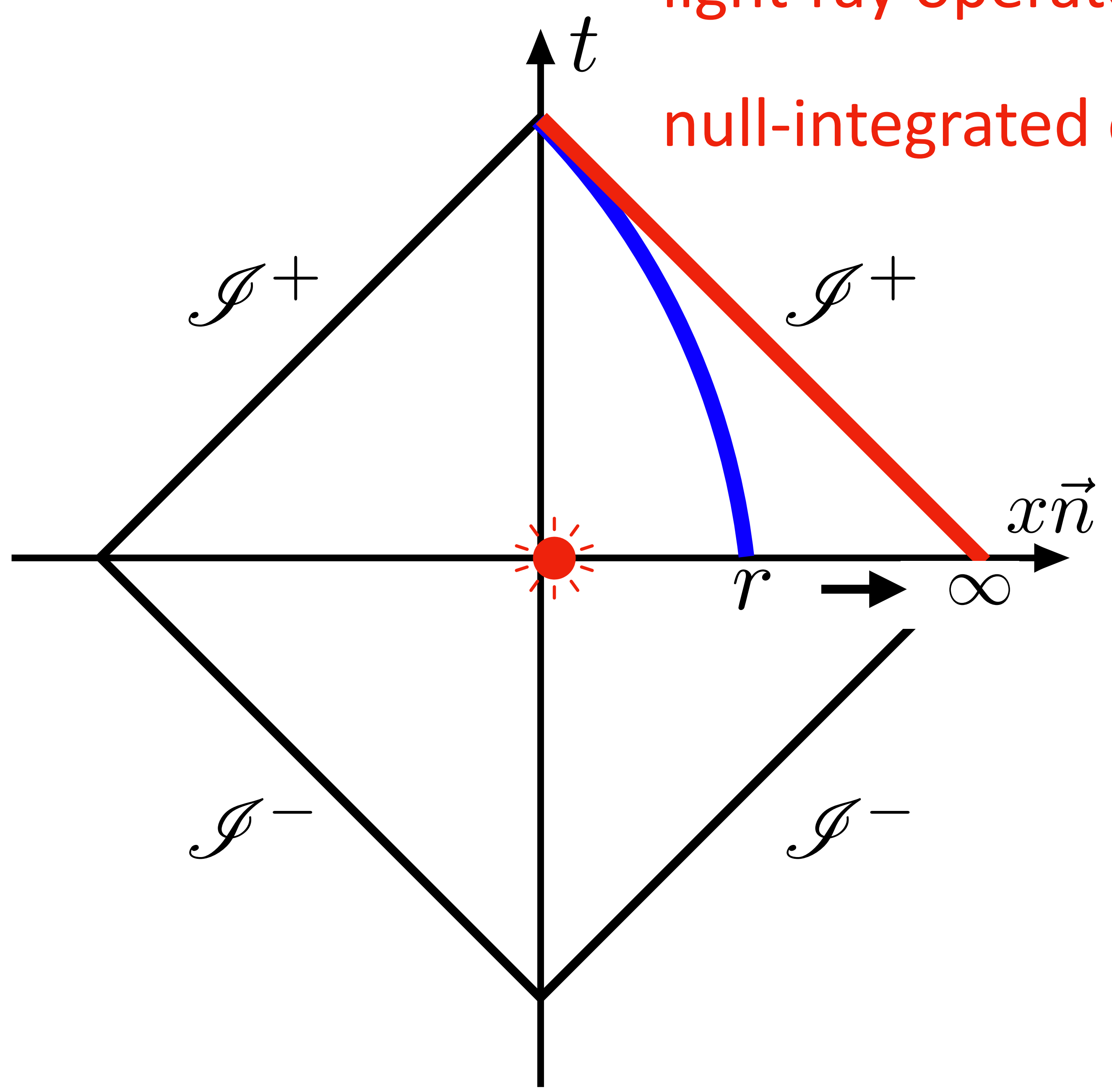
null-integrated operator



Minkowski space-time

$$ds^2 = dt^2 - dr^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

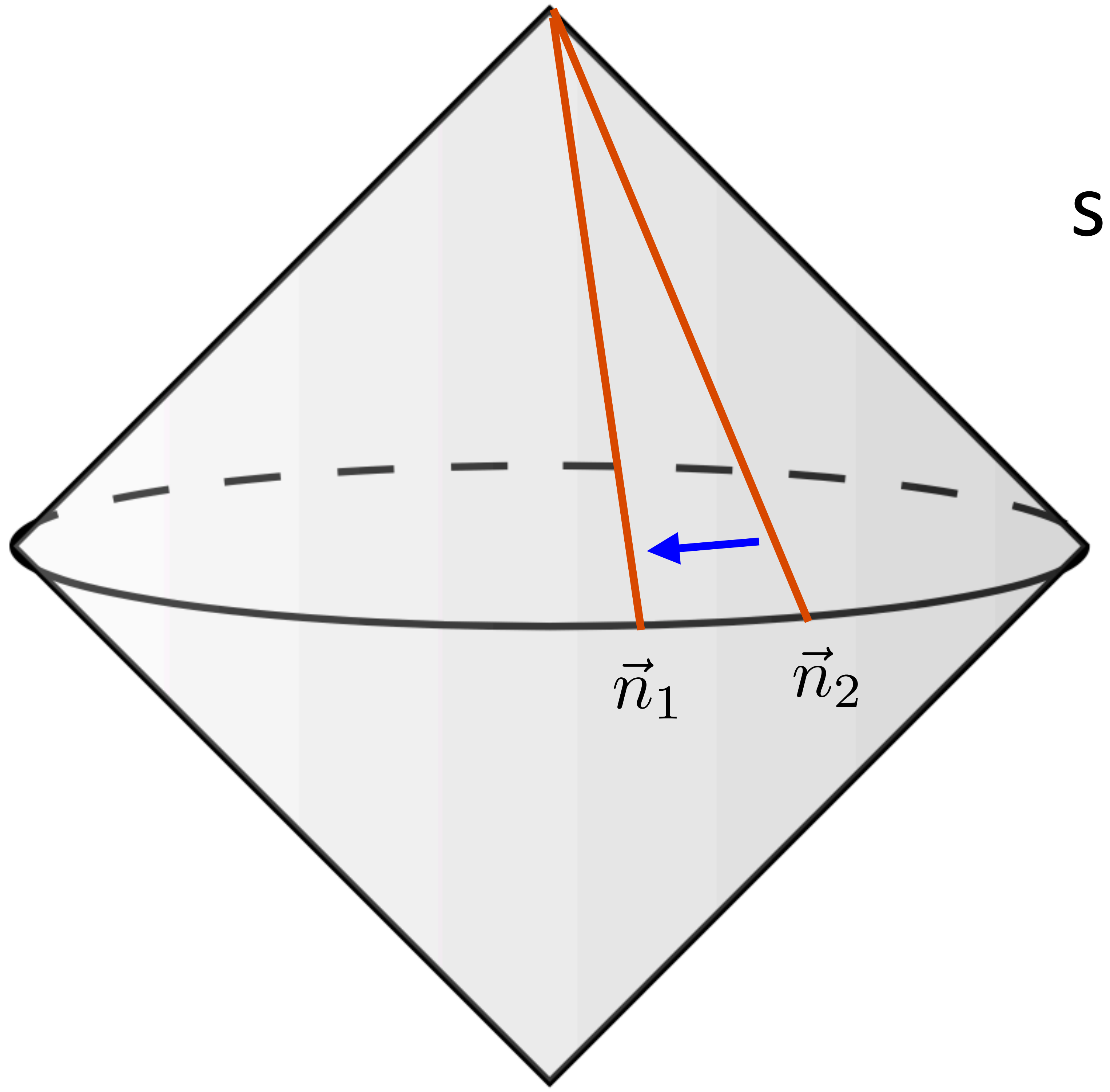




Penrose diagram of Minkowski space-time

EEC as correlator of energy flow operator

$$\int d^4x e^{-iq\cdot x} \langle \Omega | j^{\dagger}_{\text{em},\mu}(x) \mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2) j^{\mu}_{\text{em}}(0)|\Omega\rangle \qquad j^{\mu}_{\text{em}}(x) = e_q \bar{q}\gamma^{\mu} q(x)$$



small angle limit ⟷ $\vec{n}_2 \to \vec{n}_1$ ⟷ OPE of $\mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2)$

$$\lim_{\vec{n}_2\to\vec{n}_1} \mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2) = \sum_i \theta_{12}^{\gamma_i} \mathbb{O}_i(\vec{n}_1)$$

To make sense of the OPE, the two energy flow operator better be space-like separated. ✅

Then one can try to do light-transform on both side, and then establish the light-ray OPE. Cautious: not all light-ray operators can be written as light-transform of local operator.

It's convenient to write energy flow operator as light-transform of local twist 2 (collinear) spin 2 traceless symmetric tensor

$\bar{n}_\mu = (1, -\vec{n})$ $\quad A^\mu \bar{n}_\mu = A^+$

$$\mathcal{E}(\vec{n}) = \lim_{r\to\infty} r^2 \int_0^\infty dt\, \vec{n}_i T^{0i}(t, r\vec{n}) \quad \Rightarrow \quad \mathcal{E}(\vec{n}) = \lim_{r\to\infty} r^2 \int_0^\infty dt\, [O_q^{\mu_1\mu_2}(t, r\vec{n}) + O_g^{\mu_1\mu_2}(t, r\vec{n})]\bar{n}_{\mu_1}\bar{n}_{\mu_2}$$

twist τ = dimension Δ - (collinear) spin J

$$O_q^{\mu\nu} = \frac{1}{4}\bar{q}\gamma^\mu(iD^\nu)q \qquad O_g^{\mu\nu} = -\frac{1}{4}F^{\rho\mu}F_\rho{}^\nu$$

Therefore, for the local OPE, we need local Wilson operator label by various twist and spin. For twist 2 even spin, there are three different family in QCD

transverse spin-0

$$\mathcal{O}_q^{[J]} = \frac{1}{2^J}\bar{\psi}\gamma^+(iD^+)^{J-1}\psi$$

$$\mathcal{O}_g^{[J]} = -\frac{1}{2^J}F_a^{\mu+}(iD^+)^{J-2}F_a^{\mu+}$$

$$\mathcal{O}_{\tilde{g},\lambda}^{[J]} = -\frac{1}{2^J}F_a^{\mu+}(iD^+)^{J-2}F_a^{\nu+}\epsilon_{\lambda,\mu}\epsilon_{\lambda,\nu}$$

helicity ±

transverse spin-2

$$\epsilon_\pm = (0, 1, \pm i, 0)$$

odd spin vanishes

$$FD^3F = D^3(FF) - (D^3F)F - D(DF)^2$$

However, under light-transform, one can analytic continue even spin to odd spin.

Symmetries

Hofman, Maldacena 08; Kologlu, Kravchuk, Simmons-Duffin, Zhiboedov, 19

ensure finite, non-vanishing light transform

$$\mathbb{O}(\vec{n}) = \lim_{r\to\infty} r^{\Delta-J} \int_0^\infty dt\, O^{\mu_1\ldots\mu_J}(t, r\vec{n})\bar{n}_{\mu_1}\ldots\bar{n}_{\mu_J}$$

Light-transform of $O_{(\Delta,J)}$

dimension	$J-\Delta-1$	$+$	Δ	$= J-1$
collinear spin	$-\Delta+J+1$	$+$	$-J$	$= 1-\Delta$

for energy flow operator

$\Delta = 4\,, J = 2$

$$\lim_{\vec{n}_2\to\vec{n}_1} \mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2) = \sum_i \theta_{12}^{\gamma_i}\mathbb{O}_i(\vec{n}_1)$$

$\mathbb{O} = \mathbf{L}[O]$ $\quad \tau = \Delta - J$

dimension	$(2-1)$	$+$	$(2-1)$	$=$	0	$+$	$(3-1)$	
collinear spin	$(1-4)$	$+$	$(1-4)$	$=$	γ_i	$+$	$(1-\tau_i-3)$	$\Rightarrow \gamma_i = \tau_i - 4$

Only J=3 local operator appear in the OPE

$$\mathcal{E}(\vec{n}_1)\,\mathcal{E}(\vec{n}_2) \sim \sum_i c_i\, \theta^{\tau_i-4}\mathbb{O}_i(\vec{n}_2)$$

Small angle expansion reduce to twist expansion of local operator (under analytic continuation)

$$\mathcal{E}(\vec{n}_1)\ \mathcal{E}(\vec{n}_2) \sim \sum_i c_i\ \theta^{\tau_i - 4}\mathbb{O}_i(\vec{n}_2)$$

