Making Sense of Ambiguities in Renormalization Group Functions

Anders Eller Thomsen

Based on [2104.07037] with F. Herren

CHIPP, June 10th 2021

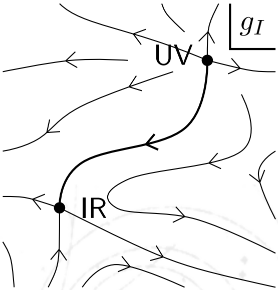
b
UNIVERSITÄT
BERN

AEC
ALBERT EINSTEIN CENTER
FOR FUNDAMENTAL PHYSICS

In the last decade:

- First computation of 4-loop gauge and 3-loop Yukawa β-functions in gauge-Yukawa theories
- New results for the structure of RG
  - Weak A-theorem
  - Weyl consistency conditions
  - Perturbative limit cycle as a result flavor symmetry
- 3-loop SM RG functions feature poles in the dimensional expansion: $\left[y_u, y_d\right] \neq 0$, $\gamma|_{\mathrm{div}} = -\gamma^\dagger|_{\mathrm{div}} \neq 0$

RG flow

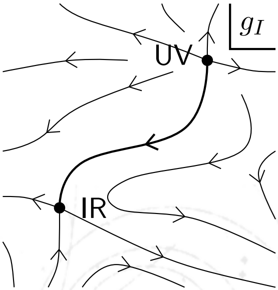


In the last decade:

- First computation of 4-loop gauge and 3-loop Yukawa β-functions in gauge-Yukawa theories
- New results for the structure of RG
  - Weak A-theorem
  - Weyl consistency conditions
  - Perturbative limit cycle as a result flavor symmetry
- 3-loop SM RG functions feature poles in the dimensional expansion: $\left[y_u, y_d\right] \neq 0$, $\gamma|_{\text{div}} = -\gamma^\dagger|_{\text{div}} \neq 0$

  Are RG functions not finite?

RG flow



Can the RG be finite given divergent RG functions?

CS Eq.: $$0 = \left(\frac{\partial}{\partial t} + \beta_I \frac{\partial}{\partial g_I} + \int \mathrm{d}^d x \, \mathcal{J}_\beta \gamma^\beta{}_\alpha \frac{\delta}{\delta \mathcal{J}_\alpha} \right) \mathcal{W}[g, \, \mathcal{J}]$$

Divergences in RG functions

Can the RG be finite given divergent RG functions?

CS Eq.: $$0 = \left(\frac{\partial}{\partial t} + \beta_I \frac{\partial}{\partial g_I} + \int \mathrm{d}^d x \, \mathcal{J}_\beta \gamma^\beta{}_\alpha \frac{\delta}{\delta \mathcal{J}_\alpha}\right) \mathcal{W}[g,\, \mathcal{J}]$$

Flavor WI: $$0 = \left((\omega\, g)_I \frac{\partial}{\partial g_I} - \int \mathrm{d}^d x \, \mathcal{J}_\beta \omega^\beta{}_\alpha \frac{\delta}{\delta \mathcal{J}_\alpha}\right) \mathcal{W}, \qquad \omega \in \mathfrak{g}_F$$

Can the RG be finite given divergent RG functions?

CS Eq.: $$0 = \left(\frac{\partial}{\partial t} + \beta_I \frac{\partial}{\partial g_I} + \int \mathrm{d}^d x \, \mathcal{J}_\beta \gamma^\beta{}_\alpha \frac{\delta}{\delta \mathcal{J}_\alpha} \right) \mathcal{W}[g,\, \mathcal{J}]$$

Flavor WI: $$0 = \left((\omega\, g)_I \frac{\partial}{\partial g_I} - \int \mathrm{d}^d x \, \mathcal{J}_\beta \omega^\beta{}_\alpha \frac{\delta}{\delta \mathcal{J}_\alpha} \right) \mathcal{W}, \qquad \omega \in \mathfrak{g}_F$$

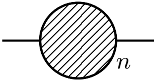
RG finiteness—For a finite RG flow

$\gamma\big|_{\mathrm{div}} \in \mathfrak{g}_F$ and $\beta_I\big|_{\mathrm{div}} = -\left(\gamma\big|_{\mathrm{div}}\, g\right)_I$