At small angle, leading contributions from twist 2 family, given below

Local Operators

transverse spin-0

$$\mathcal{O}_q^{[J]} = \frac{1}{2^J}\bar{\psi}\gamma^+(iD^+)^{J-1}\psi$$

$$\mathcal{O}_g^{[J]} = -\frac{1}{2^J}F_a^{\mu+}(iD^+)^{J-2}F_a^{\mu+}$$

transverse spin-2

$$\mathcal{O}_{\tilde{g},\lambda}^{[J]} = -\frac{1}{2^J}F_a^{\mu+}(iD^+)^{J-2}F_a^{\nu+}\epsilon_{\lambda,\mu}\epsilon_{\lambda,\nu}$$

helicity $\pm$

$$\lim_{r\to\infty} r^2 \int_0^\infty dt$$

light transform

$$\vec{\mathbb{O}}^{[J]}(\vec{n}) = \begin{bmatrix} \mathbb{O}_q^{[J]}(\vec{n}) \\ \mathbb{O}_g^{[J]}(\vec{n}) \\ \mathbb{O}_{\tilde{g},+}^{[J]}(\vec{n}) \\ \mathbb{O}_{\tilde{g},-}^{[J]}(\vec{n}) \end{bmatrix}$$

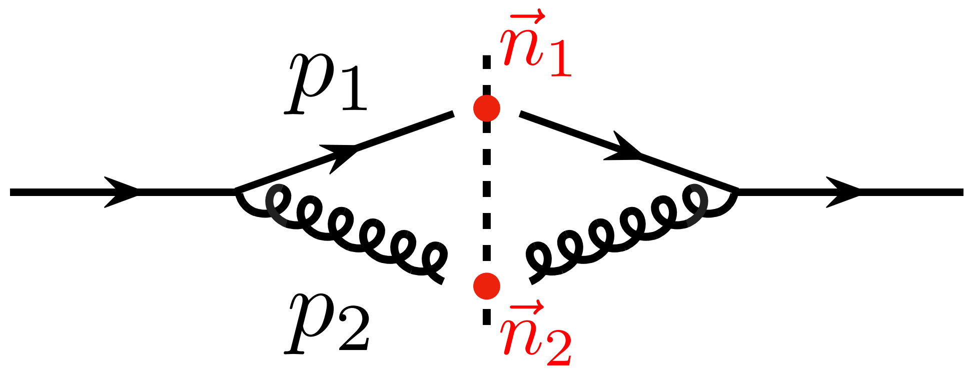
unpolarized

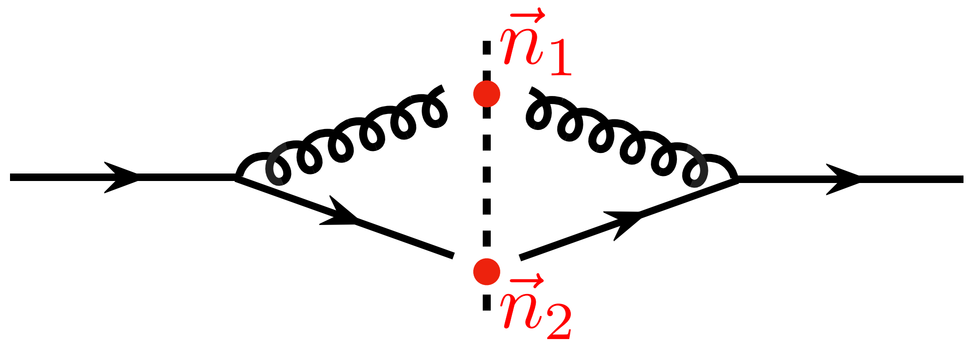
polarized

To complete the OPE, we need to calculate the OPE coefficients

$$\langle\Omega|\Phi^\dagger\mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2)\Phi|\Omega\rangle \sim \sum_i c_i\theta^{\tau_i-4}\langle\Omega|\Phi^\dagger\mathbb{O}_i(\vec{n}_2)\Phi|\Omega\rangle$$

Matching for quark operator





omit the Wilson line for simplicity,
but does not affect the tree level matching

$$\langle 0|\psi(x)\mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2)\bar{\psi}(0)|0\rangle$$

$$= g^2 C_F \int \frac{E_1^2 dE_1}{(2\pi)^3 2E_1}\frac{E_2^2 dE_2}{(2\pi)^3 2E_2} e^{-i(p_1+p_2)\cdot x} E_1 E_2 \left(\sum_\lambda \frac{\not{p}_1+\not{p}_2}{(p_1+p_2)^2}\not{\epsilon}_\lambda(p_2)\not{p}_1\not{\epsilon}_{-\lambda}(p_2)\frac{\not{p}_1+\not{p}_2}{(p_1+p_2)^2} + (2\leftrightarrow 3)\right)$$

$$\overset{\theta\to 0}{=} \frac{1}{2\pi}\frac{4}{\theta_S^2}\frac{g^2}{(4\pi)^2}C_F\int_0^1 dz\, z(1-z)\left(\frac{1+z^2}{1-z}+\frac{1+(1-z)^2}{z}\right)\int\frac{E^2 dE}{(2\pi)^3 2E}\not{p}E^2 e^{-iEn\cdot x}$$

P_{qq} P_{gq}

$$\Rightarrow -\frac{1}{2\pi}\frac{2}{\theta^2}\left[(\gamma_{qq}(2)-\gamma_{qq}(3))+(\gamma_{gq}(2)-\gamma_{gq}(3))\right]\langle\Omega|\psi(x)\,\mathbb{O}_q^{[3]}(\vec{n}_2)\bar{\psi}(0)|\Omega\rangle$$

Matching coefficient

momentum conservation: $\gamma_{qq}(2)+\gamma_{gq}(2)=0$

Matching for gluon operator

$$\epsilon_\pm = (0, 1, \pm i, 0)$$

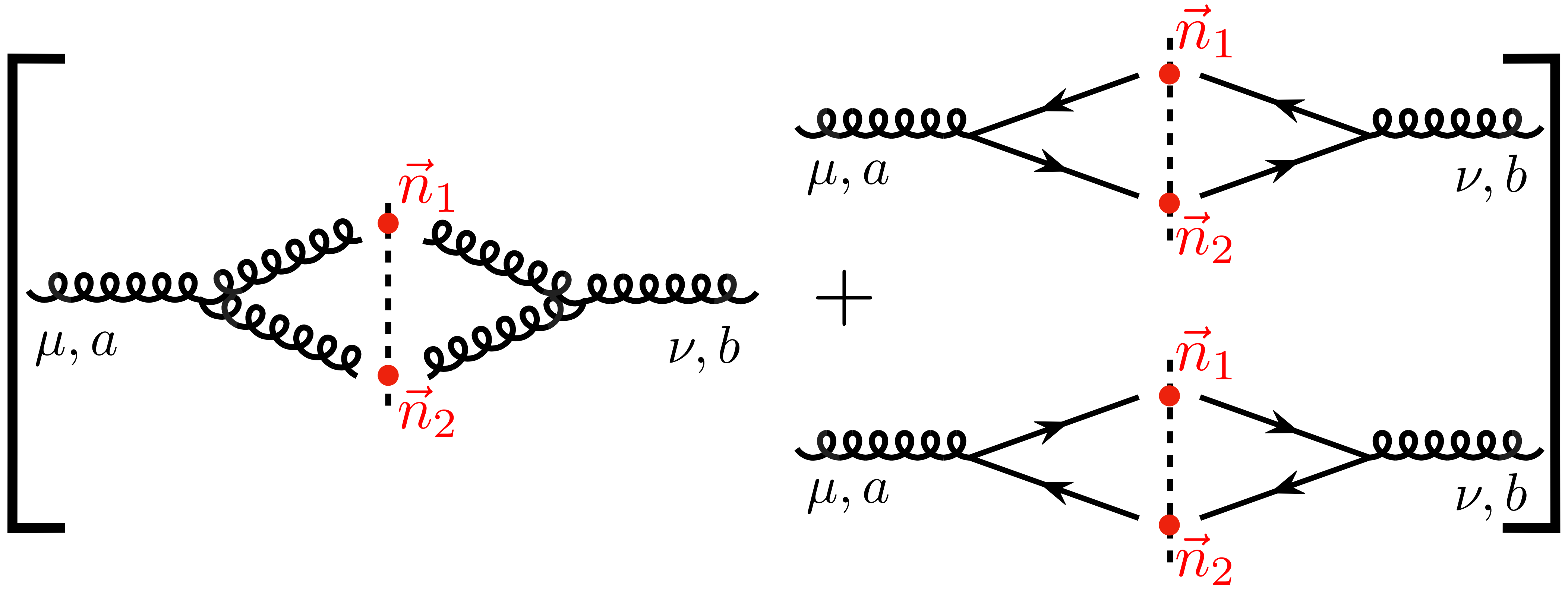
$\epsilon^\mu_+ \epsilon^\nu_- + \epsilon^\mu_- \epsilon_+ -^\nu$: $\mathbb{O}_g$ transverse spin 0

$\epsilon^\mu_+ \epsilon^\nu_+$: $\mathbb{O}_{\tilde{g},+}$ transverse spin 2

$\epsilon^\mu_- \epsilon^\nu_-$: $\mathbb{O}_{\tilde{g},-}$ transverse spin 2

$$\langle \Omega | A_b^\nu(x)\ \mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2)\ A_a^\mu(0) | \Omega \rangle$$

$$= \int \frac{E_1^2 dE_1}{(2\pi)^3 2E_1} \frac{E_2^2 dE_2}{(2\pi)^3 2E_2}\ E_1 E_2\ e^{-i(p_1+p_2)\cdot x}$$



$\theta \to 0$

$$-\frac{1}{2\pi}\frac{2}{\theta^2}\Big[c_g\ \langle \Omega | A_b^\nu(x)\ \mathbb{O}_g^{[3]}\ A_a^\mu(0) | \Omega \rangle$$

$$+ c_{\tilde{g}} \Big(e^{2i\phi} \langle \Omega | A_b^\nu(x)\ \mathbb{O}_{\tilde{g},-}^{[3]}\ A_a^\mu(0) | \Omega \rangle + e^{-2i\phi} \langle \Omega | A_b^\nu(x)\ \mathbb{O}_{\tilde{g},+}^{[3]}\ A_a^\mu(0) | \Omega \rangle \Big)\Big]$$

$\langle 1\, 2 \rangle^2 = s_{12} e^{2i\phi}$

$$c_g = (\gamma_{gg}(2) - \gamma_{gg}(3)) + 2n_f\,(\gamma_{qg}(2) - \gamma_{qg}(3))$$

$$c_{\tilde{g}} = (\gamma_{g\tilde{g}}(2) - \gamma_{g\tilde{g}}(3)) + 2n_f\,(\gamma_{q\tilde{g}}(2) - \gamma_{q\tilde{g}}(3))$$

$$\vec{\mathbb{O}}^{[J]} = \begin{pmatrix} \mathbb{O}_q^{[J]} \\ \mathbb{O}_g^{[J]} \\ \mathbb{O}_{\tilde{g},+}^{[J]} \\ \mathbb{O}_{\tilde{g},-}^{[J]} \end{pmatrix}$$

$$\boxed{\vec{\mathbb{O}}^{[J]}(\hat{n}_1)\mathcal{E}(\hat{n}_2) = -\frac{1}{2\pi}\frac{2}{\theta^2}\left[\widehat{C}_\phi(J) - \widehat{C}_\phi(J+1)\right]\vec{\mathbb{O}}^{[J+1]}(\hat{n}_1) + \text{higher twist}}$$

special case: $\mathcal{E} \sim \mathbb{O}_q^{[2]} + \mathbb{O}_g^{[2]}$

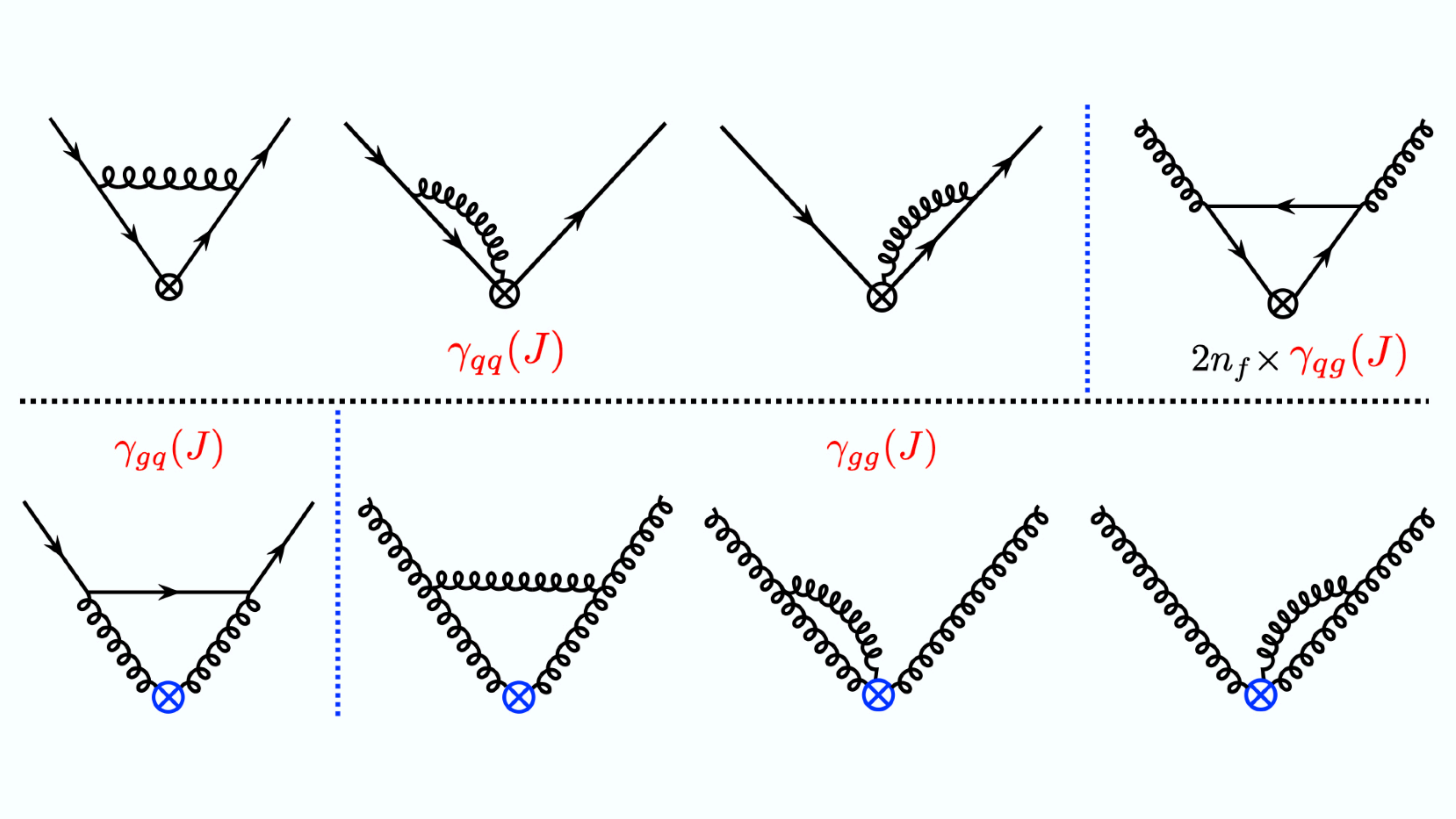
$$\widehat{C}_\phi(J) = \begin{pmatrix} \gamma_{qq}(J) & 2n_f\gamma_{qg}(J) & 2n_f\gamma_{q\tilde{g}}(J)e^{-2i\phi}/2 & 2n_f\gamma_{q\tilde{g}}(J)e^{2i\phi}/2 \\ \gamma_{gq}(J) & \gamma_{gg}(J) & \gamma_{g\tilde{g}}(J)e^{-2i\phi}/2 & \gamma_{g\tilde{g}}(J)e^{2i\phi}/2 \\ \gamma_{\tilde{g}q}(J)e^{2i\phi} & \gamma_{\tilde{g}g}(J)e^{2i\phi} & \gamma_{\tilde{g}\tilde{g}}(J) & \gamma_{\tilde{g}\tilde{g},\pm}(J)e^{4i\phi} \\ \gamma_{\tilde{g}q}(J)e^{-2i\phi} & \gamma_{\tilde{g}g}(J)e^{-2i\phi} & \gamma_{\tilde{g}\tilde{g},\pm}(J)e^{-4i\phi} & \gamma_{\tilde{g}\tilde{g}}(J) \end{pmatrix}$$

$$\vec{\mathbb{O}}^{[J]} = \begin{pmatrix} \mathbb{O}_q^{[J]} \\ \mathbb{O}_g^{[J]} \\ \mathbb{O}_{\tilde{g},+}^{[J]} \\ \mathbb{O}_{\tilde{g},-}^{[J]} \end{pmatrix}$$

$$\vec{\mathbb{O}}^{[J]}(\hat{n}_1)\mathcal{E}(\hat{n}_2) = -\frac{1}{2\pi}\frac{2}{\theta^2}\left[\widehat{C}_\phi(J) - \widehat{C}_\phi(J+1)\right]\vec{\mathbb{O}}^{[J+1]}(\hat{n}_1) + \text{higher twist}$$

special case: $\mathcal{E} \sim \mathbb{O}_q^{[2]} + \mathbb{O}_g^{[2]}$

$$\widehat{C}_\phi(J) = \begin{pmatrix} \gamma_{qq}(J) & 2n_f\gamma_{qg}(J) & 2n_f\gamma_{q\tilde{g}}(J)e^{-2i\phi}/2 & 2n_f\gamma_{q\tilde{g}}(J)e^{2i\phi}/2 \\ \gamma_{gq}(J) & \gamma_{gg}(J) & \gamma_{g\tilde{g}}(J)e^{-2i\phi}/2 & \gamma_{g\tilde{g}}(J)e^{2i\phi}/2 \\ \gamma_{\tilde{g}q}(J)e^{2i\phi} & \gamma_{\tilde{g}g}(J)e^{2i\phi} & \gamma_{\tilde{g}\tilde{g}}(J) & \gamma_{\tilde{g}\tilde{g},\pm}(J)e^{4i\phi} \\ \gamma_{\tilde{g}q}(J)e^{-2i\phi} & \gamma_{\tilde{g}g}(J)e^{-2i\phi} & \gamma_{\tilde{g}\tilde{g},\pm}(J)e^{-4i\phi} & \gamma_{\tilde{g}\tilde{g}}(J) \end{pmatrix}$$



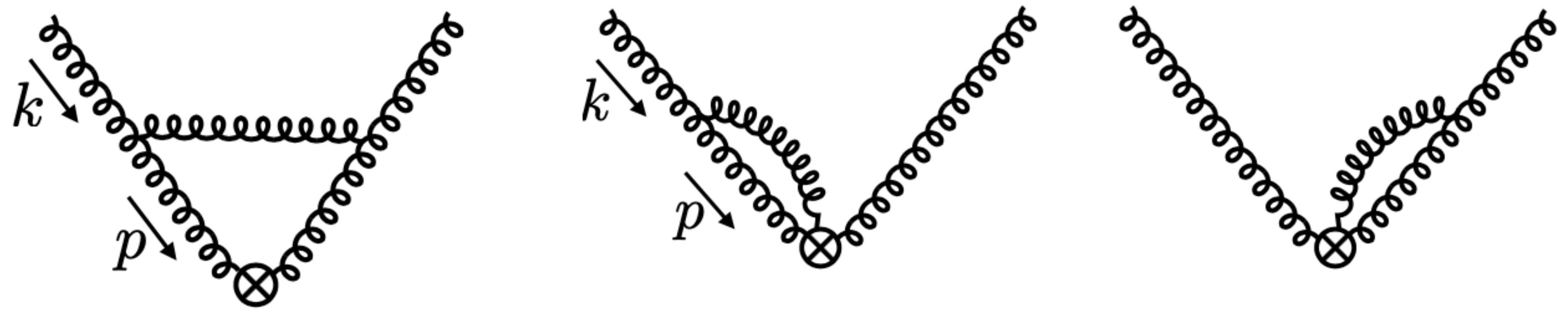
QCD 4 loops and 5 loops! Moch, Ruijl, Ueda, Vermeresen, 17; + Herzog, 18

$$\vec{\mathbb{O}}^{[J]} = \begin{pmatrix} \mathbb{O}_q^{[J]} \\ \mathbb{O}_g^{[J]} \\ \mathbb{O}_{\tilde{g},+}^{[J]} \\ \mathbb{O}_{\tilde{g},-}^{[J]} \end{pmatrix}$$

$$\boxed{\vec{\mathbb{O}}^{[J]}(\hat{n}_1)\mathcal{E}(\hat{n}_2) = -\frac{1}{2\pi}\frac{2}{\theta^2}\left[\widehat{C}_\phi(J) - \widehat{C}_\phi(J+1)\right]\vec{\mathbb{O}}^{[J+1]}(\hat{n}_1) + \text{higher twist}}$$

special case: $\mathcal{E} \sim \mathbb{O}_q^{[2]} + \mathbb{O}_g^{[2]}$

$$\widehat{C}_\phi(J) = \begin{pmatrix} \gamma_{qq}(J) & 2n_f\gamma_{qg}(J) & 2n_f\gamma_{q\tilde{g}}(J)e^{-2i\phi}/2 & 2n_f\gamma_{q\tilde{g}}(J)e^{2i\phi}/2 \\ \gamma_{gq}(J) & \gamma_{gg}(J) & \gamma_{g\tilde{g}}(J)e^{-2i\phi}/2 & \gamma_{g\tilde{g}}(J)e^{2i\phi}/2 \\ \gamma_{\tilde{g}q}(J)e^{2i\phi} & \gamma_{\tilde{g}g}(J)e^{2i\phi} & \gamma_{\tilde{g}\tilde{g}}(J) & \gamma_{\tilde{g}\tilde{g},\pm}(J)e^{4i\phi} \\ \gamma_{\tilde{g}q}(J)e^{-2i\phi} & \gamma_{\tilde{g}g}(J)e^{-2i\phi} & \gamma_{\tilde{g}\tilde{g},\pm}(J)e^{-4i\phi} & \gamma_{\tilde{g}\tilde{g}}(J) \end{pmatrix}$$



$$\gamma_{\tilde{g}\tilde{g}}(J) = \frac{g^2}{(4\pi)^2}\left(4C_A\sum_{j=1}^{J}\frac{1}{j} - \beta_0\right)$$

$$\langle k,+|O_{J,\lambda}|k,-\rangle \quad \text{or} \quad \langle k,-|O_{J,\lambda}|k,+\rangle\,,$$

$$O_{J,\lambda} = \epsilon_{\lambda,i}\epsilon_{\lambda,j}O_J^{ij} = \epsilon_{\lambda,i}\epsilon_{\lambda,j}F^{+i}(iD^+)^{J-2}F^{+j}$$

$$\vec{\mathbb{O}}^{[J]} = \begin{pmatrix} \mathbb{O}_q^{[J]} \\ \mathbb{O}_g^{[J]} \\ \mathbb{O}_{\tilde{g},+}^{[J]} \\ \mathbb{O}_{\tilde{g},-}^{[J]} \end{pmatrix}$$

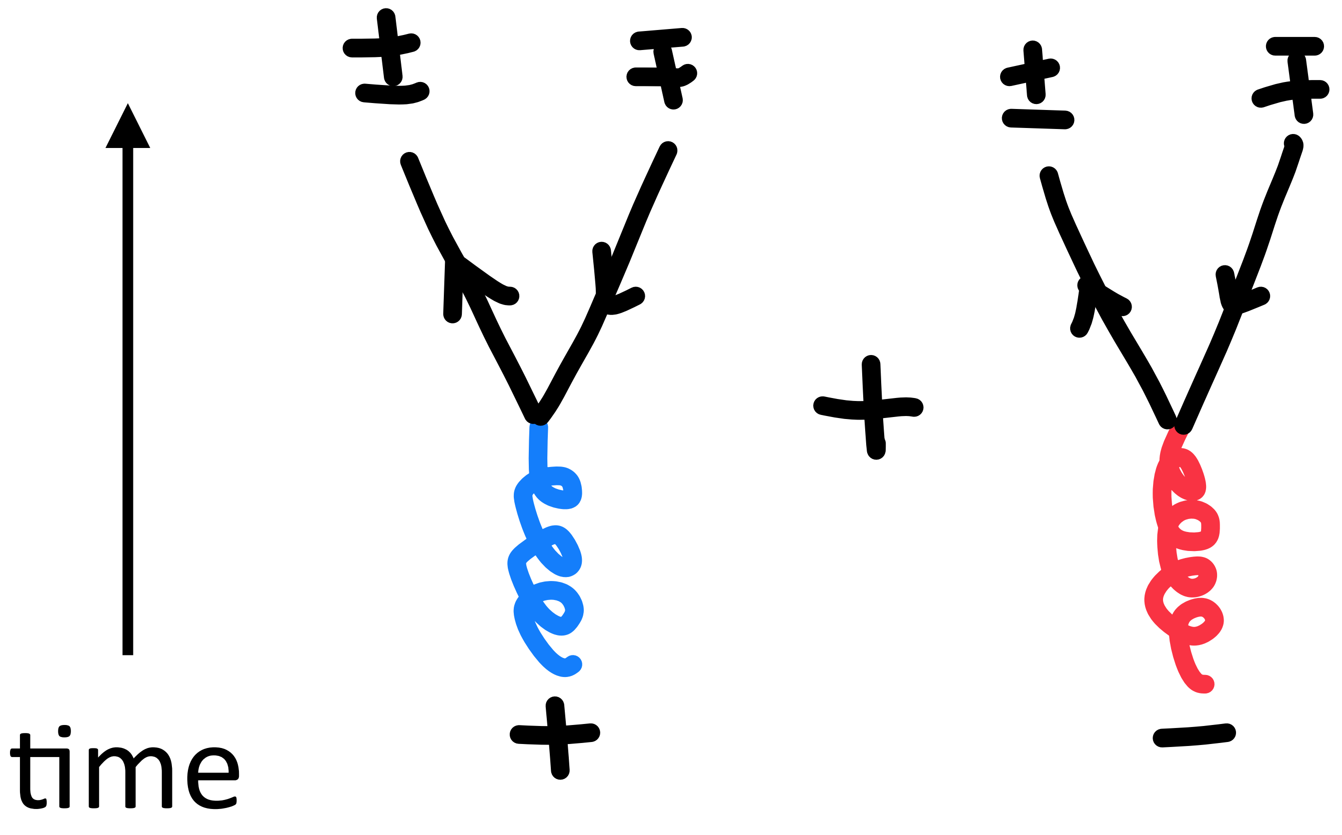
$$\boxed{\vec{\mathbb{O}}^{[J]}(\hat{n}_1)\mathcal{E}(\hat{n}_2) = -\frac{1}{2\pi}\frac{2}{\theta^2}\left[\widehat{C}_\phi(J) - \widehat{C}_\phi(J+1)\right]\vec{\mathbb{O}}^{[J+1]}(\hat{n}_1) + \text{higher twist}}$$