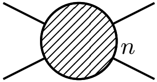
Ambiguity of the RG functions

The renormalization constants are ambiguous:

n-loop $\overline{\text{MS}}$ normalization conditions

$Z_n^\dagger + Z_n +$ 

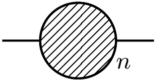
$=$ finite

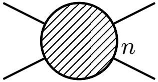
$(\delta g_I)_n + \Big($ 

$\Big)_I =$ finite

The renormalization constants are ambiguous:

n-loop $\overline{\text{MS}}$ normalization conditions

$\overbrace{Z_n^\dagger + Z_n}^{\delta Z^\dagger + \delta Z = 0} +$  $=$ finite

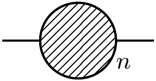
$(\delta g_I)_n + \Big($  $\Big)_I =$ finite

Ambiguity of the RG functions

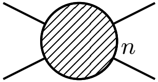
The renormalization constants are ambiguous:

n-loop $\overline{\text{MS}}$ normalization conditions

$$\underbrace{Z_n^\dagger + Z_n}_{\delta Z^\dagger + \delta Z = 0} + \quad [\text{diagram}]_n \quad = \quad \text{finite}$$



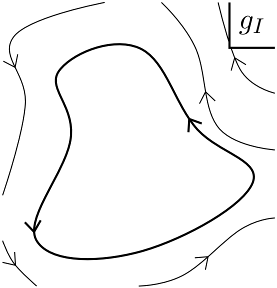
$$(\delta g_I)_n + \Big(\quad [\text{diagram}]_n \quad \Big)_I \quad = \quad \text{finite}$$



A divergent G_F rotation of the theory changes the renormalization constants and the RG functions:

$$Z \longrightarrow \exp\left[\sum_n \frac{1}{\epsilon^n} u_n(g)\right] Z \quad \Longrightarrow \quad \begin{cases} (\beta_I,\, \gamma) & \longrightarrow (\beta_I',\, \gamma') \\ \Delta_{\text{RG}} & \longrightarrow \Delta_{\text{RG}} + \Delta^F_{-\Delta\gamma} \end{cases}$$

Limit Cycle



$$\beta_I = (v\, g)_I$$
$$B_I = 0$$

i) RG functions can be divergent but the flow is finite

ii) The flavor-improved $(B_I,\, \Gamma)$ are unambiguous and finite*

iii) It is always possible to choose finite RG functions, e.g., $(B_I,\, \Gamma)$

γ-pole at the 3-loop order

Renormalization condition for 2-point functions: $(\overline{\text{MS}},\ d = 4 - \epsilon)$

$$Z^\dagger \text{—(1PI)—} Z \;+\; Z^\dagger \text{———} Z \;=\; \text{finite}, \qquad Z = \mathbb{1} + \sum_{n=1}^{\infty} \frac{z^{(n)}}{\epsilon^n}$$

with field anomalous dimension

$$\gamma = Z^{-1} \frac{\mathrm{d}}{\mathrm{d}t} Z = \sum_{n=0}^{\infty} \frac{\gamma^{(n)}}{\epsilon^n} \quad \Longrightarrow \quad \hat{\gamma}^{(0)} = -\zeta z^{(1)}, \qquad \zeta = k_I g_I \partial^I$$

In SM $\gamma^{(1)} \neq 0$ at the 3-loop order for $Z^\dagger = Z$

$$\gamma^{(1)} - \gamma^{(1)\dagger} = \left[z^{(1)},\, \zeta z^{(1)}\right]$$

$\gamma^{(1)}$ can be made to vanish with $Z' = UZ$ for some divergent rotation U.