special case: $\mathcal{E} \sim \mathbb{O}_q^{[2]} + \mathbb{O}_g^{[2]}$

$$\widehat{C}_\phi(J) = \begin{pmatrix} \gamma_{qq}(J) & 2n_f\gamma_{qg}(J) & 2n_f\gamma_{q\tilde{g}}(J)e^{-2i\phi}/2 & 2n_f\gamma_{q\tilde{g}}(J)e^{2i\phi}/2 \\ \gamma_{gq}(J) & \gamma_{gg}(J) & \gamma_{g\tilde{g}}(J)e^{-2i\phi}/2 & \gamma_{g\tilde{g}}(J)e^{2i\phi}/2 \\ \gamma_{\tilde{g}q}(J)e^{2i\phi} & \gamma_{\tilde{g}g}(J)e^{2i\phi} & \gamma_{\tilde{g}\tilde{g}}(J) & \gamma_{\tilde{g}\tilde{g},\pm}(J)e^{4i\phi} \\ \gamma_{\tilde{g}q}(J)e^{-2i\phi} & \gamma_{\tilde{g}g}(J)e^{-2i\phi} & \gamma_{\tilde{g}\tilde{g},\pm}(J)e^{-4i\phi} & \gamma_{\tilde{g}\tilde{g}}(J) \end{pmatrix}$$

Catani, Grazzini, 99

$$\hat{P}_{gg}^{\mu\nu}(z,k_\perp;\epsilon) = 2C_A\left[-g^{\mu\nu}\left(\frac{z}{[1-z]_+} + \frac{1-z}{z} + z(1-z)\right) + 2z(1-z)\left(\frac{g_\perp^{\mu\nu}}{2} - \frac{k_\perp^\mu k_\perp^\nu}{k_\perp^2}\right)\right]$$
$$+\left[ag^{\mu\nu} + b\left(\frac{g_\perp^{\mu\nu}}{2} - \frac{k_\perp^\mu k_\perp^\nu}{k_\perp^2}\right)\right]\delta(1-z)\,, \qquad (2.19)$$



interference

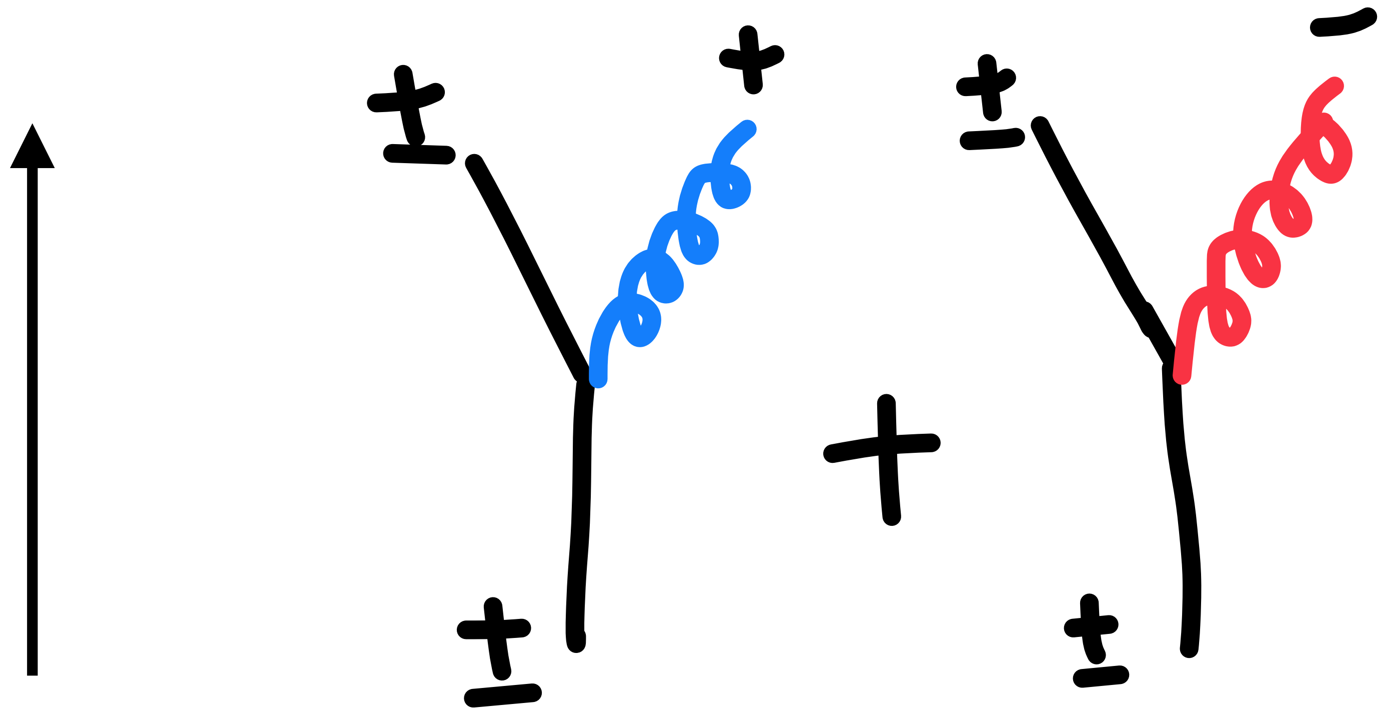
fixed by momentum conservation sum rule

$$\vec{\mathbb{O}}^{[J]} = \begin{pmatrix} \mathbb{O}_q^{[J]} \\ \mathbb{O}_g^{[J]} \\ \mathbb{O}_{\tilde{g},+}^{[J]} \\ \mathbb{O}_{\tilde{g},-}^{[J]} \end{pmatrix}$$

$$\boxed{\vec{\mathbb{O}}^{[J]}(\hat{n}_1)\mathcal{E}(\hat{n}_2) = -\frac{1}{2\pi}\frac{2}{\theta^2}\left[\widehat{C}_\phi(J) - \widehat{C}_\phi(J+1)\right]\vec{\mathbb{O}}^{[J+1]}(\hat{n}_1) + \text{higher twist}}$$

special case: $\mathcal{E} \sim \mathbb{O}_q^{[2]} + \mathbb{O}_g^{[2]}$

$$\widehat{C}_\phi(J) = \begin{pmatrix} \gamma_{qq}(J) & 2n_f\gamma_{qg}(J) & 2n_f\gamma_{q\tilde{g}}(J)e^{-2i\phi}/2 & 2n_f\gamma_{q\tilde{g}}(J)e^{2i\phi}/2 \\ \gamma_{gq}(J) & \gamma_{gg}(J) & \gamma_{g\tilde{g}}(J)e^{-2i\phi}/2 & \gamma_{g\tilde{g}}(J)e^{2i\phi}/2 \\ \gamma_{\tilde{g}q}(J)e^{2i\phi} & \gamma_{\tilde{g}g}(J)e^{2i\phi} & \gamma_{\tilde{g}\tilde{g}}(J) & \gamma_{\tilde{g}\tilde{g},\pm}(J)e^{4i\phi} \\ \gamma_{\tilde{g}q}(J)e^{-2i\phi} & \gamma_{\tilde{g}g}(J)e^{-2i\phi} & \gamma_{\tilde{g}\tilde{g},\pm}(J)e^{-4i\phi} & \gamma_{\tilde{g}\tilde{g}}(J) \end{pmatrix}$$



time

interference

$$\mathrm{Split}_{-}^{q\to gq,\mathrm{tree}}(1^+,2^+)\,\mathrm{Split}_{-}^{q\to gq,\mathrm{tree},*}(1^-,2^+) + \mathrm{Split}_{-}^{q\to gq,\mathrm{tree},*}(1^+,2^+)\,\mathrm{Split}_{-}^{q\to gq,\mathrm{tree}}(1^-,2^+) + c.c. \tag{2.34}$$

$$= -2\frac{1-z}{z}\left(\frac{1}{\langle 1\,2\rangle^2} + \frac{1}{[1\,2]^2}\right), \tag{2.35}$$

Applied to EEC

$$\mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2)$$
$$= -\frac{1}{2\pi}\frac{2}{\theta^2}\Big\{\left[(\gamma_{qq}(2)-\gamma_{qq}(3))+(\gamma_{gq}(2)-\gamma_{gq}(3))\right]\mathbb{O}_q^{[3]}+\left[(\gamma_{gg}(2)-\gamma_{gg}(3))+2n_f(\gamma_{qg}(2)-\gamma_{qg}(3))\right]\mathbb{O}_g^{[3]}$$
$$+\frac{1}{2}\left[(\gamma_{g\tilde{g}}(2)-\gamma_{g\tilde{g}}(3))+2n_f(\gamma_{q\tilde{g}}(2)-\gamma_{q\tilde{g}}(3))\right]\left(e^{2i\phi}\mathbb{O}_{\tilde{g},-}^{[3]}+e^{-2i\phi}\mathbb{O}_{\tilde{g},+}^{[3]}\right)\Big\}+\mathcal{O}(\theta^0)\,.$$

Leading log series: $\widehat{C}_\phi(J)\simeq\alpha_s(1+\alpha_s\ln\theta+\alpha_s^2\ln^2\theta+\cdots)$

transverse spin-0

$$\mathcal{O}_q^{[J]}=\frac{1}{2^J}\bar{\psi}\gamma^+(iD^+)^{J-1}\psi$$

$$\mathcal{O}_g^{[J]}=-\frac{1}{2^J}F_a^{\mu+}(iD^+)^{J-2}F_a^{\mu+}$$

transverse spin-2

$$\left[\mathcal{O}_{\tilde{g},\lambda}^{[J]}=-\frac{1}{2^J}F_a^{\mu+}(iD^+)^{J-2}F_a^{\nu+}\epsilon_{\lambda,\mu}\epsilon_{\lambda,\nu}\right]$$

CANNOT MIX

RG equation:

$$\frac{d}{d\ln\mu^2}\vec{\mathcal{O}}^{[J]}=-\widehat{\gamma}(J)\cdot\vec{\mathcal{O}}^{[J]}$$

$$\widehat{\gamma}(J)=\begin{pmatrix}\gamma_{qq}(J) & 2n_f\gamma_{qg}(J) & 0\\ \gamma_{gq}(J) & \gamma_{gg}(J) & 0\\ 0 & 0 & \gamma_{\tilde{g}\tilde{g}}(J)\mathbf{1}\end{pmatrix}$$

$$\mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2)\sim\frac{1}{\theta^{2-\gamma}}\mathbb{O}_{q,g}+\frac{1}{\theta^{2-\tilde{\gamma}}}\mathbb{O}_{\tilde{g}}+\text{twist corrections}$$

EEC -> one point correlator

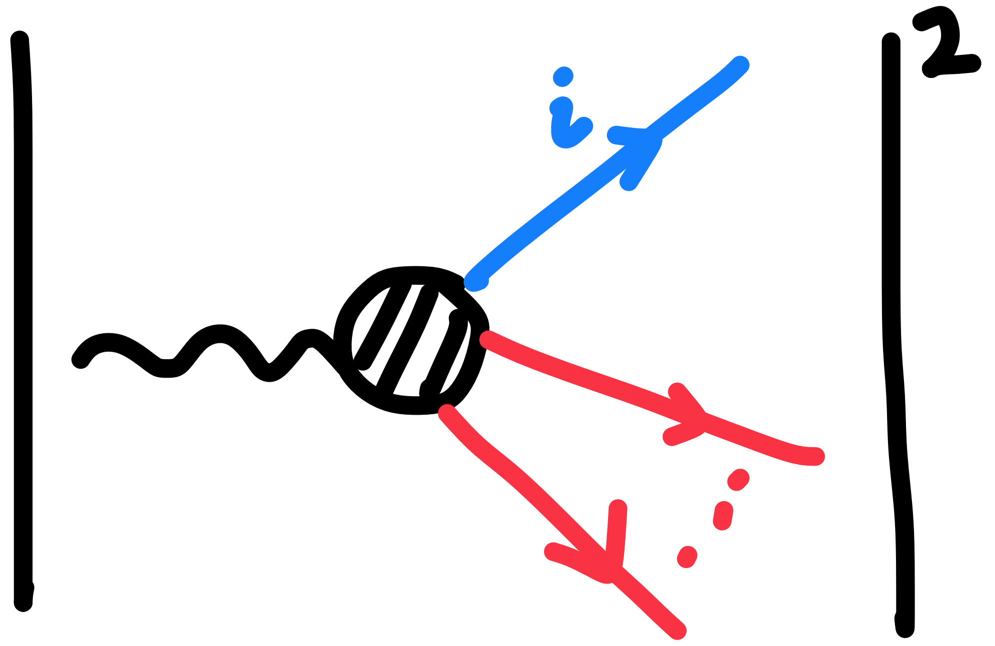
$$\int d^4x e^{-iq\cdot x}\langle\Omega|j_{\rm em}^{\dagger,\mu}(x)\mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2)j^{\nu}_{\rm em}(0)|\Omega\rangle \qquad \mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2)\sim\frac{1}{\theta^{2-\gamma}}\mathbb{O}_{q,g}+\frac{e^{\pm 2i\phi}}{\theta^{2-\tilde{\gamma}}}\mathbb{O}_{\tilde{g}}+\text{twist corrections}$$

$$\rightarrow \int d^2\vec{n}\int d^4x e^{-iq\cdot x}\langle\Omega|j_{\rm em}^{\dagger,\mu}(x)\left(\frac{1}{\theta^{2-\gamma}}\mathbb{O}_{q,g}(\vec{n})+\frac{e^{\pm 2i\phi}}{\theta^{2-\tilde{\gamma}}}\mathbb{O}_{\tilde{g}}(\vec{n})\right)j^{\nu}_{\rm em}(0)|\Omega\rangle$$

one-point correlator

1. When j^μ j^ν contain transverse polarized component (polarized e+e-, or retaining beam direction—oriented event shape), the matrix element of $O_{\tilde{g}}$ does not vanish.
2. If there is no beam polarization, and average over orientation, only the first operator survive

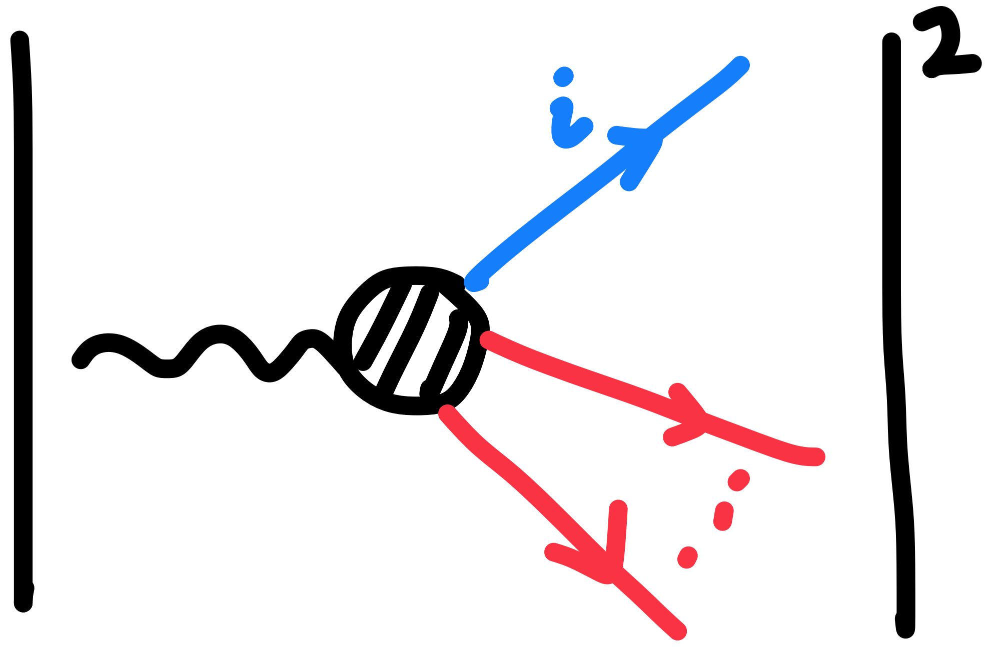
$$< \Omega|j_{\rm em}^{\dagger,\mu}\frac{1}{\theta^{J-1-\gamma}}\mathbb{O}^{[J]}(\vec{n})j^{\dagger}_{\mu,\rm em}|\Omega\rangle\Big|_{J=3}\sim\int dE_i\, E_i^{J-1}\sum_i$$



Mitov, Moch, Vogt, 06; Almasy, Moch, Vogt, 11

Space-like v.s. time-like reciprocity

$$< \Omega | j^{\dagger,\mu}_{\text{em}} \frac{1}{\theta^{J-1-\gamma}} \mathbb{O}^{[J]}(\vec{n}) j^{\dagger}_{\mu,\text{em}} |\Omega\rangle \Big|_{J=3} \sim \int dE_i \, E_i^{J-1} \sum_i$$



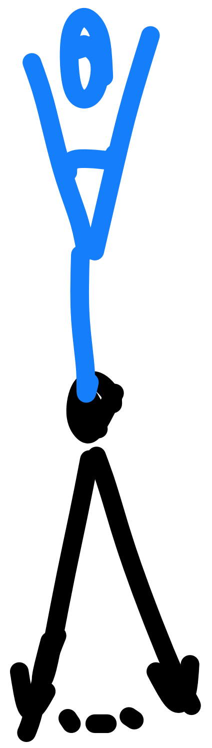
collinear unsafe observable:

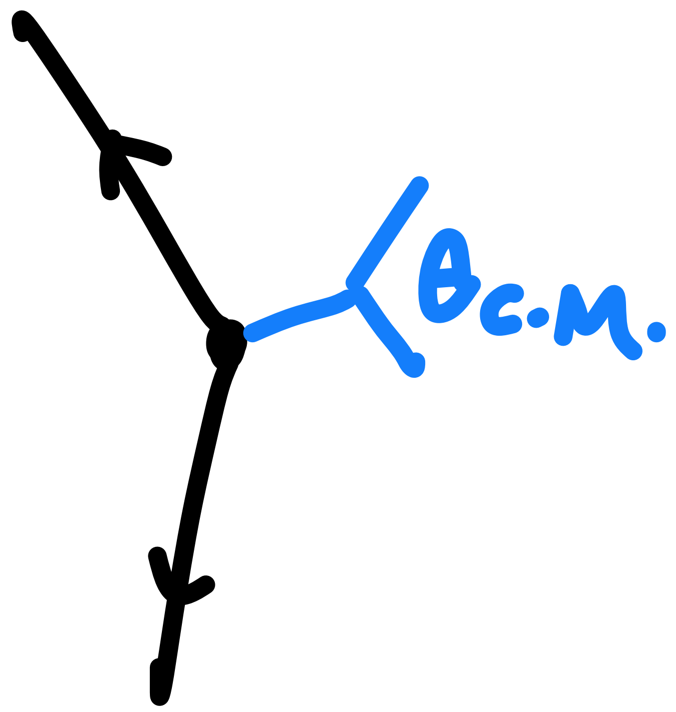
$$\frac{1}{\epsilon}\int_0^1 dz\, z^{J-1} P^T(z) = \frac{1}{\epsilon}\gamma^T(J)$$

evolution controlled by anomalous dimension of local spin J operator = **moment of space-like splitting func.**

contradiction: evolution mismatched

In the OPE, θ is defined in the frame where everything recoil against the jet is back-to-back.





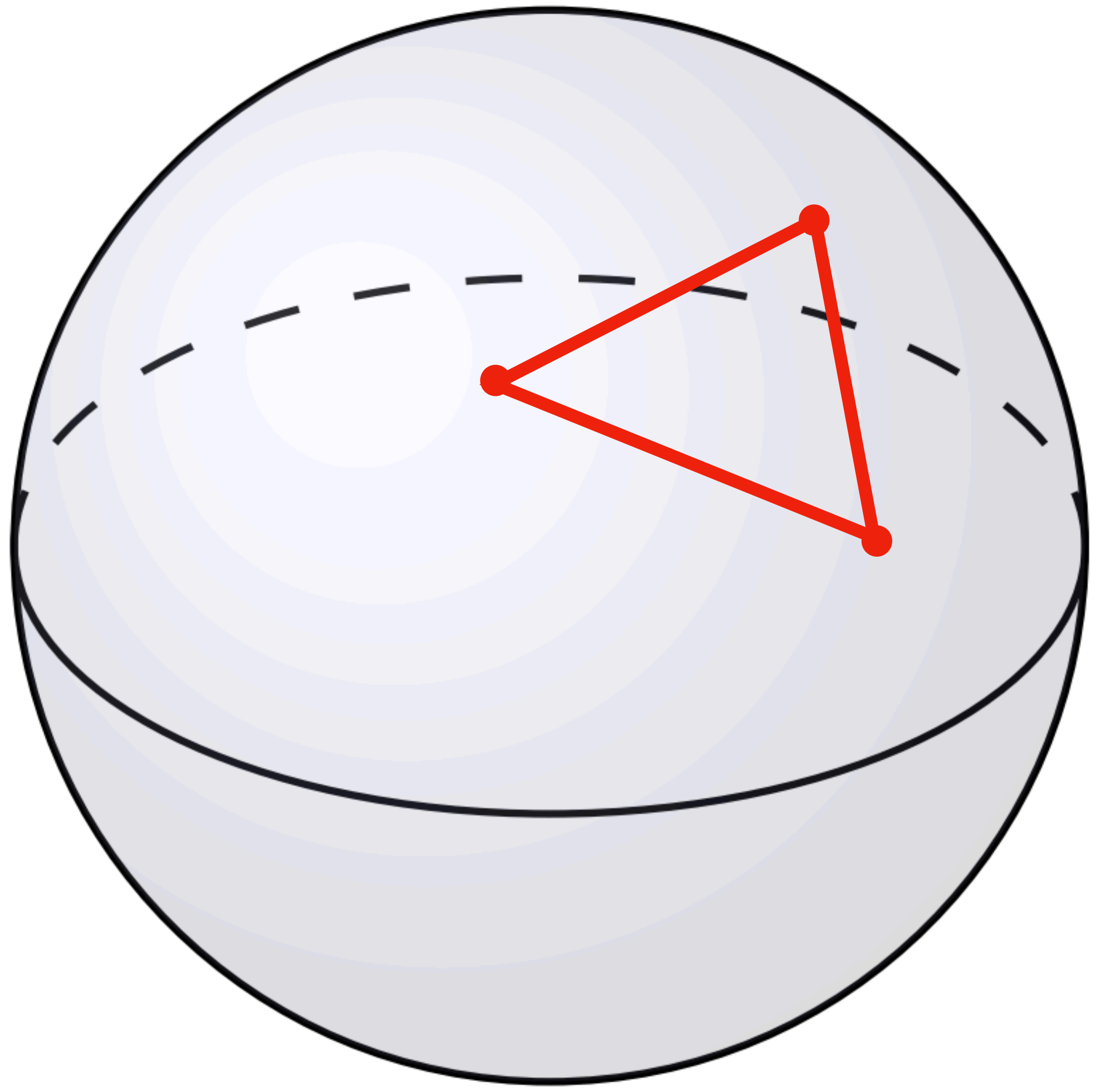
But in reality, we measure angle in the C.M. frame of the collision.