RG finiteness in the SM

At 3-loop RG divergences in the SM Herren, Mihaila, Steinhauser [1712.06614]

$$(4\pi)^6 \gamma_q^{(1)} = \frac{g_1^2}{96} \left[y_u y_u^\dagger,\, y_d y_d^\dagger \right] + \frac{1}{32} \left[y_u y_u^\dagger y_u y_u^\dagger,\, y_d y_d^\dagger \right] + \frac{1}{32} \left[y_d y_d^\dagger y_d y_d^\dagger,\, y_u y_u^\dagger \right]$$

$$(4\pi)^6 \gamma_u^{(1)} = \frac{1}{16} y_u^\dagger \left[y_d y_d^\dagger,\, y_u y_u^\dagger \right] y_u$$

$$\begin{aligned}(4\pi)^6 \beta_{y_u}^{(1)} = &- \frac{g_1^2}{96} \left[y_u y_u^\dagger,\, y_d y_d^\dagger \right] y_u - \frac{1}{32} \left[y_u y_u^\dagger y_u y_u^\dagger,\, y_d y_d^\dagger \right] y_u \\ &- \frac{1}{32} \left[y_d y_d^\dagger y_d y_d^\dagger,\, y_u y_u^\dagger \right] y_u + \frac{1}{16} y_u y_u^\dagger \left[y_d y_d^\dagger,\, y_u y_u^\dagger \right] y_u\end{aligned}$$

$\beta_{y_u}^{(1)} = -(\gamma^{(1)}\, y_u)$, $\beta_{y_u}^{(2)} = -(\gamma^{(2)}\, y_u)$, *etc.* in the SM

$$(\omega\, y_u)^i{}_j = \omega_q{}^i{}_k y_u{}^k{}_j - y_u{}^i{}_k \omega_u{}^k{}_j + \omega_h y_u{}^i{}_j$$

Renormalization ambiguity

Consider a rotation with $R \in G_F$: $y_u \longrightarrow R_q y_u R_u^\dagger$

$$\mathcal{W}[\gamma,\, g,\, \mathcal{J},\, a] = \mathcal{W}[\gamma,\, Rg,\, R\mathcal{J},\, a^R] =$$
$$\mathcal{W}_0[\gamma,\, g_0,\, \mathcal{J}_0,\, a_0] = \mathcal{W}_0[\gamma,\, Rg_0,\, R\mathcal{J}_0,\, a_0^R], \quad (Rg_0)_I = g_{0,I}(Rg)$$

Take a divergent rotation instead:

$$U = \exp\Big[-\sum_{n=1}^{\infty} \frac{1}{\epsilon^n} u^{(n)}(g) \Big], \qquad u^{(n)} \in \mathfrak{g}_F$$

$$\mathcal{W}[\gamma,\, g,\, \mathcal{J},\, a] = \mathcal{W}_0[\gamma,\, g_0,\, \mathcal{J}_0,\, a_0] = \mathcal{W}_0[\gamma,\, Ug_0,\, U\mathcal{J}_0,\, a_0^U]$$

Results in a change of Z: Ambiguity in taking $\sqrt{Z^\dagger Z}$

$$(U\mathcal{J}_0)_\alpha = \mathcal{J}_{0,\beta} U^{\dagger\beta}{}_\alpha = \mathcal{J}_\beta (Z^{-1}U^\dagger)^\beta{}_\alpha \implies \tilde{Z}^\alpha{}_\beta = U^\alpha{}_\gamma Z^\gamma{}_\beta.$$

Preferred set of RG functions

There is an infinite class of RG functions, which describe the flow:

$$(\gamma,\, \beta_I,\, \upsilon) \longrightarrow \big(\gamma + \Delta\gamma,\, \beta_I - (\Delta\gamma\, g),\, \upsilon - \Delta\gamma\big), \qquad \Delta\gamma(g) \in \mathfrak{g}_F$$

One additional consideration is

$$[T^\mu{}_\mu] = B_I[\mathcal{O}^I] - \eta_a \partial^2 [\mathcal{O}^a_M] = \beta_I[\mathcal{O}^I] + \upsilon \cdot \partial_\mu [J^\mu_F] - \eta_a \partial^2 [\mathcal{O}^a_M]$$

Baume, Keren-Zur, Rattazzi, Vitale [1312.0428]

Flavor-improved RG functions

The set $\big(\Gamma,\, B_I,\, 0\big)$ is finite and unambiguous