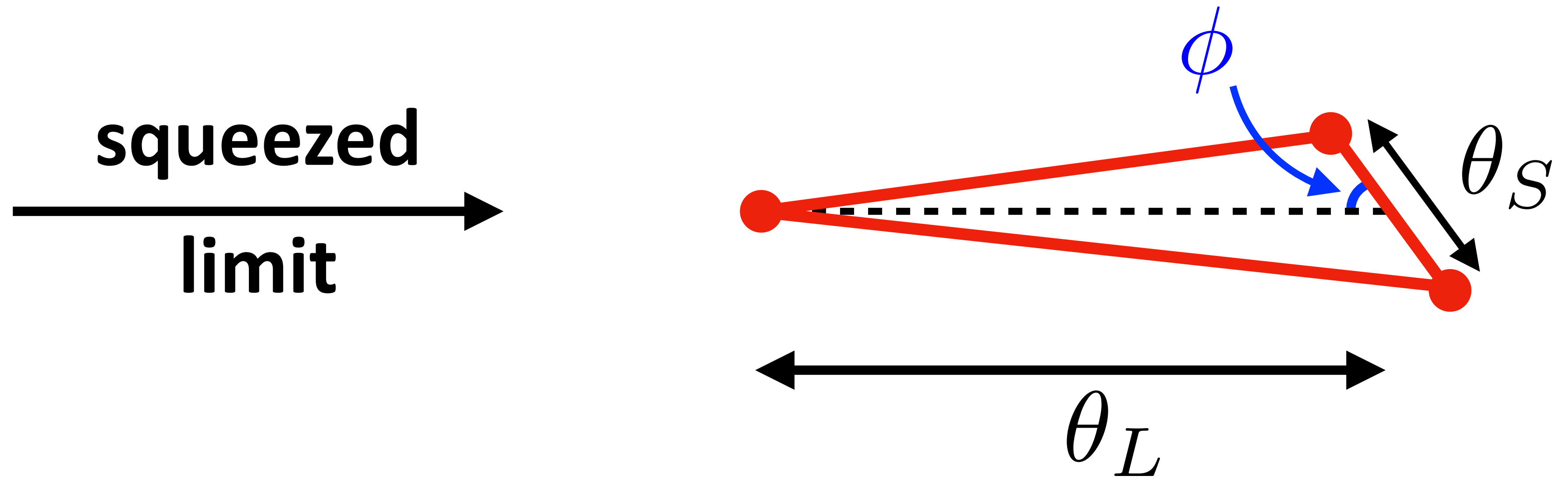
$$\theta^{\gamma} \to \theta^{\gamma}_{\text{C.M.}} \left(\frac{E_i}{Q}\right)^{\gamma}$$

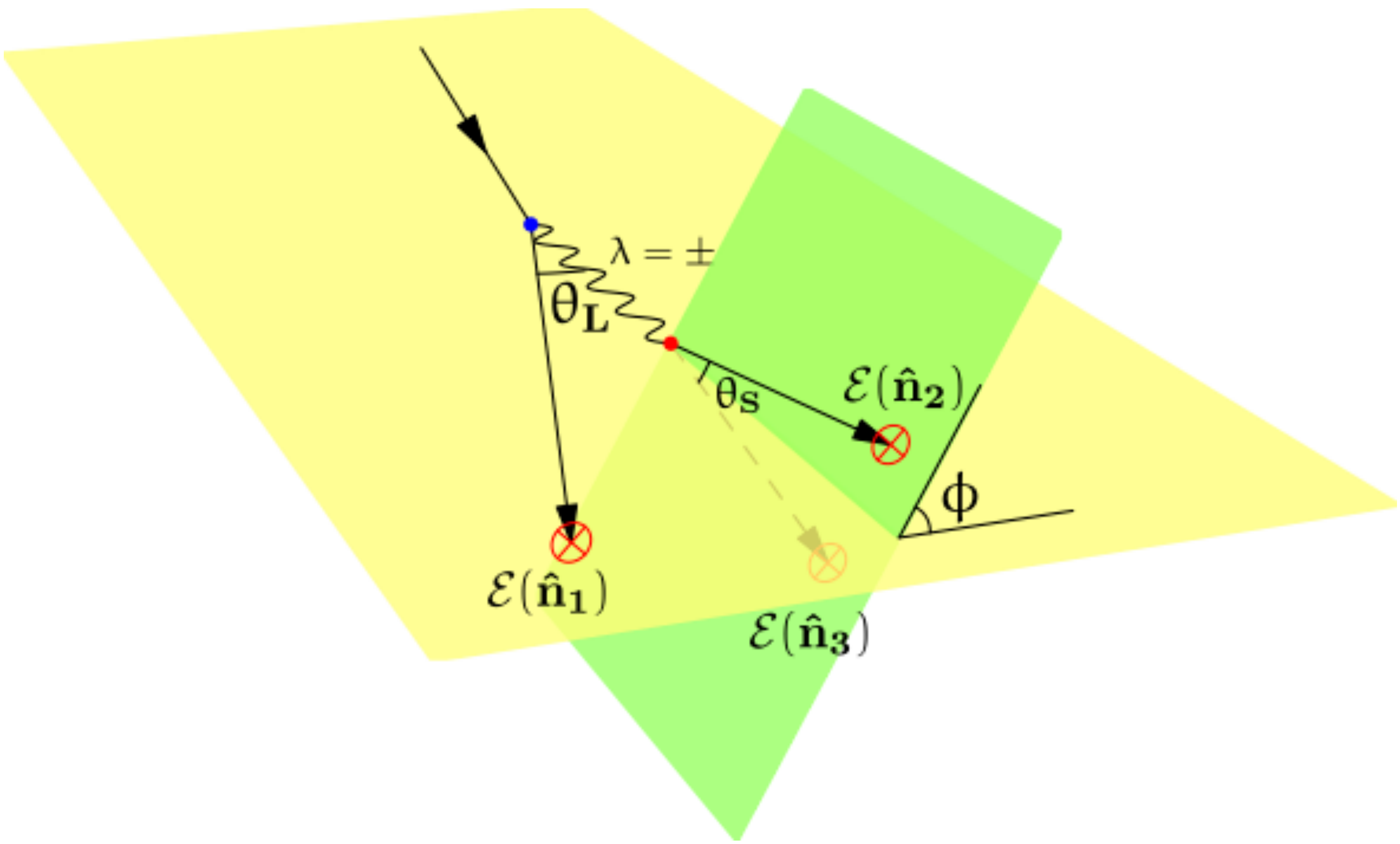
$$\int_0^1 dz\, z^{J-1+\gamma^S(J)} P^T(z) = \gamma^T(J+\gamma^S(J)) = \gamma^S(J)$$

generalized Gribov-Lipatov reciprocity

Squeeze three-point correlator







$$\frac{d^3\Sigma_i}{d\theta_L^2 d\theta_S^2 d\phi} \simeq \frac{1}{\pi}\left(\frac{\alpha_s}{4\pi}\right)^2 \frac{\mathrm{Sq}_i^{(0)}(\phi)}{\theta_L^2\theta_S^2} + \cdots$$

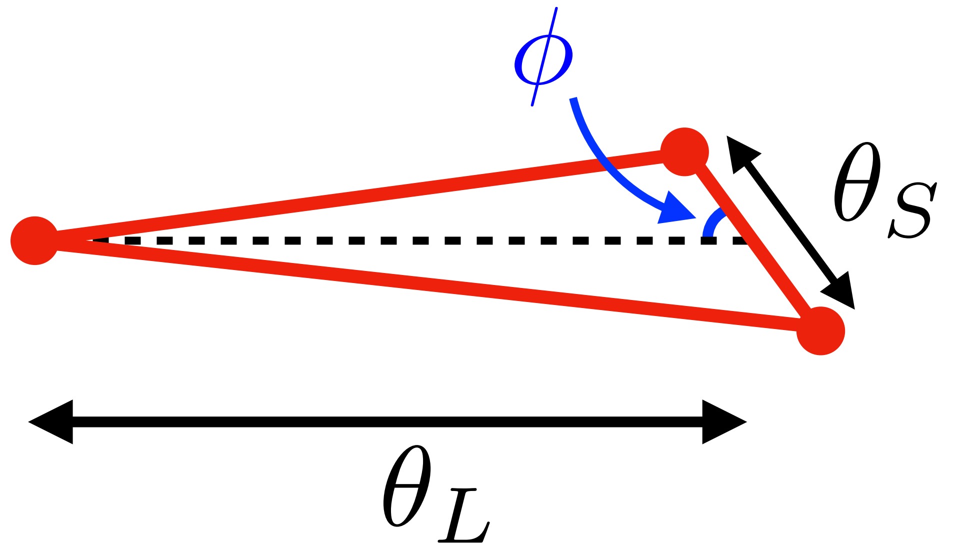
$$\mathrm{Sq}_q^{(0)}(\phi) = C_F n_f T_F\left(\frac{39 - 20\cos(2\phi)}{225}\right) + C_F C_A\left(\frac{273 + 10\cos(2\phi)}{225}\right) + C_F^2\frac{16}{5}$$

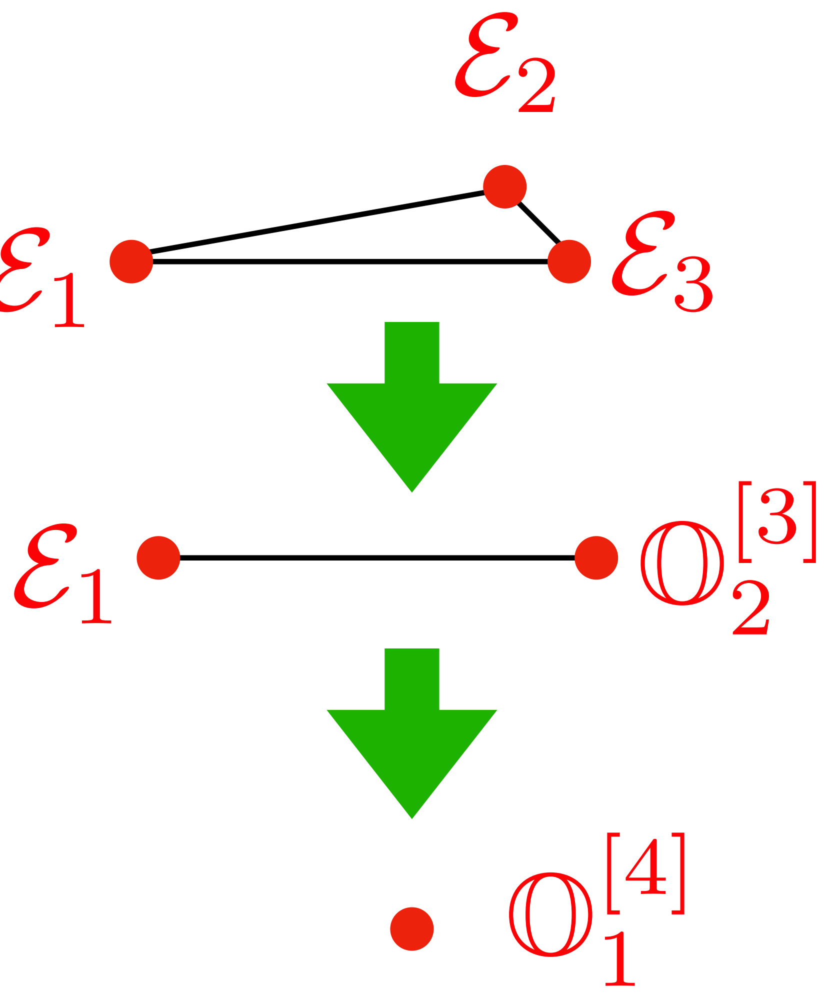
$$\mathrm{Sq}_g^{(0)}(\phi) = C_A n_f T_F\left(\frac{126 - 20\cos(2\phi)}{225}\right) + C_A^2\left(\frac{882 + 10\cos(2\phi)}{225}\right) + C_F n_f T_F\frac{3}{5}$$

Interference Effect

1. Cancellation between boson and fermion
2. The equal coefficient due to an effective N=1 supersymmetry

Sequential light-ray OPE





$$\langle \mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2)\mathcal{E}(\vec{n}_3)\rangle$$

$$\longrightarrow -\frac{1}{2\pi}\frac{2}{\theta_S^2}\vec{\mathcal{J}}\left[\underbrace{\widehat{C}_{\phi_S}(2)-\widehat{C}_{\phi_S}(3)}_{}\right]^{e^{2i\phi_S}}\left\langle \mathcal{E}(\vec{n}_1)\vec{\mathbb{O}}^{[3]}(\vec{n}_2)\right\rangle$$

$$\longrightarrow \frac{1}{(2\pi)^2}\frac{2}{\theta_S^2}\frac{2}{\theta_L^2}\vec{\mathcal{J}}\left[\widehat{C}_{\phi_S}(2)-\widehat{C}_{\phi_S}(3)\right]\left[\widehat{C}_{\phi_L}(3)-\widehat{C}_{\phi_L}(4)\right]\boxed{\left\langle \vec{\mathbb{O}}^{[4]}(\vec{n}_1)\right\rangle}$$

$$e^{2i\phi_S} \qquad \cos(2\phi) \qquad e^{-2i\phi_L}$$

Hierarchy $1 \gg \theta_L \gg \theta_S$ **fixed order result needs to be resummed**

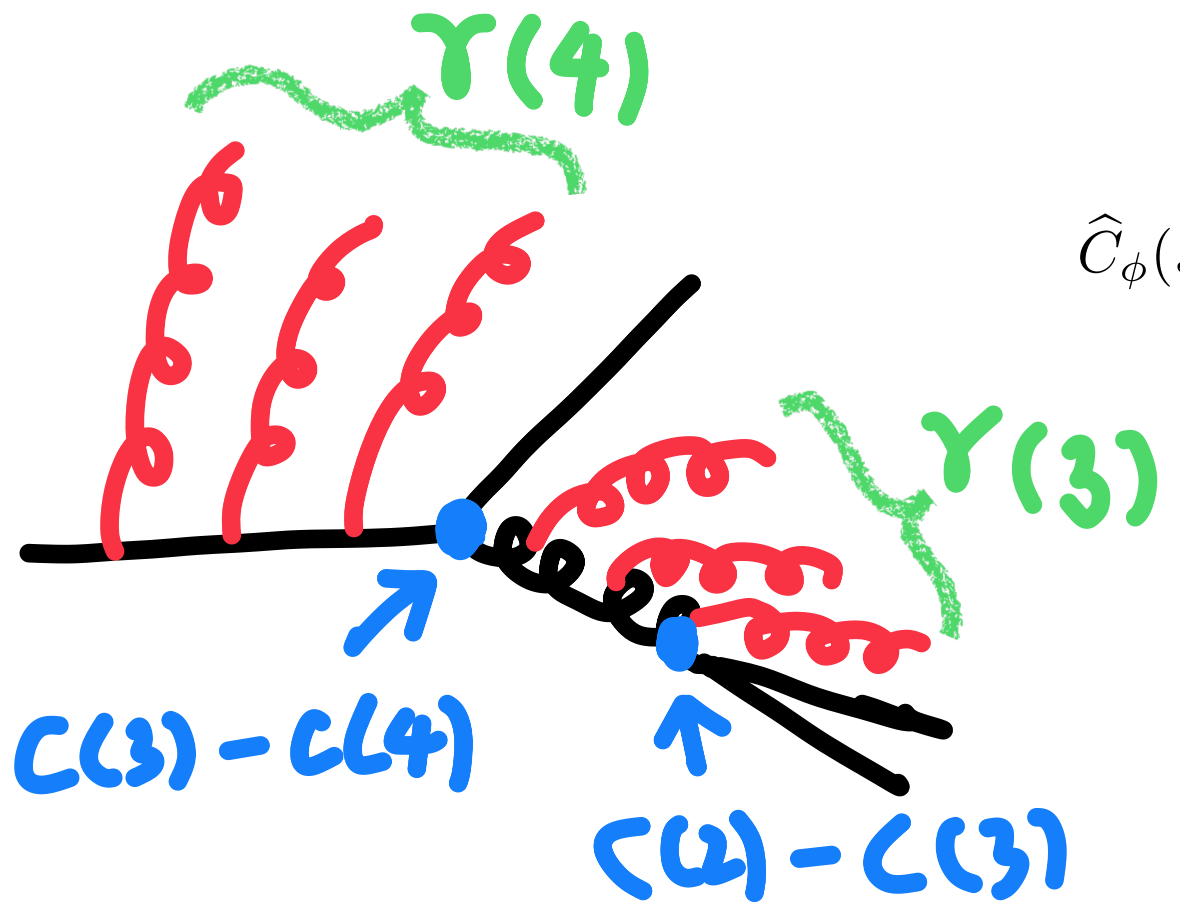
$$\langle \mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2)\mathcal{E}(\vec{n}_3)\rangle \sim C_1(\theta_S,\mu)\left\langle \mathcal{E}(\vec{n}_1)\mathbb{O}^{[3]}(\vec{n}_2)\right\rangle_\mu \sim C_1(\theta_S,\mu)C_2(\theta_L,\mu)\left\langle \mathbb{O}^{[4]}(\vec{n}_1)\right\rangle_\mu$$

$$\widehat{\gamma}(J)=\begin{pmatrix}\gamma_{qq}(J) & 2n_f\gamma_{qg}(J) & 0\\ \gamma_{gq}(J) & \gamma_{gg}(J) & 0\\ 0 & 0 & \gamma_{\tilde{g}\tilde{g}}(J)\mathbf{1}\end{pmatrix}$$

Leading log approximation

$$\widehat{\gamma}(J) = \begin{pmatrix} \gamma_{qq}(J) & 2n_f\gamma_{qg}(J) & 0 \\ \gamma_{gq}(J) & \gamma_{gg}(J) & 0 \\ 0 & 0 & \gamma_{\tilde{g}\tilde{g}}(J)\mathbf{1} \end{pmatrix}$$

$$\mathcal{E}(\hat{n}_1)\mathcal{E}(\hat{n}_2)\mathcal{E}(\hat{n}_3)$$
$$= \frac{1}{(2\pi)^2}\frac{2}{\theta_S^2}\frac{2}{\theta_L^2}\vec{\mathcal{J}}\left[\widehat{C}_{\phi_S}(2)-\widehat{C}_{\phi_S}(3)\right]\left[\frac{\alpha_s(\theta_L Q)}{\alpha_s(\theta_S Q)}\right]^{\frac{\widehat{\gamma}(3)}{\beta_0}}\left[\widehat{C}_{\phi_L}(3)-\widehat{C}_{\phi_L}(4)\right]\left[\frac{\alpha_s(Q)}{\alpha_s(\theta_L Q)}\right]^{\frac{\widehat{\gamma}(4)}{\beta_0}}\vec{\mathbb{O}}^{[4]}(\hat{n}_1)$$

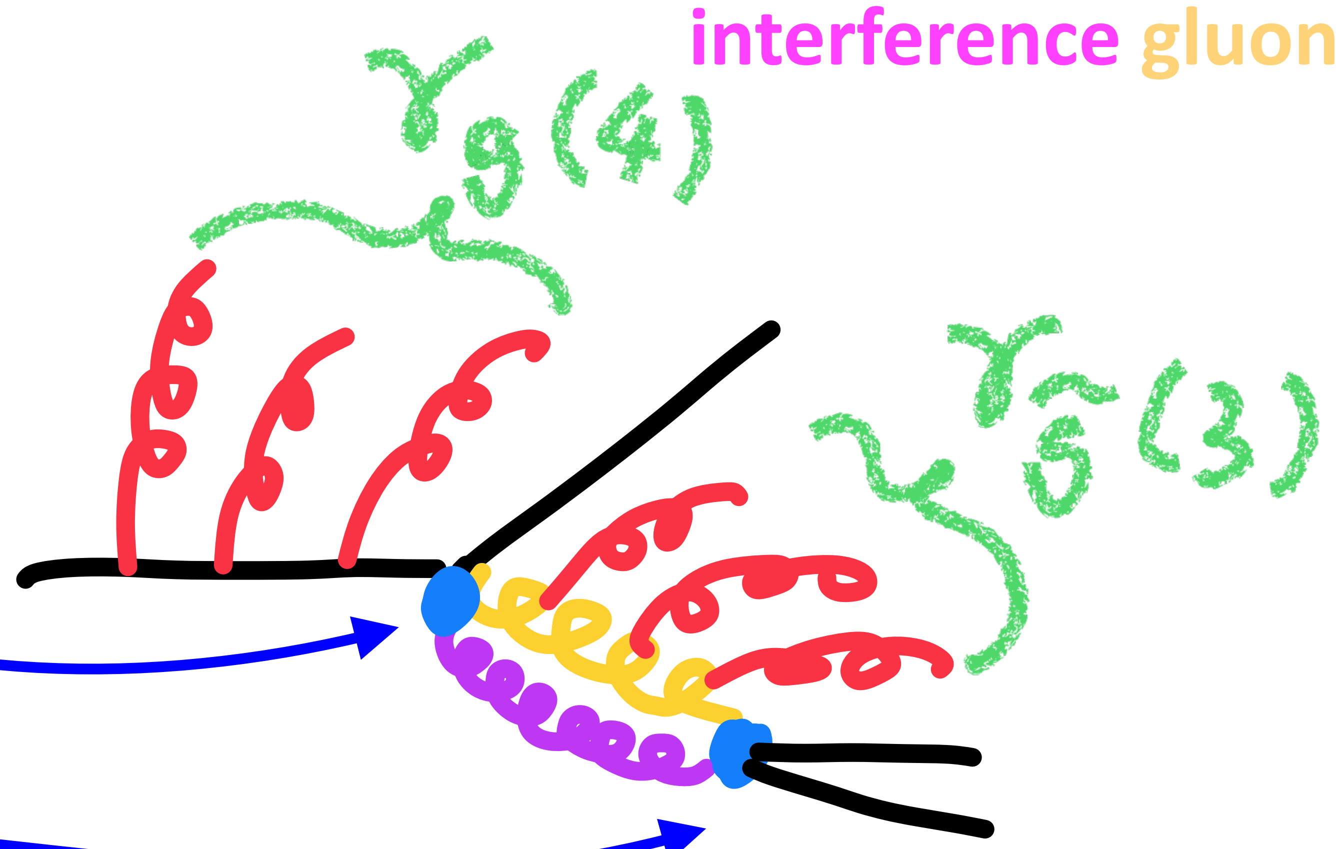


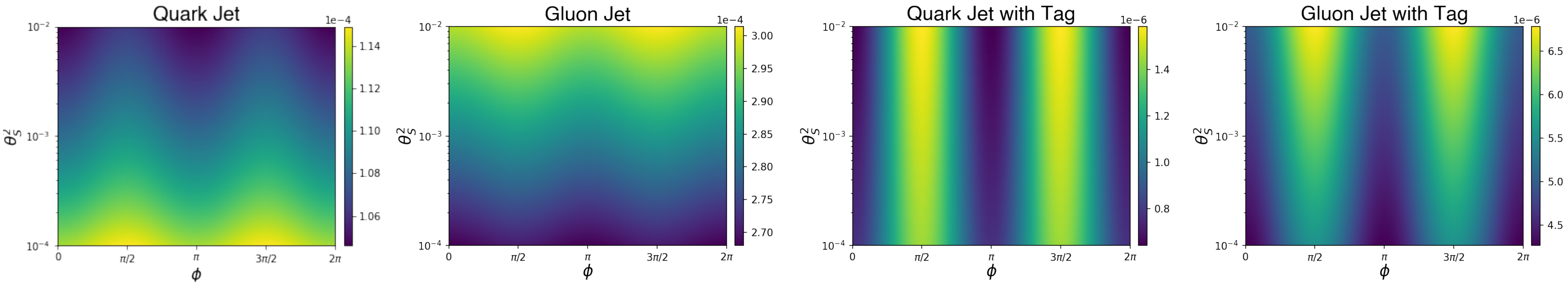
$$\widehat{C}_\phi(J) = \begin{pmatrix} \gamma_{qq}(J) & 2n_f\gamma_{qg}(J) & 2n_f\gamma_{q\tilde{g}}(J)e^{-2i\phi}/2 & 2n_f\gamma_{q\tilde{g}}(J)e^{2i\phi}/2 \\ \gamma_{gq}(J) & \gamma_{gg}(J) & \gamma_{g\tilde{g}}(J)e^{-2i\phi}/2 & \gamma_{g\tilde{g}}(J)e^{2i\phi}/2 \\ \gamma_{\tilde{g}q}(J)e^{2i\phi} & \gamma_{\tilde{g}g}(J)e^{2i\phi} & \gamma_{\tilde{g}\tilde{g}}(J) & \gamma_{\tilde{g}\tilde{g},\pm}(J)e^{4i\phi} \\ \gamma_{\tilde{g}q}(J)e^{-2i\phi} & \gamma_{\tilde{g}g}(J)e^{-2i\phi} & \gamma_{\tilde{g}\tilde{g},\pm}(J)e^{-4i\phi} & \gamma_{\tilde{g}\tilde{g}}(J) \end{pmatrix}$$

unobserved

angular-ordering

interference gluon





Conformal symmetry on the celestial sphere?

$$x_\mu \sigma^\mu = \begin{pmatrix} x^0 + x^3 & x^1 - ix^2 \\ x^1 + ix^2 & x^0 - x^3 \end{pmatrix} \qquad \det(x_\mu \sigma^\mu) = (x^0)^2 - (\vec{x})^2 = x_\mu x^\mu$$

$$L \in \mathrm{SL}(2, \mathbb{C}) \qquad \begin{pmatrix} a & b \\ c & d \end{pmatrix} \qquad ad - bc = 1$$

$$x'_\mu \sigma^\mu = L^\dagger x_\mu \sigma^\mu L \qquad x'_\mu x'^\mu = \det(x'_\mu \sigma^\mu) = \det(L^\dagger x_\mu \sigma^\mu L) = x_\mu x^\mu$$

Therefore L define a Lorentz transform $\qquad \mathrm{SL}(2, \mathbb{C})/Z_2 = SO(3,1)$

SL(2,C) is also the conformal group of Riemann sphere.

Since energy correlators are defined on celestial sphere, can there be a conformal field theory living on the celestial sphere, which describes all energy correlators?

If true, The usual Minkowski space becomes embedding space of celestial sphere.

This naive expectation does not hold true, because of the source.

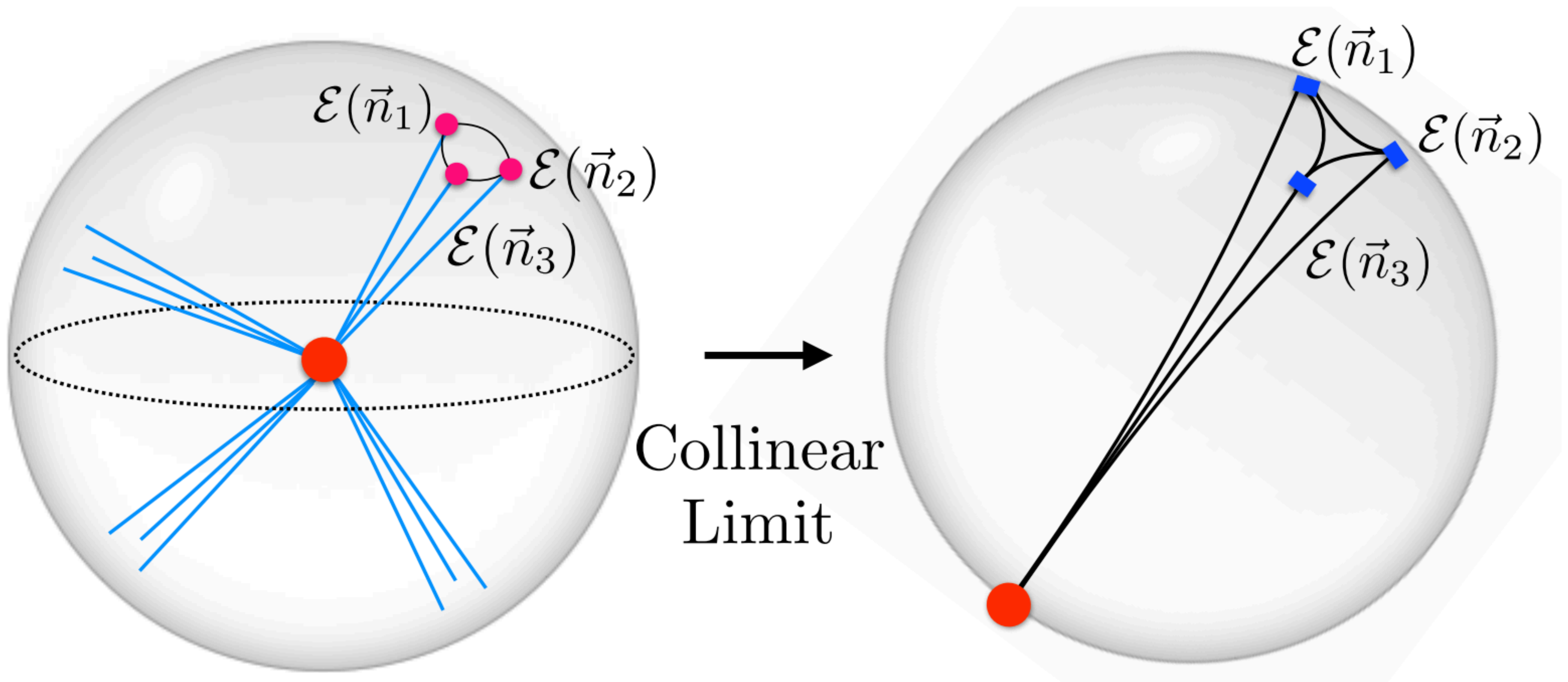
$$\int d^4x e^{-iq\cdot x} \langle\Omega| j^{\dagger}_{\mathrm{em},\mu}(x) \mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2) j^{\mu}_{\mathrm{em}}(0)|\Omega\rangle \qquad q=(q^0,0,0,0)$$

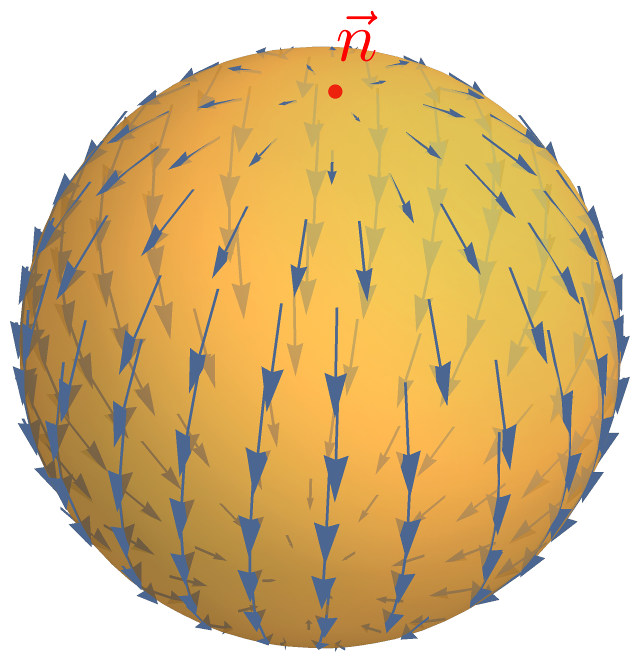
Breaks SO(3,1) to SO(3)

However, in the collinear limit, which is relevant for jet substructure

$$\int_{-\infty}^{\infty} du\, e^{-iu\bar{n}\cdot P} \langle\Omega|\bar{\chi}_n(u\bar{n})\mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2)\gamma^+\chi_n(0)|\Omega\rangle \qquad \chi_n(x) = \frac{\gamma^-\gamma^+}{4}\psi(x)W_n^{\dagger}(x)$$

Only reference to the source are null vector P and $\bar{n}$, no dependence on interior.





4D boost = dilation on sphere

4D transverse spin = spin on sphere

$$\mathbb{O}(\vec{n}) = \underbrace{\lim_{r\to\infty} r^{\Delta-J} \int_0^\infty dt}\, \underbrace{O^{\mu_1\cdots\mu_J}(t, r\vec{n})\bar{n}_{\mu_1}\ldots\bar{n}_{\mu_J}}$$

Light-transform of $O_{(\Delta,J)}$

dimension	$J-\Delta-1$	$+$	Δ	$= J-1$
collinear spin	$-\Delta+J+1$	$+$	$-J$	$= 1-\Delta$

$\mathbb{O}$(n) as an operator on the celestial sphere

δ = (celestial) dimension = - collinear spin = Δ - 1

j = (celestial) spin = transverse spin

e.g., $\mathbb{O}_{\tilde{g},+}$ has spin 2 on the sphere

$\mathbb{O}^{[J=3]}_{\tilde{g},\pm} \in \mathcal{E}\mathcal{E}$

δ = Δ-1=4

j = 2

responsible for the cos(2ϕ) modulation at leading power

$$\frac{\cos(2\phi)}{\theta_S^2}$$

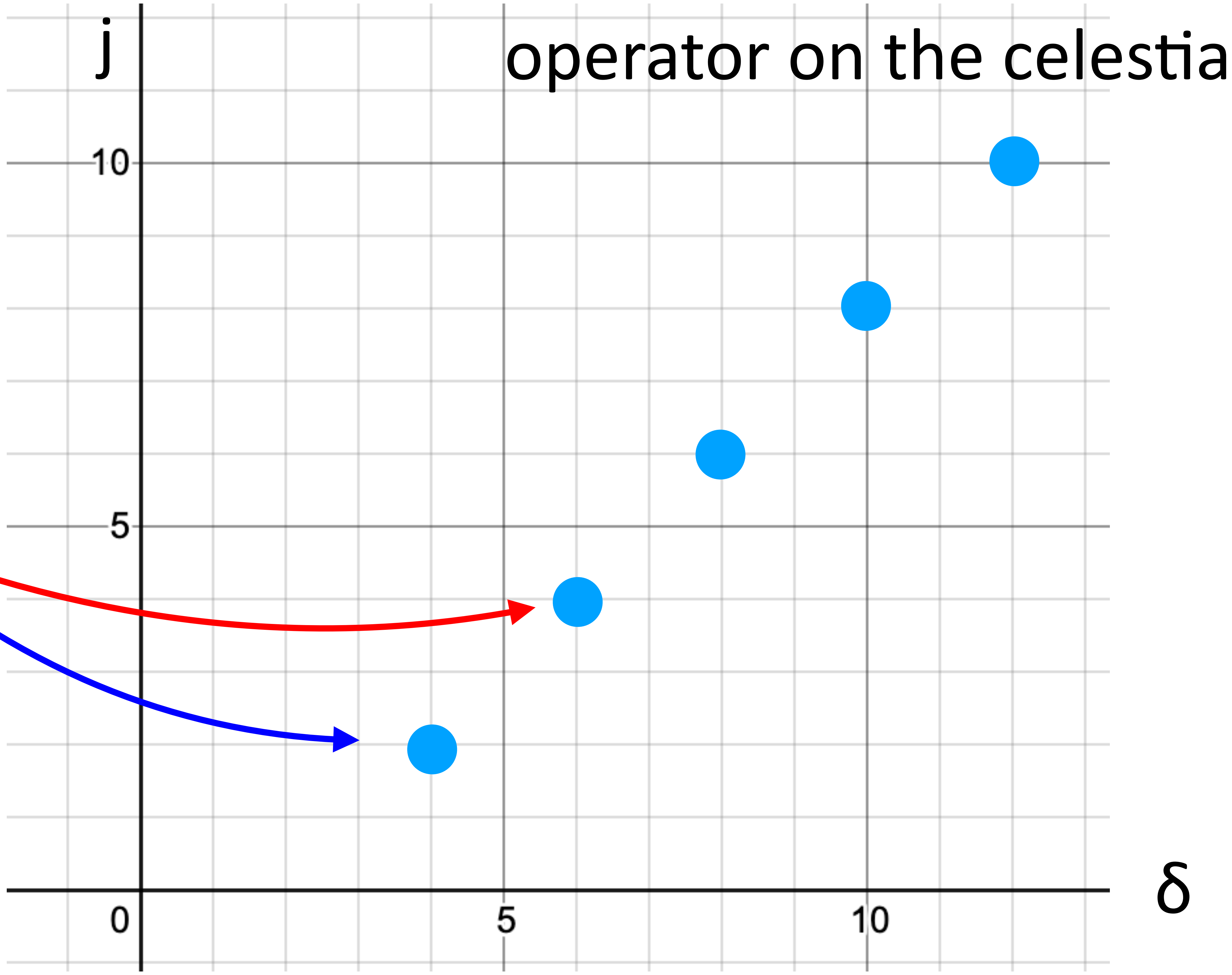
$(\epsilon_\lambda \cdot P)^2 \mathbb{O}^{[J=3]}_{\tilde{g},\lambda}$

δ = 6

j = 4

$$|\theta_S|^0 \cos(4\phi)$$

The family of higher spin (descendant) operator on the celestial sphere



Conformal partial wave

This particular family of conformal primary δ=4, j=2 can be resum by considering the action conformal casimir operator:

$$\int_{-\infty}^{\infty} du\, e^{-iu\bar{n}\cdot P} \langle \Omega | \bar{\chi}_n(u\bar{n}) [\hat{C}, \mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2)] \mathcal{E}(\vec{n}_3) \gamma^+ \chi_n(0) | \Omega \rangle \qquad \hat{C} = \frac{1}{2} M_{\mu\nu} M^{\mu\nu}$$

$M^{\mu\nu}$ generator of the 4D Lorentz group

The solution is a Gegenbauer Q function:

$$\tilde{G}_{\delta=4,j=2}(w,\bar{w}) \sim \frac{1}{\bar{w}^2}\, {}_2F_1(1, \frac{3}{2}, \frac{7}{2}; w^2) + \text{c.c.} \qquad w = \frac{\theta_S}{2} e^{i\phi}$$

Expand in power series:

$$C_F n_f T_F \left(-\frac{1}{|w|^2} \frac{\cos(2\phi)}{720} - \frac{\cos(4\phi)}{1680} - |w|^2 \frac{\cos(6\phi)}{3024} - |w|^4 \frac{\cos(8\phi)}{4752} - \cdots \right)$$

largest spin only

In full agreement with perturbative data! In this specific case, perturbative power corrections resummed by conformal block.

Summary

- Jet substructure is an ideal tool to uncover the high energy dynamics of QCD.
- Light-ray operator product expansion, originally developed for N=4 SYM, now applied to QCD successfully at LO/LL.
- Symmetry allows transparent treatment of spin interference effects from light-ray OPE.
- Hidden conformal symmetry in jet substructure. Physical 4D space-time becomes the embedding space of two-dimensional celestial sphere. Evidence: power corrections resummed by conformal block.

Thank you very much!